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Research on Trajectory Planning of On-The-Fly Observing for the Active Feed Receiver in FAST

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Abstract: On-The-Fly (OTF) observing is one of the five kinds of operation mode owned by Five-hundred-meter Aperture Spherical Radio Telescope (FAST). In order to guarantee the active feed receiver move stably during the OTF observation, its trajectory planning was dealt with in this paper. Considering the trajectory in local coordinate system is complex, the paper planned its trajectory planning in the equatorial coordinate system. For the Earth's rotation, in order to guarantee the active feed receiver to start smoothly from a standstill and to soft-stop stably in the specified time when scanning along the declination, the right ascension and declination were planned with the double S velocity profile at the same time. The resulting trajectory is ensured the continuation in position, velocity and acceleration under local coordinate system, meanwhile, considering the execute ability of the mechanism, the trajectory respect specified limits in velocity, acceleration and jerk. An example and the comparison between the original trajectory and the planned trajectory are given to illustrate the advantage of the presented approach.

Key Words: FAST telescope, trajectory planning, On-The-Fly observing, double S

1 Introduction

Five-hundred-meter Aperture Spherical Radio Telescope (FAST), which is building in Guizhou Province, southwest of China, is a Chinese mega-science project to build the largest single dish radio telescope in the world. FAST will enable astronomers to jump-start many science goals, for example, surveying the neutral hydrogen line in distant galaxies, looking for the first shining star, detecting thousands of new pulsars, hearing the possible signals from other civilizations, etc [1, 2]. The conceptual design of the FAST is shown in Fig.1. The FAST telescope mainly consists of the feed-supporting system, the active reflector system and the receiver system [2].

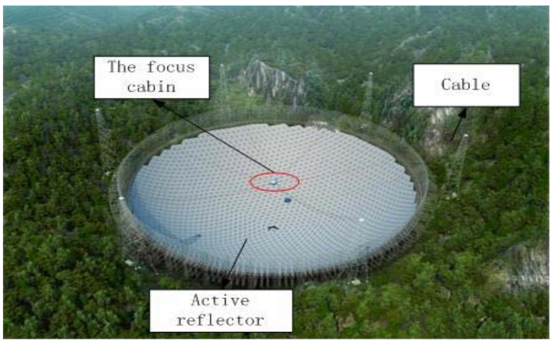

Fig. 1: Conceptual design of the FAST.

The feed-supporting system is a six-cable driven parallel manipulator, by which the focus cabin is driven. The position and pose of the active feed receiver is realized by the feed-supporting system [3]. Fig. 2 shows the mechanical structure of the focus cabin. The diameter of the focus cabin is about 13 meters. It consists of a star-frame, an A-B rotator and a Stewart manipulator. Nine feed receivers are mounted on the active platform of the Stewart manipulator, the one move on the focus surface called the active feed receiver [4, 5].

Compared with the traditional single-dish radio telescope, the FAST has no rigid connection between the active reflector and the focus cabin, replaced by a six-cable driven parallel manipulator in feed-supporting system [6, 7]. On one hand, this character saves the weight and the cost of the feed-supporting system. But on the other hand, since the cable can only sustain tension but cannot bear thrust, this character determines that the unsmooth velocity of the moving focus cabin may make the feed-supporting system more easily stimulated. As a result, the smooth motion of the telescope trajectory, i.e. the trajectory of the active feed receiver, is fatal important. So the design of the trajectory planning methods is a fundamental work to ensure the telescope move stably.

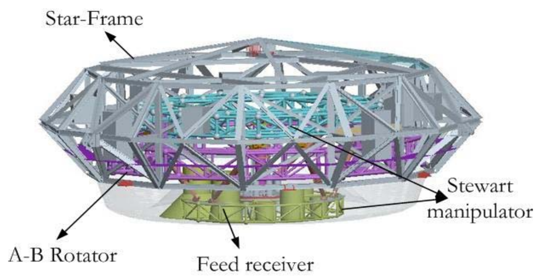

Fig. 2: Focus cabin Mechanical Structure.

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Luo Ya-bo, Zhang Yong and Zhu Li-chun [8] presented an integrated trajectory planning method for FAST. However, their method was to compensate the control error between the feed-supporting system and the active reflector system, without the consideration of the smooth of the trajectory.

According to the requirement of astronomical observations, the FAST should have the ability to complete five kinds of astronomical observation modes: drift-scan, slew, tracking, basket-weaving and On-The-Fly (OTF) observing. Among them, On-The-Fly (OTF) observing is the most complex one, which is consisted of a series of 3-dimensional curve sections with varied velocities in local coordinate system, has been widely studied and used in the traditional radio telescopes [9, 10, 11]. For the traditional radio telescopes having different structures, the smooth of the trajectory were also not considered by those papers.

Considering the new conceptual design and the special mechanical structure of the FAST, this paper focuses on designing the smooth trajectory planning method for On-The-Fly (OTF) observing of the FAST. The resulting trajectory guarantees to be continuous in position, velocity and acceleration in local coordinate system, as well as to respect the execute ability of the mechanism.

The rest of this paper is organized as follows. In section 2, the definitions of the workspace, velocity and acceleration of the active feed receiver and the description of OTF observing are given. The active feed receiver's trajectory planning algorithm is designed in section 3. In section 4, a trajectory planning example by using the planning algorithm in this paper is given, as well as a comparison between the before-plan trajectory and the planed trajectory. Finally, a conclusion and the future work are drawn in section 5.

2 Problem Descriptions

2.1 The Workspace ,Velocity and Acceleration of The Active Feed Receiver

The workspace of the active feed receiver is schematically illustrated in Fig. 3. During the observation, the active feed receiver scans on the focus surface. In order to observe the radio sources, the radio sources, the center of the active reflector and the active feed receiver should be at the same line [12]. As shown in Fig. 3, **s** is the incident direction of the radio source. The local coordinate system O-XYZ is located at the sphere center of the active reflector, X-axis points to east direction, and Y-axis points to north direction. So the incident direction in local coordinate system can be described as [8]:

$$\vec{s}=\begin{bmatrix}\cos\delta\sin H\\ \sin\varphi\cos\delta\cos H-\cos\varphi\sin\delta\\ -\cos\varphi\cos\delta\cos H-\sin\varphi\sin\delta\end{bmatrix} \tag{1}$$

Where δ represents the radio source's declination, φ represents the latitude of the sites for FAST, and H represents the hour angle, which can be described as

$$H=S_0+(t-8)(1+\mu)+\lambda-\alpha \tag{2}$$

In Eq. (2), S_0 is sidereal time, t is Beijing time, μ is a constant with value $\mu=1/365.2422$, λ is the longitude of the sites, and α is the radio source's right ascension.

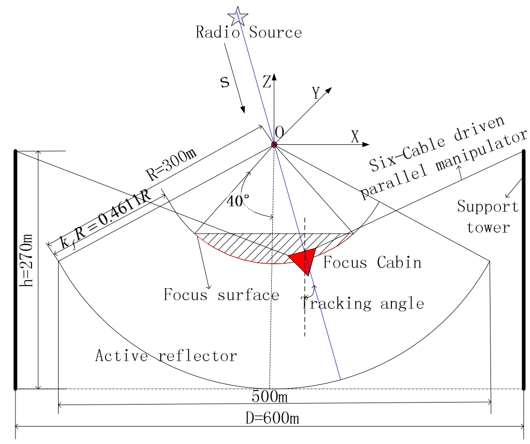


Fig. 3: Schematic illustration of the active feed receiver's workspace.

The velocity of the incident direction in local coordinate system can be shown as:

$$\dot{\vec{s}}=\begin{bmatrix}-\sin\delta\sin H\cdot\dot{\delta}+\cos\delta\cos H\cdot\dot{H}\\ \sin\varphi(-\sin\delta\cos H\cdot\dot{\delta}-\cos\delta\sin H\cdot\dot{H})-\cos\varphi\cos\delta\cdot\dot{\delta}\\ -\cos\varphi(-\sin\delta\cos H\cdot\dot{\delta}-\cos\delta\sin H\cdot\dot{H})-\sin\varphi\cos\delta\cdot\dot{\delta}\end{bmatrix} \tag{3}$$

Where $\dot{\delta}$ is the time derivative of the δ, $\dot{H}$ is the time derivative of Eq. (2), which can be described as:

$$\dot{H}=(1+\mu)-\dot{\alpha} \tag{4}$$

Where $\dot{\alpha}$ is the time derivative of the α.

Accordingly, the acceleration of the incident direction in local coordinate system can be shown as:

$$\ddot{\vec{s}}=\begin{bmatrix}-\cos\delta\sin H\cdot\dot{\delta}^2-2\sin\delta\cos H\cdot\dot{\delta}\cdot\dot{H}-\sin\delta\sin H\cdot\ddot{\delta}-\cos\delta\sin H\cdot\dot{H}^2+\cos\delta\cos H\cdot\ddot{H}\\ \sin\varphi(-\cos\delta\cos H\cdot\dot{\delta}^2+2\sin\delta\sin H\cdot\dot{\delta}\cdot\dot{H}-\sin\delta\cos H\cdot\ddot{\delta}-\cos\delta\cos H\cdot\dot{H}^2-\cos\delta\sin H\cdot\ddot{H})-\cos\varphi(-\sin\delta\cdot\dot{\delta}^2+\cos\delta\cdot\ddot{\delta})\\ -\cos\varphi(-\cos\delta\cos H\cdot\dot{\delta}^2+2\sin\delta\sin H\cdot\dot{\delta}\cdot\dot{H}-\sin\delta\cos H\cdot\ddot{\delta}-\cos\delta\cos H\cdot\dot{H}^2-\cos\delta\sin H\cdot\ddot{H})-\sin\varphi(-\sin\delta\cdot\dot{\delta}^2+\cos\delta\cdot\ddot{\delta})\end{bmatrix} \tag{5}$$

Where $\ddot{\delta}$ is the time derivative of the $\dot{\delta}$, $\ddot{H}$ is the time derivative of Eq. (4), which can be described as:

$$\ddot{H}=-\ddot{\alpha} \tag{6}$$

So the position, velocity and accelerate of the active feed receivers can be shown as:

$$\vec{P}(t) = (1-k_f)r\cdot\vec{s} \quad (7)$$

$$\vec{V}(t) = (1-k_f)r\cdot\dot{\vec{s}} \quad (8)$$

$$\vec{A}(t) = (1-k_f)r\cdot\ddot{\vec{s}} \quad (9)$$

Where k_f is a constant (see Fig. 3), r is the radius of curvature of the active reflector. The Eq. (7) to Eq. (9) maps the parameters in equatorial coordinate system to local coordinate system.

2.2 OTF Observing Description

On-The-Fly (OTF) observing is a technique to perform mapping observations efficiently with single-dish radio telescopes [11]. In OTF observing, the telescope is driven smoothly and rapidly across a region of sky, while the receiver data and the position of the feed receivers are recorded continuously [9, 10].

In FAST, the OTF observing is a kind of operation mode owned by FAST, in which the active feed receiver should scan back and forth along the right ascension or the declination, while the active feed receiver should keep tracking. The scanning field of sky is a rectangular in equatorial coordinate system, but a series of 3-dimensional curve sections with varied velocities in local coordinate system. Depending on the difference of the scanning direction, the OTF observing can divided into two categories: scanning along the right ascension and scanning along the declination.

The path in equatorial coordinate system when scanning along the declination is schematically illustrated in Fig. 4, where (α_1,δ_1) is the start right ascension and declination, (α_2,δ_2) is the end right ascension and declination, θ is the scanning interval angle between the adjacent scanning rows, and ω is the scanning velocity of the scanning rows.

Accordingly, the path in equatorial coordinate system when scanning along the right ascension is shown in Fig. 5.

Referring to Fig. 4, the distance along the scanning line is $(\delta_2-\delta_1)$, so the time to scan one line is

$$t_{scan\_de} = (\delta_2-\delta_1)/\omega \quad (10)$$

The length of the map is $(\alpha_2-\alpha_1)$, so the number of scanning lines in the map is given by

$$N_{line\_de} = (\alpha_2-\alpha_1)/\theta \quad (11)$$

Accordingly, referring to Fig. 5, the time to scan one line is

$$t_{scan\_ra} = (\alpha_1-\alpha_2)/\omega \quad (12)$$

The number of scanning lines is given by

$$N_{line\_ra} = (\delta_2-\delta_1)/\theta \quad (13)$$

Theoretically, when scanning along the right ascension, the active feed receiver should scan along the declination line with the constant velocity ω in equatorial coordinate system, after t_{scan} seconds, the active feed receiver skips to the next scanning line at once, and it arrives to the end point after scan N_{line} lines. Given the start point, the end point, the scanning velocity, the interval angle between adjacent scanning line and the start time, the right ascension and declination at the special time is confirmed. Substituting the right ascension and declination at the special time into Eq. (7) yields the position in the local coordinate system. But these yield position cannot be used in engineering, since the velocity and accelerate isn't continuous and smooth, which may generate efforts and stresses on the mechanical system that may result detrimental or generate undesired vibration effects. What's more, there is no constraint on acceleration of the trajectory. The trajectory may over the limit of the acceleration which the telescope mechanical structure can bear. So it is necessary to plan the trajectory for telescope.

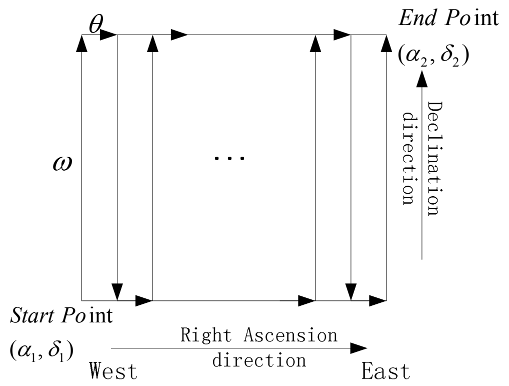


Fig. 4: A schematic illustration of OTF observing when scanning along declination.

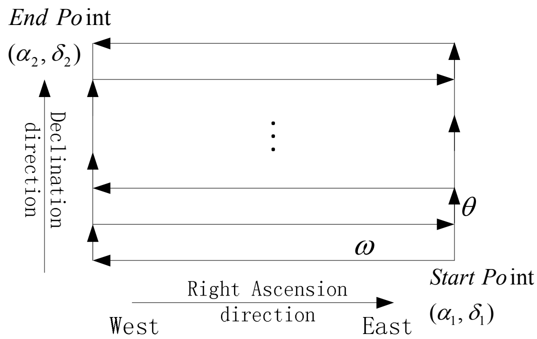


Fig. 5: A schematic illustration of OTF observing when scanning along right ascension.

3 Trajectory Planning Algorithm

As described in section 2, the velocity and accelerate of the theoretical trajectory is discontinuous, and the acceleration hasn't been limited, so the trajectory of the telescope must be planned.

There are two coordinate systems in the trajectory representation: the equatorial coordinate system and the local coordinate system. So there are two ways to planning the trajectory accordingly. One way is to plan it in local coordinate system, and another way is in equatorial coordinate system.

In local coordinate system, there are three axis components should be planned at the same time, while there is only one component (right ascension or declination) need to be considered at every time in equatorial coordinate system. So it is easier to planning trajectory in equatorial coordinate system than in local coordinate system.

In order to guarantee the continuous of the position, velocity, acceleration, and limit the acceleration, the double S velocity profile is selected [13]. Given the start point, the start velocity of the path, the end point, the end velocity of the path, the max velocity, the max acceleration, and the max jerk, the double S velocity profile generates a trajectory to the desired end point, as well as returns the velocity and acceleration of the interpolate points. The generated trajectory is guaranteed to be smooth in position, velocity and to be continuous in acceleration [13, 14].

In order to guarantee the active feed receiver move stable during the OTF observing, the velocity should be smooth and the acceleration should be continuous when the telescope skips to the adjacent scanning line. In the following approach, the velocity of the beginning point and the stopping point (see Fig. 6) are planned to be zero in local coordinate system, while the active feed receiver uses the slew operation mode to switch from the current stopping point to the next beginning point as soon as possible during the skip period. The slew operation mode is a mode which planned in local coordinate system using double S velocity profile algorithm to guarantee the telescope move stably.

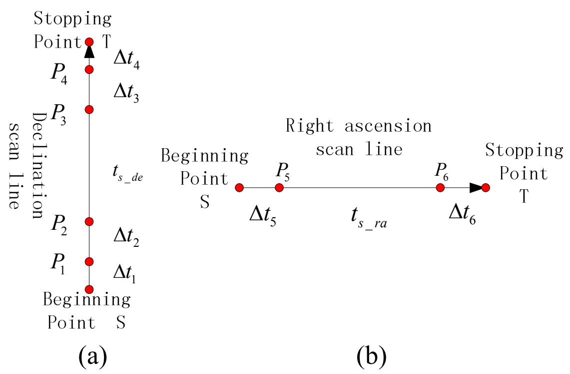


Fig. 6: Schematic illustration of plans of scanning lines. (a) Scan along declination lines, (b) Scan along right ascension lines.

When the telescope scans along the right ascension line, the declination keeps a constant. According to Eq. (8), the velocity of the active feed receiver will be zero if the scanning velocity of the right ascension is earth's rotation speed $(1+\mu)$. So set the speed of the right ascension to $(1+\mu)$ at the beginning point and the stopping point. The scanning time t_{scan_ra} is divided into three parts (see Fig. 6 (b)): "approach-run" time Δt_5, "soft-stop" time Δt_6, and the effective scanning time t_{s_ra}. The velocity of point P_5 and point P_6 keep the scanning velocity ω.

Using double S velocity profile plan the "approach-run" and "soft-stop" period trajectory. While the time Δt_5 and Δt_6 is self adapt obtained by searching the time using double S algorithm, in the condition of $P_5 = S + \omega \cdot \Delta t_5$ and Δt_5 is equal to the time the double S algorithm returned. The pseudo code for finding the approach-run time Δt_5 is shown as follows:

Step 1: initialize variables.
  Init Δt_5;
  $v_0 = 1+\mu; v_1 = \omega;$
Step 2: set constraints.
  Set v_{max}, a_{max}, j_{max};
Step 3: find the suitable approach-run time Δt_5.
  $CC = \Delta t_5;$
  $P_5 = S + \omega \cdot \Delta t_5$
  $cc = doubleS(S, v_0, P_5, v_1, v_{max}, a_{max}, j_{max});$
  While $cc != CC$
    Modify Δt_5;
    $CC = \Delta t_5;$
    $P_5 = S + \omega \cdot \Delta t_5$
    $cc = doubleS(S, v_0, P_5, v_1, v_{max}, a_{max}, j_{max});$
  End

Where v_0 represents the velocity of the beginning point S, v_1 represents the velocity of the stopping point P_5. v_{max}, a_{max} and j_{max} represent max velocity, acceleration and jerk of the "approach-run" period respectively, cc represents the time the double S algorithm returned, *doubleS* is a function which realizes the double S velocity profile [13]. The "soft-stop" time Δt_6 can be obtained using the same algorithm.

While when the telescope scans along the declination line, the right ascension should keep as a constant theoretically, but the velocity of the active feed receiver will not be zero at the beginning point S (see Fig. 6 (a)) according to Eq. (8).

The ascension and declination should be planned at the same time to guarantee the active feed receiver to start smoothly from a standstill and stop stably.

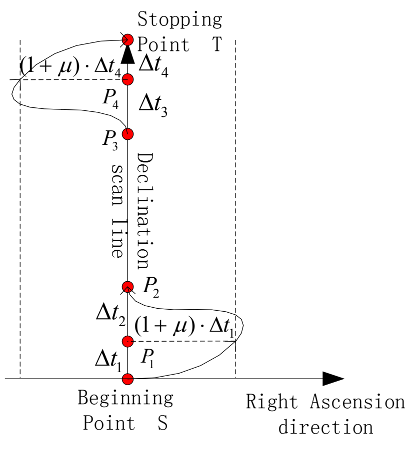


Fig. 7: Schematic illustration of the path of declination and right ascension in the equatorial coordinate system when scanning along declination.

As shown in Fig. 6 (a), the scanning time t_{scan_de} has divided into five parts: the declination's approach-run time Δt_1, during which the declination approach-run to scan velocity, while its right ascension keeps the earth rotation velocity $(1+\mu)$; the right ascension's soft-stop time Δt_2, during which the right ascension soft-stop from the earth's rotation speed, while its declination keeps the scanning velocity ω ; the effective scanning time t_{s_ra}, during which the right ascension keeps a constant value, the declination runs with the scanning velocity; the right ascension's reverse approach-run time Δt_3 , during which the right ascension approach-run to the earth's rotation speed, while its declination keeps the scanning velocity ω ; the declination's soft-stop time Δt_4 , during which the declination soft-stop from the scanning speed, while its right ascension keeps the earth rotation velocity $(1+\mu)$.

So the schematic illustration of the path of declination and right ascension in the equatorial coordinate system is shown is Fig. 7. When the active feed receiver arrives to point P_1, the right ascension has offset $(1+\mu)\cdot\Delta t_1$ along the right ascension direction. Respectively, $(1+\mu)\cdot\Delta t_1$ has offset along the reverse direction when the active feed receiver arrives to point P_4 .

The divided time Δt_1 to Δt_4 can be obtained from the algorithm described before. After the divided times are obtained, using double S velocity profile plans the declination and right ascension. Define

$$v_{earth} = (1+\mu) \tag{14}$$

The plans for declination and right ascension at the divided time are list in Table 1.Where α_i represent the right ascension of the ith scanning line.

Table 1: Algorithms for different period when scanning along declination

Periods	Algorithms
Δt_1	Right ascension: keep earth's rotation speed v_{earth}.
	Declination: $doubleS(\delta_1, 0, \delta_1+\omega\cdot\Delta t_1, \omega, v_{\max}, a_{\max}, j_{\max})$.
Δt_2	Right ascension: $doubleS(\alpha_i + v_{earth}\cdot\Delta t_1, v_{earth}, \alpha_i, 0, v_{\max}, a_{\max}, j_{\max})$.
	Declination: keep the scanning speed ω .
Δt_3	Right ascension: $doubleS(\alpha_i, 0, \alpha_i - v_{earth}\cdot\Delta t_4, v_{earth}, v_{\max}, a_{\max}, j_{\max})$.
	Declination: keep the scanning speed ω .
Δt_4	Right ascension: keep earth's rotation speed v_{earth}.
	Declination: $doubleS(\delta_2 - \omega\cdot\Delta t_4, \omega, \delta_2, 0, v_{\max}, a_{\max}, j_{\max})$.

Inserting the position, velocity and acceleration of the declination and right ascension for the special time which obtained above to Eq. (7) to Eq. (9), the trajectory of the active feed receiver in local coordinate system will be obtained.

4 Experimental Results

4.1 Experiment setup

As described in section 3, the algorithm used in scanning along the right ascension is contained in the algorithms which used in scanning along the declination mode, so in this section, only the simulation for scanning along declination is listed.

According to the characters of FAST, the common parameters of the FAST are given in Table. 2. In order to satisfy the demand of the OTF observing, Table. 3 show the parameters for scanning along the declination.

Table 2: Common parameters for FAST

Symbols	Quantities	Values
φ	Latitude of the site	25°39'12.379324"
λ	Longitude of the site	106°51'22.598015"
S_0	Sidereal time	0 (h)
t_{start}	Start scanning time (Beijing time)	0 (h)
r	radius of curvature of the main reflector	300 (m)
$v_{\max\_ra}$	Max velocity along right ascension	$30.0821°/h$
$a_{\max\_ra}$	Max acceleration along right ascension	$1.0830\times10^{4}°/h^2$
$j_{\max\_ra}$	Max jerk along right ascension	$3.8986\times10^{6}°/h^3$
$v_{\max\_slew}$	Max velocity of slew operation mode	0.4 (m/s)

Table 3: Special parameters for scanning along the declination

Symbols	Quantities	Values
(α_1, δ_1)	Start right ascension and start declination	(9.8°, 2°)
(α_2, δ_2)	End right ascension and end declination	(10.15°, 3°)
θ	Scan interval angle	$1°/120$
ω	Scanning velocity	$15.0411°/h$
$v_{\max\_de}$	Max velocity along declination	$30.0821°/h$
$a_{\max\_de}$	Max acceleration along declination	$1.0830\times10^{4}°/h^2$
$j_{\max\_de}$	Max jerk along declination	$3.8986\times10^{6}°/h^3$

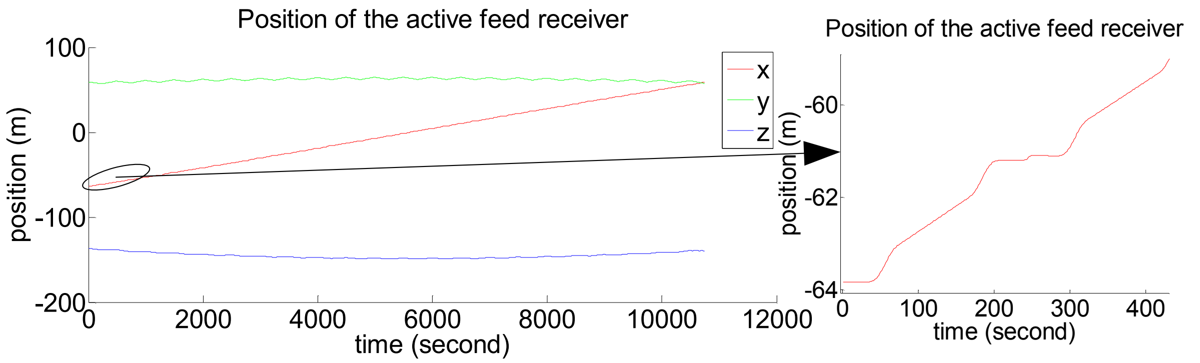
4.2 Simulation Experiments

Using the planning algorithm described in section 3, with the parameters list in Table 2 and Table 3, the planning trajectory is show in Fig. 8.

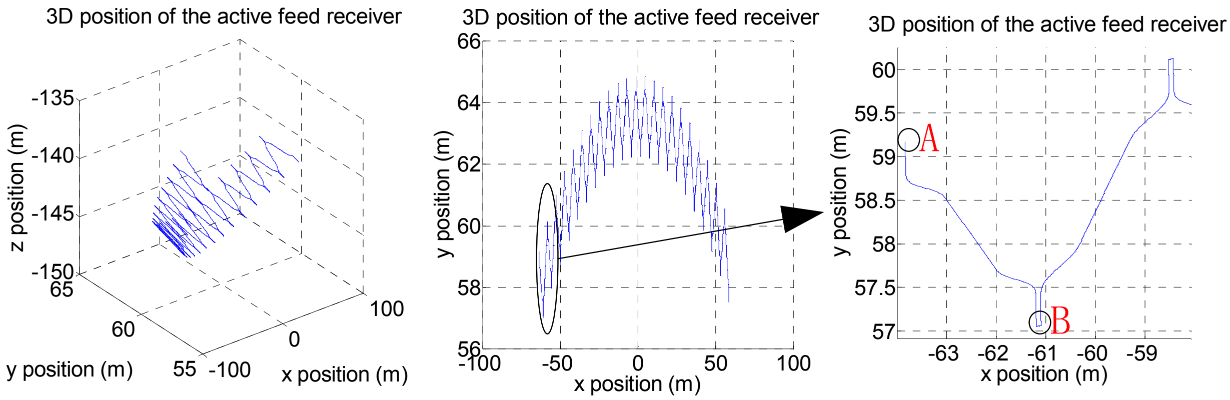
The position curves with using the proposed approach are shown in Fig. 8 (a) and Fig. 8 (b). In Fig. 8 (a), red represents the X-axis component in local coordinate system, green represents Y-axis, and blue represents Z-axis, the picture in right is an enlarged partial view of the left. In Fig. 8 (b), the left image is a 3-dimensional space panorama for the trajectory, the middle image is a view in XOY plane, and the right one is an enlarged partial view of the middle image. With the time going on, the position curves move smoothly during each scanning line. The active feed receiver is in a standstill status when it arrives in point A and point B (see Fig. 8 (b)). While, without using the planning algorithm, the velocity of the active feed receiver at point A and point B are normal scanning velocity. What's worse, the direction of the velocity reverses at once at point A and point B, so the trajectory before planning can't be used in the engineering.

The velocity curves is shown in Fig. 8 (c), red represents the X-axis component in local coordinate system, green represents Y-axis, and blue represents Z-axis, the picture in right is an enlarged partial view of the left. The velocity curves move smoothly, as well as respect specified limit in velocity. The velocities of point A and point B in Fig. 8 (b) are the point C and point D in Fig. 8 (c) respectively, their velocities are zero, thus guarantee the telescope move stably. The curve from point D to point E corresponds to the scanning interval period which using the slew operation mode planning the trajectory, it velocity moves smoothly as well.

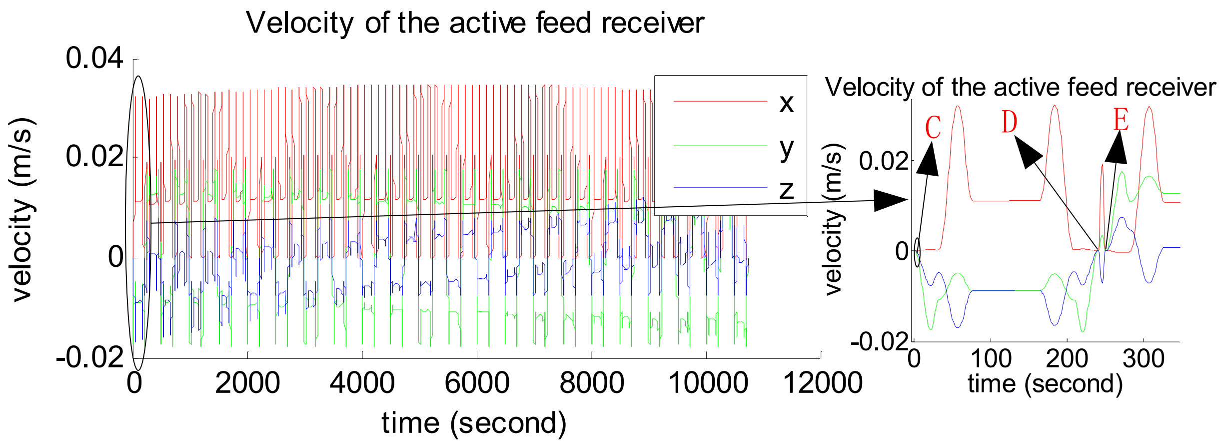
The acceleration curves is shown in Fig. 8 (d), red represents the X-axis component in local coordinate system, green represents Y-axis, and blue represents Z-axis, the picture in right is an enlarged partial view of the left. It curve guarantee to be continuous, as well as to respect specified limits in acceleration and jerk.



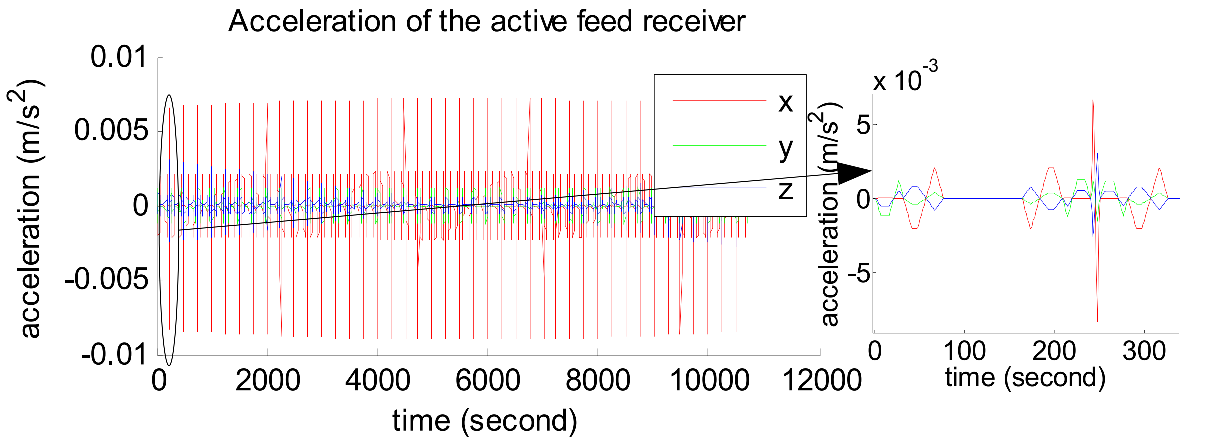
(a) Position for each axis component in local coordinate system



(b) Position for 3-dimensional space in local coordinate system.



(c) Velocity for each axis component in local coordinate system



(d) Acceleration for each axis component in local coordinate system

Fig. 8: Planned trajectory of the OTF observing, scanning along the declination.

5 Conclusion and future work

This paper addressed an algorithm to plan the trajectory of On-The-Fly (OTF) observing for FAST. The resulting trajectory guarantees to be continuous in position, velocity, and acceleration, as well as to respects the specified limits in velocity, acceleration, and jerk. The generated trajectory can be used in engineering to improve the telescope's motion performance.

Finally, making the telescope soft-stop, and then slewing to the next scanning line is not the most effective way to switch to the next scanning line, it wastes much time during the soft-stop and approach-run period, and waste much effective scanning area as a result. So maximizing the effective scanning area is the subject of continuing work.

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