Hybrid Sorting Method for Successive Cancellation List Decoding of Polar Codes

Huan Li
University of Chinese Academy of Sciences
Institute of Automation, Chinese Academy of Sciences
Beijing
lihuan2013@ia.ac.cn

Chen Guo
Institute of Automation, Chinese Academy of Sciences
Beijing
chen.guo@ia.ac.cn

Donglin Wang
Institute of Automation, Chinese Academy of Sciences
Beijing
donglin.wang@ia.ac.cn

ABSTRACT

This paper proposes a hybrid metric sorting method (HMS) of successive cancellation list decoders for polar codes, which plays a critical role in decoding process. We review the state-of-the-art metric sorting methods and combine the advantages of them to generate the proposed method. Due to the optimized architecture, the proposed HMS method reduces the number of comparing stages effectively with little increase in comparisons. Evaluation results show that about 25 percent of comparing stages can be removed by HMS, compared with state-of-the-art methods. The proposed method enjoys a latency reduction for hardware implementation.

CCS Concepts

• **Security and privacy → Information-theoretic techniques**

Keywords

Polar codes, successive cancellation list decoding, metric sorting.

1. INTRODUCTION

Polar codes are a class of error-correction codes, which were first invented by Arikan in 2009 [1]. Since incredible channel capacity can be achieved on a memoryless channel when the code length tends to infinity, polar codes draw much attention of researchers. Successive cancellation (SC) method was proposed in [1] with satisfied decoding performance when code length is large. However, the performance degrades quickly with short and medium codes. To overcome the shortcoming and further enhance the performance, successive cancellation list (SCL) decoding method was proposed in [2], which brought undesirable complexity. The main difference between SCL and SC methods is that L path candidates are allowed to survive instead of one in SC. For each information (non-frozen) bit, L path candidates are extended into 2L and L paths with the smallest path metrics can be survived [3][4]. Therefore, sorting method needs to be applied to select the paths with the smallest metrics, which plays an important role in SCL decoding.

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In recent years, researchers proposed many sorting methods based on parallel implementation to reduce the delay caused by metrics sorting step for SCL decoding. [5] proposed a full bitonic sorter (FBS), which could be utilized to solve the general sorting problem. [6] introduced Pruned bitonic sorter (PBS) based on FBS, which reduces the unnecessary comparisons. [5] also proposed simplified bubble sorter (SBS) based on general bubble sorter to further remove the comparisons. [7] put forward Odd-even sorter (OES). It eliminated unnecessary many comparisons because the odd paths were sorted before, which effectively reduces the latency in hardware. [8] proposed two-step sorting method. Since the output paths were not sorted but the input ones were sorted, the sorting process in the last step was completed while the LLR was calculating in the current step.

In this paper, we propose an enhanced sorting method for SCL of polar codes called the hybrid metrics sorting (HMS), which is able to reduce the comparing stages as well as the latency. We first analyze the advantages of the state-of-the-art sorting methods. We propose the HMS architecture by combining the advantages. Only half of the survived paths are sorted and the rest paths are unsorted. Unnecessary comparing stages are removed, since the sorting process of the L survival paths are eliminated. We prove the feasibility of architecture in detail. Number of comparing stages and comparisons for the proposed method and existing methods are compared. Results show that HMS method removes 30 percent of comparing stages, compared with state-of-the-art OES. The number of the reduced stages increases as the number of survived paths decreases.

The rest of the paper is constructed as follows. The sorting problem for SCL decoding of polar codes and existing solutions are briefly introduced in Section II. Section III proposes HMS method and provides the proof of feasibility. Section IV compares the number of comparing stages and comparisons of different method. Then the results are analyzed. Finally, conclusions are drawn in Section V.

2. Backgrounds

This section formulates the sorting problem for SCL decoding and introduces the existing sorting methods.

2.1 Problem formulation

Unlike the SC decoding, the SCL algorithm estimates a bit considering both its possible values 0 and 1. At every step of SCL decoding corresponded with information bit, there are L parent path candidates. Each of the parent paths extends to two child paths. In LLR-based implementation, the path metrics can be obtained after calculating the LLR corresponding to the information bit. There are at most 2L paths after the extending. In order to limit the increase of complexity, only the L paths with the smallest metrics are retained while others are removed. Except for the initial several information bits, the general sorting problem for SCL is selecting the L paths with the smallest metrics among 2L candidates. Let $\boldsymbol{m} = [m_0, m_1, \cdots, m_{2L-1}]$ represent the metrics of the 2L child path candidates after extending and $\boldsymbol{n} = [n_0, n_1, \cdots, n_{2L-1}]$ denote the metrics of the L parent candidates obtained by the last step dealing with the information bit. However, according to the updating criteria of SCL decoding, the relations between the parent paths and the child paths can be represented as (1)(2), where $\boldsymbol{a} = [a_0, a_1, \cdots, a_{2L-1}]$ and $a_l > 0$.

$$m_{2l} = n_l, \text{ for } l = 0, 1, \cdots L-1 \tag{1}$$

$$m_{2l+1} = n_l + a_l, \text{ for } l = 0, 1, \cdots L-1 \tag{2}$$

According to equation (1)(2), one property of sorting problem between the child candidates can be expressed as (3), which represents that the even metrics are smaller than the corresponding odd one.

$$m_{2l} \le m_{2l+1}, \text{ for } l = 0, 1, \cdots, L-1 \tag{3}$$

Moreover, another property is satisfied as (4) if the L parent candidates have been sorted at the last step.

$$m_{2l} \le m_{2l+2}, \text{ for } l = 0, 1, \cdots, L-2 \tag{4}$$

In order to further remove the unnecessary comparisons and comparing stages to reduce the latency, these two properties need to be applied adequately. For example, m_{2l-1} and m_{2l} has a fixed relation, based on which comparisons between these two elements can be eliminated at all.

2.2 Existing sorting methods

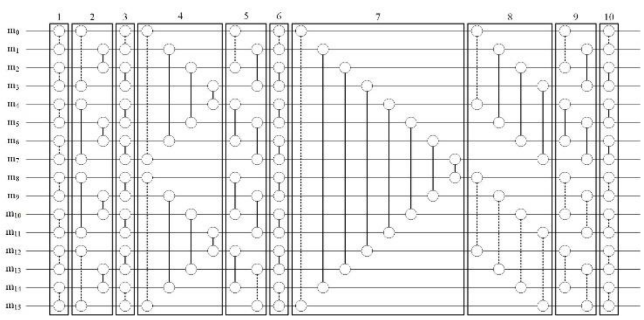

Figure 1. L=8, PBS requires all the comparisons except dotted ones while FBS required all of them

Firstly, we review general full bitonic sorter, which sorts the 2L candidates and selects the smallest L paths. Considering the elements as arbitrary real numbers, the adjacent elements are regarded as one group and all the groups sort separately. The combination of the two groups is the parallel comparisons between every two elements with a same sum of serial number. Since the metrics for SCL decoding enjoy the properties in (3)(4), several comparisons and comparing stages can be removed in FBS. PBS was proposed to simplify the FBS by pruning the known relations between the candidates. The comparisons of sorting the greatest L paths are removed, which reduces the complexity effectively. Figure 1 shows the architecture of FBS and PBS. The dotted lines represent the comparisons removed by PBS. The number of comparing stages S and comparisons C required by PBS is represented as (5).

$$\begin{cases} S^{PBS}(L) = \frac{1}{2}(\log_2 L + 1)(\log_2 L + 2) - 1 \\ C^{PBS}(L) = \left(\frac{L}{2} - 1\right)\log_2 L\,(\log_2 L + 2) - 1 \end{cases} \tag{5}$$

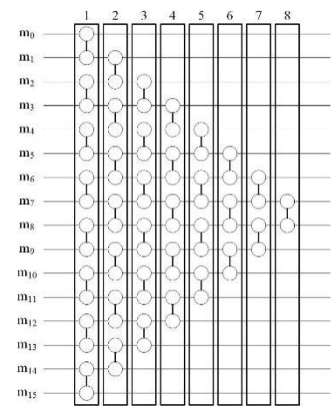

Figure 2. L=8, SBS architecture

The simplified bubble sorter (SBS) was proposed in [5]. When L is small, the number of sorting stages of SBS is low, since the relations between adjacent elements are already known. The stages increase linearly as the L grows, thus it is not suitable when L is large. Figure 2 shows the SBS architecture and the number of stages and comparisons required is represented as (6).

$$\begin{cases} S^{SBS}(L) = L - 1 \\ C^{SBS}(L) = \frac{L}{2}(L - 1) \end{cases} \tag{6}$$

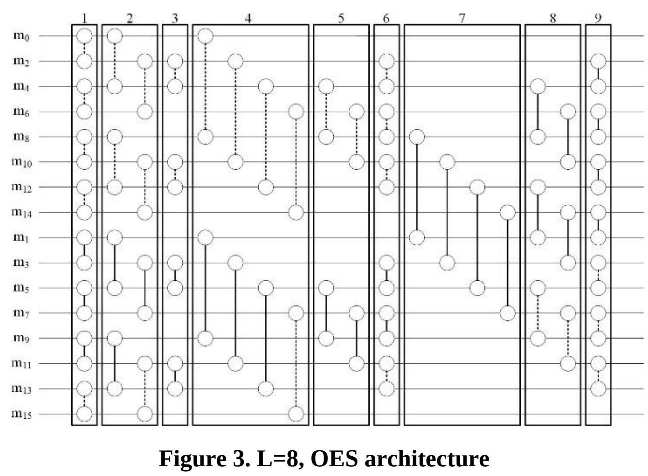

Figure 3. L=8, OES architecture

Odd-even sorting method was proposed in [6]. It divides all the candidates into odd and even group. Since the even group are sorted in the last step, the comparisons are removed at all according to equation (4). The odd group is sorted as arbitrary real numbers. The combination of the two group only considers the selection of L paths with the smallest metrics. Several comparisons can be eliminated by OES. Figure 3 shows the architecture and the number of stages and comparisons required is represented as (7)

$$\begin{cases} S^{OES}\left(L\right) = \frac{1}{2}\left(\log_2 L + 1\right)\left(\log_2 L + 2\right) - 1 \\ C^{OES}\left(L\right) = \log_2 L\left(\frac{L}{4}\log_2 L - 1\right) + \frac{7L}{4} - 2 \end{cases} \quad (7)$$

3. Hybrid metrics sorting method

To further reduce unnecessary comparing stages and the latency, we demonstrate our HMS method.

3.1 Analysis of existing sorting architecture

The number of comparing stages of OES method is the same with PBS method, but the comparisons are lower. The reason is that OES method makes full use of the properties. But it depends that the parent candidates have been sorted before the extending process. Much more comparisons and comparing stages are paid to sort the L paths with the smallest metrics. However, if the comparisons are removed, OES architecture cannot be utilized since it depends on the relations already known. But the full use of the known relations needs to be kept. Although PBS takes higher complexity, the combining operation is the fast way to pick-out the candidates without sorting, which can be utilized to reduce the comparing stages.

3.2 HMS method

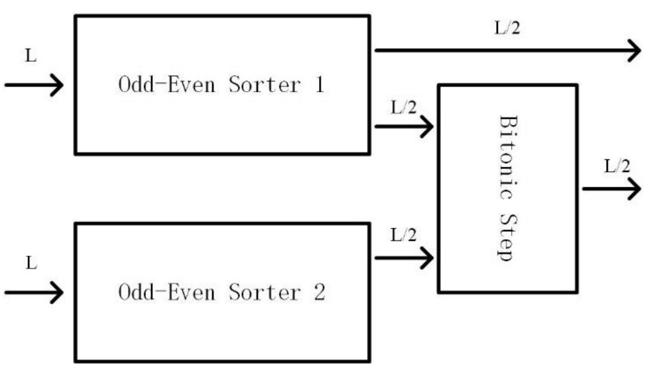

Figure 4. General HMS architecture

Based on the analysis above, a general architecture of HMS is shown in figure 4. As can be seen, the HMS is combined by two OES and one bitonic architecture. The 2L child candidates are divided into two groups corresponding to two OES. When the two groups are sorted, the L/2 paths from sorter 1 with the smallest metrics are kept. The L/2 paths from sorter 1 with the greatest metrics and L/2 paths from sorter 2 with the smallest metrics are exported to the bitonic step to select the other L/2 candidates. At next step, the candidates kept before and their child candidates are exported into sorter 1, while the candidates selected by bitonic step and their child candidates are exported into sorter 2. It is obviously that the kept L/2 paths are sorted and others are unsorted. If the kept L/2 paths are the paths with the smallest metrics among the 2L paths, the output L/2 smallest paths from sorter 1 are the smallest among L survival paths. When L = 8, the architecture of HMS is shown in figure 5.

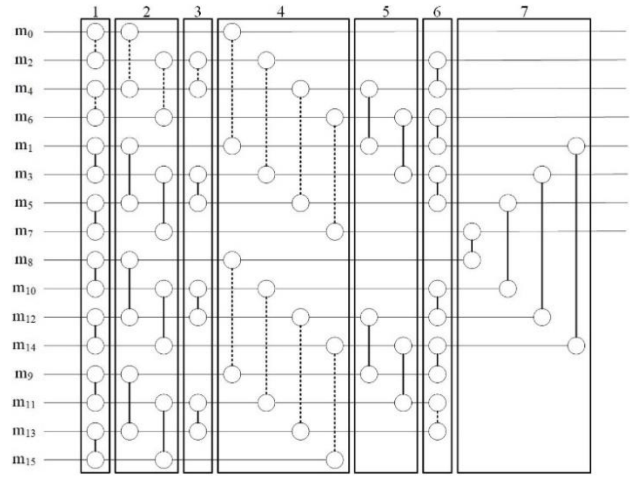

Figure 5. L=8, HMS architecture

Now we prove that the L/2 paths from sorter 1 are the smallest L/2 paths among L survival paths. We consume that all the L/2 smallest candidates are sorted while other L/2 output paths are unsorted. The property (4) can be replaced by the following equations.

$$m_{2l} \le m_{2l+2}, \text{for } l = 0, 1, \cdots, \frac{L}{2} - 1 \quad (8)$$

$$m_{L-2} \le m_{2l+2}, \text{for } l = \frac{L}{2}, \frac{L}{2} + 1, \cdots, L - 1 \quad (9)$$

The input elements of sorter 1 are $[m_0, m_1, \cdots, m_{L-1}]$ and sorter 2 is $[m_L, m_{L+1}, \cdots, m_{2L-1}]$. Let $p_0 \le p_1 \le \cdots p_{L-1}$ denote the sorted result of sorter 1 and $q_0 \le q_1 \le \cdots q_{L-1}$ denote the sorted result of sorter 2, respectively. Based on (3)(8)(9), (10)(11) can be simply obtained.

$$p_0 \le p_1 \le \cdots p_{L/2-1} \le m_{L-2} \quad (10)$$

$$m_{L-2} \le q_0 \le q_1 \le \cdots q_{L-1} \quad (11)$$

According to (10)(11), any element of sorter 2 is larger than m_{L-2}, while any element of p_l is smaller than m_{L-2}, it can be concluded than any element of p_l is smaller than any element of sorter 2. Based on that, since $p_0 \le p_1 \le \cdots p_{L/2-1}$ are the L/2 smallest numbers of sorter 1, they are the are the L/2 smallest numbers of sorter 1 merging sorter 2. Therefore, it is proved that $p_0 \le p_1 \le \cdots p_{L/2-1}$ are the smallest paths of $\boldsymbol{m} = [m_0, m_1, \cdots, m_{2L-1}]$.

In addition, several comparisons of sorter 2 are removed because only L/2 smallest paths are required in the bitonic step. All the comparisons corresponded to the L/2 largest metrics are unnecessary. The number of comparing stages and comparisons required by HMS is presented as (12).

$$\begin{cases} S^{HMS}\left(L\right)=\dfrac{1}{2}\log_2 L\left(\log_2 L+1\right) \\ C^{HMS}\left(L\right)=\log_2 L\left(\dfrac{3L}{8}\log_2 L-\dfrac{L}{8}\right)+\dfrac{3L}{4}-2 \end{cases} \quad (12)$$

4. Evaluation

In this section, we compare the number of comparing stages and comparisons required by HMS with existing methods. The results show the superiority of HMS with reduction of stages. The number of stages relates to the latency in hardware implementation, since all the comparisons in the same stage can be implemented in parallel.

Table 1. The number of comparisons and stages for various method with different L

Stages	PBS	SBS	OES	HMS
L=2	1	1	1	1
L=4	5	3	5	3
L=8	9	7	9	6
L=16	14	15	14	10
L=32	20	31	20	15
Com.	**PBS**	**SBS**	**OES**	**HMS**
L=2	1	1	1	1
L=4	9	6	7	7
L=8	46	28	17	29
L=16	169	120	86	99
L=32	526	496	249	303

Table 1 summarizes the numbers of comparing stages for various sorting networks with different L, i.e., the pruned bitonic method [5], the simplified bubble method [5], the odd-even method [7], and the hybrid sorting method. The numbers for the proposed architecture are computed by using (12). The number of stages required by HMS is the smallest among the sorting methods. When L=8, 1/3 stages can be removed by HMS, compared with PBS and OES, while 1/4 stages can be eliminated when $L=32$. When $L\leq 16$, the number of stagers for SBS is lower than PBS and OES, but it is still larger than HMS method. Since the comparisons can be implemented in parallel in the same stages, latency will be reduced for SCL sorting.

The number of comparisons is also listed in Table 1. As can be seen, with the same L, the number of comparisons required by HMS is lower than PBS and SBS when $L\geq 4$, but it is little bit more than OES. Then gap between HMS and OES is 1/5 when L=32, while they are almost the same when $L\leq 8$.

In order to guarantee the performance and reduce the complexity, L in SCL decoding is always set to 8,16,32. Considering the stages reduced by HMS, a better performance and lower latency are enjoyed by HMS but with a little increase in comparisons.

5. Conclusions

In this paper, an enhanced metric sorting method of successive cancellation list decoders for polar codes is proposed to reduce the latency. The state-of-the-art metric sorting methods are reviewed and analysis of existing methods is provided. Through combining the advantages of OES and PBS, we propose a new sorting called HMS to further reduce the complexity and latency. The feasibility of the proposed HMS is proved in detail. The optimized architecture reduces the number of comparing stages effectively with little increase of comparisons. Comparison results show that about 25 percent stages can be removed when L=32, compared with state-of-the-art methods. The number increases as the number of lists decreases. It can be concluded that proposed HMS method enjoys a latency reduction in hardware implementation.

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