A Low-Complexity Min-Sum Decoding Algorithm for LDPC Codes

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Abstract—**This paper proposes a low-complexity LDPC decoding algorithm with simplified check nodes updating. The proposed algorithm simplifies the result of the second-minimum in check nodes based on the first-minimum computation instead of computing it directly. In order to obtain the approximate result, effective corrected coefficients are utilized, which can reduce the complexity by eliminating the complex computations. Complexity analysis is provided and the results indicate that the complexity of the proposed algorithm is much lower than the general NMS algorithm and MS-based simplified algorithms. Simulation results show that the performance of the proposed algorithm can closely match the NMS algorithm with the same number of iterations.**

Keywords: LDPC decoding, low-complexity, Min-Sum, Simplified check nodes.

I. INTRODUCTION

Due to the incredible error-correcting capability, which is close to Shannon limit [1], low-density parity check (LDPC) codes draw much more attention of researchers in recent years. Based on that, LDPC codes have been adopted in many standards, such as wireless local area network (IEEE 802.11n) [2], wireless personal area network millimeter-wave-based alternative (IEEE 802.15.3c) [3], mobile broadband wireless access systems (IEEE 802.16e) [4]. In addition, it is confirmed that LDPC codes will be adopted in 5G system for enhanced mobile broadband (eMBB) scenario. It indicates that LDPC codes will occupy an important position in wireless communication systems in the future.

LDPC codes are linear block codes with a sparse parity-check matrix. A Tanner graph can be utilized to describe the matrix [5]. Two kinds of nodes, variable nodes and check nodes, are included in the graph. An edge exists between a variable node and a check node corresponding to the matrix. Belief propagation (BP) algorithm, also called sum-product (SP) algorithm [6], can make the performance of LDPC codes close to Shannon limit in iterative way. Information is transmitted repeatedly between check nodes and variable nodes through edges. Although SP algorithm enjoys the best performance, complex computations such as logarithms are required, which causes difficulties for hardware implementation. In order to reduce the complexity, min-sum (MS) algorithm is proposed to simplify the complex computations [7], which transforms the logarithms in check nodes into comparisons and summations. Performance loss cannot be ignored since errors exist in MS compared with SP algorithm. In order to reduce the errors and improve the performance, two modified algorithms, called normalized-MS (NMS) and offset-MS (OMS), are proposed in [8][9]. NMS algorithm utilizes a multiplication coefficient to modify the check nodes results in MS, while a summation coefficient is used in OMS algorithm. MS and these two simplified algorithms above require the computations of first-minimum and second-minimum in each row of the matrix. Many attempts have been made to further reduce the complexity. The single-minimum MS (smMS) algorithm was proposed in [10]. It only found the first-minimum, and the second-minimum is estimated by applying a summation coefficient on the first-minimum. This algorithm exhibits unavoidable performance loss due to the fixed coefficient, which is not applicable in different scenarios. [11] [12] proposed variable-wise MS (vWMS) algorithm. It divided the check nodes into two parts and the minimum results are computed separately. If the minimum results are the same, the computation of second-minimum is omitted. However, if the results are different, a summation coefficient is required to obtain the second-minimum, which brings some performance loss. [13] proposed relaxed MS (rMS) algorithm, which utilized the check nodes results in different iterations to reduce the performance loss, but the complexity is not reduced because the summation coefficient is used.

In this paper, a low-complexity LDPC decoding algorithm based on MS is proposed. The computation of second-minimum is eliminated and the approximate result is obtained by applying corrected coefficients. In addition, corrected coefficients simplify the whole computation of second-minimum. The results of complexity analysis show that the proposed algorithm enjoys low complexity compared with NMS and MS-based algorithms. Simulation results of NMS and MS-based simplified algorithms including proposed algorithm are provided. The performance of simplified algorithm is almost same as NMS algorithm with the same number of iterations in different scenarios.

The rest of the paper is constructed as follows. LDPC codes and the MS-based algorithms are briefly introduced in Section II. Section III proposes the low-complexity algorithm and the complexity analysis is provided. In Section IV, simulation results are analyzed. Finally, conclusions are drawn in Section V.

II. LDPC Codes and Decoding Algorithms

A binary LDPC code is a linear block code with a sparse $M \times N$ parity-check matrix $\boldsymbol{H}$. where M represents the number of check nodes and N indicates the number of variable nodes. A bipartite Tanner graph [19] can be utilized to describe matrix $\boldsymbol{H}$ with M check nodes and N variable nodes on both sides. If $\boldsymbol{H}(m,n)$ is nonzero, an edge exists between the check-node m and the variable-node n . Structured LDPC codes, such as QC-LDPC codes, are generated by partitioning matrix $\boldsymbol{H}$ into $M_b \times N_b$ square submatrices of size $z \times z$, so $M = M_b \times z$ and $N = N_b \times z$. Since structured codes enjoy productive performance and lower complexity, they become the most popular type in industrial standards.

Let $N_m = \{n : \boldsymbol{H}(m,n) = 1\}$ denote the set of neighbor nodes that connects to check node m and $M_n = \{m : \boldsymbol{H}(m,n) = 1\}$ represent the neighbor nodes that connects to variable node n . Meanwhile, let $N_{m/n}$ denote the set N_m except variable node n and $M_{n/m}$ represent the set M_n except check node m . v_n^i represents the n-th node decision information in i-th iteration. v_{mn} and u_{mn} denote the information transmitted from check-node n to variable-node m and from variable-node m to check-node n , respectively.

A. Min-Sum Algorithm

MS algorithm is an iterative algorithm based on the information transmission between check nodes and variable nodes. The MS algorithm is carried out during $i = 1,2,3 \ldots I_{\max}$ iterations as follows in detail.

Firstly, assign the initial value of v_{mn} by the input information.

$$v_{mn} = v_n^0 \tag{1}$$

Secondly, update the value of u_{mn} .

$$u_{mn} = \prod_{n' \in N_{m \setminus n}} \operatorname{sgn}\left(u_{mn'}\right) \min_{n' \in N_{m \setminus n}} \left|v_{mn'}\right| \tag{2}$$

Thirdly, update the value of v_{mn} .

$$v_{mn} = v_n^0 + \sum_{m' \in M_{n \setminus m}} u_{m'n} \tag{3}$$

Then, update the n-th node decision information on i-th iteration v_n^i .

$$v_n^i = \sum_{m \in M_n} u_{mn} \tag{4}$$

At last, make the hard decision.

$$z_n = \begin{cases} 0, v_n^i > 0 \\ 1, v_n^i \le 0 \end{cases} \tag{5}$$

When hard decision results of all checks nodes satisfy the following equations or the maximum number of iterations $I_{\max}$ is reached, the iteration process stops.

$$\boldsymbol{HZ} = \boldsymbol{0} \tag{6}$$

B. NMS and OMS algorithms

NMS and OMS are two modified algorithms based on general MS algorithm. Since the value of u_{mn} in MS is larger than corresponding value in BP algorithm, scaling coefficients are utilized in NMS and OMS to reach the value of u_{mn} in BP algorithm to improve the performance. A multiplication coefficient α and a summation coefficient β is used in NMS and OMS algorithms, respectively. The result of u_{mn} can be obtained by (7) for NMS and (8) for OMS.

$$u_{mn} = \alpha \cdot \prod_{n' \in N_{m \setminus n}} \operatorname{sgn}\left(v_{mn'}\right) \min_{n' \in N_{m \setminus n}} \left|v_{mn'}\right| \tag{7}$$

$$u_{mn} = \prod_{n' \in N_{m \setminus n}} \operatorname{sgn}\left(v_{mn'}\right) \left(\min_{n' \in N_{m \setminus n}} \left|v_{mn'}\right| - \beta \right) \tag{8}$$

C. Recent MS-based Simplified algorithms

As can be seen, MS and the two modified algorithms require the computations of first-minimum and second-minimum results in each row. Based on that, the computation in (2) can be transformed into (9) and (10).

$$u_{mn} = \begin{cases} \operatorname{sgn}\left(u_{mn}\right) \min_{n' \in N_m} \left|v_{mn'}\right|, v_{mn} \ne \min_{n' \in N_m} \left|v_{mn'}\right| \\ \operatorname{sgn}\left(u_{mn}\right) \min_{n' \in N_{m \setminus n}} \left|v_{mn'}\right|, v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right| \end{cases} \tag{9}$$

$$\operatorname{sgn}\left(u_{mn}\right) = \operatorname{sgn}\left(v_{mn}\right) \prod_{n' \in N_m} \operatorname{sgn}\left(v_{mn'}\right) \tag{10}$$

Many attempts have been made to simplify the computation of the second-minimum value of u_{mn} . The most common method is utilizing a summation corrected coefficient on the first minimum to approximate the second minimum. The estimation result can be obtained as the following form, where the corrected coefficient f^i is corresponding to the number of iterations i .

$$u_{mn} = \begin{cases} \operatorname{sgn}\left(u_{mn}\right) \min_{n' \in N_m} \left|v_{mn'}\right|, v_{mn} \ne \min_{n' \in N_m} \left|v_{mn'}\right| \\ \operatorname{sgn}\left(u_{mn}\right) \left(\min_{n' \in N_m} \left|v_{mn'}\right| - f^i \right), v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right| \end{cases} \tag{11}$$

III. Low-Complexity Decoding Algorithms

All the existing MS-based simplified algorithms utilize a summation corrected coefficient to obtain the approximate result of the second-minimum. Since the coefficient only depends on the number of iterations, it is possible that the approximate result is much more different from the true value. Based on that, large errors might be introduced. In some cases, the iteration fails to converge. By analyzing the behavior of the first and second minimum values on each iteration of NMS algorithm, it can be concluded that first-minimum and second-minimum are in the same order of magnitude. Furthermore, the difference between the first and second minimum values increase as the number of iterations increased. Based on that, the check-node update operation in proposed algorithm is shown in (12)

$$u_{mn} = \begin{cases} \operatorname{sgn}\left(u_{mn}\right) \min_{n' \in N_m} \left|v_{mn'}\right|, v_{mn} \neq \min_{n' \in N_m} \left|v_{mn'}\right| \\ \operatorname{sgn}\left(u_{mn}\right)\left(\gamma^i \cdot \min_{n' \in N_m} \left|v_{mn'}\right|\right), v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right| \end{cases} \quad (12)$$

Generally, one multiplication is much more complex than one summation, which indicates that the computation above is difficult for hardware implementation. However, the corrected coefficient is associated with the scaling coefficient in NMS method. The value of the corrected coefficient is represented by equation (13)

$$\gamma^i = \begin{cases} 1/\alpha, i \le 5 \\ 2/\alpha, 5 < i \le 10 \\ 4/\alpha, i > 10 \end{cases} \quad (13)$$

It can be found that the multiplication result of correction factor and scaling factor is shown in (14). Based on that, the check-node update operation can be obtained in equation (15)

$$\gamma^i \cdot \alpha = \begin{cases} 1, i \le 5 \\ 2, 5 < i \le 10 \\ 4, i > 10 \end{cases} \quad (14)$$

$$u_{mn} = \begin{cases} \alpha \cdot \operatorname{sgn}\left(u_{mn}\right) \min_{n' \in N_m} \left|v_{mn'}\right|, v_{mn} \neq \min_{n' \in N_m} \left|v_{mn'}\right| \\ \operatorname{sgn}\left(u_{mn}\right)\left(\gamma^i \cdot \min_{n' \in N_m} \left|v_{mn'}\right|\right), \\ v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right|, i \le 5 \\ 2\operatorname{sgn}\left(u_{mn}\right)\left(\gamma^i \cdot \min_{n' \in N_m} \left|v_{mn'}\right|\right), \\ v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right|, 5 < i \le 10 \\ 4\operatorname{sgn}\left(u_{mn}\right)\left(\gamma^i \cdot \min_{n' \in N_m} \left|v_{mn'}\right|\right), \\ v_{mn} = \min_{n' \in N_m} \left|v_{mn'}\right|, i > 10 \end{cases} \quad (15)$$

As can be seen from equation (15), the computation of the second-minimum is transformed into a shift of first-minimum before multiplying the scaling coefficient, which can be obtained fast for hardware implementation.

Then the number of shifters, comparators, and multipliers are analyzed to evaluate the complexity of the proposed algorithm and other MS-based algorithms. Let d_v denote the average number of variable node in each row and d_c denote the average number of variable node in each column. To obtain the first-minimum of u_{mn}, $d_v - 1$ times of comparisons are required. Next, $d_v - 2$ times of comparisons are required to find the second-minimum. Furthermore, we need to use d_v times of comparisons to decide whether to select the first-minimum or second-minimum for each variable node. Based on that, $2n$ times of multiplications are required in NMS algorithm to modify the result. In the proposed algorithm, since the computation of second-minimum is omitted and can be obtained in (14), only one shift operation is required. Similarly, the amount of other computations can be analyzed for proposed algorithm and other MS-based algorithms.

TABLE I. COMPLEXITY OF MS-BASED DECODING ALGORITHMS

Ms Based	Com.	Mul.	Shifters	Adders
NMS	$3m(d_v - 1)$	$2n$	-	n
smMS	$2md_v - 1$	$2n$	-	n
vwMS	$2md_v$	$2n$	-	n
rMS	$2md_v - 1$	$2n$	-	n
proposed	$2md_v - 1$	n	n	-

Table I compares the computation complexity of the proposed algorithm with several different MS-based algorithms. As can be seen, the number of comparators required is almost same for all the MS-based simplified algorithms, since the second-minimum is not obtained by comparators. The number of multipliers required for proposed algorithm is half of the other simplified algorithms because the relationship between the corrected coefficient and the scaling coefficient. Furthermore, one shift operation is less complex than one summation, which indicates that the computation complexity of proposed algorithm is less than others, considering the shifters and adders. It can be concluded that the performance of proposed algorithm enjoys lower complexity compared with NMS and other MS-based decoding algorithms.

IV. SIMULATION RESULTS

In this section, the performance of LDPC codes with different decoding algorithms is simulated with MS-based simplified algorithms and NMS algorithm as a standard. The (648,324) and (648,540) QC-LDPC code in Verizon 5G standard are utilized for evaluating the bit-error-rate (BER) performance of all the decoding algorithms with different signal-noise-rate (SNR). Additive white Gaussian channel (AWGN) noise is considered as the channel model. The maximum number of iterations is set to 10 in all simulations. QPSK is used as the modulation scheme. The scaling coefficient is set to $4/5$ in all the simulations.

Figure 1 compares the simulation results of the proposed algorithm compared with standard NMS algorithm. With the same number of iterations, the performance loss is nearly

0.1dB when the code rate is $1/2$. When the code rate goes up to $5/6$, the performance loss is nearly 0.15dB. Based on that, we can conclude that the proposed algorithm enjoys nearly same performance compared with NMS.

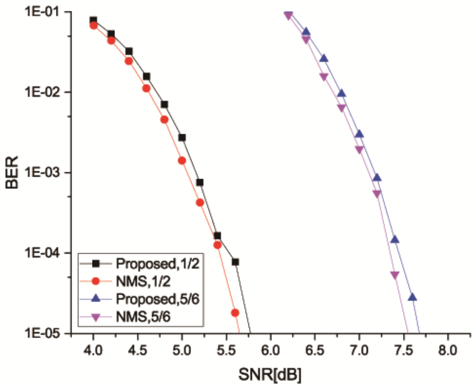

Figure 1. BER perfomance of proposed and NMS algorithms

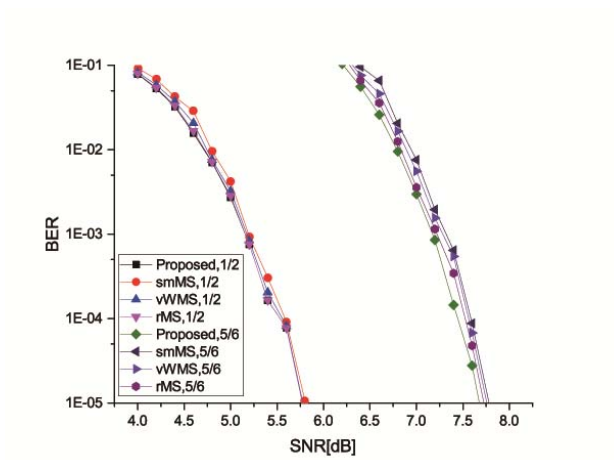

Figure 2. BER perfomance of proposed and existing simplified algorithms

Figure 2 compares simulation performance of the proposed algorithm and other MS-based simplified algorithms. As can be seen, the performance is almost same when the code rate is $1/2$. However, when the code rate goes up to $5/6$, the performance loss of addition-based simplified algorithms is nearly 0.2dB compared with the proposed algorithm. The reason is that the high code rate is more sensitive to noise, and fixed addition corrected coefficients make the estimation of second-minimum is inaccurate. Based on that, the performance loss of addition-based simplified algorithms is large in some cases, which can be avoided by the proposed algorithm. Therefore, we can conclude that the proposed algorithm enjoys better flexibility.

V. CONCLUTIONS

This paper proposes a low-complexity LDPC decoding algorithm based on MS. The computation of second-minimum is transformed into correction of first-minimum. The corrected coefficient enjoys a great relationship to scaling coefficient in popular NMS algorithm. The proposed algorithm simplifies the whole computation of the second-minimum. Computation complexity is analyzed and results show that the complexity of proposed algorithm is much lower than NMS and MS-based algorithms. Performance of NMS and MS-based simplified algorithms are simulated and results show that the performance of proposed algorithm and NMS algorithm are almost same. The proposed algorithm enjoys better performance than other MS-based algorithms with high code rate.

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