A Low-Complexity Detection Method Based on Iteration for Massive MIMO Systems

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***Abstract*—For uplink multi-user massive MIMO systems with hundreds of antennas at the base station (BS), minimum mean square error (MMSE) signal detection method is near-optimal due to the asymptotically orthogonal channel, but it involves matrix inversion with high complexity. In this paper, a low-complexity signal detection method in an iterative way is proposed to avoid the complicated matrix inversion. The convergence proof and complexity analysis of the proposed method are provided in detail. The analysis shows that the proposed method can effectively reduce the complexity by one order of magnitude. Simulation results demonstrate that the proposed method outperforms recently proposed Neumann series approximation method. Near-optimal performance of the MMSE method can be achieved with only a small number of iterations.**

Keywords-massive MIMO; signal detection; iteration; matrix blocking

I. INTRODUCTION

As a key technology of 5G, massive multiple-input multiple-output (MIMO) can take advantage of the large-scale antenna array to increase the space diversity gain and space multiplexing gain, which greatly enhance the spectrum and energy efficiency [1][2]. However, the increased number of antennas and supported transport streams makes massive MIMO systems faced with several challenging problems for baseband algorithm in practice. One of which is the signal detection in the uplink system.

Nowadays, MIMO detection of LTE-A system usually adopts linear detection algorithm, such as zero forcing (ZF) method or minimum mean square error (MMSE) method, based on the cholesky decomposition to obtain the matrix inversion [3]. The computational complexity is $O\left(K^3\right)$, where K represents the number of data streams transmitted on the same time-frequency resource. Compared with the traditional maximum likelihood (ML) algorithm based on a posteriori probability, the above linear detection methods enjoy better performance and lower complexity [4]. However, since the number of data streams supported by a massive MIMO system will be much larger than the current LTE-A system, the complexity cannot meet implementation requirements. Therefore, the research of massive MIMO linear detection method has become a hotspot. The research on low-complexity detection method mainly focuses on the simplification of large-scale matrix inversion. Neumann series approximation (NSA) method is proposed in [5][6], which transforms the inversion of the matrix into a series of matrix-vector multiplications. However, numbers of series items are required to approximate the performance of MMSE method. SOR iterative method is proposed in [7][8], which can achieve better performance with a small number of iterations. However, in order to reduce the complexity, SOR method can only be realized in serial, which indicates that it is not conducive to implement in hardware in parallel. Richardson iterative method is proposed in [9]. The iterative parameter calculation is too complex, and the performance is poor when the number of iterations is small. The amount of divisions is not considered in the above methods, but the delay caused by divisions is not negligible.

In this paper, a low-complexity near-optimal iterative detection method is adopted to avoid complex matrix inversion and reduce the complexity. First of all, the construction of the iterative matrix is proposed in detail. Secondly, the convergence of the proposed method is proved based on the characteristics of the channel matrix of massive MIMO system. Then the computational complexity is analyzed combined with the operation process. Analysis shows that the proposed method can reduce the complexity by one order of magnitude comparing with MMSE method. Through the performance simulation of the massive MIMO system, it is verified that the proposed method can approximate the MMSE method with a small number of iterations, which is half of the NSA method.

The rest of the paper is structured as follows. Section II briefly introduces the massive MIMO system model. In Section III, the optimization method based on iteration is proposed. The convergence proof and complexity analysis of the proposed method are also provided. Simulation results and analysis are shown in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a multi-user massive MIMO system, where the base station is equipped with N antennas and receives K data streams from different users transmitted on the same time-frequency resource, assuming that each user is a single antenna transmission. In such system, we usually have $N \gg K$ in such system.

According to Fig.1, a system model of a multi-user massive MIMO system can be described as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{N} \tag{1}$$

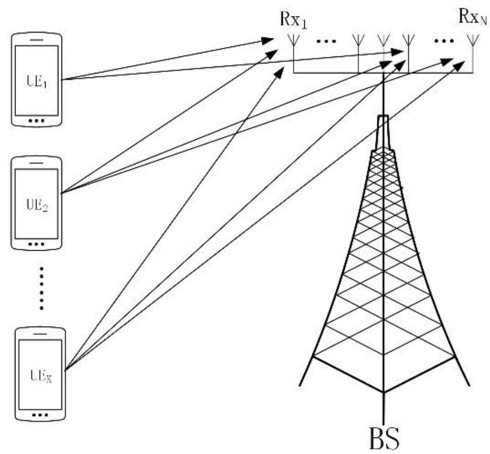

Figure 1. Multi-user massive MIMO in the uplink.

Let $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \cdots & \boldsymbol{x}_K \end{bmatrix}^{\mathrm{T}}$ denote the transmitted signals, where $\boldsymbol{x}_k$ represents the transmitted signal from the k-th user. $\boldsymbol{H} \in \mathbb{C}^{N \times K}$ denotes the channel matrix, and $\boldsymbol{N} = \begin{bmatrix} n_1 & n_2 & \cdots & n_N \end{bmatrix}^{\mathrm{T}}$ denotes the noise of N different receiving antennas, whose entries are independently and identically distributed (i.i.d) and follow the distribution $\boldsymbol{N}\left(0, \delta^2\right)$. The received signals at all the antennas can be expressed in a vector form as $\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{y}_1 & \boldsymbol{y}_2 & \cdots & \boldsymbol{y}_N \end{bmatrix}^{\mathrm{T}}$, where $\boldsymbol{y}_n$ denotes the received signal from antenna n.

According to reference [3], estimate of the transmitted signal vector $\hat{\boldsymbol{X}}$ can be expressed as (2) when using MMSE method

$$\hat{\boldsymbol{X}} = \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \delta^2 \boldsymbol{I}_x\right)^{-1} \boldsymbol{H}^{\mathrm{H}}\boldsymbol{Y} \tag{2}$$

Let $\boldsymbol{b} = \boldsymbol{H}^{\mathrm{H}}\boldsymbol{Y}$ denote the matched-filter, and $\boldsymbol{A} = \boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma^2 \boldsymbol{I}_x$ denotes the MMSE filtering matrix. The estimate of transmitted signal can be obtained as

$$\hat{\boldsymbol{X}} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{3}$$

The traditional MMSE method base on cholesky decomposition first evaluates the inversion of the matrix $\boldsymbol{A}$ and then multiplies the matched filter to estimate the transmitted signal. Note that the computation of the matrix inversion $\boldsymbol{A}^{-1}$ requires high complexity of $O\left(K^3\right)$. In order to obtain higher system capacity, K is usually 8, 16 or higher for massive MIMO systems, which makes matrix inversion difficult to implement for hardware. Therefore, the equivalent conversion of the inverse expression is required. The results of MMSE method can be approximated by a low complexity solution.

III. AN ITERATIVE OPTIMIZATION ALGORITHM

In order to avoid the complex matrix inversion, (3) can be transformed into the following system of linear equations (4), thus $\hat{\boldsymbol{X}}$ can be estimated by iterative method.

$$\boldsymbol{A}\hat{\boldsymbol{X}} = \boldsymbol{b} \tag{4}$$

When using iterative method, $\boldsymbol{A}$ is divided into a sum of two matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, where $\boldsymbol{P}$ is a nonsingular matrix.

$$\boldsymbol{A} = \boldsymbol{P} + \boldsymbol{Q} \tag{5}$$

The equation above can be obtained as

$$\boldsymbol{X} = -\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{X} + \boldsymbol{P}^{-1}\boldsymbol{b} \tag{6}$$

The solution of the system of linear equations in iterative way can be expressed as

$$\begin{aligned} \boldsymbol{X}_{k+1} &= -\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{X}_k + \boldsymbol{P}^{-1}\boldsymbol{b} \\ s.t.\ \lim_{x \to \infty}\left(-\boldsymbol{P}^{-1}\boldsymbol{Q}\right)^k &= 0 \end{aligned} \tag{7}$$

As can be seen from the above expression, we need to find a nonsingular $\boldsymbol{P}$, which satisfies the convergence condition when using iterative method. At the same time, we need to take into account the computational complexity. It is required that the inversion of $\boldsymbol{P}$ can be simply obtained and the number of iterations required for convergence is small. Commonly used iterative methods mainly include Jacobi iterative method and Gauss-Seidel iterative method [10]. In Jacobi method, $\boldsymbol{P} = diag\left(\boldsymbol{A}\right)$. In Gauss-Seidel method, $\boldsymbol{P} = diag\left(\boldsymbol{A}\right) + tril\left(\boldsymbol{A}\right)$.

In this paper, a new method of constructing iteration matrix is proposed. Matrix $\boldsymbol{P}$ is constructed by splicing several block matrices, which are obtained by partitioning the matrix $\boldsymbol{A}$, and the computational complexity can be reduced to $O\left(K^2\right)$ by utilizing the property of block matrix inversion, and the operation can be implemented in hardware in parallel. In addition, by using the properties of the 2×2 matrix inversion, the number of division can be reduced by half compared with the existing algorithms.

Proposed matrix $\boldsymbol{P}$ is constructed as follows. Firstly, dividing $\boldsymbol{A}$ into several 4×4 sub-matrices. The number of sub-matrices in each line after blocking is $M = K/4$. The original matrix $\boldsymbol{A}$ is divided into $M \times M$ sub-matrices. For M sub-matrices on the diagonal of $\boldsymbol{A}$, they are divided into four 2×2 sub-matrices. $\boldsymbol{P}$ is the result of combining the three sub-matrices at the upper left in all the sub-matrices on the diagonal. The construction results of $\boldsymbol{P}$ matrix can be expressed as

$$p_{i,j} = \begin{cases} a_{i,j}, j-3 \le i \le j, & i \bmod 4 = 1 \\ a_{i,j}, j-2 \le i \le j+1, & i \bmod 4 = 2 \\ a_{i,j}, j-1 \le i \le j, & i \bmod 4 = 3 \\ a_{i,j}, j \le i \le j+1, & i \bmod 4 = 0 \\ 0, \text{others} & \end{cases} \tag{8}$$

where $p_{i,j}$ and $a_{i,j}$ donate the values of the i-th row and j-th column elements of $\boldsymbol{P}$ and $\boldsymbol{A}$, respectively. Figure 2 shows the $\boldsymbol{P}$ when the matrix dimension is 8.

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & 0 & 0 & 0 & 0 \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,3} & a_{3,4} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{4,3} & a_{4,4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{5,5} & a_{5,6} & a_{5,7} & a_{5,8} \\ 0 & 0 & 0 & 0 & a_{6,5} & a_{6,6} & a_{6,7} & a_{6,8} \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{7,7} & a_{7,8} \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{8,7} & a_{8,8} \end{bmatrix}$$

Figure 2. The iterative matrix $\boldsymbol{P}$ (K=8,M=4,m=2).

In order to verify the feasibility of the proposed method, the convergence is proved in Section III-A. The computational complexity is analyzed to explain the superiority of the proposed method in Section III-B.

A. Convergence Proof

Let λ_n denote an eigenvalue of matrix $-\boldsymbol{P}^{-1}\boldsymbol{Q}$, and $\boldsymbol{v}^{\mathrm{T}}$ denotes the eigenvector corresponding to the eigenvalue λ_n which satisfies (9)

$$-\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{v}^{\mathrm{T}} = \lambda_n \boldsymbol{v}^{\mathrm{T}} \tag{9}$$

According to (5), expression (9) can be expressed as

$$\boldsymbol{v}\left(\boldsymbol{P} - \boldsymbol{A}\right)\boldsymbol{v}^{\mathrm{T}} = \lambda_n \boldsymbol{v}\boldsymbol{P}\boldsymbol{v}^{\mathrm{T}} \tag{10}$$

Utilizing to the property of conjugate transpose matrix

$$\boldsymbol{v}\left(\boldsymbol{P}^{\mathrm{H}} - \boldsymbol{A}\right)\boldsymbol{v}^{\mathrm{T}} = \lambda_n \boldsymbol{v}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{v}^{\mathrm{T}} \tag{11}$$

According to (10)(11)

$$\boldsymbol{v}\left(\boldsymbol{P} + \boldsymbol{P}^{\mathrm{H}} - 2\boldsymbol{A}\right)\boldsymbol{v}^{\mathrm{T}} = \lambda_n \boldsymbol{v}\left(\boldsymbol{P} + \boldsymbol{P}^{\mathrm{H}}\right)\boldsymbol{v}^{\mathrm{T}} \tag{12}$$

Note $\boldsymbol{C} = \boldsymbol{P} - \boldsymbol{Q}^{\mathrm{H}}$, According to (5)

$$\begin{aligned} \boldsymbol{P} + \boldsymbol{P}^{\mathrm{H}} - 2\boldsymbol{A} &= \boldsymbol{C} - \boldsymbol{A} \\ \boldsymbol{P} + \boldsymbol{P}^{\mathrm{H}} &= \boldsymbol{C} + \boldsymbol{A} \end{aligned} \tag{13}$$

According to (13), (12) can be expressed as

$$\left(1 - \lambda_n\right)\boldsymbol{v}\boldsymbol{C}\boldsymbol{v}^{\mathrm{T}} = \left(1 + \lambda_n\right)\boldsymbol{v}\boldsymbol{A}\boldsymbol{v}^{\mathrm{T}} \tag{14}$$

Unlike the small-scale MIMO system, massive MIMO systems enjoy a special property that the column vectors of the channel matrix are asymptotically orthogonal. Base on that, $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ is a strictly diagonally dominant matrix. At the same time, $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ is a Hermite matrix, and diagonal elements of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ are positive, then $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ is a positive definite matrix

Considering $\boldsymbol{A} = \boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma^2\boldsymbol{I}_x$, σ^2 is greater than or equal to 0, thus $\boldsymbol{A}$ is a strictly diagonally dominant matrix and is a positive definite matrix. According to the definition of strictly diagonally dominant matrix, the expression (15) is satisfied.

$$\left\| a_{j,j} \right\| > \sum_{i=1, i\neq j}^{K} \left\| a_{j,i} \right\| \tag{15}$$

According to the property of positive definite matrix, the expression (16) is satisfied.

$$\boldsymbol{v}\boldsymbol{A}\boldsymbol{v}^{\mathrm{T}} > 0 \tag{16}$$

Since $\boldsymbol{C} = \boldsymbol{P} - \boldsymbol{Q}^{\mathrm{H}}$, which keeps all the diagonal elements of $\boldsymbol{A}$, (15) is satisfied. Thus $\boldsymbol{C}$ is a strictly diagonally dominant matrix. Meanwhile, because the diagonal elements are positive numbers, $\boldsymbol{C}$ is a Hermite positive definite matrix, which satisfies (17)

$$\boldsymbol{v}\boldsymbol{C}\boldsymbol{v}^{\mathrm{T}} > 0 \tag{17}$$

According to the expression (14)(16)(17), the expression (18) is satisfied.

$$\left(1 - \lambda_n\right)\left(1 + \lambda_n\right) > 0 \tag{18}$$

According to (18), it can be obtained that $\left|\lambda_n\right| < 1$, which satisfies the convergence condition , thus the constructed matrix can make the iterative process converged.

B. Computational Complexity Analysis

According to the above derivation process, using the matrix constructed in this paper, the detected signal can be obtained in an iterative way of the following form.

$$\boldsymbol{X}_{k+1} = \boldsymbol{P}^{-1}\left(\boldsymbol{b} - \boldsymbol{Q}\boldsymbol{X}_k\right) \tag{19}$$

The computational complexity in terms of required number of multiplications and divisions is analyzed, since the computational complexity is dominated by these two operations. First of all, it can be found from (19) that in the k-th iteration, the proposed method involves one multiplication of the $K \times K$ matrix $\boldsymbol{Q}$ and the $K \times 1$ vector $\boldsymbol{X}_k$. Since the number of nonzero elements in $\boldsymbol{Q}$ is $\left(K - 3\right) \times K$, the required number of multiplications to compute $\left(\boldsymbol{b}-\boldsymbol{Q}\boldsymbol{X}_k\right)$ is $\left(K-3\right)\times K$. Secondly, note $\boldsymbol{c}^k=\left[\boldsymbol{c}^{k,1}\ \ \boldsymbol{c}^{k,2}\ \ \cdots\ \ \boldsymbol{c}^{k,\mathrm{M}}\right]^{\mathrm{T}}=\boldsymbol{b}-\boldsymbol{Q}\boldsymbol{X}_k$, where $\boldsymbol{c}^{k,1}$ denotes the l-th 4×1 vector. Similarly, let $\boldsymbol{P}_l$ denote the l-th 4×4 diagonal matrix of $\boldsymbol{A}$. Based on that, we can compute $\boldsymbol{P}_l^{-1}\boldsymbol{c}^{k,1}$ in parallel instead of $\boldsymbol{P}^{-1}\boldsymbol{c}^k$. $\boldsymbol{P}_l^{-1}$ can be obtained by the following expression, where $\boldsymbol{A}_l$, $\boldsymbol{B}_l$, $\boldsymbol{C}_l$ donate three 2×2 matrix.

$$\boldsymbol{P}_l^{-1}=\begin{bmatrix}\boldsymbol{A}_l & \boldsymbol{B}_l\\ \boldsymbol{0} & \boldsymbol{C}_l\end{bmatrix}^{-1}=\begin{bmatrix}\boldsymbol{A}_l^{-1} & -\boldsymbol{A}_l^{-1}\boldsymbol{B}_l\boldsymbol{C}_l^{-1}\\ \boldsymbol{0} & \boldsymbol{C}_l^{-1}\end{bmatrix} \tag{20}$$

Based on that, $\boldsymbol{P}_l^{-1}\boldsymbol{c}^{k,1}$ can be obtained as (20), where $\boldsymbol{c}_{k,l}^1$, $\boldsymbol{c}_{k,l}^2$ donate two 2×1 vector.

$$\boldsymbol{P}_l^{-1}\boldsymbol{c}^{k,1}=\begin{bmatrix}\boldsymbol{A}_l^{-1} & -\boldsymbol{A}_l^{-1}\boldsymbol{B}_l\boldsymbol{C}_l^{-1}\\ \boldsymbol{0} & \boldsymbol{C}_l^{-1}\end{bmatrix}\begin{bmatrix}\boldsymbol{c}_{k,l}^1\\ \boldsymbol{c}_{k,l}^2\end{bmatrix}= \begin{bmatrix}\boldsymbol{A}_l^{-1}\left[\boldsymbol{c}_{k,l}^1-\boldsymbol{B}_l\left(\boldsymbol{C}_l^{-1}\boldsymbol{c}_{k,l}^2\right)\right]\\ \boldsymbol{C}_l^{-1}\boldsymbol{c}_{k,l}^2\end{bmatrix} \tag{21}$$

It can be found from (21) that it involves three multiplications of 2×2 matrix and 2×1 vector as well as two inversions of 2×2 matrix. The required number of multiplications is 12. Taking into account the relationship between $\boldsymbol{P}_l$ and $\boldsymbol{P}$, the required number of multiplications to compute $\boldsymbol{P}^{-1}\boldsymbol{c}^k$ is $3K$. Therefore, K^2 times of multiplications are required for each iteration.

The inversion of 2×2 matrix can be obtained as (22)

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} \tag{22}$$

The 2×2 matrix in this paper is a Hermite matrix. Based on that, the required number of multiplications is 4, and one division is also required. Therefore, the multiplication and division times brought by all 2×2 matrix inversions are $2K$ and $K/2$. As the initial vector takes $\boldsymbol{0}$ for the first iteration, $3K$ times of multiplications are required.

TABLE I. COMPLEXITY COMPARASION

	Proposed method		NSA method	
iteration	multiplications	divisions	multiplications	divisions
2	K^2+5K	K/2	$2K^2$-K	K
3	$2K^2$+5K		K^3	
4	$3K^2$+5K		$2K^3$-K^2	
5	$4K^2$+5K		$3K^3$-$2K^2$	

Table I compares the complexity of the NSA and the proposed method based on iteration. Note that the complexity of MMSE method based on cholesky decomposition is $O\left(K^3\right)$. It can be concluded from Table I that he complexity of the NSA can be reduced from $O\left(K^3\right)$ to $O\left(K^2\right)$ when the number of iterations is less than 3 However, a large number of iterations is required to ensure the approximation performance by NSA method. It can be observed that the complexity of proposed method is $O\left(K^2\right)$ with any number of iterations. At the same number of iterations, the complexity of proposed method is lower than NSA method.

Additionally, the proposed method only requires K/2 times of divisions, which is half of the NSA method and cholesky decomposition. Since the division operation is difficult for hardware implementation, the proposed method can further reduce the delay.

IV. SIMULATION RESULTS

In this section, the detection performance of the massive MIMO system with different number of iterations is simulated and compared with the NSA method. The simulation is based on the LTE-A uplink system, the number of antennas at base station is 64, and the number of users is 8, assuming that each user transmits 1 stream of data. Turbo encoding is used, and 16QAM is adopted for the modulation scheme.

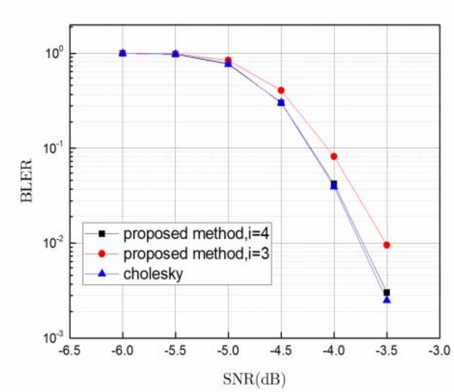

Figure 3. The simulation results of proposed method

Figure 3 shows the comparison results of the proposed method with different numbers of iterations and the results of MMSE method as a standard. As we can see, with the increase of the number of iterations, the performance of the optimization algorithm is improved. When the number of iteration is 3, the performance gap between the proposed method and the standard result is 0.2dB. The performance of the proposed method is almost same as the standard result when the number of iteration is 4.

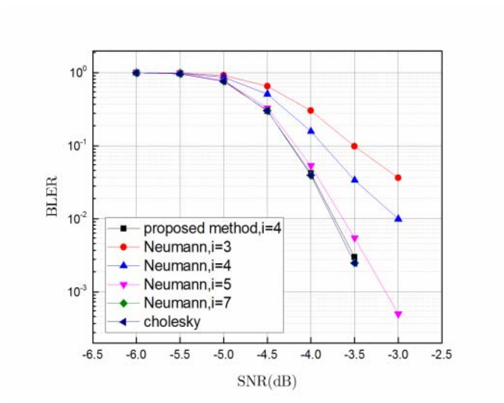

Figure 4. The performance comparison between the proposed method and NSA method

Figure 4 shows the performance comparison between the proposed method and NSA method. As can be seen from the figure, both the two methods can approach the standard results after several times of iterations. 7 times of iterations is required for NSA method and only 4 times of iterations is required for the proposed method. Taking into account the complexity of both methods, it can be concluded that the proposed method converge much faster than NSA method. The proposed method can achieve almost same performance of MMSE method with low-complexity.

V. CONCLUSIONS

In this paper, we propose a low-complexity detection method to avoid the complicated matrix inversion, which transforms the inversion into an iterative linear solution. Based on that, we construct the iteration matrix by blocking the original complex matrix. By ultizing the channel characteristics of massive MIMO systems, the convergence of the proposed method is proved in detail. The complexity analysis shows that the proposed method can reduce the complexity by one order of magnitude comparing with the traditional MMSE method. In addition, the complexity of proposed method is lower than the NSA method and half of the divisions can be reduced. Simulation results shows that with only a small number of iterations, the proposed method can achieve the near-optimal performance of the MMSE method. The number of iterations required to achieve the near-optimal performance is near half comparing with the NSA method. Additionally, the detection algorithm can be used in other communication field of signal processing, such as downlink linear precoding process of massive MIMO system.

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