



Immersion and invariance adaptive control with σ -modification for uncertain nonlinear systems

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Abstract

A novel adaptive control with σ -modification for uncertain nonlinear systems is proposed in the paper. The application of conventional adaptive control is severely limited by the problems of construction of Lyapunov function and parameter drift because of non-parametric uncertainties. The proposed adaptive control that is on the basis of the immersion and invariance theory and σ -modification can be used to deal with these problems to some extent. It turns out to be a structured design method without requiring a Lyapunov function in the design level and robust to non-parametric uncertainties. Moreover, constrained command filter backstepping is adopted to meet the amplitude and rate constraints on the states and actuators. The uniformly ultimately bounded stability of the closed-loop system has been analyzed by Lyapunov theory with parametric and non-parametric uncertainties of the controlled model. To demonstrate the design flexibility, the method is applied to the position tracking control system design of a mass-damper-spring system and the flight control system design of a scramjet-powered air-breathing hypersonic vehicle. Finally, the effectiveness of the proposed adaptive control method is illustrated by numerical simulations.

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1. Introduction

Adaptive control (AC) is an effective control method for a class of uncertain plants with unknown constant or slow-varying parameters. Since appearing in 1950s, it has gained

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sustained attention and extensive research in various fields, such as flight control [1–4], robot control [5,6] and industrial control [7], especially in recent decades [8–10]. In sharp contrast, applications of AC in practical control engineering has not gained enough attention. It is evident that there is a gap between theoretical research and engineering practice.

One of the most widely studied adaptive control methods is the so-called Lyapunov synthesis method which needs a construction of the Lyapunov function. But as known, there is not a generic or systematic technique to choose a Lyapunov function. Particularly, for some complex nonlinear systems, it could be much difficult. The common practice is empirical and trial-and-error at present. Moreover, AC is sensitive to non-parametric uncertainties, including unmodeled dynamics, disturbances, and measurement noises, which not only lead to performance degradation but also cause some unpredictable instability phenomena, such as parameter drift [11]. The primary reason for the instability is that adaptive controller is inherently nonlinear, which means the sensitiveness is unavoidable.

Both the difficulties of the design and the sensitiveness to non-parametric uncertainties of the closed-loop systems greatly limit the application of existing adaptive control methods. Fortunately, much work has been done to overcome the gap. One remarkable work among them is called immersion and invariance (I&I) adaptive control [12], which bypasses the design issue of choosing Lyapunov function in the conventional AC. In the I&I adaptive control, there is no need to find a Lyapunov function at the adaptive law design level, but it is convenient to construct one for stability analysis of the closed-loop system. One of the remarkable innovation of the method is the introduction of an additional term β in the adaptive law which not only provides an extra design freedom to shape the dynamics of parameter estimation error but also plays a key role in the stability analysis. Due to the flexibility of adaptive law design, I&I adaptive control obtains great attention. In [13], an I&I adaptive control system is designed for a hypersonic vehicle model for cruise control with the assumption that all the aerodynamic parameters are unknown. Similarly, it has been applied to missile control [14], quadrotor UAV control [15] and so on [16].

To address the unpredictable instability phenomena of conventional AC, robust adaptive control as a new field has been gradually formed since the 1980s [17]. Its main idea is to modify the adaptive law to tolerate the non-parametric uncertainties. Three modification methods have been widely discussed [18]. The first is referred to as dead-zone [19] which introduces an interval for the adaptive law so that it only updates parameters when the tracking error is larger than the upper bound of disturbance, otherwise it is switched-off. It is easy to understand and effective, but the set of the dead-zone needs the assumption that the bound of the modeling error is known which is hard to be satisfied. Moreover, the discontinuity of adaptive law resulting from the switch may cause some other theoretical problems. The second is called σ -modification [20], which subtracts a proportional item of the parameter estimate from the original pure integral adaptive law. The algorithm can guarantee all signals of the closed-loop system bounded in the presence of unknown bounded modeling error. The third is named e_1 -modification [21], which subtracts a term that is the product of the absolute value of the tracking error and the parameter estimate from the conventional adaptive law. Theoretically speaking, e_1 -modification has some better properties than σ -modification, but it is difficult to analyze.

The methods mentioned above investigate the two difficulties of AC application respectively. It appears that little work has been done to solve the two difficulties together and no attempt has been made to design I&I adaptive laws with σ -modification. Motivated by this, a new I&I adaptive control method with σ -modification is proposed in this paper trying to deal

with the two difficulties of AC application together. The method, based on constrained command filter backstepping framework, creatively introduces σ -modification into I&I adaptive control, which guarantees the stability of parameter update law in the existence of non-parametric uncertainties. The most important contribution of the paper is that the proposed method is free of the knowledge of Lyapunov function in the adaptive law design process and robust to non-parametric uncertainties, which could facilitate the application of AC.

The mass-damper-spring (MDS) system is one of the most common controlled objects in control theory, used for validation of new control methods and comparison of different methods [22,23]. The position control of the MDS system is a classical control problem. The model of the MDS system can be viewed as a generalized pendulum, which has a wide range of applications in practice, such as RLC circuits. It is a good example to show the design procedure of the proposed adaptive control method by applying it to the position control problem. The flight control system design or autopilots design for aircraft is the origin of AC and one of the main directions of AC research and application [23]. Scramjet-powered air-breathing hypersonic vehicles (AHVs) is a research focus in aerospace field in recent years [24]. However, the complicated dynamic characteristics of the vehicles pose great challenges to the design of flight control systems [25,26]. Therefore, the proposed adaptive control is applied to the AHVs flight control system design, which on the one hand proposes a new AHVs flight control system, on the other hand, is an effective validation of the method.

It is noteworthy that the universal approximation property based adaptive control methods including adaptive neural network control and adaptive fuzzy control have gained more and more attention and research for uncertain nonlinear systems in recent years. In [27], an adaptive neural impedance control is developed for a n -link robotic manipulator, which does not require the knowledge of robotic dynamics. An adaptive network control is designed for a flapping wing micro aerial vehicle to deal with the model uncertainties and enhance the system robustness in [28,29]. An adaptive neural control is developed to address the output tracking problem for a class of stochastic nonlinear time-delay systems with multiple constraints in [30]. For a class of nonlinear systems with sampled and delayed measurements, an adaptive fuzzy backstepping control is presented in [31]. This kind of adaptive methods with function approximation ability requires less model information, which greatly simplifies the control system design process. However, there also exist some difficulties in constructing basis functions and setting initial values of adaptive parameters for these methods. Besides, They cannot make full use of the existing model information in some certain applications, such as flight control system design. In practice, a large number of ground tests will be done to obtain a nominal model of a vehicle by system identification. But aerodynamic parameters of the model will change dramatically in the whole flight envelope. Therefore, an adaptive flight control system with parameter update laws based on the nominal model can not only take full advantage of the model information but also improve model estimation accuracy. They are two different types of adaptive control methods possessing different advantages/disadvantages and application scenarios for the function approximation based adaptive methods and the parameter estimation based ones, respectively. And the focus of this paper is the improvement and application of the parameter estimation based adaptive control.

The rest of the paper is organized as follows. The model and control objective are described in the next section. In Section 3, the details about the proposed I&I adaptive control method are discussed, including design process and stability analysis. Furthermore, two academic examples are studied with the proposed method in Section 4. The first case is the position tracking control system design of a MDS system with unknown mass to study the tuning of

the σ parameter and its effect on the adaptive law. The second case is the cruise flight control system design of a longitudinal model of an AHV with aerodynamic parameter uncertainty to illustrate the effectiveness and robustness of the proposed method. Finally is the conclusion and prospect of the paper.

2. Problem formulation

Consider the n th-order parametric feedback nonlinear system described by

$$\begin{cases} \dot{x}_1 = f_1(z_1) + g_1(z_1)x_2 + \Delta_1 \\ \dot{x}_i = f_i(z_i) + g_i(z_i)x_{i+1} + \Delta_i, \quad i = 2, \dots, n-1 \\ \dot{x}_n = f_n(z_n) + g_n(z_n)u + \Delta_n, \\ y = x_1, \end{cases} \quad (1)$$

where $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $\mathbf{z}_i = [x_1, \dots, x_i]^T$ and $\mathbf{\Delta} = [\Delta_1, \dots, \Delta_n]^T$ denotes model errors. The control objective is to design an adaptive controller for system (1) such that the output y tracks the desired reference command x_c , while all signals of the closed-loop system are bounded in the presence of modeling errors. Before further discussion, the following basic assumptions are presented.

Assumption 1. There exist additive modeling errors Δ_i in the system (1), and

$$\|\Delta_i\| \leq \eta_i, \quad i = 1, \dots, n,$$

where η_i are unknown positive constants and $\|\cdot\|$ denotes the Euclidian norm of a vector.

Notice that it only requires the modeling errors to be bounded, but the specific boundary values are not needed. This is easy to be satisfied.

Assumption 2. $f_i(\cdot)$ and $g_i(\cdot)$ are smooth functions, $g_i(\cdot) \neq 0$ and

$$\begin{aligned} f_i(z_i) &= \theta_{i1}^T \phi_{i1}(z_i) \\ g_i(z_i) &= \theta_{i2}^T \phi_{i2}(z_i), \quad i = 1, \dots, n, \end{aligned}$$

where $\phi_{i1}(z_i)$ and $\phi_{i2}(z_i)$ are known smooth function vectors, and θ_{i1} and θ_{i2} are unknown constant vectors.

This is the so-called linear parametrization condition which is very common in modeling and system identification. However, not all modeling errors can be parametrized. Because non-parametric uncertainties are unavoidable, which is presented as Assumption 1.

3. Main results

3.1. I&I adaptive control with σ -modification

To begin with, define the estimates of θ_{i1} , θ_{i2} as $\hat{\theta}_{i1} + \beta_{i1}$, $\hat{\theta}_{i2} + \beta_{i2}$, respectively. Obviously, they are different from the estimators based on the Lyapunov synthesis adaptive approach in terms of the additional terms $\beta_{(\cdot)}$ which allow for shaping the dynamics of the estimation errors and whose specific forms will be represented in the following pages. Then,

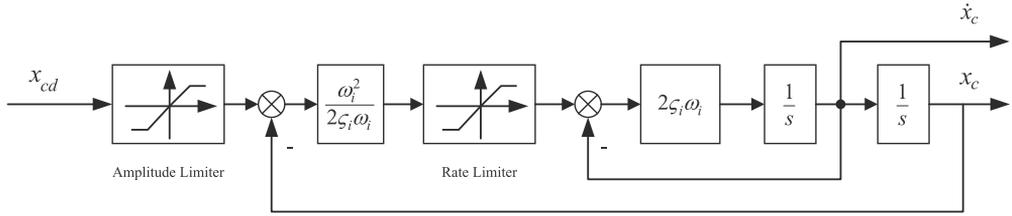


Fig. 1. Constraint command filter.

the parameter estimation errors can be defined as

$$\begin{aligned} \tilde{\theta}_{i1} &\triangleq \hat{\theta}_{i1} + \beta_{i1} - \theta_{i1} \\ \tilde{\theta}_{i2} &\triangleq \hat{\theta}_{i2} + \beta_{i2} - \theta_{i2}, \quad i = 1, \dots, n. \end{aligned} \tag{2}$$

Step 1: Treating x_2 as an intermediate virtual control variable to make x_1 track x_c for Eq. (1). Define the tracking error as $\tilde{x}_1 \triangleq x_1 - x_c$ whose dynamics is

$$\dot{\tilde{x}}_1 = \theta_{11}^T \phi_{11}(x_1) + \theta_{12}^T \phi_{12}(x_1)x_2 - \dot{x}_c + \Delta_1. \tag{3}$$

To get the desired virtual control command, the stabilizing signal can be designed as

$$x_{2cd} = \left[-k_1 \tilde{x}_1 - (\hat{\theta}_{11} + \beta_{11})^T \phi_{11} + \dot{x}_c \right] \left[(\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \right]^{-1}. \tag{4}$$

Because of the state constraint, the desired virtual command cannot be guaranteed to be reached all the time, especially in the transient process. Thus, the feasible virtual control command x_{2c} and its time derivative \dot{x}_{2c} can be generated by passing x_{2cd} through the constraint command filter shown as Fig. 1 [32].

On account of the existence of the command filter, the stability of the tracking error \tilde{x}_1 becomes implicit. So the compensated error which takes the error caused by the filter into account is introduced [32] as

$$\bar{x}_1 \triangleq \tilde{x}_1 - \xi_1, \tag{5}$$

where

$$\dot{\xi}_1 \triangleq -k_1 \xi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12}(x_{2c} - x_{2cd} + \xi_2), \tag{6}$$

is the auxiliary filtering error. It can be seen that the compensated tracking error would converge to the actual tracking error if the constraints on the virtual control were not in effect. Differentiating Eq. (5) gives the dynamics of \bar{x}_1

$$\dot{\bar{x}}_1 = -k_1 \bar{x}_1 - \tilde{\theta}_1^T \Phi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \bar{x}_2 + \Delta_1, \tag{7}$$

where $\tilde{\theta}_1 \triangleq [\tilde{\theta}_{11}^T, \tilde{\theta}_{12}^T]^T$, $\hat{\theta}_1 \triangleq [\hat{\theta}_{11}^T, \hat{\theta}_{12}^T]^T$, $\beta_1 \triangleq [\beta_{11}^T, \beta_{12}^T]^T$, $\Phi_1 \triangleq [\phi_{11}^T, \phi_{12}^T x_2]^T$ and \bar{x}_2 is the compensated tracking error of x_2 , which will be defined in the next step.

Substituting Eq. (7) into the derivative of Eq. (2), the dynamics of the parametric estimation errors are

$$\dot{\tilde{\theta}}_1 = \dot{\hat{\theta}}_1 + \frac{\partial \beta_1}{\partial \bar{x}_1} \left[-k_1 \bar{x}_1 - \tilde{\theta}_1^T \Phi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \bar{x}_2 + \Delta_1 \right]. \tag{8}$$

Considering that traditional adaptive control is sensitive to non-parametric uncertainties, the σ -modification is introduced into I&I adaptive control. This guarantees convergence of

adaptive law in the presence of bounded non-parametric uncertainties while simplifies the design of the adaptive law. The impact of the σ term will be shown in the stability analysis. The I&I adaptive law with σ -modification can be designed as

$$\begin{cases} \dot{\hat{\theta}}_1 &= \frac{\partial \beta_1}{\partial \bar{x}_1} \left[k_1 \bar{x}_1 - (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \bar{x}_2 \right] - r_1 \sigma_1 (\hat{\theta}_1 + \beta_1) \\ \frac{\partial \beta_1}{\partial \bar{x}_1} &= r_1 \Phi_1, \end{cases} \tag{9}$$

where r_1 is a diagonal positive definite matrix and σ_1 is a small positive constant. Note that the nonlinear function β_1 should be chosen to guarantee the stability of the estimate errors dynamics. This is a great advantage of the I&I adaptive approach over the other adaptive methods. The extra degree of freedom in designing adaptive laws, i.e., the selection of β_1 , allows the construction of the $\tilde{\theta}_1$ dynamics, which could yield an improvement of the closed-loop performance. It is in sharp contrast with the classical nonlinear adaptive control which has little knowledge of the behavior of the estimation errors.

The term including σ_1 in the adaptive law is brought in by σ -modification which ensures the update law bounded in the existence of modeling errors. There exist both parametric and non-parametric uncertainties in the model used for control system design in control engineering. However, both the conventional and I&I adaptive control may excite parameter drift phenomenon in adaptive law and result in instability of the closed-loop system under non-parametric uncertainties. It must be carefully addressed in practical adaptive control system design. Therefore, the I&I adaptive control with σ -modification is designed here. And the important role of σ -modification will be shown in the stability analysis section.

Substituting Eq. (9) into Eq. (8), the dynamics of the estimation errors can be rewritten as

$$\dot{\tilde{\theta}}_1 = -r_1 \Phi_1 \tilde{\theta}_1^T \Phi_1 - r_1 \sigma_1 (\hat{\theta}_1 + \beta_1) + r_1 \Phi_1 \Delta_1. \tag{10}$$

Step i: Taking x_{i+1} ($i = 2, \dots, n - 1$) as an intermediate virtual control variable to make x_i track x_{ic} . Define the tracking error as $\tilde{x}_i \triangleq x_i - x_{ic}$ whose dynamics is

$$\dot{\tilde{x}}_i = \theta_{i1}^T \phi_{i1} + \theta_{i2}^T \phi_{i2} x_{i+1} - \dot{x}_{ic} + \Delta_i. \tag{11}$$

So the stabilizing function is

$$x_{i+1,cd} = \left[-k_i \tilde{x}_i - (\hat{\theta}_{i1} + \beta_{i1})^T \phi_{i1} + \dot{x}_{ic} - (\hat{\theta}_{i-1,2} + \beta_{i-1,2})^T \phi_{i-1,2} \tilde{x}_{i-1} \right] \left[(\hat{\theta}_{i2} + \beta_{i2})^T \phi_{i2} \right]^{-1}. \tag{12}$$

Pass $x_{i+1,cd}$ through the command filter to produce the magnitude and rate limited command signal $x_{i+1,c}$ and its derivative $\dot{x}_{i+1,c}$.

The corresponding compensated tracking error is defined as

$$\tilde{x}_i := \tilde{x}_i - \xi_i, \tag{13}$$

where

$$\dot{\xi}_1 \triangleq -k_1 \xi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} (x_{1+1,c} - x_{1+1,cd} + \xi_{1+1}).$$

And its dynamics can be arranged as

$$\dot{\tilde{x}}_i = -k_i \tilde{x}_i - \tilde{\theta}_i^T \Phi_i - (\hat{\theta}_{i-1,2} + \beta_{i-1,2})^T \phi_{i-1,2} \tilde{x}_{i-1} + (\hat{\theta}_{i2} + \beta_{i2})^T \phi_{i2} \tilde{x}_{i+1} + \Delta_i, \tag{14}$$

where $\tilde{\theta}_i \triangleq [\tilde{\theta}_{i1}^T, \tilde{\theta}_{i2}^T]^T$, $\hat{\theta}_i \triangleq [\hat{\theta}_{i1}^T, \hat{\theta}_{i2}^T]^T$, $\beta_i \triangleq [\beta_{i1}^T, \beta_{i2}^T]^T$, and $\Phi_i \triangleq [\phi_{i1}^T, \phi_{i2}^T x_{i+1}]^T$.

The dynamics of the estimate errors are

$$\dot{\hat{\theta}}_i = \dot{\theta}_i + \frac{\partial \beta_i}{\partial \bar{x}_i}(\dot{\bar{x}}_i). \tag{15}$$

Substituting Eq. (14) into Eq. (15), the I&I adaptive control with σ -modification can be designed as

$$\begin{cases} \dot{\hat{\theta}}_i = \frac{\partial \beta_i}{\partial \bar{x}_i} \left[k_i \bar{x}_i + (\hat{\theta}_{i-1,2} + \beta_{i-1,2})^T \phi_{i-1,2} \bar{x}_{i-1} - (\hat{\theta}_{i2} + \beta_{i2})^T \phi_{i2} \bar{x}_{i+1} \right] - r_i \sigma_i (\hat{\theta}_i + \beta_i) \\ \frac{\partial \beta_i}{\partial \bar{x}_i} = r_i \Phi_i. \end{cases} \tag{16}$$

Substituting Eqs. (16) and (14) into Eq. (15), the dynamics of the estimate errors are rewritten as

$$\dot{\hat{\theta}}_i = -r_i \Phi_i \tilde{\theta}_i^T \Phi_i - r_i \sigma_i (\hat{\theta}_i + \beta_i) + r_i \Phi_i \Delta_i \tag{17}$$

Step n: Design the state feedback control law u to make the state x_n track virtual command x_{nc} . Define the tracking error as $\tilde{x}_n = x_n - x_{nc}$ whose dynamics are

$$\dot{\tilde{x}}_n = \theta_{n1}^T \phi_{n1}(x) + \theta_{n2}^T \phi_{n2}(x)u - \dot{x}_{nc} + \Delta_n. \tag{18}$$

Then, to get the desired control signal, the controller can be designed as

$$u_{cd} = \left[-k_n \tilde{x}_n - (\hat{\theta}_{n1} + \beta_{n1})^T \phi_{n1} + \dot{x}_{nc} - (\hat{\theta}_{n-1,2} + \beta_{n-1,2})^T \phi_{n-1,2} \bar{x}_{n-1} \right] \left[(\hat{\theta}_{n2} + \beta_{n2})^T \phi_{n2} \right]^{-1}. \tag{19}$$

Taking account of the physical limits of actuators, the desired control signal may be unavailable. The feasible control signal u_c can be generated by passing u_{cd} through the constraint command filter, i.e. , $u = u_c$.

Define the compensated tracking error as

$$\bar{x}_n := \tilde{x}_n - \xi_n, \tag{20}$$

where

$$\dot{\xi}_n \triangleq -k_2 \xi_n + (\hat{\theta}_{n2} + \beta_{n2})^T \phi_{n2}(u_c - u_{cd}). \tag{21}$$

The dynamics of compensated tracking error can be written as

$$\dot{\bar{x}}_n = -k_n \bar{x}_n - \tilde{\theta}_n^T \Phi_n - (\hat{\theta}_{n-1,2} + \beta_{n-1,2})^T \phi_{n-1,2} \bar{x}_{n-1} + \Delta_n, \tag{22}$$

where $\tilde{\theta}_n \triangleq [\tilde{\theta}_{n1}^T, \tilde{\theta}_{n2}^T]^T$, $\hat{\theta}_n \triangleq [\hat{\theta}_{n1}^T, \hat{\theta}_{n2}^T]^T$, $\beta_n \triangleq [\beta_{n1}^T, \beta_{n2}^T]^T$, and $\Phi_n \triangleq [\phi_{n1}^T, \phi_{n2}^T u]^T$.

The dynamics of the estimation errors are

$$\dot{\hat{\theta}}_n = \dot{\theta}_n + \frac{\partial \beta_n}{\partial \bar{x}_n} \dot{\bar{x}}_n. \tag{23}$$

Substituting Eq. (22) into Eq. (23), the I&I σ -modification adaptive law can be designed as

$$\begin{cases} \dot{\hat{\theta}}_n = \frac{\partial \beta_n}{\partial \bar{x}_n} \left[k_n \bar{x}_n + (\hat{\theta}_{n-1,2} + \beta_{n-1,2})^T \phi_{n-1,2} \bar{x}_{n-1} \right] - r_n \sigma_n (\hat{\theta}_n + \beta_n) \\ \frac{\partial \beta_n}{\partial \bar{x}_n} = r_n \Phi_n \end{cases} \tag{24}$$

Substituting Eqs. (24) and (22) into Eq. (23), the dynamics of estimation error can be rewritten as

$$\dot{\tilde{\theta}}_n = -r_n \Phi_n \tilde{\theta}_n^T \Phi_n - r_n \sigma_n (\hat{\theta}_n + \beta_n) + r_n \Phi_n \Delta_n. \tag{25}$$

3.2. Stability analysis

By now, the design of the I&I adaptive control system with σ -modification is accomplished. The stability property of the closed-loop system will be analyzed subsequently. The conclusion can be summarized as follows.

Theorem 1. *If the n th-order nonlinear plant (1) meets Assumption 1 and Assumption 2, then the closed-loop adaptive control system is global uniformly ultimately bounded stable with the control laws (4), (12), (19) and adaptive laws (9), (16), (24).*

Proof. The dynamics of the closed-loop system are described by Eqs. (7), (10), (14), (17), (22) and (25). Considering the modeling errors, we get

$$\begin{cases} \dot{\tilde{x}}_1 = -k_1 \tilde{x}_1 - \tilde{\theta}_1^T \Phi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \tilde{x}_2 + \Delta_1 \\ \dot{\tilde{\theta}}_1 = -r_1 \Phi_1 \tilde{\theta}_1^T \Phi_1 - r_1 \sigma_1 (\hat{\theta}_1 + \beta_1) + r_1 \Phi_1 \Delta_1 \\ \vdots \\ \dot{\tilde{x}}_i = -k_i \tilde{x}_i - \tilde{\theta}_i^T \Phi_i - (\hat{\theta}_{i-1,2} + \beta_{i-1,2})^T \phi_{i-1,2} \tilde{x}_{i-1} + (\hat{\theta}_{i2} + \beta_{i2})^T \phi_{i2} \tilde{x}_{i+1} + \Delta_i \\ \dot{\tilde{\theta}}_i = -r_i \Phi_i \tilde{\theta}_i^T \Phi_i - r_i \sigma_i (\hat{\theta}_i + \beta_i) + r_i \Phi_i \Delta_i \\ \vdots \\ \dot{\tilde{x}}_n = -k_n \tilde{x}_n - \tilde{\theta}_n^T \Phi_n - (\hat{\theta}_{n-1,2} + \beta_{n-1,2})^T \phi_{n-1,2} \tilde{x}_{n-1} + \Delta_n \\ \dot{\tilde{\theta}}_n = -r_n \Phi_n \tilde{\theta}_n^T \Phi_n - r_n \sigma_n (\hat{\theta}_n + \beta_n) + r_n \Phi_n \Delta_n. \end{cases} \tag{26}$$

To analyse the stability of the closed-loop system, consider a composite Lyapunov function

$$W(\tilde{x}_1, \dots, \tilde{x}_n, \tilde{\theta}_1, \dots, \tilde{\theta}_n) = \frac{1}{2} \sum_{j=1}^n \tilde{x}_j^2 + \frac{1}{2} \sum_{j=1}^n k_j^{-1} \tilde{\theta}_j^T r_j^{-1} \tilde{\theta}_j,$$

whose derivative along the system (26) is

$$\begin{aligned}
 \dot{W} &= \sum_{j=1}^n \bar{x}_j \dot{\bar{x}}_j + \sum_{j=1}^n k_j^{-1} \tilde{\theta}_j^T r_1^{-j} \dot{\tilde{\theta}}_j \\
 &= \bar{x}_1 \left[-k_1 \bar{x}_1 - \tilde{\theta}_1^T \Phi_1 + (\hat{\theta}_{12} + \beta_{12})^T \phi_{12} \bar{x}_2 + \Delta_1 \right] \\
 &\quad + \sum_{i=2}^{n-1} \bar{x}_i \left[-k_i \bar{x}_i - \tilde{\theta}_i^T \Phi_i - (\hat{\theta}_{i-1,2} + \beta_{i-1,2})^T \phi_{i-1,2} \bar{x}_{i-1} + (\hat{\theta}_{i2} + \beta_{i2})^T \phi_{i2} \bar{x}_{i+1} + \Delta_i \right] \\
 &\quad + \bar{x}_n \left[-k_n \bar{x}_n - \tilde{\theta}_n^T \Phi_n - (\hat{\theta}_{n-1,2} + \beta_{n-1,2})^T \phi_{n-1,2} \bar{x}_{n-1} + \Delta_n \right] \\
 &\quad + \sum_{j=1}^n k_j^{-1} \tilde{\theta}_j^T r_j^{-1} \left[-r_j \Phi_j \tilde{\theta}_j^T \Phi_j - r_j \sigma_j (\hat{\theta}_j + \beta_j) + r_i \Phi_i \Delta_i \right] \\
 &\leq - \sum_{j=1}^n k_j \bar{x}_j^2 - \sum_{j=1}^n \tilde{\theta}_j^T \Phi_j \bar{x}_j + \sum_{j=1}^n \bar{x}_j \Delta_j - \sum_{j=1}^n k_j^{-1} (\tilde{\theta}_j^T \Phi_j)^2 - \sum_{j=1}^n k_j^{-1} \sigma_j \tilde{\theta}_j^T (\hat{\theta}_j + \beta_j) \\
 &\quad + \sum_{j=1}^n k_j^{-1} \tilde{\theta}_j^T \Phi_j \Delta_j.
 \end{aligned}$$

By inequalities

$$\begin{aligned}
 \bar{x}_j \Delta_j &\leq \frac{1}{2} \bar{x}_j^2 + \frac{1}{2} \eta_j^2 \\
 \tilde{\theta}_j^T \Phi_j \Delta_j &\leq \frac{1}{2} (\tilde{\theta}_j^T \Phi_j)^2 + \frac{1}{2} \eta_j^2 \\
 -\tilde{\theta}_j^T \Phi_j \bar{x}_j &\leq \frac{k_j}{2} \bar{x}_j^2 + \frac{k_j^{-1}}{2} (\tilde{\theta}_j^T \Phi_j)^2 \\
 -\tilde{\theta}_j^T (\hat{\theta}_j + \beta_j) &\leq -\frac{1}{2} \|\tilde{\theta}_j\|^2 + \frac{1}{2} \|\theta_j\|^2,
 \end{aligned} \tag{27}$$

one can get

$$\dot{W} \leq -\frac{1}{2} \sum_{j=1}^n (k_j - 1) \bar{x}_j^2 - \frac{1}{2} \sum_{j=1}^n k_j^{-1} \sigma_j \|\tilde{\theta}_j\|^2 + \frac{1}{2} \sum_{j=1}^n k_j^{-1} \sigma_j \|\theta_j\|^2 + \frac{1}{2} \sum_{j=1}^n (k_j^{-1} + 1) \eta_j^2. \tag{28}$$

When $k_j > 1, j = 1, \dots, n$, we obtain

$$\dot{W} \leq -\alpha W + M, \tag{29}$$

where $M = \frac{1}{2} \sum_{j=1}^n k_j^{-1} \sigma_j \|\theta_j\|^2 + \frac{1}{2} \sum_{j=1}^n (k_j^{-1} + 1) \eta_j^2$ and $\alpha = \min \left[k_j - 1, \frac{\sigma_j}{\lambda_{\max}(r_j^{-1})} \right]$. The symbol $\lambda_{\max}(\cdot)$ means the maximum eigenvalue of the matrix. It implies that for $W \geq W_0 = M/\alpha, \dot{W} \leq 0$. Thus, the closed loop adaptive control system is uniformly ultimately bounded, namely, $\bar{x}_j, \tilde{\theta}_j, \hat{\theta}_j \in \mathcal{L}_\infty$. \square

Remark 1. The stability analysis demonstrates the stability of the compensated tracking errors, but there is no conclusion about the states or output tracking errors directly. Actually, provided no saturation occurs, the actual tracking errors will converge to compensated tracking errors, thus it is also uniformly ultimately bounded [33]. Even when the constraints are in effect, the stability of the parameter update laws can be guaranteed, because the compensated tracking errors not the actual tracking errors are used in the adaptive laws, which is

one of the advantages of the command filter backstepping [34]. Based on the command filter backstepping framework, the proposed method does not have the problem of “explosion of terms” in standard backstepping. And the magnitude/rate constraints of intermediate virtual controls are taken into account in the controller design, which is significant for controller implementation.

Remark 2. It can be observed that the proposed adaptive control method has structured design process with amplitude and rate constraints of actuators taken into account. Particularly, the introduction of σ -modification makes the adaptive law away from the problem of parameter drift. Therefore, the proposed method overcomes the obstacles to the application of traditional adaptive control methods as mentioned in Section 1, which has great potential in engineering.

4. Simulation

To validate the method proposed above, two numerical simulations are performed in the section.

4.1. Mass-damper-spring system control

Consider a MDS system modeled as

$$m\ddot{x} = u + d - k_p x - k_v \dot{x} + \Delta \quad (30)$$

where m , k_p and k_v are unknown positive constant, d is an unknown constant disturbance, and Δ denotes non-parametric uncertainties. The objective is to design an adaptive state feedback control law making the output x track the reference command x_d .

To design the control system, rewrite the model into a state-space form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1^T \phi_1 + \theta_2^T \phi_2 u + \Delta, \end{aligned} \quad (31)$$

where $x_1 = x$, $x_2 = \dot{x}$, $\theta_1 = [k_p/m, k_v/m, d/m]^T$, $\phi_1 = [-x_1, -x_2, 1]^T$, $\theta_2 = 1/m$, and $\phi_2 = 1$.

According to Section 3, firstly, the control law can be chosen as

$$\begin{aligned} x_{2cd} &= -k_1 \tilde{x}_1 + \dot{x}_d \\ u_{cd} &= \frac{-k_2 \tilde{x}_2 - (\hat{\theta}_1 + \beta_1)^T \phi_1 + \dot{x}_{2c} - \tilde{x}_1}{\hat{\theta}_2 + \beta_2} \end{aligned} \quad (32)$$

where k_1 and k_2 are positive constant to be designed.

Secondly, the adaptive law is designed as

$$\begin{cases} \dot{\hat{\theta}} = \frac{\partial \beta}{\partial \tilde{x}_2} (k_2 \tilde{x}_2 + \tilde{x}_1) - r\sigma (\hat{\theta} + \beta) \\ \frac{\partial \beta}{\partial \tilde{x}_2} = r\Phi, \end{cases} \quad (33)$$

where $r = \text{diag}[r_1 I_{3 \times 3}, r_2]$, σ is a positive constant, $\tilde{\theta} = [\tilde{\theta}_1, \tilde{\theta}_2]^T$ and $\Phi = [\phi_1^T, u_c]^T$.

The definitions of other variables are listed as below.

$$\begin{aligned}
 \tilde{x}_1 &\triangleq x_1 - x_d \\
 \bar{x}_1 &\triangleq \tilde{x}_1 - \xi_1 \\
 \dot{\xi}_1 &\triangleq -k_1\xi_1 + x_{2c} - x_{2cd} + \dot{\xi}_2, \quad \xi_1(0) = 0 \\
 \bar{x}_2 &\triangleq x_2 - x_{2c} \\
 \bar{x}_2 &\triangleq \tilde{x}_2 - \xi_2 \\
 \dot{\xi}_2 &\triangleq -k_2\xi_2 + (\hat{\theta}_2 + \beta_2)^T (u_c - u_{cd}), \quad \xi_2(0) = 0 \\
 \tilde{\theta}_1 &\triangleq \hat{\theta}_1 + \beta_1 - \theta_1 \\
 \tilde{\theta}_2 &\triangleq \hat{\theta}_2 + \beta_2 - \theta_2,
 \end{aligned} \tag{34}$$

where x_{2c} and u_c are generated by passing x_{2cd} and u_{cd} through constrained command filters, respectively.

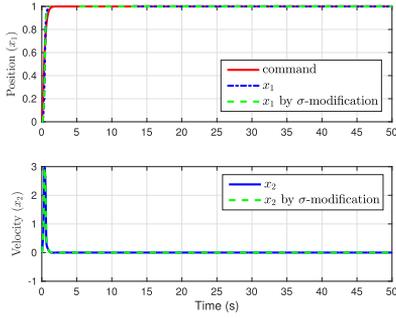
With the control law Eq. (32) and the adaptive law Eq. (33), the closed-loop system dynamics can be written as

$$\begin{aligned}
 \dot{\bar{x}}_1 &= -k_1\bar{x}_1 + \bar{x}_2 \\
 \dot{\bar{x}}_2 &= -k_2\bar{x}_2 - \tilde{\theta}^T \Phi - \bar{x}_1 + \Delta \\
 \dot{\tilde{\theta}} &= -r\Phi\tilde{\theta}^T \Phi - r\sigma(\hat{\theta} + \beta) + r\Phi\Delta.
 \end{aligned} \tag{35}$$

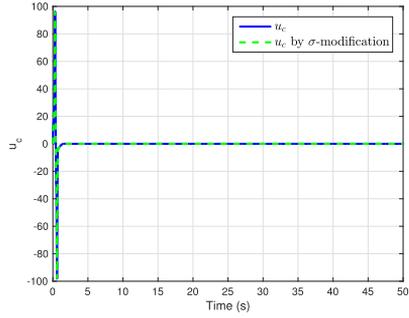
The initial states are set as $(x_1(0), x_2(0)) = (0, 0)$, and the parameters of the model are chosen as $(m, k_p, k_v, d) = (5, 1, 1, 1)$. The initial values of the estimates are set as $(\hat{\theta}_1, \hat{\theta}_2)|_{t=0} = (1, 1, 1, 2)$, and reference command $x_d = 1$ which will be passed through a command prefilter. The parameters of the control system are selected as $k_1 = 10$, $k_2 = 5$, $r_1 = 0.1$, $r_2 = 0.01$, $\sigma_1 = 2$, $\sigma_2 = 0.1$.

To illustrate the effectiveness and robustness of the designed MDS control system, two groups of comparative simulations are conducted between the proposed I&I adaptive control with σ -modification and the pure I&I adaptive control [35]. Note that when $\sigma_i = 0$, $i = 1, \dots, n$, the proposed I&I adaptive control with σ -modification degenerates to the pure I&I adaptive control. In the first case, only parameter uncertainties are considered, i.e. $\Delta = 0$. The results of the simulation are listed in Fig. 2. Fig. 2(a) is the time histories of the states which show that under the control of both methods the system output x_1 tracks the reference signal fast and accurately, and the internal state x_2 is smooth. Fig. 2(b) shows that both of the control outputs not only meet the amplitude and rate constraints but also are continuous and smooth. And all the parameter estimates of both methods have a stable adaptive behavior shown as Fig. 2(c) and (d), although the convergence processes of $\hat{\theta}_1$ are different. The simulation demonstrates the good tracking performance of the designed adaptive control system compared with the pure I&I adaptive control.

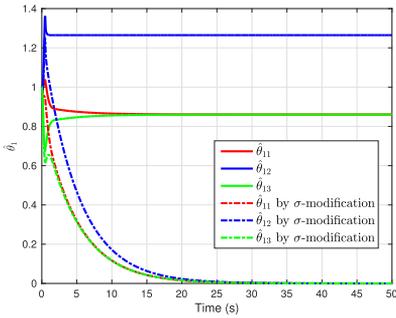
In the second case, the process noise is considered besides the parametric uncertainties, where the Δ is set as a zero-mean Gaussian white noise (GWN) whose variance is 500, to further illustrate the robustness of the designed adaptive control system. The corresponding simulation results are shown in Fig. 3. There exists severe oscillation in position tracking by pure I&I adaptive control especially at the 24th second. By contrast, the results of I&I adaptive control with σ -modification are still accurate and smooth shown as Fig. 3(a). Although the outputs of both control methods chatter seriously, the one with σ -modification is much



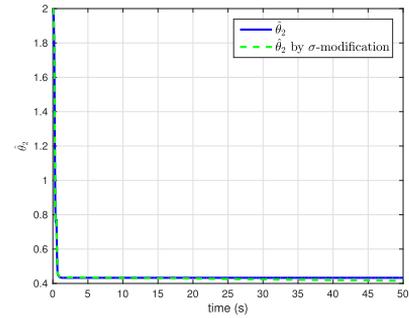
(a) Response of the states



(b) Response of the actuator



(c) Response of parameter estimation $\hat{\theta}_1$



(d) Response of parameter estimation $\hat{\theta}_2$

Fig. 2. Simulation results with parametric uncertainties in MDS system.

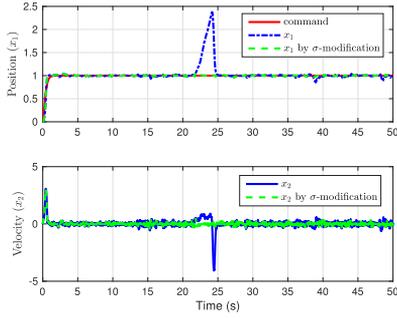
lighter in both frequency and amplitude shown as Fig. 3(b). It is also worth noting that the parameter adjustment process is still a smooth convergence behavior under the control with σ -modification, but obvious oscillation and sudden changes can be observed in Fig. 3(c) and (d). Therefore, the proposed I&I adaptive control with σ -modification has stronger robustness to modeling errors and significant improvement both in control performance and adaptive process.

4.2. Hypersonic vehicle control

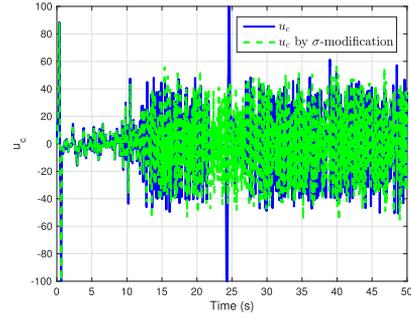
By convention, two models of the longitudinal dynamics of AHVs will be involved in the control system design. The full model [25], including complex couplings and structural flexibility, is employed to evaluate the closed-loop control system in simulation. And a control-oriented simplified model [36], is used for control system design. The control-oriented model can be described by Fiorentini et al. [37]

$$\dot{V} = \frac{T \cos \alpha - D}{m} - g \sin \gamma \tag{36a}$$

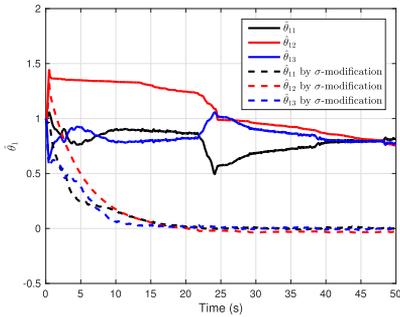
$$\dot{h} = V \sin \gamma \tag{36b}$$



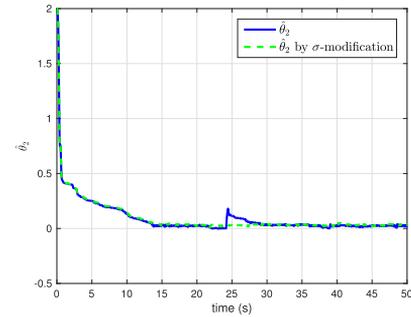
(a) Response of the states



(b) Response of the actuator



(c) Response of parameter estimation $\hat{\theta}_1$



(d) Response of parameter estimation $\hat{\theta}_2$

Fig. 3. Simulation results with parametric uncertainties and process noise in MDS system.

$$\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{g \cos \gamma}{V} \tag{36c}$$

$$\dot{\alpha} = -\frac{L + T \sin \alpha}{mV} + Q + \frac{g \cos \gamma}{V} \tag{36d}$$

$$\dot{Q} = \frac{M}{I_{yy}} \tag{36e}$$

$$\ddot{\eta}_i = -2\zeta_i \omega_i \dot{\eta}_i - \omega_i^2 \eta_i + N_i, \quad i = 1, 2, 3. \tag{36f}$$

The expressions of the aerodynamic forces, pitch moment, generalized forces, and the corresponding aerodynamic coefficients can be referred to [35]. The control objective is to design robust adaptive control system for AHVs to achieve stable tracking of velocity and altitude reference trajectories in the presence of significant parametric and non-parametric uncertainties.

To begin with, the model Eq. (36) is decomposed into velocity subsystem and altitude subsystem from the control viewpoint. Furthermore, the altitude system is taken as a sequential

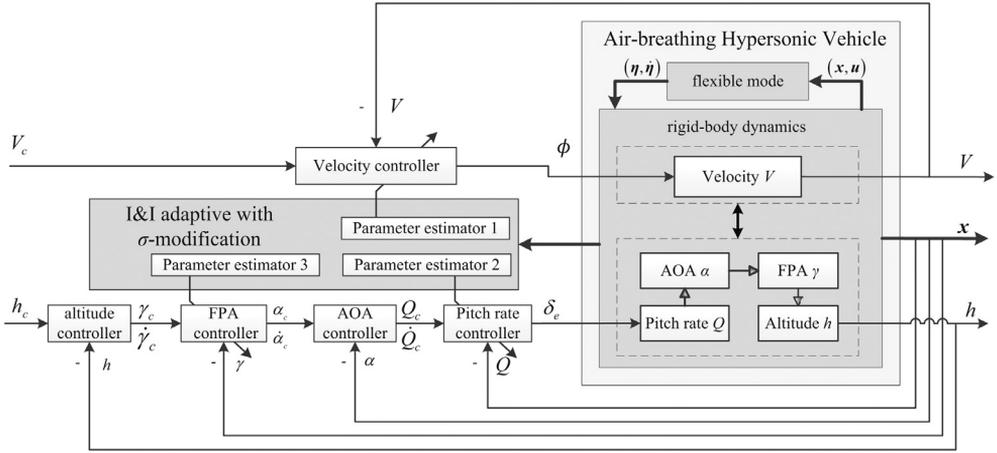


Fig. 4. Block diagram of the AHV control system.

loop structure, and the backstepping method is applied to controller design of each order subsystem. The block diagram of the control system structure is shown as Fig. 4. Note that each controller has an amplitude and rate constraints filter [32] as output to take the physical constraints of states into account.

For velocity subsystem, define the velocity tracking error as $\tilde{V} \triangleq V - V_c$. Substituting the expresses of aerodynamic forces and moment into Eq. (36a), we can arrange the dynamics of velocity tracking error as

$$\dot{\tilde{V}} = \theta_{v1}^T \varphi_{v1} + \theta_{v2}^T \varphi_{v2} \phi - g \sin \gamma - \dot{V}_c + \Delta_v, \quad (37)$$

where $\theta_{v1} \in \mathbb{R}^9$, $\theta_{v2} \in \mathbb{R}^4$ are vectors of unknown parameters and

$$\varphi_{v1} = \frac{\bar{q}S}{m} [\alpha^3 \cos \alpha, \alpha^2 \cos \alpha, \alpha \cos \alpha, \cos \alpha, -\alpha^2, -\alpha, -\delta_e^2, -\delta_e, -1]^T$$

$$\theta_{v1} = [C_T^3, C_T^2, C_T^1, C_T^0, C_D^{\alpha^2}, C_D^\alpha, C_D^{\delta_e^2} + k_{ec}^2 C_D^{\delta_e^2}, C_D^{\delta_e} + k_{ec} C_D^{\delta_e}, C_D^0]^T$$

$$\varphi_{v2} = \bar{q}S \cos \alpha [\alpha^3, \alpha^2, \alpha, 1]^T / m$$

$$\theta_{v2} = [C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^\phi]^T.$$

The estimates of θ_{v1} and θ_{v2} are defined as $\hat{\theta}_{v1} + \beta_{v1}$ and $\hat{\theta}_{v2} + \beta_{v2}$, respectively. Then, the estimation errors can be defined as

$$\tilde{\theta}_v \triangleq \hat{\theta}_v + \beta_v - \theta_v, \quad (38)$$

where $\tilde{\theta}_v = [\tilde{\theta}_{v1}^T, \tilde{\theta}_{v2}^T]^T$, $\hat{\theta}_v = [\hat{\theta}_{v1}^T, \hat{\theta}_{v2}^T]^T$, $\beta_v = [\beta_{v1}^T, \beta_{v2}^T]^T$ and $\theta_v = [\theta_{v1}^T, \theta_{v2}^T]^T$.

Thus, the control law can be designed as

$$\phi_{cd} = [-k_v \tilde{V} - (\hat{\theta}_{v1} + \beta_{v1})^T \varphi_{v1} + g \sin \gamma + \dot{V}_c] [(\hat{\theta}_{v2} + \beta_{v2})^T \varphi_{v2}]^{-1}, \quad (39)$$

where k_v is a positive constant. Pass ϕ_{cd} through the constraint command filter to produce the feasible control command ϕ_c . It can be assumed that $\phi = \phi_c$ as ϕ_c is physically realizable by the actuator.

The adaptive law with σ -modification is designed as

$$\begin{cases} \dot{\hat{\theta}}_v = \frac{\partial \beta_v}{\partial \bar{V}}(k_v \bar{V}) - r_v \sigma_v (\hat{\theta}_v + \beta_v) \\ \frac{\partial \beta_v}{\partial \bar{V}} = r_v \Phi_v, \end{cases} \quad (40)$$

where $\Phi_v = [\varphi_{v1}^T, \varphi_{v2}^T \phi]^T$, σ_v is a small constant, r_v is a positive diagonal matrix and $r_v = \text{diag}[r_{v1} I_{9 \times 9}, r_{v2} I_{4 \times 4}]$, $r_{v1} > 0$, $r_{v2} > 0$. And the definition of the compensated velocity tracking error are

$$\bar{V} \triangleq \tilde{V} - \xi_v, \quad (41)$$

where

$$\dot{\xi}_v \triangleq -k_v \xi_v + (\hat{\theta}_{v2} + \beta_{v2})^T \varphi_{v2} (\phi_c - \phi_{cd}). \quad (42)$$

With the control law Eq. (39) and the adaptive law Eq. (40), the closed-loop dynamics of the velocity subsystem are written as

$$\begin{aligned} \dot{\bar{V}} &= -k_v \bar{V} - \tilde{\theta}_v^T \Phi_v + \Delta_v \\ \dot{\tilde{\theta}}_v &= -r_v \Phi_v \tilde{\theta}_v^T \Phi_v - r_v \sigma_v (\hat{\theta}_v + \beta_v) + r_v \Phi_v \Delta_v. \end{aligned} \quad (43)$$

For the altitude subsystem, it is a 4th-order system, whose dynamics can be rewritten as

$$\begin{aligned} \dot{h} &= V \sin \gamma + \Delta_h \\ \dot{\gamma} &= \theta_{\gamma 1}^T \varphi_{\gamma 1} + \theta_{\gamma 2}^T \varphi_{\gamma 2} \alpha - \frac{g}{V} \cos \gamma + \Delta_\gamma \\ \dot{\alpha} &= Q - \dot{\gamma} + \Delta_\alpha \\ \dot{Q} &= \theta_{q1}^T \varphi_{q1} + \theta_{q2}^T \varphi_{q2} \delta_e + \Delta_q, \end{aligned} \quad (44)$$

where $\theta_{\gamma 1} \in \mathbb{R}^{10}$, $\theta_{\gamma 2} \in \mathbb{R}$, $\theta_{q1} \in \mathbb{R}^{11}$, $\theta_{q2} \in \mathbb{R}$ are unknown constants and

$$\begin{aligned} \varphi_{\gamma 1} &= \frac{\bar{q}S}{mV} [\delta_e, 1, \alpha^3 \phi \sin \alpha, \alpha^2 \phi \sin \alpha, \alpha \phi \sin \alpha, \phi \sin \alpha, \alpha^3 \sin \alpha, \alpha^2 \sin \alpha, \alpha \sin \alpha, \sin \alpha]^T \\ \theta_{\gamma 1} &= [C_L^{\delta_e} + k_{ec} C_L^{\delta_e}, C_L^0, C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0]^T \\ \varphi_{\gamma 2} &= \bar{q}S / (mV), \quad \theta_{\gamma 2} = C_L^\alpha \\ \varphi_{q1} &= \frac{\bar{q}S}{I_{yy}} [z_T \phi \alpha^3, z_T \phi \alpha^2, z_T \phi \alpha, z_T \phi, z_T \alpha^3, z_T \alpha^2, z_T \alpha, z_T, \bar{c} \alpha^2, \bar{c} \alpha, \bar{c}]^T \\ \theta_{q1} &= [C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0, C_M^{\alpha^2}, C_M^\alpha, C_M^0]^T \\ \varphi_{q2} &= \bar{q} \bar{c} S / I_{yy}, \quad \theta_{q2} = C_M^{\delta_e} + k_{ec} C_M^{\delta_e}. \end{aligned}$$

The estimates of unknown constants are defined as $\hat{\theta}_{\gamma 1} + \beta_{\gamma 1}$, $\hat{\theta}_{\gamma 2} + \beta_{\gamma 2}$, $\hat{\theta}_{q1} + \beta_{q1}$ and $\hat{\theta}_{q2} + \beta_{q2}$, respectively. Thus the estimation errors are

$$\tilde{\theta}_\gamma \triangleq \hat{\theta}_\gamma + \beta_\gamma - \theta_\gamma \quad (45a)$$

$$\tilde{\theta}_q \triangleq \hat{\theta}_q + \beta_q - \theta_q, \quad (45b)$$

where $\tilde{\theta}_\gamma = [\tilde{\theta}_{\gamma 1}^T, \tilde{\theta}_{\gamma 2}^T]^T$, $\hat{\theta}_\gamma = [\hat{\theta}_{\gamma 1}^T, \hat{\theta}_{\gamma 2}^T]^T$, $\beta_\gamma = [\beta_{\gamma 1}^T, \beta_{\gamma 2}^T]^T$, $\theta_\gamma = [\theta_{\gamma 1}^T, \theta_{\gamma 2}^T]^T$, $\tilde{\theta}_q = [\tilde{\theta}_{q 1}^T, \tilde{\theta}_{q 2}^T]^T$, $\hat{\theta}_q = [\hat{\theta}_{q 1}^T, \hat{\theta}_{q 2}^T]^T$, $\beta_q = [\beta_{q 1}^T, \beta_{q 2}^T]^T$ and $\theta_q = [\theta_{q 1}^T, \theta_{q 2}^T]^T$.

The control objective of the subsystem is to design elevator δ_e to achieve altitude trajectory tracking. In view of Eq. (44), the first equation, i.e., altitude dynamics, does not satisfy the parametric strict-feedback form, but it is deterministic, namely, there are no uncertain parameters between altitude and FPA. Therefore, the common practice is to design a command converter which converts altitude reference command to FPA reference directly [13]. Then, the altitude subsystem that is 4th-order originally can be reduced to a 3rd-order system to which the backstepping will be applied because it is parametric strict-feedback.

Step 1: Define the PFA tracking error as $\tilde{\gamma} \triangleq \gamma - \gamma_c$, whose dynamics is

$$\dot{\tilde{\gamma}} = \theta_{\gamma 1}^T \varphi_{\gamma 1} + \theta_{\gamma 2}^T \varphi_{\gamma 2} \alpha - \frac{g}{V} \cos \gamma - \dot{\gamma}_c + \Delta_\gamma. \tag{46}$$

To get the nominal virtual control command, the stabilizing function can be designed as

$$\alpha_{cd} = \left[-k_\gamma \tilde{\gamma} - (\hat{\theta}_{\gamma 1} + \beta_{\gamma 1})^T \varphi_{\gamma 1} + \frac{g \cos \gamma}{V} + \dot{\gamma}_c \right] \left[(\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \right]^{-1}, \tag{47}$$

where $k_h > 0$. Pass α_{cd} through the command filter to produce the magnitude and rate limited command signal α_c and its derivative $\dot{\alpha}_c$.

The I&I adaptive law with σ -modification is designed as

$$\begin{cases} \dot{\hat{\theta}}_\gamma = \frac{\partial \beta_\gamma}{\partial \tilde{\gamma}} \left[k_\gamma \tilde{\gamma} - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \tilde{\alpha} \right] - r_\gamma \sigma_\gamma (\hat{\theta}_\gamma + \beta_\gamma) \\ \frac{\partial \beta_\gamma}{\partial \tilde{\gamma}} = r_\gamma \Phi_\gamma, \end{cases} \tag{48}$$

where $\Phi_\gamma = [\varphi_{\gamma 1}^T, \varphi_{\gamma 2}^T \alpha]^T$, σ_γ is a small constant, r_γ is a positive diagonal matrix and $r_\gamma = \text{diag}[r_{\gamma 1} I_{10 \times 10}, r_{\gamma 2}]$, $r_{\gamma 1} > 0$, $r_{\gamma 2} > 0$. And the compensated FPA tracking error is defined as

$$\bar{\gamma} = \tilde{\gamma} - \xi_\gamma, \tag{49}$$

where

$$\dot{\xi}_\gamma = -k_\gamma \xi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} (\alpha_c - \alpha_{cd} + \xi_\alpha), \tag{50}$$

and ξ_α will be defined in the next section.

With the stabilizing function Eq. (47) and the adaptive law Eq. (48), the dynamics of the compensated tracking error $\bar{\gamma}$ and the estimation errors can be rewritten as

$$\begin{cases} \dot{\bar{\gamma}} = -k_\gamma \bar{\gamma} - \tilde{\theta}_\gamma^T \Phi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \tilde{\alpha} + \Delta_\gamma \\ \dot{\tilde{\theta}}_\gamma = -r_\gamma \Phi_\gamma \tilde{\theta}_\gamma^T \Phi_\gamma - r_\gamma \sigma_\gamma (\hat{\theta}_\gamma + \beta_\gamma) + r_\gamma \Phi_\gamma \Delta_\gamma. \end{cases} \tag{51}$$

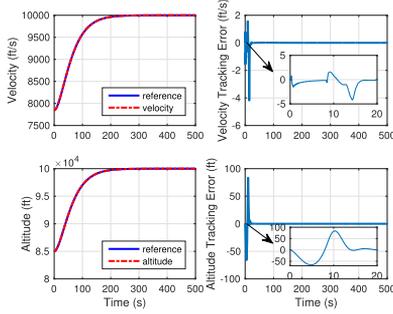
Step 2: Define the AOA tracking error as $\tilde{\alpha} \triangleq \alpha - \alpha_c$, whose dynamics is

$$\dot{\tilde{\alpha}} = Q - \dot{\gamma} - \dot{\alpha}_c + \Delta_v. \tag{52}$$

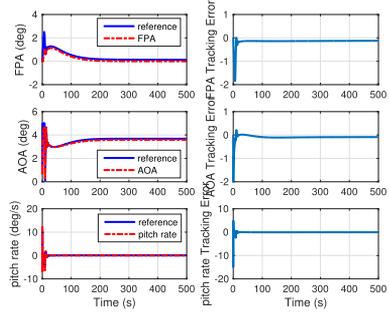
The nominal virtual command trajectory for the pitch rate can be designed as

$$Q_{cd} = -k_\alpha \tilde{\alpha} + \dot{\gamma} + \dot{\alpha}_c - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\gamma}, \tag{53}$$

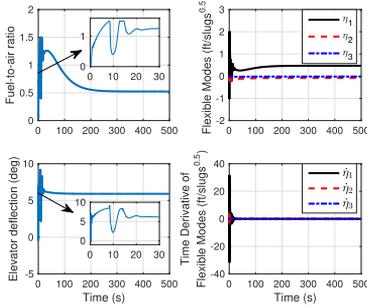
where $k_\alpha > 0$. The feasible virtual command Q_c and its derivate \dot{Q}_c can be produced by passing Q_{cd} through the magnitude and rate constraints filter.



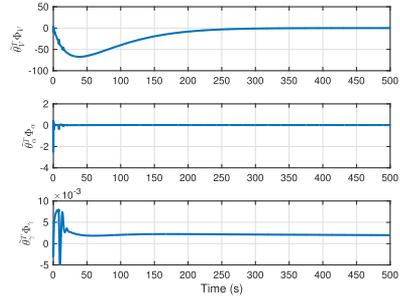
(a) Response of the velocity and altitude



(b) Response of the intermediate states



(c) Response of actuators and flexible modes



(d) Response of the adaptive terms

Fig. 5. Simulation results with parametric uncertainties in AHV control.

Define the compensated AOA tracking error as

$$\bar{\alpha} = \tilde{\alpha} - \xi_{\alpha}, \quad (54)$$

where

$$\dot{\xi}_{\alpha} = -k_{\alpha}\xi_{\alpha} + Q_c - Q_{cd} + \xi_q, \quad (55)$$

and ξ_q will be defined in the next section. With the nominal virtual command Eq. (53), we get the dynamics of the compensated AOA tracking error as

$$\dot{\bar{\alpha}} = -k_{\alpha}\bar{\alpha} - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\gamma} + \bar{Q} + \Delta_{\alpha}. \quad (56)$$

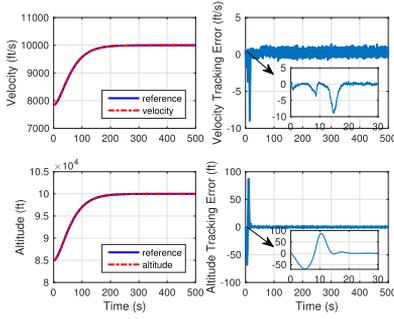
Step 3: Define the pitch rate tracking error $\tilde{Q} \triangleq Q - Q_c$, whose dynamics is

$$\dot{\tilde{Q}} = \theta_{q1}^T \varphi_{q1} + \theta_{q2}^T \varphi_{q2} \delta_e - \dot{Q}_c. \quad (57)$$

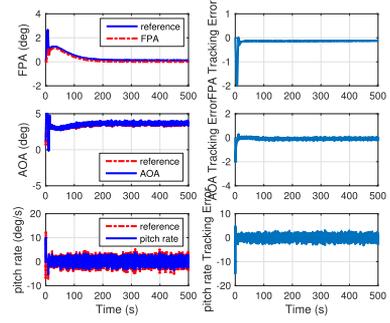
To get the nominal reference command for the elevator, the controller is designed as

$$\delta_{ecd} = \left[-k_q \tilde{Q} - (\hat{\theta}_{q1} + \beta_{q1})^T \varphi_{q1} + \dot{Q}_c - \bar{\alpha} \right] \left[(\hat{\theta}_{q2} + \beta_{q2})^T \varphi_{q2} \right]^{-1}, \quad (58)$$

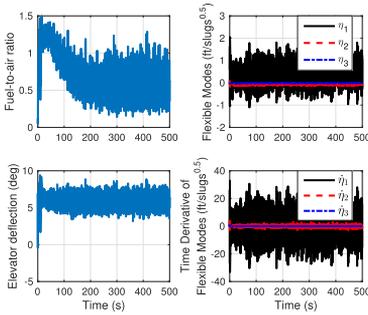
where $k_q > 0$. Pass δ_{ecd} to generate the achievable control signal δ_{ec} through a magnitude and rate constraints filter which is the model of the elevator. As δ_{ec} is feasible by physical



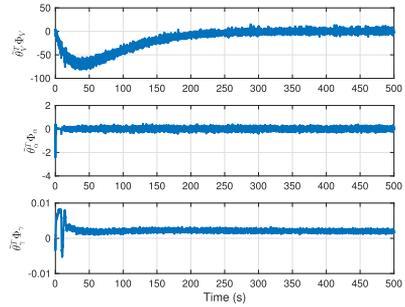
(a) Response of the velocity and altitude



(b) Response of the intermediate states



(c) Response of actuators and flexible modes



(d) Response of the adaptive terms

Fig. 6. Simulation results with parametric uncertainties and process noises in AHV control.

actuator, it is reasonable to assume that $\delta_e = \delta_{ec}$.

The I&I adaptive law with σ -modification can be designed as

$$\begin{cases} \dot{\hat{\theta}}_q = \frac{\partial \beta_q}{\partial \bar{Q}} (k_q \bar{Q} + \bar{\alpha}) - r_q \sigma_q (\hat{\theta}_q + \beta_q) \\ \frac{\partial \beta_q}{\partial \bar{Q}} = r_q \Phi_q, \end{cases} \quad (59)$$

where $\Phi_q = [\varphi_{q1}^T, \varphi_{q2}^T \delta_e]^T$, σ_q is a small constant, r_q is a positive diagonal matrix and $r_q = \text{diag}[r_{q1} I_{11 \times 11}, r_{q2}]$, $r_{q1} > 0$, $r_{q2} > 0$. And the compensated pitch rate tracking error is defined as

$$\bar{Q} = \tilde{Q} - \xi_q, \quad (60)$$

where

$$\dot{\xi}_q = -k_q \xi_q + (\hat{\theta}_{q2} + \beta_{q2})^T \varphi_{q2} (\delta_{ec} - \delta_{ecd}). \quad (61)$$

With the nominal command Eq. (58) and the adaptive law Eq. (59), the dynamics of the compensated tracking error \bar{Q} and the estimation errors can be rewritten as

$$\begin{aligned}\dot{\bar{Q}} &= -k_q \bar{Q} - \bar{\theta}_q^T \Phi_q - \bar{\alpha} + \Delta_q \\ \dot{\bar{\theta}}_q &= -r_q \Phi_q \bar{\theta}_q^T \Phi_q - r_q \sigma_q (\hat{\theta}_q + \beta_q) + r_q \Phi_q \Delta_q.\end{aligned}\quad (62)$$

To validate the performance of the designed AHV control system, two numerical simulations are conducted. One is performed with significant parametric uncertainties, to verify the tracking performance of the control system. The other is with significant parameter uncertainties and process noises, to test the robustness of the system. The initial flying states are set as $\mathbf{x}_0 = [V_0, h_0, \gamma_0, \alpha_0, Q_0]^T = [7846.4 \text{ ft/s}, 85000 \text{ ft}, 0 \text{ rad}, 0.0219 \text{ rad}, 0 \text{ rad/s}]^T$ with $\boldsymbol{\eta}_0 = [0.594, 0, -0.0976, 0, -0.0335, 0]^T$ and the initial control inputs are $\mathbf{u}_0 = [0.12, 0.12 \text{ rad}]^T$ [37]. It's assumed that all aerodynamic coefficients, i.e., $C_T^{(\cdot)}$, $C_M^{(\cdot)}$, $C_L^{(\cdot)}$ and $C_D^{(\cdot)}$ are uncertain which are modeled as $C_j = C_j^* (1 + \Delta C_j)$, where C_j^* and ΔC_j are nominal values and uncertainties, respectively. In the paper, a maximum variation for each coefficient is set within 40%, namely, $|\Delta C_j| \leq 0.4$. To increase fidelity, the dynamic characteristics of actuators are taken into account.

The simulation results of the first case are listed as Fig. 5, where a parameter uncertain condition of $\Delta C_L = \Delta C_T = -40\%$ and $\Delta C_D = \Delta C_M = 40\%$ are considered. Note that it's a quite tough situation for flight control. Fig. 5(a) illustrates that the closed-loop system has good velocity and altitude tracking characteristics. The internal states, including FPA, AOA and pitch rate, also can track the intermediate virtual command fast and smoothly shown as Fig. 5(b). The output of the actuators are smooth and meet the constraints while the flexible modes are stable shown as Fig. 5(c). Fig. 5(d) shows that each adaptive term converges to one small neighborhood of zero, which agrees with the σ -modification.

The results of the second case are presented as Fig. 6, where model process noises are considered in Eqs. (36a) and (36b) together with the same parametric uncertainties as the first case. And the process noises are set as additive zero-mean Gaussian white noise whose variance is 100. With the significant parametric and non-parametric uncertainties, the closed-loop system still represents good tracking performance shown as Fig. 6(a). There exists chattering in intermediate states, actuators, and flexible modes because of the model process noises shown as Fig. 6(b) and (c). Though there exists slight jitter, the adaptive estimators converge to a small neighborhood of zero and are free of parameter drift shown as Fig. 6(d). The simulation demonstrates that the I&I adaptive control with σ -modification achieves good command tracking performance and strong robustness to both parametric and non-parametric uncertainties in AHVs control.

5. Conclusion

Focusing on the problems of conventional adaptive control, a new I&I adaptive control with σ -modification is presented in the paper. Based on I&I theory and σ -modification, the method is free of Lyapunov function and robust to non-parametric uncertainties. The structured design process and consideration of states and actuators constraints make it easy to design and apply. Theoretical analysis and simulation results prove the effectiveness and robustness of the proposed method. To improve the method control performance further, future work will include introducing finite-time control into the proposed adaptive control method to improve anti-disturbance performance, and integrating the adaptive fuzzy or neural network method to approximate and compensate the non-parametric uncertainties [7,38,39].

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