

Optimal Share Reporting Strategies for Blockchain Miners in PPLNS Pools*

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Abstract—With the increasing difficulty of solo mining in blockchain mining, pool mining has become more and more popular, and most of the miners would like to join a mining pool and contribute their computational power to the pool. When the pool finds a valid block and get the reward from the blockchain network, it will distribute the reward to its miners according to its reward mechanism. In practice, the Pay-Per-Last- N -Shares (PPLNS) mechanism is one of the most commonly used mechanisms by pools, and the pool adopting PPLNS mechanism will distribute the reward to the miners whose reported shares are in the last N shares, according to their proportion of the number of shares in the last N shares. In the PPLNS mechanism, different reporting strategies may bring different rewards for miners. Thus, how to report their found shares to the pool has become an important issue faced by the miners. In this paper, we study the share reporting problem faced by the miners in PPLNS pools, and establish a share reporting optimization model for the miners. We also study the effect of the parameter N in the PPLNS mechanism on the optimal reporting strategies of the miners. With the computational experiments approach, we design experiments to evaluate our proposed share reporting strategies. This work is the first attempt to study the share reporting issue faced by miners in PPLNS pools, and it can provide useful managerial insights for miners when making their share reporting decisions in such pools.

Index Terms—share reporting strategy, blockchain mining, PPLNS reward, pool mining, computational experiments approach

I. INTRODUCTION

Since the blockchain technology was proposed by Satoshi Nakamoto [1], blockchain mining has attracted more and more people to participate, to compete for the lucrative cryptocurrency rewards through contributing their computational power to the blockchain network [9, 10, 11, 12, 13, 14].

In blockchain mining, miners are competing to solve the cryptographic puzzle with their computational power, and only the miner who finds the new valid block will get the cryptocurrency reward of the block from the blockchain network. Since the cryptocurrency rewards is extremely lucrative, blockchain

mining has attracted more and more miners, which made the difficulty for finding a new block continuously increase. Thus, it becomes more and more difficult for an individual miner to find a new block, which made pool mining more and more popular. In pool mining, all the miners in the pool will contribute their computational power in mining the new block together, which can increase the winning probability for the pool greatly. When one of the miners finds a new valid block and reports it to the pool, the pool operator will broadcast the new block to the blockchain network, and get the reward of the block. For distributing the rewards among its miners, one of the most widely used mechanism adopted by pool operator is the Pay-Per-Last- N -Shares (PPLNS) mechanism, which is a mechanism to distribute the reward according to the last N shares reported by the miners [2, 4, 16].

As can be seen in the PPLNS mechanism, if the shares reported by the miner are not in the last N shares, the miner can not get any reward from the pool. As such, the share reporting issue has become an important problem faced by miners when they participate in pool mining in a pool adopted PPLNS mechanism, since different reporting strategies may get different rewards for the miners. However, in the literature, the share reporting problem has not got any attention from researchers.

This paper aims to study the share reporting issue faced by the miners in PPLNS pools, and to our best knowledge, our work is the first attempt to study this issue. We establish a share reporting optimization model for the miners to maximize the expected reward of the miners. We also study the effect of different share reporting strategies on the reward of the miner under different values of N , and find the optimal share reporting strategies for the miner under different N . By utilizing the computational experiments approach, we design some computational experiments to evaluate our proposed share reporting strategies, and the results show that our proposed share reporting strategy outperform two baseline strategies.

The rest of this paper is organized as follows. In Section II, we present our research problem, and establish our share

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reporting optimization model. In Section III, we design some computational experiments to evaluate our proposed share reporting strategy. Section IV concludes our paper.

II. OPTIMAL REPORTING STRATEGIES OF MINERS

A. Problem Statement

In this section, we study the optimal reporting strategies of miners in PPLNS pools. Suppose the pool adopts the PPLNS mechanism, and it will distribute the rewards after K rounds, according to the last N shares reported by the miners. The block reward in each round keeps the same, denoted as R , and δ portion of the reward will be reserved by the pool. Each round contains exactly M shares, with the M th share as a full solution. In the M shares of each round, there are exactly two shares reported by the miner.

For the miner, there are two feasible strategies to report his/her found shares to the pool, namely Packaging Strategy and Random Strategy, which can be described as follows:

- **Packaging Strategy:** In this strategy, the miner will report the two shares simultaneously. For simplicity, we denote such strategy as s_1 .
- **Random Strategy:** In this strategy, the miner will report the two shares randomly. For simplicity, we denote such strategy as s_2 .

As shown in Figure 1 and Figure 2, there are $M - 1$ cases and $\frac{M(M-1)}{2}$ cases for the positions of the shares of the miner in Packaging Strategy and Random Strategy, respectively. For simplicity, we always assume that the positions of the two shares are the same in the K rounds for both the two strategies.

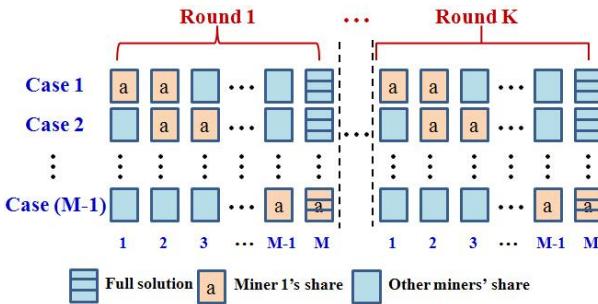


Fig. 1. An illustration of the $M - 1$ cases in Packaging Strategy

Thus, how to choose the reporting strategies for the miner to maximize his/her expected revenue has become an important decision issue faced by the miner. In this section, we mainly study the reporting strategies for the miners, and in the following, we first study the expected rewards of the miner under the two reporting strategies.

B. Rewards in Packaging Strategy

In the M shares, there are two shares submitted by the miner, which are submitted to the pool simultaneously, and their positions in the M shares are random, with a probability p_i to be in the i th and $(i+1)$ th positions, where $1 \leq i \leq M-1$. As such, there are $M-1$ cases for the positions of these shares,

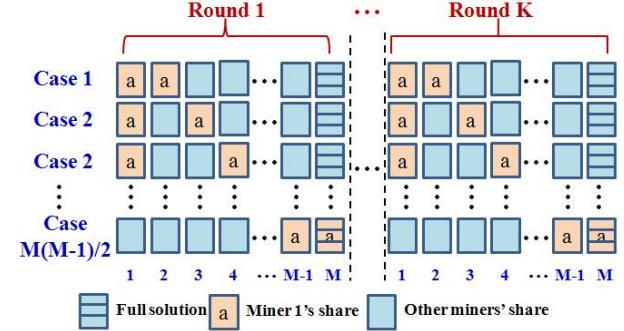


Fig. 2. An illustration of the $\frac{M(M-1)}{2}$ cases in Random Strategy

and in case i , the two shares appears in the i th and $i + 1$ th positions of the M shares in each round. For simplicity, we assume that the probability of the shares submitted by the miner appeared in each position is the same, i.e., $p_i = \frac{1}{M-1}$, and the position keeps the same in the K rounds.

Since the pool reserves δ portion of the reward, the total rewards for the miners in the K rounds are $(1 - \delta)KR$, and the reward of the miner is

$$V_{s_1, N, i} = r_{s_1, N, i}(1 - \delta)KR \quad (1)$$

for case i , where $r_{s_1, N, i}$ is the proportion of the reward of the miner in case i under N . For each case, the values of $r_{s_1, N}$ under any $N = (k-1)M + j$, $j = 1, 2, \dots, M$, $k = 1, 2, \dots, K$, are given in Table I.

TABLE I
THE VALUES OF $r_{s_1, N, i}$ UNDER DIFFERENT N

Case	1	2	\dots	$M-2$	$M-1$
$N=1$	0	0	\dots	0	1
$N=2$	0	0	\dots	$\frac{1}{2}$	$\frac{2}{2}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$N=M$	$\frac{2}{M}$	$\frac{2}{M}$	\dots	$\frac{2}{M}$	$\frac{2}{M}$
$N=M+1$	$\frac{2}{M+1}$	$\frac{2}{M+1}$	\dots	$\frac{2}{M+1}$	$\frac{3}{M+1}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$N=KM-1$	$\frac{2K-1}{KM-1}$	$\frac{2K}{KM-1}$	\dots	$\frac{2K}{KM-1}$	$\frac{2K}{KM-1}$
$N=KM$	$\frac{2K}{KM}$	$\frac{2K}{KM}$	\dots	$\frac{2K}{KM}$	$\frac{2K}{KM}$

From Table I, for case i and any $N = (k-1)M + j$, $j = 1, 2, \dots, M$, $k = 1, 2, \dots, K$, we have

$$r_{s_1, N} = \begin{cases} \frac{2(k-1)}{N}, & 1 \leq i \leq M-j-1, \\ & 1 \leq j \leq M-2 \\ \frac{2k-1}{N}, & i = M-j, 1 \leq j \leq M-1 \\ \frac{2k}{N}, & M-j+1 \leq i \leq M-1, \\ & 2 \leq j \leq M, \end{cases} \quad (2)$$

which can be regarded as a discrete random variable with the following probability distribution

$$r_{s_1,N} = \begin{cases} \frac{2(k-1)}{N}, & \text{with probability } \frac{M-j-1}{M-1}, \\ & 1 \leq j \leq M-2 \\ \frac{2k-1}{N}, & \text{with probability } \frac{1}{M-1}, \\ & 1 \leq j \leq M-1 \\ \frac{2k}{N}, & \text{with probability } \frac{j-1}{M-1}, \\ & 2 \leq j \leq M, \end{cases} \quad (3)$$

Thus, the reward $V_{s_1,N}$ is also a discrete random variable, and the expected reward of the miner in Packaging Strategy is

$$\begin{aligned} E[V_{s_1,N}] &= E[r(s_1, N)](1 - \delta)KR \\ &= \left(\frac{2(k-1)}{N} \frac{M-j-1}{M-1} I(1 \leq j \leq M-2) \right. \\ &\quad \left. + \frac{2k-1}{N} \frac{1}{M-1} I(1 \leq j \leq M-1) \right. \\ &\quad \left. + \frac{2k}{N} \frac{j-1}{M-1} I(2 \leq j \leq M) \right) (1 - \delta)KR \end{aligned} \quad (4)$$

where $I(m_1 \leq j \leq m_2)$ is an indicator function defined as follows

$$I(m_1 \leq j \leq m_2) = \begin{cases} 1, & \text{if } m_1 \leq j \leq m_2 \\ 0, & \text{other.} \end{cases} \quad (5)$$

In the following, we compute the value of $E[V_{s_1,N}]$ for different j . For $j = 1$, we have

$$\begin{aligned} E[V_{s_1,N}] &= \frac{2(k-1)(M-j-1)+2k-1}{N(M-1)} (1 - \delta)KR \\ &= \frac{2[(k-1)M+j]-2j-2(k-1)j+1}{N(M-1)} (1 - \delta)KR \\ &= \frac{2N-2k+1}{N(M-1)} (1 - \delta)KR. \end{aligned} \quad (6)$$

For $2 \leq j \leq M-1$, we have

$$\begin{aligned} E[V_{s_1,N}] &= \frac{2(k-1)(M-j-1)+2k-1+2k(j-1)}{N(M-1)} (1 - \delta)KR \\ &= \frac{2N-2k+1}{N(M-1)} (1 - \delta)KR. \end{aligned} \quad (7)$$

For $j = M$, we have

$$\begin{aligned} E[V_{s_1,N}] &= \frac{2k}{N} \frac{j-1}{M-1} (1 - \delta)KR \\ &= \frac{2}{M} (1 - \delta)KR \end{aligned} \quad (8)$$

Thus, we have

$$E[V_{s_1,N}] = \begin{cases} \frac{2N-2k+1}{N(M-1)} (1 - \delta)KR, & \text{if } 1 \leq j \leq M-1 \\ \frac{2}{M} (1 - \delta)KR, & \text{if } j = M, \end{cases} \quad (9)$$

C. Rewards in Random Strategy

In this section, we compute the expected reward of the miner in Random Strategy. For $N = (k-1)M + j$, $r_{s_2,N}$ can only take the value of $\frac{2(k-1)}{N}$, $\frac{2k-1}{N}$ and $\frac{2k}{N}$. For the case of $r_{s_2,N} = \frac{2(k-1)}{N}$, it means no share of the miner in the k th round is in the last j shares of the k th round, i.e., the position of the share could be chosen from $(k-1)M+1, (k-1)M+2, \dots, (k-$

$1)M+(M-j)$. Thus, the probability of $r_{s_2,N} = \frac{2(k-1)}{N}$ can be computed by

$$\Pr \left\{ r_{s_2,N} = \frac{2(k-1)}{N} \right\} = \frac{C_{M-j}^2}{C_M^2} = \frac{(M-j)(M-j-1)}{M(M-1)}. \quad (10)$$

For the case of $r_{s_2,N} = \frac{2(k-1)+1}{N}$, it means that in the k th round, there is only one share for the miner in the last j shares, and another one is in the first $M-j$ shares. Thus, the probability of $r_{s_2,N} = \frac{2(k-1)+1}{N}$ can be computed by

$$\Pr \left\{ r_{s_2,N} = \frac{2(k-1)+1}{N} \right\} = \frac{C_{M-j}^1 C_j^1}{C_M^2} = \frac{2j(M-j)}{M(M-1)}. \quad (11)$$

For the case of $r_{s_2,N} = \frac{2k}{N}$, it means that in the k th round, both the two shares for the miner are in the last j shares. Thus, the probability of $r_{s_2,N} = \frac{2k}{N}$ can be computed by

$$\Pr \left\{ r_{s_2,N} = \frac{2k}{N} \right\} = \frac{C_j^2}{C_M^2} = \frac{j(j-1)}{M(M-1)}. \quad (12)$$

Thus, $r_{s_2,N}$ is a random variable with the following distributions

$$r_{s_2,N} = \begin{cases} \frac{2(k-1)}{N}, & \text{with probability } \frac{(M-j)(M-j-1)}{M(M-1)}, \\ & 1 \leq j \leq M-2 \\ \frac{2k-1}{N}, & \text{with probability } \frac{2j(M-j)}{M(M-1)}, \\ & 1 \leq j \leq M-1 \\ \frac{2k}{N}, & \text{with probability } \frac{j(j-1)}{M(M-1)}, \\ & 2 \leq j \leq M, \end{cases} \quad (13)$$

and its expected value is

$$\begin{aligned} E[r_{s_2,N}] &= \left(\frac{2(k-1)}{N} \frac{(M-j)(M-j-1)}{M(M-1)} I(1 \leq j \leq M-2) \right. \\ &\quad \left. + \frac{2k-1}{N} \frac{2j(M-j)}{M(M-1)} I(1 \leq j \leq M-1) \right. \\ &\quad \left. + \frac{2k}{N} \frac{j(j-1)}{M(M-1)} I(2 \leq j \leq M) \right) (1 - \delta)KR. \end{aligned} \quad (14)$$

In the following, we compute the value of $E[r_{s_2,N}]$ for different j . For $j = 1$, we have

$$\begin{aligned} E[r_{s_2,N}] &= \frac{2(k-1)}{N} \frac{(M-j)(M-j-1)}{M(M-1)} + \frac{2k-1}{N} \frac{2j(M-j)}{M(M-1)} \\ &= \frac{2}{M}. \end{aligned} \quad (15)$$

For $j = 2, \dots, M-2$, we have

$$\begin{aligned} E[r_{s_2,N}] &= \frac{2(k-1)}{N} \frac{(M-j)(M-j-1)}{M(M-1)} \\ &\quad + \frac{2k-1}{N} \frac{2j(M-j)}{M(M-1)} + \frac{2k}{N} \frac{j(j-1)}{M(M-1)} \\ &= \frac{2}{M}. \end{aligned} \quad (16)$$

For $j = M-1$, we have

$$\begin{aligned} E[r_{s_2,N}] &= \frac{2k-1}{N} \frac{2j(M-j)}{M(M-1)} + \frac{2k}{N} \frac{j(j-1)}{M(M-1)} \\ &= \frac{2}{M}. \end{aligned} \quad (17)$$

For $j = M$, we have

$$E[r_{s_2,N}] = \frac{2k}{N} \frac{j(j-1)}{M(M-1)} = \frac{2k}{N} = \frac{2}{M}. \quad (18)$$

Thus, we have

$$E[r_{s_2,N}] = \frac{2}{M}, \quad (19)$$

from which we can obtain the expected reward of the miner in Random Strategy is

$$E[V_{s_2, N}] = E[r_{s_2, N}](1 - \delta)KR = \frac{2}{M}(1 - \delta)KR. \quad (20)$$

D. Expected Reward based Optimal Reporting Strategies

Based on the analysis in the above sections, we establish the following expectation model to optimize the miner's reporting decisions under different values of N

$$\max_{s \in \{s_1, s_2\}} E[V(s, N)] \quad (21)$$

In the following, we find the optimal strategies for different N . By comparing the expected value in (9) and (20), we have

$$E[V(s_1, N)] - E[V(s_2, N)] = \frac{2N-2k+1}{N(M-1)} - \frac{2}{M} = \frac{2j-M}{NM(M-1)}, \quad (22)$$

for $j = 1, 2, \dots, M-1$ and

$$E[V(s_1, N)] - E[V(s_2, N)] = 0 \quad (23)$$

for $j = M$. Thus, when $2j - M \geq 0$, i.e., $\frac{M}{2} \leq j \leq M-1$, we have $E[V(s_1, N)] \geq E[V(s_2, N)]$, when $2j - M < 0$, i.e., $1 \leq j < \frac{M}{2}$, we have $E[V(s_1, N)] < E[V(s_2, N)]$, and when $j = M$, we have $E[V(s_1, N)] = E[V(s_2, N)]$.

Therefore, for any $N = (k-1)M + j$, $j = 1, 2, \dots, M$, if $1 \leq j < \frac{M}{2}$, the optimal strategy is Random Strategy, if $\frac{M}{2} \leq j \leq M-1$, the optimal strategy is Packaging Strategy, and if $j = M$, both Packaging Strategy and Random Strategy are the optimal strategy.

E. An Illustrative Example

To better illustrate our proposed new strategy, we present an example with $M = 5$ and $K = 2$, and in each round, there are two shares reported by the miner. Obviously, we have $N = KM = 10$. For Packaging Strategy and Random Strategy, there are 4 cases and 10 cases of the positions of the two shares of the miner in the two rounds, as shown in Fig. 3 and Fig. 4, respectively.

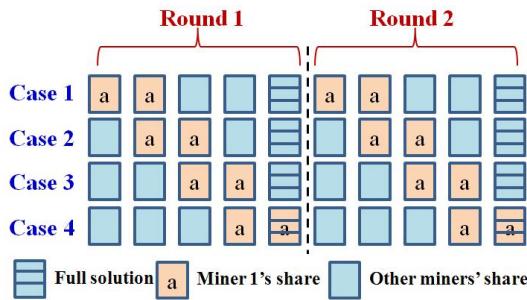


Fig. 3. An illustration of the positions for the shares of the miner in the two rounds in Packaging Strategy

According to (9) and (20), the expected rewards of the miner in Packaging Strategy and Random Strategy are

$$\begin{aligned} E[V_{s_1, N}] &= E[r_{s_1, N}](1 - \delta)KR \\ &= \begin{cases} \frac{2N-2k+1}{4N}(1 - \delta)R, & \text{if } 1 \leq j \leq 4 \\ \frac{2}{5}(1 - \delta)KR, & \text{if } j = 5 \end{cases} \end{aligned}$$

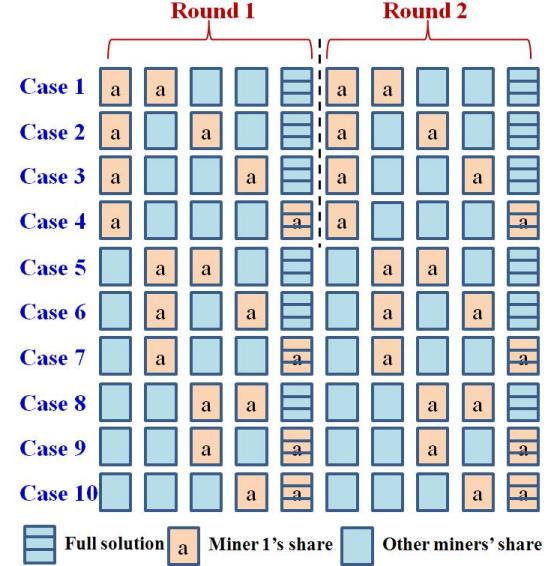


Fig. 4. An illustration of the positions for the shares of the miner in the two rounds in Random Strategy

and

$$E[V_{s_2, N}] = E[r_{s_2, N}](1 - \delta)KR = \frac{2}{5}(1 - \delta)KR,$$

respectively. By comparing the values of $E[r_{s_1, N}]$ and $E[r_{s_2, N}]$ under different N , we can obtain the optimal strategy under each value of N , as given in Table II.

TABLE II
VALUES OF $E[r_s, N]$ OF THE MINER UNDER THE TWO STRATEGIES AND
THE OPTIMAL STRATEGIES FOR DIFFERENT N

N	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$
$E[r_{s_1, N}]$	1/4	3/8	5/12	7/16	8/20
$E[r_{s_2, N}]$	2/5	2/5	2/5	2/5	2/5
s^*	s_2	s_2	s_1	s_1	s_1, s_2
N	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$
$E[r_{s_1, N}]$	9/24	11/28	13/32	15/36	16/40
$E[r_{s_2, N}]$	2/5	2/5	2/5	2/5	2/5
s^*	s_2	s_2	s_1	s_1	s_1, s_2

III. COMPUTATIONAL EXPERIMENTS

In this section, we aim to evaluate our proposed reporting strategies for the miner in PPLNS pools with the computational experiments approach [3, 5, 6, 7, 8, 15]. For comparison purpose, we denote our new proposed strategy as New

Strategy, and Packaging Strategy and Random Strategy as two baseline strategies.

A. Experimental Scenario

We consider an experimental scenario that a miner is mining in a pool adopted PPLNS mechanism. In the pool, each round contains $M = 5$ shares, in which two shares are found by the miner, and the pool will distribute the rewards after $K = 2$ rounds, according to the last N shares submitted to the pool, where N can take the values of $1, 2, \dots, 15$. For each feasible N , there are 4 cases and 10 cases for the positions of the shares of the miner in the Packaging strategy and Random Strategy, respectively, and these cases are shown in Fig. 3 and Fig. 4. Moreover, we assume that the reward for each block in the same, and the proportion of the service fees reserved by the pool is fixed.

B. Results and Analysis

For general conclusions, we run 1000 independent experiments for each N , and the number of times of each case in the Packaging Strategy and the Random Strategy are given in Fig. 5 and Fig. 6, respectively, and the proportion of reward for each case in the Packaging Strategy and the Random Strategy under different N are given in Fig. 7 and Fig. 8, respectively.

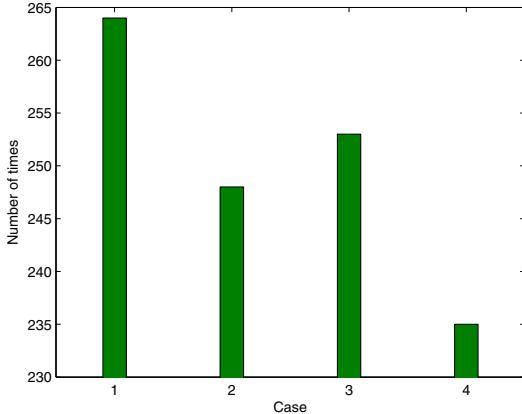


Fig. 5. The number of times for each case in the 1000 experiments in the Packaging Strategy

With our proposed model, we can obtain our proposed new share reporting strategies for the miner under different N , as shown in Fig. 9. Comparisons of the average rewards for the three strategies under different N are given in Fig. 10.

From Fig. 5–Fig. 10, we can draw the following conclusions:

- (1) Our proposed new strategy is to adopt the packaging reporting strategy for $N = 3, 4, 8, 9$, and the random reporting strategy for $N = 1, 2, 6, 7$. For $N = 5, 10$, since the Packaging Strategy and the Random Strategy can get the same reward, our new strategy has two choices for $N = 5, 10$.
- (2) For each N , our new strategy can get a reward which is higher than or equal to that of the Packaging Strategy,

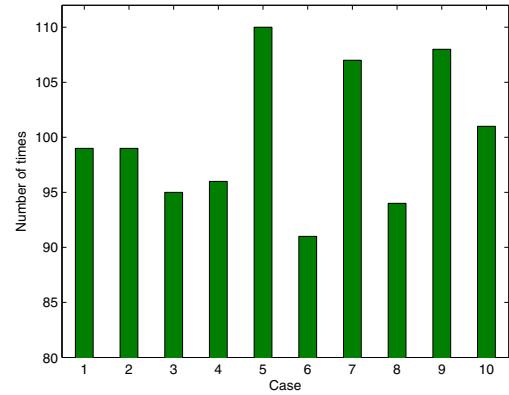


Fig. 6. The number of times for each case in the 1000 experiments in the Random Strategy

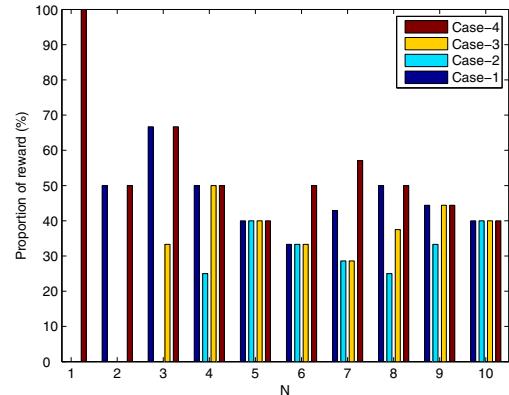


Fig. 7. Comparisons of proportion of reward for each case in the Packaging Strategy under different N

which illustrates that our proposed new strategy is better than the Packaging Strategy.

- (3) For each N , our new strategy can get a reward which is higher than or equal to that of the Random Strategy, which illustrates that our proposed new strategy is better than the Random Strategy.

IV. CONCLUSIONS AND FUTURE WORK

This paper studied the share reporting issues faced by the miners in mining pools adopted the PPLNS reward mechanism. Considering different share reporting strategies can greatly affect the rewards of the miners, we established a share reporting optimization model for the miners, which can help the miners find their optimal share reporting strategies under different values of N . We also designed some computational experiments to evaluate our proposed share reporting strategies, and the experimental results showed the superiorities of our proposed strategy comparing with two baseline strategies.

In our future work, we aim to extend this paper from the following aspects: 1) Studying the share reporting strategies in the case that there are multiple shares of the miner in a round; b) Exploring the joint optimization strategy of pool selection and share reporting for the miners when there are multiple pools with different reward mechanisms.

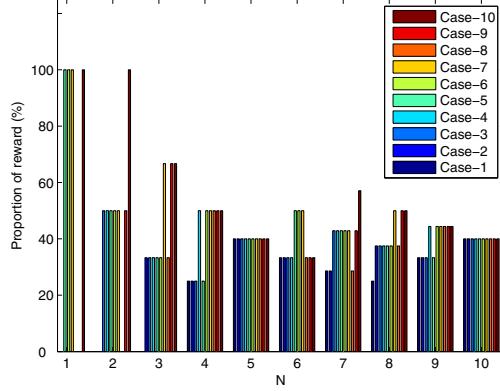


Fig. 8. Comparisons of proportion of reward for each case in the Random Strategy under different N

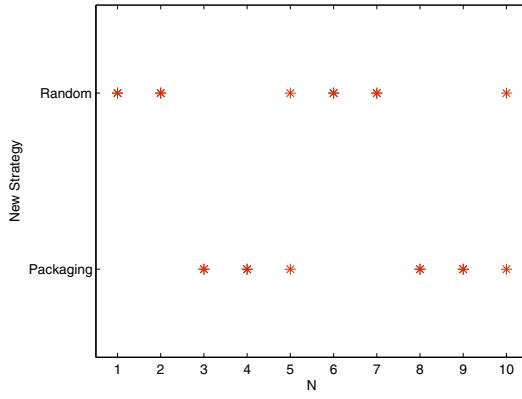


Fig. 9. Our new reporting strategies for the miner under different N

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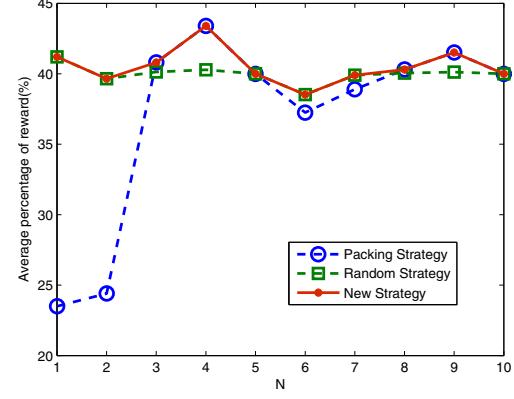


Fig. 10. Comparisons of the average rewards for the three strategies under different N