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Mixed H_∞ and Passive Depth Control for Autonomous Underwater Vehicles with Fuzzy Memorized Sampled-Data Controller

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Abstract This paper investigates the depth control problem for autonomous underwater vehicles (AUVs) by developing a novel fuzzy memorized sampled-data controller. In particular, the mixed H_∞ and passive performance index is considered for disturbances in more practical underwater environments. By means of the Lyapunov–Krasovskii functional method, sufficient criteria are established such that the desired depth can be stabilized while satisfying the prescribed mixed H_∞ and passive performance. Then, the desired fuzzy memorized controller is designed in terms of linear matrix inequalities (LMIs). Finally, an illustrative example is provided for demonstrating the feasibility and effectiveness of our derived results.

Keywords Autonomous underwater vehicles · Mixed H_∞ and passive control · Fuzzy memorized controller · Sampled-data controller

1 Introduction

The past decade has witnessed a growing interest in autonomous underwater vehicles (AUVs) due to their various applications in sea inspection, mapping, localization and so on [1–3]. With the rapid development of artificial intelligence and network technology, substantial progresses have been made toward both academic researches and practical applications of AUVs. As a result, a large number of effective control methods for AUVs have been well developed with encouraging and remarkable results [4–6]. It is worth mentioning that the AUV dynamics are always nonlinear and coupled, which lead to considerable difficulties in the analysis and synthesis in the design procedures. Fortunately, the fuzzy modeling techniques have been successfully applied to simplify the AUV models and the corresponding effective control methods have been reported in the literature and the references therein [7–10]. In particular, the T–S fuzzy models have great benefits in flexibility describing the complex nonlinear systems by relative simple rules, which can be a promising approach to deal with the control problems of AUVs.

On another research front line, great efforts have been devoted to the networked control systems (NCSs). By utilizing the communication network and the digital controllers, more efficiency and facility with less costs can be obtained with NCSs [11, 12]. However, it is noted that owing to practical network environments, the sampling periods of NCSs are always kept varying [13–15]. In order to deal with the time-varying sampling period, many effective approaches have been applied including the well-known input delay approaches [16, 17] and the impulsive approaches [18]. It should be pointed out that for the fuzzy NCSs, there still exist certain challenges since the fuzzy memberships of the sensors are sampled with the transmitted signals, which could be different from the system states [19–22].

In view of the aforementioned discussions and the nonlinear dynamics of the AUVs, in this paper, we deal with the depth control problem for AUVs with fuzzy

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models and effective sampled-data control schemes. More precisely, the Takagi–Sugeno (T–S) fuzzy depth control model is introduced for considering the high nonlinearity of the AUVs. Based on the T–S fuzzy model, the mixed H_∞ and passive performance is also introduced in the design procedure. In comparison with the existing literature, our main contributions can be summarized as twofold:

(1) To the authors' best knowledge, this paper makes one of the first attempts to address the depth control problem of AUVs by the mixed H_∞ and passive performance index, which can provide more control flexibilities than the common H_∞ and the passive performances in the disturbance attenuation issues. As is well known, the depth control is a very significant issue for AUVs during the underwater missions, since unstable depth control may damage or even destroy the AUVs. Therefore, how to achieve the depth control for AUVs while considering the practical underwater disturbances is an urgent yet challenging issue, which also motivates us for this study.

(2) A novel fuzzy memorized sampled-data depth controller is developed with memory coefficient for the AUVs. By utilizing the past sampled-data information in memory, the control performance can be improved while the disturbances can be attenuated. It should be pointed out that one most important merit of this technique is that the conservation for the established sufficient conditions can be considerably decreased.

The outline of this paper is arranged as follows. Section 2 introduces T–S fuzzy model of the AUV depth control problem with some necessary preliminaries and develops the fuzzy memorized sampled-data controller. In Sect. 3, by constructing an appropriate Lyapunov–Krasovskii function, delay-dependent sufficient criteria are established for satisfying the prescribed mixed H_∞ and passive performance. Based on the derived results, the desired controller gains are designed via the linear matrix inequality (LMI) method. Section 4 provides an illustrative example for showing the feasibility and effectiveness of our proposed method. Finally, the concluding remarks are given in Sect. 5.

Notation The notations are standard in this paper. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the space of $m \times n$ real matrices, respectively. $A - B \succ 0$ ($A - B \prec 0$) denotes that $A - B$ is positive definite (negative definite). $\mathrm{col}_N\{x_i\}$ presents $[x_1^T, x_2^T, \ldots, x_N^T]^{\mathrm{T}}$. $\mathcal{L}_2[0,\infty)$ denotes the space of square-integrable vector functions over $[0,\infty)$. $*$ is used for representing the ellipsis in symmetric block matrices and $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. If not explicitly stated, all matrices are assumed to have compatible dimensions.

2 Problem Formulation and Preliminaries

2.1 T–S Fuzzy Model of AUV Depth Control

Given the coordinate frames depicted in Fig. 1 and consider the AUV with the following depth control dynamics [23]:

$$\begin{bmatrix} \dot{z}_e(t) \\ \dot{\theta}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 & u(t) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \dfrac{M_{uq}u(t)}{I_{yy} - M_{\dot{q}}} \end{bmatrix} \begin{bmatrix} z_e(t) \\ \theta(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{M_{uu}u^2(t)}{I_{yy} - M_{\dot{q}}} \end{bmatrix} \delta(t), \tag{1}$$

where $z_e(t) = z(t) - z_d$ with z_d being the desired depth of the AUV, $\theta(t)$ represents the pitch angle in earth-fixed frame, $q(t)$ is the angular rate, $\delta(t)$ stands for the control input, $u(t)$ denotes the surge speed of the AUV, $M_{uq}, I_{yy}, M_{\dot{q}}$ and M_{uu} are the coefficients.

By denoting $x(t) = [z_e(t), \theta(t), q(t)]^{\mathrm{T}}$ and considering the external disturbance, system (1) can be represented by

$$\dot{x}(t) = Ax(t) + B\delta(t) + B_w w(t), \tag{2}$$

where

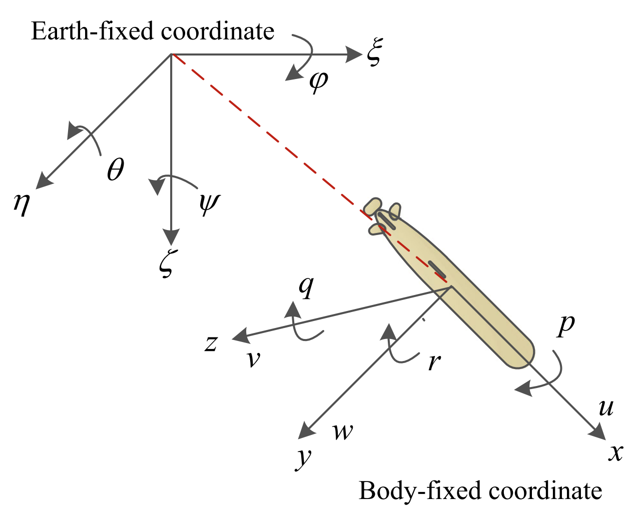

Fig. 1 Earth-fixed frame and body-fixed frame for AUV

$$A = \begin{bmatrix} 0 & u(t) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \dfrac{M_{uq}u(t)}{I_{yy} - M_{\dot{q}}} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{M_{uu}u^2(t)}{I_{yy} - M_{\dot{q}}} \end{bmatrix},$$

and $w(t)$ denotes the disturbance and B_w is a constant matrix.

Since $u(t)$ is not a constant parameter, the following T–S fuzzy model can be given for system (2):

Rule i: IF $u(t)$ is U_i, THEN

$$\dot{x}(t) = A_i x(t) + B_i \delta(t) + B_{wi} w(t),$$

where $i = 1, 2, \ldots, r$ and r is the number of IF-THEN rules, U_i is a fuzzy set corresponding to $u(t)$.

Additionally, by utilizing the center average fuzzifier, product inference, and center average defuzzifier, it follows that

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(u(t))[A_i x(t) + B_i \delta(t) + B_{wi} w(t)],$$

where

$$\sum_{i=1}^{r} \lambda_i(u(t)) = 1,$$

$$\lambda_i(u(t)) = \frac{\mu_i(u(t))}{\sum_{i=1}^{r} \mu_i(u(t))},$$

and $\mu_i(u(t))$ is the membership function of $u(t)$ in U_i.

2.2 Fuzzy Memorized Sampled-Data Controller

In the networked scenario, it is assumed that the sampler is time-driven according to a unified sampling sequence: $0 = t_0 < t_1 < \cdots < t_k < \cdots$, and $t_k \to \infty$ as $t \to \infty$, the controller and the actuator are event-driven with zero-order hold (ZOH). Then, $x(t)$ is measured as $x(t_k)$. Since the sampling period may be time-varying in practical networks, we define the sampling period as $T := t_{k+1} - t_k$ with $T \le \bar{h}, \bar{h} > 0$.

Consequently, we develop the following fuzzy memorized sampled-data controller for depth control based on the idea of parallel distributed compensation (PDC), where the controller would share the same premise parts with the system:

Rule i: IF $u(t)$ is U_i, THEN

$$\delta(t) = (1 - \rho) K_{1i} x(t_k) + \rho K_{2i} x(t_k - \tau),$$

where $0 \le \rho \le 1$ represents the memory coefficient, τ denotes the constant delay, K_{1i} and K_{2i} are the controller gains to be designed later.

Similarly, it can be obtained that

$$\delta(t) = \sum_{i=1}^{r} \lambda_i(u(t_k)) \left[(1 - \rho) K_{1i} x(t_k) + \rho K_{2i} x(t_k - \tau)\right]. \tag{3}$$

Remark 1 Based on the basic sampled-data controller, we introduce the memory mechanism with the past sampled-data information as $\sum_{i=1}^{r} \lambda_i(u(t_k))(K_{2i}x(t_k - \tau))$, which can bring better control performance.

Remark 2 It is worth mentioning that $u(t)$ is sampled as $u(t_k)$ during the control procedure, which does not require that the controller and the AUV model share the same premise variables. This sampled-data scheme can provide more relax analysis and synthesis results, and is more practical in the real-world network environment.

Remark 3 Compared with the results in [24], the tuning parameter ρ is introduced in our designed fuzzy memorized sampled-data controller for more design flexibility. In this sense, our proposed memorized sampled-data controller can be turned to common sampled-data controllers with or without time delay by choosing $\rho = 0$ or $\rho = 1$, respectively.

With the control input in (3), the overall closed-loop depth control dynamics of the AUV can be inferred by

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} & \lambda_i(u(t)) \lambda_j(u(t_k)) \big[A_i x(t) \\ & + (1 - \rho) B_i K_{1j} x(t_k) \\ & + \rho B_i K_{2j} x(t_k - \tau) + B_{wi} w(t)\big]. \end{aligned} \tag{4}$$

2.3 Control Objective

In order to deal with the depth control problem with disturbance, the following mixed H_∞ and passive performance is given.

Definition 1 (*Mixed H_∞ and passive performance*) System (4) is said to achieve the mixed H_∞ and passive performance, if there exists a scalar κ such that for any $t_p > 0$ and any nonzero $w(t) \in \mathcal{L}_2$

$$\begin{aligned} & \int_0^{t_p} \left(-\gamma^{-1} \kappa x^T(s) x(s) + 2(1 - \kappa) x^T(s) w(s)\right) \mathrm{d}s \\ & \quad \ge -\gamma \int_0^{t_p} w^T(s) w(s) \mathrm{d}s, \end{aligned}$$

where $\kappa \in [0, 1]$ is the tuning parameter.

Remark 4 Note that the mixed H_∞ and passive performance is a more general case of the well-known H_∞ and the passivity cases. By choosing appropriate κ, the disturbance attenuation problem can be well solved accordingly. More details with the relevant performance indexes can be found in [25–27] and the references therein.

The aim of this paper is to design a desired fuzzy memorized sampled-data controller for the AUV depth control, such that the prescribed mixed H_∞ and passive performance can be satisfied.

Before proceeding further, the following lemmas are important for subsequent analysis.

Lemma 1 [28] *For any matrix $\mathcal{M} > 0$, scalars $\tau > 0, \tau(t)$ satisfying $0 \le \tau(t) \le \tau$, vector function $\dot{x}(t) : [-\tau, 0] \to \mathbb{R}^n$ such that the concerned integrations are well defined, then*

$$-\tau \int_{t-\tau}^{t} \dot{x}^T(s)\mathcal{M}\dot{x}(s)\mathrm{d}s \le \zeta^T(t)\Omega\zeta(t),$$

where

$$\zeta(t) = \left[x^T(t), x^T(t-\tau(t)), x^T(t-\tau)\right]^{\mathrm{T}},$$
$$\Omega = \begin{bmatrix} -\mathcal{M} & \mathcal{M} & 0 \\ * & -2\mathcal{M} & \mathcal{M} \\ * & * & -\mathcal{M} \end{bmatrix}.$$

Lemma 2 [28] *Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$, where $\mathcal{S}_1^T = \mathcal{S}_1$ and $\mathcal{S}_2^T = \mathcal{S}_2 > 0$, then $\mathcal{S}_1 + \mathcal{S}_3^T \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$ if and only if*

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^T \\ * & -\mathcal{S}_2 \end{bmatrix} < 0,$$

or

$$\begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ * & \mathcal{S}_1 \end{bmatrix} < 0.$$

3 Main Results

In this section, delay-dependent sufficient criteria are derived for the AUV depth control in the form of LMIs, based on which the desired fuzzy memorized sampled-data controller gains are designed by matrix transforms.

Theorem 1 *For given parameters $\bar{h}, \rho, \tau$ and κ, the resulting closed-loop AUV depth control system (4) can achieve the mixed H_∞ and passive performance with the given controller gains, if there exist matrices $P \succ 0, Q_1 \succ 0, Q_2 \succ 0$, $R_1 \succ 0$ and $R_2 \succ 0$, such that the following LMIs hold with $i, j = 1, 2, \ldots, r$:*

$$\Pi_{ij} := \begin{bmatrix} \Pi_{1ij} & \Pi_{2ij} \\ * & \Pi_{3ij} \end{bmatrix} \prec 0, \tag{5}$$

where

$$\Pi_{1ij} := \begin{bmatrix} \Pi_{11ij} & \Pi_{12ij} \\ * & \Pi_{13ij} \end{bmatrix},$$
$$\Pi_{11ij} := \begin{bmatrix} \Pi_{111ij} & (1-\rho)PB_iK_{1j} + R_1 \\ * & -2R_1 \end{bmatrix},$$
$$\Pi_{12ij} := \begin{bmatrix} 0 & \rho PB_iK_{2j} + R_2 \\ R_1 & 0 \end{bmatrix},$$
$$\Pi_{13ij} := \begin{bmatrix} -Q_1 - R_1 & 0 \\ * & -2R_2 \end{bmatrix},$$
$$\Pi_{111ij} := PA_i + A_i^T P + Q_1 + Q_2 - R_1 - R_2,$$
$$\Pi_{2ij} := \begin{bmatrix} \Pi_{21ij} & \Pi_{22ij} \\ \Pi_{23ij} & \Pi_{24ij} \end{bmatrix},$$
$$\Pi_{21ij} := \begin{bmatrix} 0 & PB_{wi} - (1-\kappa)I & hA_i^T R_1 \\ 0 & 0 & (1-\rho)hK_{1j}^T B_i^T R_1 \end{bmatrix},$$
$$\Pi_{22ij} := \begin{bmatrix} (\bar{h}+\tau)A_i^T R_2 & I \\ (1-\rho)(\bar{h}+\tau)K_{1j}^T B_i^T R_2 & 0 \end{bmatrix},$$
$$\Pi_{23ij} := \begin{bmatrix} 0 & 0 & 0 \\ R_2 & 0 & \rho hK_{2j}^T B_i^T R_1 \end{bmatrix},$$
$$\Pi_{24ij} := \begin{bmatrix} 0 & 0 \\ \rho(\bar{h}+\tau)K_{2j}^T B_i^T R_2 & 0 \end{bmatrix},$$
$$\Pi_{3ij} := \begin{bmatrix} \Pi_{31ij} & \Pi_{32ij} \\ * & \Pi_{33ij} \end{bmatrix},$$
$$\Pi_{31ij} := \begin{bmatrix} -Q_2 - R_2 & 0 & 0 \\ * & -\gamma I & hB_{wi}^T R_1 \\ * & * & -R_1 \end{bmatrix},$$
$$\Pi_{32ij} := \begin{bmatrix} 0 & 0 \\ (\bar{h}+\tau)B_{wi}^T R_2 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\Pi_{33ij} := \begin{bmatrix} -R_2 & 0 \\ * & -\gamma\kappa^{-1} \end{bmatrix}.$$

Proof First, by utilizing the input delay approach, system (4) can be further rewritten as

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(u(t))\lambda_j(u(t_k))\big[A_i x(t) \\ + (1-\rho)B_iK_{1j}x(t-d(t)) \\ + \rho B_iK_{2j}x(t-d(t)-\tau) + B_{wi}w(t)\big], \end{aligned} \tag{6}$$

where $d(t) := t - t_k$ denotes the virtual delay satisfying $0 \le d(t) < \bar{h}$.

Choose the following Lyapunov–Krasovskii function:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{7}$$

where

$$\begin{aligned}
V_1(t) &:= x^T(t)Px(t),\\
V_2(t) &:= \int_{t-\bar{h}}^{t} x^T(\varphi)Q_1x(\varphi)\mathrm{d}\varphi\\
&\quad + \int_{t-\bar{h}-\tau}^{t} x^T(\varphi)Q_2x(\varphi)\mathrm{d}\varphi,\\
V_3(t) &:= \bar{h}\int_{-\bar{h}}^{0}\int_{t+\varphi}^{t} \dot{x}^T(\eta)R_1\dot{x}(\eta)\mathrm{d}\eta\mathrm{d}\varphi\\
&\quad + (\bar{h}+\tau)\int_{-\bar{h}-\tau}^{0}\int_{t+\varphi}^{t} \dot{x}^T(\eta)R_2\dot{x}(\eta)\mathrm{d}\eta\mathrm{d}\varphi.
\end{aligned}$$

Taking the time derivative of $V(t)$ along the solution of system (4) yields

$$\begin{aligned}
\dot{V}_1(t) &= 2x^T(t)P\dot{x}(t)\\
&= 2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))x^T(t)P\big[A_ix(t)\\
&\quad + (1-\rho)B_iK_{1j}x(t-d(t))\\
&\quad + \rho B_iK_{2j}x(t-d(t)-\tau) + B_{wi}w(t)\big],\\
\dot{V}_2(t) &= x^T(t)Q_1x(t) - x^T(t-\bar{h})Q_1x(t-\bar{h})\\
&\quad + x^T(t)Q_2x(t) - x^T(t-\bar{h}-\tau)Q_2\times\\
&\quad x(t-\bar{h}-\tau),\\
\dot{V}_3(t) &= \bar{h}^2\dot{x}^T(t)R_1\dot{x}(t) - \bar{h}\int_{t-\bar{h}}^{t}\dot{x}^T(\varphi)R_1\dot{x}(\varphi)\mathrm{d}\varphi\\
&\quad + (\bar{h}+\tau)^2\dot{x}^T(t)R_2\dot{x}(t)\\
&\quad - (\bar{h}+\tau)^2\int_{t-\bar{h}-\tau}^{t}\dot{x}^T(\varphi)R_2\dot{x}(\varphi)\mathrm{d}\varphi.
\end{aligned}$$

Then, it can be obtained by Lemma 1 that

$$\begin{aligned}
&-\bar{h}\int_{t-\bar{h}}^{t}\dot{x}^T(\varphi)R_1\dot{x}(\varphi)\mathrm{d}\varphi\\
&\le \begin{bmatrix} x(t)\\ x(t-d(t))\\ x(t-\bar{h})\end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 & 0\\ * & -2R_1 & R_1\\ * & * & -R_1\end{bmatrix}\\
&\quad\times\begin{bmatrix} x(t)\\ x(t-d(t))\\ x(t-\bar{h})\end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
&-(\bar{h}+\tau)\int_{t-\bar{h}-\tau}^{t}\dot{x}^T(\varphi)R_2\dot{x}(\varphi)\mathrm{d}\varphi\\
&\le \begin{bmatrix} x(t)\\ x(t-d(t)-\tau)\\ x(t-\bar{h}-\tau)\end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 & 0\\ * & -2R_2 & R_2\\ * & * & -R_2\end{bmatrix}\\
&\quad\times\begin{bmatrix} x(t)\\ x(t-d(t)-\tau)\\ x(t-\bar{h}-\tau)\end{bmatrix}.
\end{aligned}$$

In addition, it can be derived that

$$\begin{aligned}
\dot{x}^T(t)R_1\dot{x}(t) &= \vartheta^T(t)\\
&\times\begin{bmatrix}
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))A_i^T\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))(1-\rho)K_{1j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))\rho K_{2j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))B_{wi}^T
\end{bmatrix} R_1\\
&\times\begin{bmatrix}
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))A_i^T\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))(1-\rho)K_{1j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))\rho K_{2j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))B_{wi}^T
\end{bmatrix}^{\mathrm{T}} \vartheta(t),
\end{aligned}$$

and

$$\begin{aligned}
\dot{x}^T(t)R_2\dot{x}(t) &= \vartheta^T(t)\\
&\times\begin{bmatrix}
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))A_i^T\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))(1-\rho)K_{1j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))\rho K_{2j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))B_{wi}^T
\end{bmatrix} R_2\\
&\times\begin{bmatrix}
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))A_i^T\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))(1-\rho)K_{1j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))\rho K_{2j}^TB_i^T\\
0\\
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))B_{wi}^T
\end{bmatrix}^{\mathrm{T}} \vartheta(t),
\end{aligned}$$

where

$$\begin{aligned}
\vartheta(t) = \big[&x^T(t), x^T(t-d(t)), x^T(t-\bar{h}), x^T(t-d(t)-\tau),\\
&x^T(t-\bar{h}-\tau), w(t)\big]^{\mathrm{T}}.
\end{aligned}$$

Consequently, denote

$$J = \dot{V}(t) + \gamma^{-1}\kappa x^T(t)x(t) - 2(1-\kappa)x^T(t)w(t) - \gamma w^T(t)w(t),$$

then it can be verified by Lemma 2 that

$$J \le \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(u(t))\lambda_j(u(t_k))\vartheta^T(t)\Pi_{ij}\vartheta(t), \quad i,j = 1,2,\ldots,r.$$

Therefore, one has that if $\Pi_{ij} < 0$ holds, then $J < 0$, which implies that the mixed H_∞ and passive performance can be satisfied under zero initial conditions. This completes the proof. □

Remark 5 The established sufficient conditions are in the form of strict LMIs, and the corresponding computational complexity is related to the fuzzy rules design. Thus, both the fuzzy rules and the computational complexity should be considered in the design procedure.

Remark 6 Note that in the sampled-data scheme, the symmetric character

$$\lambda_i(u(t))\lambda_j(u(t_k)) = \lambda_i(u(t_k))\lambda_j(u(t))$$

could be false in most cases. As a consequence, the derived results are formulated in the form of $\Pi_{ij} \prec 0, i,j = 1,2,\ldots,r$. Although some initial attempt has been made in [29], the relevant results are only applicable for the fixed sampling periods, which is still restrictive in practical applications. In our future work, we will try to further reduce the conservatism of our derived results in this paper.

Remark 7 It should be pointed out that the design scheme of the fuzzy memorized sampled-data controller in this paper can be conveniently applied to the common T–S fuzzy control systems with commonality and effectiveness.

Based on the established feasible results in Theorem 1, the following theorem is provided for the fuzzy memorized sampled-data controller gains.

Theorem 2 *For given parameters $\bar{h}, \rho, \tau$ and κ, the resulting closed-loop AUV depth control system (4) can achieve the mixed H_∞ and passive performance, if there exist matrices $\tilde{P} \succ 0, \tilde{Q}_1 \succ 0, \tilde{Q}_2 \succ 0, \tilde{R}_1 \succ 0, \tilde{R}_2 \succ 0$ and matrices $\tilde{K}_{1j}$ and $\tilde{K}_{2j}$, such that the following LMIs hold with $i,j = 1,2,\ldots,r$:*

$$\tilde{\Pi}_{ij} := \begin{bmatrix} \tilde{\Pi}_{1ij} & \tilde{\Pi}_{2ij} \\ * & \tilde{\Pi}_{3ij} \end{bmatrix} \prec 0, \tag{8}$$

where

$$\tilde{\Pi}_{1ij} := \begin{bmatrix} \tilde{\Pi}_{11ij} & \tilde{\Pi}_{12ij} \\ * & \tilde{\Pi}_{13ij} \end{bmatrix},$$

$$\tilde{\Pi}_{11ij} := \begin{bmatrix} \tilde{\Pi}_{111ij} & \tilde{\Pi}_{112ij} \end{bmatrix},$$

$$\tilde{\Pi}_{111ij} := \begin{bmatrix} A_i\tilde{P} + \tilde{P}A_i^T + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2 \\ * \end{bmatrix},$$

$$\tilde{\Pi}_{112ij} := \begin{bmatrix} (1-\rho)B_i\tilde{K}_{1j} + \tilde{R}_1 \\ -2\tilde{R}_1 \end{bmatrix},$$

$$\tilde{\Pi}_{12ij} := \begin{bmatrix} 0 & \rho B_i\tilde{K}_{2j} + \tilde{R}_2 & 0 \\ \tilde{R}_1 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Pi}_{13ij} := \begin{bmatrix} -\tilde{Q}_1 - \tilde{R}_1 & 0 & 0 \\ * & -2\tilde{R}_2 & \tilde{R}_2 \\ * & * & -\tilde{Q}_2 - \tilde{R}_2 \end{bmatrix},$$

$$\tilde{\Pi}_{2ij} := \begin{bmatrix} \tilde{\Pi}_{21ij} & \tilde{\Pi}_{22ij} \end{bmatrix},$$

$$\tilde{\Pi}_{21ij} := \begin{bmatrix} B_{wi} - (1-\kappa)\tilde{P} & h\tilde{P}A_i^T \\ 0 & (1-\rho)h\tilde{K}_{1j}^TB_i^T \\ 0 & 0 \\ 0 & \rho h\tilde{K}_{2j}^TB_i^T \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Pi}_{22ij} := \begin{bmatrix} (\bar{h}+\tau)\tilde{P}A_i^T & \tilde{P} \\ (1-\rho)(\bar{h}+\tau)\tilde{K}_{1j}^TB_i^T & 0 \\ 0 & 0 \\ \rho(\bar{h}+\tau)\tilde{K}_{2j}^TB_i^T & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Pi}_{3ij} := \begin{bmatrix} -\gamma I & hB_{wi}^T & (\bar{h}+\tau)B_{wi}^T & 0 \\ * & \tilde{R}_1 - 2\tilde{P} & 0 & 0 \\ * & * & \tilde{R}_2 - 2\tilde{P} & 0 \\ * & * & * & -\gamma\kappa^{-1} \end{bmatrix}.$$

Moreover, the desired controller gains can be obtained by

$$K_{1j} = \tilde{K}_{1j}\tilde{P}^{-1},$$
$$K_{2j} = \tilde{K}_{2j}\tilde{P}^{-1}.$$

Proof Let $\tilde{P} = P^{-1}, \tilde{Q}_1 = P^{-1}Q_1P^{-1}, \tilde{Q}_2 = P^{-1}Q_2P^{-1}, \tilde{R}_1 = P^{-1}R_1P^{-1}, \tilde{R}_2 = P^{-1}R_2P^{-1}, \tilde{K}_{1j} = K_{1j}\tilde{P}$ and $\tilde{K}_{2j} = K_{2j}\tilde{P}$. Pre- and post-multiply both sides of Π_{ij} in Theorem 1 with $\mathrm{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I\}$ and its transpose. Since $-\tilde{P}\tilde{R}_1^{-1}\tilde{P} \le \tilde{R}_1 - 2\tilde{P}$ and $-\tilde{P}\tilde{R}_2^{-1}\tilde{P} \le \tilde{R}_2 - 2\tilde{P}$ holds, then the proof can be obtained directly. □

Remark 8 Once the parameters $\bar{h}, \rho, \tau$ and κ are given, the minimum allowable value of γ can be obtained by solving the following convex optimization problem:

$$\min_{P,Q_1,Q_2,R_1,R_2,} \gamma,$$
$$\text{subjectto}\, \Pi_{ij} \prec 0.$$

4 Illustrative Example

In this section, the numerical example is presented to verify the effectiveness of our obtained theoretical results.

Consider the depth control problem of the AUV with following modified T–S fuzzy rules [30]:

Rule 1: IF $u(t)$ is about 2 m/s, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 \delta(t) + B_{w1} w(t),$$

Rule 2: IF $u(t)$ is about 3 m/s THEN

$$\dot{x}(t) = A_2 x(t) + B_2 \delta(t) + B_{w2} w(t),$$

Rule 3: IF $u(t)$ is about 4 m/s, THEN

$$\dot{x}(t) = A_3 x(t) + B_3 \delta(t) + B_{w3} w(t),$$

where

$$A_1 = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -0.4802 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ -2.9532 \end{bmatrix}, \quad B_{w1} = 0.1I,$$
$$A_2 = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -0.7203 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ -6.6447 \end{bmatrix}, \quad B_{w2} = 0.2I,$$
$$A_3 = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -0.9604 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ -11.8127 \end{bmatrix}, \quad B_{w3} = 0.3I,$$

As depicted in Fig. 2, the corresponding fuzzy memberships are set by:

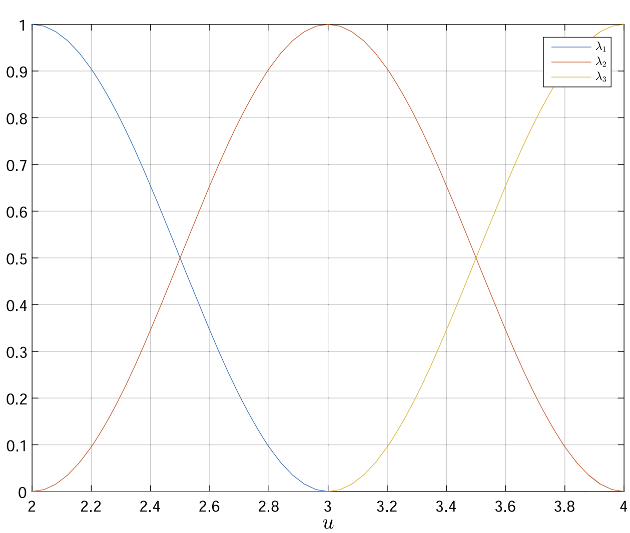

Fig. 2 The fuzzy rule membership

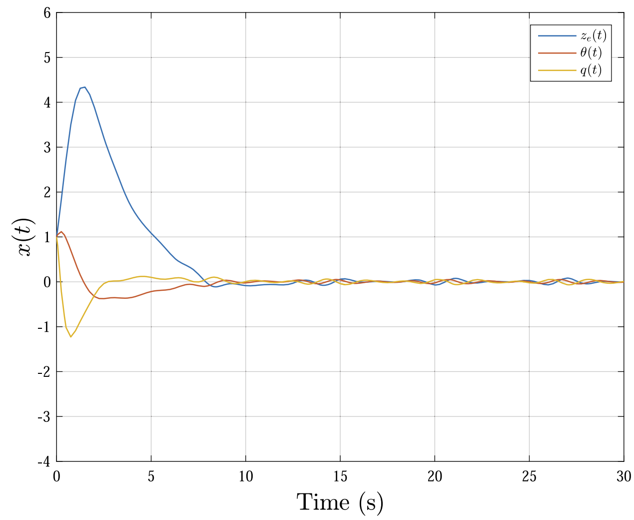

Fig. 3 The state response of $x(t)$

By solving the established LMIs in Theorem 2, the feasible solutions of the controller gains can be calculated as

$$K_{11} = [0.1005, 0.8147, 0.7943],$$
$$K_{21} = [0.0567, 0.4611, 0.4431],$$
$$K_{12} = [0.0617, 0.5115, 0.4603],$$
$$K_{22} = [0.0346, 0.2896, 0.2575],$$
$$K_{13} = [0.0438, 0.3706, 0.3032],$$
$$K_{23} = [0.0244, 0.2097, 0.1702].$$

In the simulation, the parameters are set as $\bar{h} = 0.25, \rho = 0.6, \tau = 0.2, \kappa = 0.5$ and $\gamma = 5$, with the above controller gains. Moreover, the disturbance $w(t)$ is assumed to be $0.3\sin(10/\pi t)$. It can be seen from Fig. 3 that our designed fuzzy memorized sampled-data controller can stabilize the desired depth with the prescribed mixed H_∞ and passive performance, which supports our theoretical results.

5 Conclusions

In this paper, we have studied the fuzzy depth control problem of AUVs in the presence of disturbance. A novel fuzzy memorized sampled-data controller is developed and the mixed H_∞ and passive performance is introduced to deal with the disturbance. Sufficient conditions are derived via the Lyapunov–Krasovskii method for guaranteeing the desired performance, and the corresponding controller design procedure is given. Simulation results of the numerical example demonstrates the effectiveness of our proposed method. Our future studies would focus on extending our results to the cases with limited communication resources.

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