

Robust Adaptive Control for a Class of Nonlinear Systems Based on Interval Type-2 Fuzzy Logic System and Small Gain Approach*

Xinlong Tao, Jianqiang Yi, *Senior Member, IEEE*, Xiangmin Tan, Zhen Liu and Chao Han

Abstract— In this paper, a novel robust adaptive control scheme for a class of uncertain nonlinear systems based on interval type-2 fuzzy logic system (IT2-FLS) and small gain approach is proposed. An interval type-2 Takagi-Sugeno-Kang fuzzy logic system (IT2-TSK-FLS) is employed to approximate the unknown dynamics of such a system. Based on the small gain theorem, a composite feedback form of the system is then established and a novel robust adaptive control law is developed, which can ensure all the signals in the close-loop system are uniformly ultimately bounded (UUB). Throughout the whole control scheme, only one parameter needs to be adapted online, which is different from most of existing IT2-FLSs. Numerical simulations demonstrate the robustness of our proposed scheme against uncertainties as well as the superiorities of IT2-TSK-FLS.

I. INTRODUCTION

To date, numerous efforts have been made on adaptive nonlinear control design and great progress has been witnessed. In order to better deal with the unknown dynamics of systems, fuzzy logic systems (FLSs) and neural networks (NNs), which can be regarded as universal approximators, have been incorporated into various adaptive control schemes since 1990s [1]. Recently, Zhou et al. [2] proposed an adaptive fuzzy control scheme for a class of nonlinear systems in nonstrict-feedback form subject to unmodeled dynamics and input saturation. Tong and Li [3] investigated the adaptive fuzzy output feedback tracking control problems in the presence of unknown dead zones. In [4], NNs were employed to approximate the model uncertainties, and a novel state constrained adaptive neural controller based on barrier Lyapunov function was developed. To handle the wind effects on flexible hypersonic flight vehicle, Xu et al. [5] proposed a disturbance observer based neural control scheme which can keep the states uniformly ultimately bounded (UUB). For MIMO nonlinear systems, Shi [6] developed a novel indirect adaptive fuzzy control scheme which can simultaneously deal with the possible singularity problem, the parameter initialization problem as well as the unknown control direction.

Actually, the FLSs mentioned above are all type-1 FLSs (T1-FLSs), which employ type-1 fuzzy sets (T1-FSs) to handle uncertainties. However, it is not reasonable to use an accurate membership function (MF) for something uncertain

[7]. Therefore, type-2 fuzzy set (T2-FS), which is the extension of T1-FS, together with corresponding type-2 FLS (T2-FLS), appears. Compared with traditional T1-FLS and NN, T2-FLS is a better choice when facing uncertain parameters, unmodeled dynamics and measurement noises due to its much more complex structure [8]. For sake of lower computational costs in practice, interval type-2 FLS (IT2-FLS) has raised more and more attention. In [9], Zhao and Dian employed an IT2-TS fuzzy model to represent nonlinear systems subject to parameter uncertainties, and the simulation results illustrated that IT2-FLS can perform better than T1-FLS. Considering the unknown flexible dynamics and the changes of fuel levels of hypersonic vehicles, Gao et al. [10] developed a novel indirect adaptive IT2 fuzzy sliding mode controller to keep all the signals in close-loop system bounded. However, in most of existing IT2-FLSs, too many parameters need to be adapted online, which may be time-consuming in reality. The similar question was first discussed in [11] and [12]. However, few results on this question can be found for IT2-FLS so far, which motivates this study.

In this paper, a tracking control problem for a class of nonlinear systems is investigated. Both unknown dynamics and uncertain disturbances of such a system are taken into consideration. First, an IT2 Takagi-Sugeno-Kang FLS (IT2-TSK-FLS) is utilized to approximate the unknown dynamics of the system. Then, following the idea of the small gain theorem, a new composite feedback form of the system can be formulated. A novel robust adaptive control law together with its stability analysis is explored afterwards. Finally, a pole-balancing robot system is adopted to illustrate the effectiveness of our proposed control scheme. The highlights of this study can be organized as follows. First, to the best of our knowledge, it is the first time to combine IT2-FLS with small gain theorem in robust adaptive control design, and simulation results demonstrate the superiorities of IT2-FLS over type-1 case. Second, different from most of existing IT2-FLS, no matter how many rules we define, only one parameter needs to be adapted online, which can greatly relieve the computation burden of the system. Last, we need no information about the system function or input gain function for control design, which can lead to a great convenience in practical applications.

The rest of the paper is organized as follows. Section II briefly introduces the preliminaries of this study, which can help readers better understand the results we obtain. Section III states the robust adaptive control design in details. Section IV presents the numerical simulations, after which we draw our conclusions.

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Xinlong Tao, Jianqiang Yi and Chao Han are with University of Chinese Academy of Sciences, Beijing 100049 China, and with Institute of Automation, Chinese Academy of Sciences, Beijing 100190 China (e-mail: {taoxinlong2014, jianqiang.yi, hanchao2013}@ia.ac.cn).

Xiangmin Tan and Zhen Liu are with Institute of Automation, Chinese Academy of Sciences, Beijing 100190 China (e-mail: {xiangmin.tan, liuzhen}@ia.ac.cn).

II. PRELIMINARIES

A. Problem Formulation

In this paper, the following n th-order uncertain nonlinear system is considered:

$$\begin{aligned}\dot{x}_i &= x_{i+1}, \quad 1 \leq i \leq n-1 \\ \dot{x}_n &= f(\mathbf{x}) + g(\mathbf{x})u + d(\mathbf{x}, t) \\ y &= x_1\end{aligned}\quad (1)$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbf{R}^n$ stands for the state vector, $u \in \mathbf{R}$ denotes the control input, and $y \in \mathbf{R}$ represents the output of the system. $f(\mathbf{x})$ is the unknown smooth nonlinear system function with $f(\mathbf{0}) = 0$, while $g(\mathbf{x})$ is the unknown smooth nonlinear input gain function. $d(\mathbf{x}, t)$ denotes the uncertain disturbance of the system.

Throughout this paper, the following assumptions are made:

Assumption 1: There exists an unknown positive constant g_{\min} such that $|g(\mathbf{x})| \geq g_{\min} > 0$, which means the input gain function $g(\mathbf{x})$ is strictly either positive or negative. Without loss of generality, we further assume that $g(\mathbf{x}) \geq g_{\min} > 0$.

Assumption 2: $|d(\mathbf{x}, t)|$ is bounded. In other words, there exists an unknown positive constant d_{\max} such that $|d(\mathbf{x}, t)| \leq d_{\max}$.

Assumption 3: The reference command y_c together with its derivatives up to the n th order are bounded.

The objective of this study is to develop a robust controller such that the output of the system y can track the reference command y_c , meanwhile all the states of the system keep bounded.

B. Brief Descriptions of IT2-TSK-FLS

To deal with the unknown nonlinear dynamics in system (1), IT2-TSK-FLS is employed in this paper. According to [13], when the antecedents are IT2-FSs and the consequents are crisp numbers, the IT2-TSK-FLS is referred as the IT2-A2-C0-TSK-FLS. In this case, the s th rule of the rule base has the following form:

$$\begin{aligned}\text{Rule } s: & \text{ If } x_{f1} \text{ is } \tilde{F}_1^s \text{ and } \dots \text{ and } x_{fn} \text{ is } \tilde{F}_n^s \\ & \text{ Then } y^s = a_0^s + a_1^s x_{f1} + \dots + a_n^s x_{fn}, \quad s = 1, 2, \dots, M\end{aligned}\quad (2)$$

where $\mathbf{x}_f = [x_{f1}, x_{f2}, \dots, x_{fn}]^T \in \mathbf{R}^n$ is the crisp input vector and y^s is the s th consequent. \tilde{F}_m^s ($m = 1, 2, \dots, n$) is the antecedent IT2-FS, with the upper membership function (UMF) $\bar{\mu}_{\tilde{F}_m^s}(x_{fm})$ and lower membership function (LMF) $\underline{\mu}_{\tilde{F}_m^s}(x_{fm})$ respectively. Then, if singleton fuzzification and product inference are employed, the degree of firing $f^s(\mathbf{x}_f)$ can be expressed as follows:

$$\begin{aligned}f^s(\mathbf{x}_f) &\in [\underline{f}^s(\mathbf{x}_f), \bar{f}^s(\mathbf{x}_f)] \\ &= \left[\prod_{m=1}^n \underline{\mu}_{\tilde{F}_m^s}(x_{fm}), \prod_{m=1}^n \bar{\mu}_{\tilde{F}_m^s}(x_{fm}) \right].\end{aligned}\quad (3)$$

With the employment of Begian-Melek-Mendel (BMM) type reduction algorithm [14], the crisp output Y can be finally obtained:

$$Y(\mathbf{x}_f) = m_1 \frac{\sum_{s=1}^M \underline{f}^s(\mathbf{x}_f) y^s}{\sum_{s=1}^M \underline{f}^s} + m_2 \frac{\sum_{s=1}^M \bar{f}^s(\mathbf{x}_f) y^s}{\sum_{s=1}^M \bar{f}^s} \quad (4)$$

where $m_1 + m_2 = 1$. In this paper, we choose $m_1 = m_2 = 0.5$. Let

$$\underline{\xi}^s(\mathbf{x}_f) = \underline{f}^s(\mathbf{x}_f) / \sum_{s=1}^M \underline{f}^s(\mathbf{x}_f) \quad (5)$$

$$\bar{\xi}^s(\mathbf{x}_f) = \bar{f}^s(\mathbf{x}_f) / \sum_{s=1}^M \bar{f}^s(\mathbf{x}_f) \quad (6)$$

denote the boundary fuzzy basic functions (BFBFs),

$$\underline{\xi}(\mathbf{x}_f) = [\underline{\xi}^1(\mathbf{x}_f), \underline{\xi}^2(\mathbf{x}_f), \dots, \underline{\xi}^M(\mathbf{x}_f)]^T \quad (7)$$

$$\bar{\xi}(\mathbf{x}_f) = [\bar{\xi}^1(\mathbf{x}_f), \bar{\xi}^2(\mathbf{x}_f), \dots, \bar{\xi}^M(\mathbf{x}_f)]^T \quad (8)$$

denote the BFBF vectors, while $\mathbf{y} = [y^1, y^2, \dots, y^M]^T$. Then, (4) can be rewritten in the following compact form:

$$\begin{aligned}Y(\mathbf{x}_f) &= 0.5 \underline{\xi}^T(\mathbf{x}_f) \mathbf{y} + 0.5 \bar{\xi}^T(\mathbf{x}_f) \mathbf{y} \\ &= \xi^T(\mathbf{x}_f) \mathbf{y} = \xi^T(\mathbf{x}_f) \mathbf{A}_x \bar{\mathbf{x}}\end{aligned}\quad (9)$$

where $\xi(\mathbf{x}_f) = 0.5(\underline{\xi}(\mathbf{x}_f) + \bar{\xi}(\mathbf{x}_f))$, $\bar{\mathbf{x}} = [1, \mathbf{x}_f]^T$, and

$$\mathbf{A}_x = \begin{bmatrix} a_0^1 & a_1^1 & \dots & a_n^1 \\ a_0^2 & a_1^2 & \dots & a_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ a_0^M & a_1^M & \dots & a_n^M \end{bmatrix}. \quad (10)$$

The following lemma demonstrates the IT2-TSK-FLS's capability of approximating a real continuous function on a compact domain.

Lemma 1 [15]: The IT2-A2-C0-TSK-FLS can uniformly approximate any function $f(\mathbf{x}_f)$ which is continuous in $C^r[-1, 1]$ with an arbitrarily small approximation error bound. In other words, for $\forall \varepsilon > 0$, there exists an IT2-A2-C0-TSK-FLS as (9) such that

$$\sup_{\mathbf{x}_f \in C^r[-1, 1]} |f(\mathbf{x}_f) - \xi^T(\mathbf{x}_f) \mathbf{A}_x \bar{\mathbf{x}}| < \varepsilon. \quad (11)$$

C. ISpS-Lyapunov Function and Small Gain Theorem

To help readers better understand the results of this study, here we briefly introduce the input-to-state practically stable (ISpS) Lyapunov function and the small gain theorem.

Definition 1 [11]: For a system $\dot{\mathbf{x}} = f(\mathbf{x}, u)$ and a C^1 function V , if

- there exist functions α_1 and α_2 of class K_∞ such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbf{R}^n \quad (12)$$

- there exist functions α_3 and α_4 of class K and a positive constant d such that

$$\frac{\partial V}{\partial x}(x)f(x, u) \leq -\alpha_3(\|x\|) + \alpha_4(\|u\|) + d \quad (13)$$

then function V is said to be an ISpS-Lyapunov function of this system.

Lemma 2 [11]: Consider two ISpS systems in the following composite feedback form

$$\sum_{\hat{z}\omega} \begin{cases} \dot{x} = f(x, \omega) \\ \dot{\hat{z}} = H(x) \end{cases} \quad (14)$$

$$\sum_{\omega\hat{z}} \begin{cases} \dot{y} = g(y, \hat{z}) \\ \dot{\omega} = K(y, \hat{z}) \end{cases} \quad (15)$$

Particularly, for each ω in the L_∞ supremum norm, each \hat{z} in the L_∞ supremum norm, each $x \in \mathbf{R}^n$ and each $y \in \mathbf{R}^m$, all the solutions $X(x; \omega, t)$ and $Y(y; \hat{z}, t)$ are defined on $[0, \infty]$ and satisfy, for almost all $t \geq 0$,

$$\|H(X(x; \omega, t))\| \leq \beta_\omega(\|x\|, t) + \gamma_z(\|\omega_t\|_\infty) + d_1 \quad (16)$$

$$\|K(Y(y; \hat{z}, t))\| \leq \beta_\xi(\|y\|, t) + \gamma_\omega(\|\hat{z}_t\|_\infty) + d_2 \quad (17)$$

where β_ω and β_ξ are of class KL , γ_z and γ_ω are of class K , d_1 and d_2 are two positive constants. Under these conditions, if

$$\gamma_z(\gamma_\omega(s)) < s \text{ or } \gamma_\omega(\gamma_z(s)) < s, \quad \forall s > 0 \quad (18)$$

then the solution of the composite systems (14) and (15) is ISpS.

III. CONTROL DESIGN

In this section, a novel robust adaptive control design based on the IT2-A2-C0-TSK-FLS and small gain approach will be proposed for system (1). Let $e_1 = x_1 - y_c$ denote the tracking error, then (1) can be transformed into the following form:

$$\begin{aligned} \dot{e}_i &= e_{i+1}, \quad 1 \leq i \leq n-1 \\ \dot{e}_n &= f(x) + g(x)u + d(x, t) - y_c^{(n)} \end{aligned} \quad (19)$$

where $e = [e_1, e_2, \dots, e_n]^T$. Moreover, if $k = [k_1, k_2, \dots, k_n]^T$ are set as the coefficients of Hurwitz polynomial $p^n + k_n p^{n-1} + \dots + k_2 p + k_1$ which can lead to the exponentially stable dynamics, (19) can be further transformed into the following compact form:

$$\dot{e} = Ae + b[f(x) + g(x)u + d(x, t) - y_c^{(n)} + k^T e] \quad (20)$$

where

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & \dots & -k_n \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \quad (21)$$

Since the detail expressions of $f(x)$ and $g(x)$ are both unavailable for us, first an IT2-A2-C0-TSK-FLS is applied to approximate the unknown nonlinear term $f(x)$. According to Lemma 1, let $y_c = [y_c, \dot{y}_c, \dots, y_c^{(n-1)}]^T$, then $f(x)$ can be expressed as follows:

$$\begin{aligned} f(x) &= \xi^T(x)A_x \bar{x} + \varepsilon = \xi^T(x)a_x^0 + \xi^T(x)A'_x x + \varepsilon \\ &= \xi^T(x)a_x^0 + \xi^T(x)A'_x e + \xi^T(x)A'_x y_c + \varepsilon \end{aligned} \quad (22)$$

where $a_x^0 = [a_0^1, a_0^2, \dots, a_0^M]^T$ and

$$A'_x = \begin{bmatrix} a_1^1 & a_2^1 & \dots & a_n^1 \\ a_1^2 & a_2^2 & \dots & a_n^2 \\ \vdots & \vdots & & \vdots \\ a_1^M & a_2^M & \dots & a_n^M \end{bmatrix}. \quad (23)$$

Substituting (22) into (20), we can obtain

$$\begin{aligned} \dot{e} &= Ae + b[\xi^T(x)a_x^0 + \xi^T(x)A'_x e + \xi^T(x)A'_x y_c + \varepsilon \\ &\quad + g(x)u + d(x, t) - y_c^{(n)} + k^T e] \\ &= Ae + b[g(x)u + k^T e + D] + b\xi^T(x)A'_x e \end{aligned} \quad (24)$$

where $D = \xi^T(x)a_x^0 + \xi^T(x)A'_x y_c + \varepsilon + d(x, t) - y_c^{(n)}$. Besides,

$$\begin{aligned} \|D\| &\leq \|\xi(x)\| \|a_x^0\| + \|\xi(x)\| \|A'_x\| \|y_c\| + \varepsilon + d_{\max} - d_c \\ &\leq \theta \psi(x) \end{aligned} \quad (25)$$

with $\psi(x) = 1 + \|\xi(x)\|$, $|y_c^{(n)}| \leq d_c$, and $\theta = \max(\varepsilon, d_{\max}, d_c, \|a_x^0\| + \|A'_x\| \|y_c\|)$.

In order to utilize the small gain theorem, according to Lemma 2, two subsystems in the composite feedback form as (14) and (15) must be established. If we let $c_\theta = \|A'_x\| = \lambda_{\max}^{1/2}(A_x'^T A_x')$ and $A_x^m = A'_x / c_\theta$ such that $\|A_x^m\| \leq 1$, then (24) becomes

$$\dot{e} = Ae + b[g(x)u + k^T e + D] + c_\theta b \xi^T(x) A_x^m e. \quad (26)$$

Moreover, by choosing $\hat{z} = H(e) = e$ and $\omega = K(\hat{z}) = A_x^m e$, the following composite feedback form of (26) can be formulated:

$$\sum_{\hat{z}\omega} \begin{cases} \dot{e} = Ae + b[g(x)u + k^T e + D] + c_\theta b \xi^T(x) \omega \\ \dot{\hat{z}} = H(e) = e \end{cases} \quad (27)$$

$$\sum_{\omega\hat{z}} \omega = K(\hat{z}) = A_x^m \hat{z} = A_x^m e. \quad (28)$$

The control law can be designed as follows:

$$u = \left(-\hat{\lambda} \left[\mathcal{G}(x) - (e^T k)^T (e^T k) \right] b^T P e \right) \quad (29)$$

where $A^T P + P A^T = -Q$ with $\lambda_{\min}(Q) > 2$, λ_{\min} denotes the minimum eigenvalue, while

$$\mathcal{G}(x) = \psi^2(x) / (4\rho^2) + \xi^T(x) \xi(x) / (4\gamma^2) \quad (30)$$

in which $\gamma < 1$, ρ is a designed parameter, and the adaptive law

$$\dot{\hat{\lambda}} = \kappa \left[\mathcal{G}(x) e^T P b b^T P e - \sigma (\hat{\lambda} - \lambda_0) \right] \quad (31)$$

where $\kappa > 0$, $\sigma > 0$, λ_0 is the predefined constant. Then, the main results of this study are given as follows.

Theorem 1: Consider n th-order uncertain nonlinear system (1) together with the robust adaptive control input (29) and the adaptive law (31). If Assumptions 1-3 are satisfied, then the composite feedback system (26) is ISpS. Besides, all the signals in the close-loop system are UUB.

Proof: To prove that the composite feedback system (26) is ISpS, we can first prove that subsystems (27) and (28) satisfy conditions (16) and (17) respectively, then we can obtain the results through the small gain theorem.

For subsystem (27) together with control input (29) and adaptive law (31), consider the following Lyapunov function candidate:

$$V = \frac{1}{2} e^T P e + \frac{1}{2} g_{\min} \kappa^{-1} \tilde{\lambda}^2 \quad (32)$$

where $\tilde{\lambda} = \lambda^* - \hat{\lambda}$. The time derivative of V along the system trajectory is

$$\begin{aligned} \dot{V} = & \frac{1}{2} e^T (A^T P + P A) e - g_{\min} \kappa^{-1} \tilde{\lambda} \dot{\tilde{\lambda}} \\ & + e^T P b \left[g(x) u + k^T e + D \right] + c_\theta e^T P b \xi^T(x) \omega. \end{aligned} \quad (33)$$

We separately deal with each term in (33). First, since $g_{\min} > 0$, by using Young's inequality, we can obtain the following result:

$$\begin{aligned} e^T P b k^T e &= g_{\min} e^T P b k^T e / g_{\min} \\ &\leq g_{\min} \left[\|e^T k b^T P e\|^2 + 1 / (4g_{\min}^2) \right] \\ &= g_{\min} \|e^T k b^T P e\|^2 + 1 / (4g_{\min}). \end{aligned} \quad (34)$$

Besides, according to (25), the following inequality can be formulated through the Schwarz inequality:

$$\begin{aligned} e^T P b D &\leq \|e^T P b\| \|D\| \leq \|e^T P b\| \theta \psi(x) \\ &\leq \theta^2 \psi^2(x) e^T P b b^T P e / (4\rho^2) + \rho^2. \end{aligned} \quad (35)$$

The last term of (33) can be transformed into:

$$c_\theta e^T P b \xi^T(x) \omega = c_\theta e^T P b \xi^T(x) \omega - \gamma^2 \|\omega\|^2 + \gamma^2 \|\omega\|^2 \quad (36)$$

$$\begin{aligned} &= -\gamma^2 \|c_\theta \xi(x) b^T P e / (2\gamma^2) + \omega\|^2 + c_\theta^2 e^T P b \xi^T(x) \xi(x) \cdot \\ &\quad b^T P e / (4\gamma^2) + \gamma^2 \|\omega\|^2 \\ &\leq c_\theta^2 e^T P b \xi^T(x) \xi(x) b^T P e / (4\gamma^2) + \gamma^2 \|\omega\|^2. \end{aligned}$$

Thus, combining (35) and (36), we can obtain

$$\begin{aligned} &e^T P b D + c_\theta e^T P b \xi^T(x) \omega \\ &\leq \theta^2 \psi^2(x) e^T P b b^T P e / (4\rho^2) + \rho^2 + c_\theta^2 e^T P b \xi^T(x) \xi(x) \cdot \\ &\quad b^T P e / (4\gamma^2) + \gamma^2 \|\omega\|^2 \\ &\leq g_{\min} \lambda^* \mathcal{G}(x) e^T P b b^T P e + \gamma^2 \|\omega\|^2 + \rho^2 \\ &= g_{\min} \tilde{\lambda} \mathcal{G}(x) e^T P b b^T P e + g_{\min} \hat{\lambda} \mathcal{G}(x) e^T P b b^T P e + \gamma^2 \|\omega\|^2 \\ &\quad + \rho^2 \end{aligned} \quad (37)$$

where $\lambda^* = \max(g_{\min}^{-1} \theta^2, g_{\min}^{-1} c_\theta^2)$. Moreover, according to (30), $\mathcal{G}(x) > 0$ is always satisfied. Therefore,

$$\begin{aligned} &e^T P b \left[g(x) u + k^T e \right] \\ &= e^T P b g(x) \left(-\hat{\lambda} \left[\mathcal{G}(x) - (e^T k)^T (e^T k) \right] b^T P e + e^T P b k^T e \right) \\ &\leq -g_{\min} \hat{\lambda} \mathcal{G}(x) e^T P b b^T P e - g_{\min} \|e^T k b^T P e\|^2 \\ &\quad + g_{\min} \|e^T k b^T P e\|^2 + 1 / (4g_{\min}) \\ &= -g_{\min} \hat{\lambda} \mathcal{G}(x) e^T P b b^T P e + 1 / (4g_{\min}). \end{aligned} \quad (38)$$

Substituting (37) and (38) into (33), \dot{V} can be rewritten as follows:

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e - g_{\min} \kappa^{-1} \tilde{\lambda} \dot{\tilde{\lambda}} - g_{\min} \hat{\lambda} \mathcal{G}(x) e^T P b b^T P e \\ & + 1 / (4g_{\min}) + g_{\min} \tilde{\lambda} \mathcal{G}(x) e^T P b b^T P e \\ & + g_{\min} \hat{\lambda} \mathcal{G}(x) e^T P b b^T P e + \gamma^2 \|\omega\|^2 + \rho^2 \\ = & -\frac{1}{2} e^T Q e + g_{\min} \kappa^{-1} \tilde{\lambda} \left[-\dot{\tilde{\lambda}} + \kappa \mathcal{G}(x) e^T P b b^T P e \right] \\ & + \gamma^2 \|\omega\|^2 + \rho^2 + 1 / (4g_{\min}). \end{aligned} \quad (39)$$

If (31) is chosen for $\dot{\tilde{\lambda}}$, noting that

$$\tilde{\lambda} (\hat{\lambda} - \lambda_0) \leq -\frac{1}{2} \tilde{\lambda}^2 + \frac{1}{2} (\lambda^* - \lambda_0)^2 \quad (40)$$

then (39) follows that:

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2} e^T Q e + g_{\min} \tilde{\lambda} \sigma (\hat{\lambda} - \lambda_0) + \gamma^2 \|\omega\|^2 + \rho^2 \\ & + 1 / (4g_{\min}) \\ \leq & -\frac{1}{2} e^T Q e - \frac{\sigma}{2} g_{\min} \tilde{\lambda}^2 + \frac{\sigma}{2} g_{\min} (\lambda^* - \lambda_0)^2 \\ & + \gamma^2 \|\omega\|^2 + \rho^2 + 1 / (4g_{\min}) \\ = & -\frac{1}{2} e^T Q e - \frac{\sigma}{2} g_{\min} \tilde{\lambda}^2 + \gamma^2 \|\omega\|^2 + d_1 \end{aligned} \quad (41)$$

where

$$d_1 = \frac{\sigma}{2} g_{\min} (\lambda^* - \lambda_0)^2 + \rho^2 + \frac{1}{4g_{\min}} \quad (42)$$

is a bounded term. By choosing $\lambda_{\min}(\mathbf{Q}) > 2$, we can get the following result:

$$\dot{V} \leq -\|\mathbf{e}\|^2 + \gamma^2 \|\boldsymbol{\omega}\|^2 + d_1. \quad (43)$$

Thus, according to Definition 1, V is an ISpS-Lyapunov function of subsystem (27), with $\alpha_1(s) \leq V(s) \leq \alpha_2(s)$, $\alpha_3(s) = s^2$ and $\alpha_4(s) = \gamma^2 s^2$ of class K_∞ . At the same time, subsystem (27) is proved to be ISpS. Furthermore, we can obtain a nonlinear L_∞ gain $\gamma_z(s)$ of subsystem (27) satisfying [16]:

$$\gamma_z(s) = \alpha_1^{-1} \circ \alpha_2 \circ \alpha_3^{-1} \circ \alpha_4(s), \quad \forall s > 0 \quad (44)$$

where \circ represents the composition operator between two functions.

For subsystem (28), it is easy to obtain a similar form to (17):

$$\|\boldsymbol{\omega}\| \leq \|A_x^m\| \|\mathbf{e}\| = \gamma_1 \|\mathbf{e}\| = \gamma_1 \|\hat{\mathbf{z}}\| \quad (45)$$

Therefore, the gain function of subsystem (28) is $\gamma_\omega(s) = \gamma_1 s$.

According to (18), if we make $\gamma_z(\gamma_\omega(s)) < s$ satisfied, that is, $\gamma_1 < 1$, then the composite feedback system is ISpS. Furthermore, since $\gamma_1 = \|A_x^m\| \leq 1$ is always true, by choosing $\gamma < 1$, the composite feedback system (26) can be ensured to be ISpS.

Next, substituting (45) into (41), we have

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} - \frac{\sigma}{2} g_{\min} \tilde{\lambda}^2 + \gamma_1^2 \gamma^2 \|\mathbf{e}\|^2 + d_1 \\ &\leq -\frac{1}{2} \mathbf{e}^T (\mathbf{Q} - 2\mathbf{I}) \mathbf{e} - \frac{\sigma}{2} g_{\min} \tilde{\lambda}^2 + (\gamma_1^2 \gamma^2 - 1) \|\mathbf{e}\|^2 + d_1 \\ &\leq -c_1 V + d_1 \end{aligned} \quad (46)$$

where $c_1 = \min(\lambda_{\min}(\mathbf{Q} - 2\mathbf{I})/\lambda_{\max}(\mathbf{P}), \sigma\kappa)$, λ_{\max} denotes the maximum eigenvalue. Thus, all the signals in the close-loop system are UUB. This completes the proof.

Remark 1: Although $f(\mathbf{x})$ and $g(\mathbf{x})$ are both unknown for control design, only $f(\mathbf{x})$ is approximated by the IT2-A2-C0-TSK-FLS. That is, we do not need another IT2-FLS to approximate $g(\mathbf{x})$ in our control scheme, which simplifies the design in practical.

Remark 2: Different from the works in [1] and [10], we do not adapt the consequent parameters of our IT2-A2-C0-TSK-FLS online. Instead, only one new developed parameter $\hat{\lambda}$ is adapted, which can greatly lessen the computation burden of the controller.

IV. NUMERICAL SIMULATIONS

In this section, the following pole-balancing robot system model is applied to illustrate the effectiveness of our proposed method:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(\mathbf{x}) + g(\mathbf{x})u + d(\mathbf{x}, t) \\ y &= x_1 \end{aligned} \quad (47)$$

where x_1 is the angular position from the equilibrium position, x_2 is the angular velocity, while

$$f(\mathbf{x}) = \frac{g \sin x_1 - \frac{m l x_2^2 \cos x_1 \sin x_1}{m_c + m}}{l \left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \right)} \quad (48)$$

$$g(\mathbf{x}) = \frac{\frac{\cos x_1}{m_c + m}}{l \left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \right)} \quad (49)$$

are supposed to be unavailable for control designing. $d(\mathbf{x}, t) = 0.5(x_1^2 + x_2^2) \sin t$ is the uncertain disturbance of the system. $m = 0.1 \text{ kg}$, $m_c = 1 \text{ kg}$, $l = 0.5 \text{ m}$ and u represent the mass of the vehicle, the mass of the pendulum, half of the length of the pendulum and the applied force, respectively. The reference command is given as $y_c = 0.3 \sin t$.

Since our theorem has been proposed in the previous section, the applied force and the adaptive law can be developed as (29) and (31) respectively. Here, all the antecedent fuzzy sets in the IT2-A2-C0-TSK-FLS are set as Gaussian IT2-FSs, with the LMF and UMF are

$$\underline{\mu}_{\tilde{F}_i^s}(x_i) = \exp\left(-\left(x_i - m_{\tilde{F}_i^s}\right)^2 / \left(2\sigma_{\tilde{F}_i^s}^2\right)\right) \quad (50)$$

$$\bar{\mu}_{\tilde{F}_i^s}(x_i) = \exp\left(-\left(x_i - m_{\tilde{F}_i^s}\right)^2 / \left(2\bar{\sigma}_{\tilde{F}_i^s}^2\right)\right), \quad i = 1, 2 \quad (51)$$

respectively. x_1 and x_2 each has 5 antecedent fuzzy sets, with their centers $m_{\tilde{F}_i^s}$ being evenly spaced in $[-\pi/6, \pi/6] \times [-\pi/6, \pi/6]$. Thus, there are 25 rules in total. Besides, $\sigma_{\tilde{F}_i^s} = \pi / (26\sqrt{2})$ and $\bar{\sigma}_{\tilde{F}_i^s} = \pi / (23\sqrt{2})$ are chosen. Other parameters in our control scheme are set as follows: $\mathbf{k} = [1, 2]^T$, $\mathbf{Q} = 30\mathbf{I}$, $\rho = 0.5$, $\gamma = 0.5$, $\kappa = 10$, $\sigma = 0.01$ and $\lambda_0 = 0$.

By choosing $x_1(0) = 0.2$, $x_2(0) = 0$ and $\hat{\lambda}(0) = 0$ as the initial states, the simulation results of our proposed method are shown in Fig. 1-3. Fig. 1 shows the successful tracking performances of the angular position and angular velocity of the pole-balancing robot system. Fig. 2 depicts the changes of

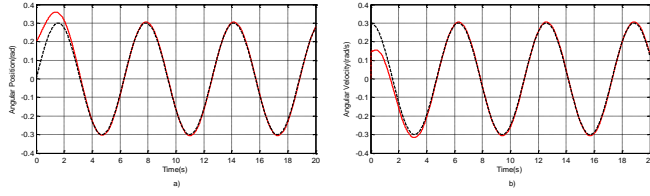


Figure 1. Responses of a) angular position and b) angular velocity. (Solid red line: actual states of system. Dashed black line: reference commands.)

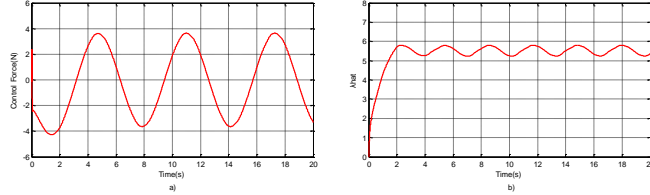


Figure 2. Applied force and adaptive parameter $\hat{\lambda}$.

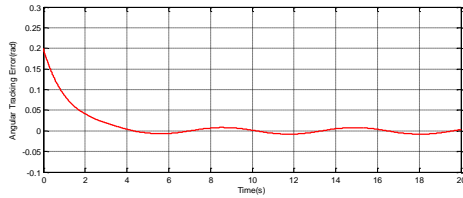


Figure 3. Angular tracking error.

TABLE I. PERFORMANCE CRITERIA

| Type of FLS | Performance Criteria | | |
|-------------|----------------------|----------------|----------------|
| | ISE | IAE | ITAE |
| IT2-FLS | 0.02453 | 0.31505 | 1.20935 |
| T1-FLS | 0.02456 | 0.31578 | 1.21546 |

the applied force and adaptive parameter $\hat{\lambda}$, which demonstrates the convergence of the close-loop system. The angular tracking error can be seen in Fig. 3. We can see that, the angular tracking error keeps around 0 after about 4 seconds, which indicates a satisfying performance through our method.

Finally, to verify the advantages of IT2-FLS over T1-FLS, a counterpart controller employing T1-TSK-FLS instead is used as a comparison. Meanwhile, integral of square error (ISE), integral of the absolute value of the error (IAE) and integral of the time multiplied by the absolute value of the error (ITAE) are utilized to evaluate the performances of the two systems. The results can be seen in Table I. We can find that, although the T1-FLS successfully shows its robustness and good performance against uncertainties, all the criteria of our proposed method are even better, which demonstrates the superiorities of IT2-FLS.

V. CONCLUSIONS

In this study, a novel robust adaptive control scheme based on IT2-FLS and small gain approach is proposed for a class of uncertain nonlinear systems. The IT2-A2-C0-TSK-FLS is applied to approximate the unknown dynamics of such a system, while the small gain theorem is utilized to develop the robust adaptive control scheme. Although the dynamic

functions of the system are unavailable for us, the proposed method can still keep all the signals in the close-loop system UUB. Meanwhile, only one parameter needs to be adapted online, which is easy to realize in applications. Simulation results verify the effectiveness and superiority of our proposed control scheme. In our future work, the proposed method will be promoted to MIMO nonlinear systems. Besides, measurement noises will be taken into consideration, and anti-noise robust adaptive control design for unknown nonlinear systems will be investigated.

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