Time-varying Formation Tracking Control for Multi-UAV Systems with Nonsingular Fast Terminal Sliding Mode

Tianyi Xiong[1,2], Zhiqiang Pu[1,2], Jianqiang Yi[1,2]

1. Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China
E-mail: {xiongtianyi2016, zhiqiang.pu, jianqiang.yi}@ia.ac.cn

2. University of Chinese Academy of Sciences Beijing 100049, China

Abstract: This paper deals with the time-varying formation tracking control problem for multiple unmanned aerial vehicle (multi-UAV) systems, where the UAVs are required to form a predefined time-varying formation while tracking the trajectory of the leader. A finite-time formation tracking control law is designed to realize time-varying formation tracking by utilizing nonsingular fast terminal sliding mode (NFTSM), which can speed up the convergence rate and be free from singularity problem. The finite-time stability and robustness of the close-loop system is proven using Lyapunov stability theory. Finally, numerical simulation results verify the effectiveness of the proposed control law.

Key Words: multi-UAV system, formation tracking control, nonsingular fast terminal sliding mode, finite time

1 INTRODUCTION

In recent years, formation control for unmanned aerial vehicle (UAV) systems have drawn significant attention from related research communities and become a research hotspot. This is partially due to the potential and wide applications of multi-UAV formation in a variety of areas, such as target seeking, telecommunication relay, environment monitoring, and so on. As a matter of fact, how to realize and maintain the desired formation through control approaches is the primary consideration for multi-UAV systems, which has been studied a lot during past decades. Three typical formation control methods are leader-follower based approaches [1], behavior based methods [2], and virtual structure based approaches [3]. With the development of consensus theory for multi-agent systems, increasing researchers have extended consensus approaches to solve formation control problem and numerous results have been obtained [4-10].

As we know, formations in the great majority of the existing results are time-invariant. However, these formations often cannot meet the practical requirements in various applications. Furthermore, it should be pointed out that the results for time-invariant formation cannot be directly applied to time-varying formations as the derivative of the formation information in latter formations may increase complexity of control law design and system stability analysis [4-6]. Therefore, it is more meaningful to study the time-varying formation control. Besides, the formation may also need to track the trajectory generated by the virtual/real leader to perform a task. Thus, the time-varying formation tracking problem arises, where a group of UAVs are expected to keep the desired time-varying formation while tracking the trajectory of the leader.

Sliding mode control (SMC) is a robust nonlinear control method, which has been widely used in many fields, such as robots formation, satellites formation control, and so on [9]. However, linear SMC can only realize asymptotical stability. On the contrary, finite-time stability has superiorities on faster response rate, higher accuracy, and stronger robustness against uncertainties [10]. Thus, terminal sliding mode (TSM) control emerged to guarantee finite-time convergence by employing a nonlinear surface. Noting that conventional TSM control may suffer from singular problem and slow convergence speed, nonsingular fast terminal sliding mode (NFTSM) control method have been further proposed to simultaneously solve such two problems [11][12]. To the best of our knowledge, time-varying formation tracking control based on NFTSM approach for multi-UAV systems is still open, which this paper exactly investigates.

Compared with the previous relevant results, the main contributions of this paper can be summarized as follows. Firstly, a consensus-based control scheme with NFTSM is proposed to achieve time-varying formation tracking in finite time as well as to avoid singularity and speed up convergence. Secondly, the finite-time stability of the close-loop system is analyzed by applying Lyapunov stability theory. Finally, numerical simulations are established to verify the effectiveness of the proposed control law.

The rest of the paper is organized as follows. Preliminaries and the problem formulation are given in Section 2. The design of NFTSM-based control law to realize time-varying formation tracking is presented in Section 3. The finite-time stability analysis is studied in Section 4. Numerical simulation results are presented in Section 5 to illustrate the

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effectiveness of the proposed control law. Conclusions are drawn in Section 6.

2 PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, the basic concepts on graph theory and notations will be firstly introduced, and then a detailed description of the time-varying formation problem will be presented.

2.1 Graph theory and notations

In this paper, the multi-UAV system comprising N UAVs are regarded as a multi-agent system, while the interaction network are modelled by a directed graph denoted by $G=(V,E,A)$, where $V=\{\upsilon_1,\upsilon_2,\ldots,\upsilon_N\}$, $E\subseteq V\times V$, and $A=[a_{ij}]\in R^{N\times N}$ are the set of nodes, the set of edges, and the associated adjacency matrix, respectively. In addition, $I=\{1,2,\ldots,N\}$ represents the number of the nodes. If $(\upsilon_j,\upsilon_i)\in E, j\neq i$, then node υ_i can receive the information from node υ_j, and vice versa. Moreover, $a_{ij}>0, j\neq i$ if and only if $(\upsilon_j,\upsilon_i)\in E, j\neq i$, and a_{ij}=0 otherwise. Besides, a_{ii}=0 . The neighbor set of node υ_i is denoted by $N_i=\{\upsilon_j\in V:(\upsilon_j,\upsilon_i)\in E\}$. Let $D=diag\{d_1,d_2,\cdots,d_N\}$ $\in R^{N\times N}$ denotes the in-degree matrix, where $d_i=\sum_{\upsilon_j\in N_i}a_{ij}$. Define $L=D-A$ as the Laplacian matrix of the graph G. A direct path from node υ_i to υ_j is a sequence of successive edges in the form of $\{(\upsilon_i,\upsilon_k),(\upsilon_k,\upsilon_l),\cdots,(\upsilon_m,\upsilon_j)\}$. A directed graph G is said to have a spanning tree, if there is at least one node (called the root) having at least a directed path to every other nodes. Furthermore, suppose that there is a virtual leader denoted as node υ_0, then we have an augmented graph $\bar{G}$ whose node set is $\bar{V}=\{\upsilon_0,V\}$. The connected weight from the ith UAV to the virtual leader is denoted by b_i, and $b_i>0$ if and only if there exits an edge from the leader to the ith UAV.

Throughout this paper, the following notations are used. Let $\mathbf{0}$ be zero column vector with appropriate dimension; $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ denotes the Euclidian 2-norm of a vector; I represents the identity matrix with appropriate dimension; $\otimes$ denotes Kronecker product.

2.2 Problem description

The dynamics of an UAV can be classified into the trajectory dynamics and the attitude dynamics. Since the dynamics of attitude are much faster than those of trajectory, the formation control can be decoupled into an inner-loop control and an outer-loop control, where the inner-loop controller stabilizes the attitude and the outer-loop controller drives the UAV towards the desired position [6][13]. The two-loop control structure schematic diagram for a UAV is depicted in Fig. 1 [5]. In this paper, the formation control is mainly concerned with the position and velocity in the outer-loop where the UAV can be regarded as a point-mass system, while the inner-loop can be controlled by the PD controller [6]. The outer-loop dynamics of UAV i can be approximately described as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=u_i(t)+d_i,\end{cases} \tag{1}$$

where $x_i(t)\in R^{\mathrm{m}}$ and $v_i(t)\in R^{\mathrm{m}}$ are the position and velocity of the i th UAV; $u_i(t)\in R^{\mathrm{m}}$ and $d_i\in R^{\mathrm{m}}$ represent the control input vector and bounded external disturbance of the i th UAV, respectively.

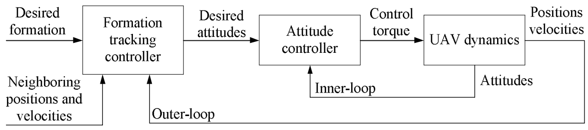


Fig. 1. The two-loop control structure scheme for an UAV.

In this paper, the expected time-varying formation is specified by a vector $h(t)=[h_1^T(t),h_2^T(t),\ldots,h_N^T(t)]^T$, where $h_i(t)=[h_{ix}^T(t),h_{iv}^T(t)^T]^T\in R^{2\mathrm{m}}$ is a piece-wise continuously differentiable function vector with $h_{ix}(t)\in R^{\mathrm{m}}$, $h_{iv}(t)\in R^{\mathrm{m}}$ and $h_{iv}(t)$=$\dot{h}_{ix}(t)$.

Denote the position and velocity of the virtual leader we need to track as $x_0(t)\in R^{\mathrm{m}}$ and $v_0(t)\in R^{\mathrm{m}}$ with $v_0(t)$=$\dot{x}_0(t)$. Furthermore, we can define formation tracking error $\delta_i(t)=[\bar{x}_i(t),\bar{v}_i(t)]^T$ for the ith UAV, where $\bar{x}_i(t)$=$x_i(t)-x_0(t)-h_{ix}(t)$ and $\bar{v}_i(t)$=$v_i(t)-v_0(t)-h_{iv}(t)$ are formation tracking position and velocity errors, respectively. For simplicity, let $m=1$ without loss of generality, since the analysis and the results can be easily extended to higher dimensional case by using the Kronecker product.

Definition 2.1. The multi-UAV system (1) is said to achieve time-varying formation if for any given bounded initial states $[x_i(0),v_i(0)]^T$, the formation tracking error δ_i satisfies

$$\lim_{t\to\infty}\delta_i(t)=\mathbf{0},\ i=1,2,\ldots,N. \tag{2}$$

Remark 2.1. The assumption of the existence of the virtual leader is reasonable, because the trajectory of the virtual leader we need to track can be regarded as that of a specified real UAV with no external control inputs whose dynamics are described by

$$\begin{cases}\dot{x}_0(t)\text{=}v_0(t)\\ \dot{v}_0(t)\text{=}\mathrm{g}(t),\end{cases} \tag{3}$$

where $\mathrm{g}(t)$ is a continuously differentiable function. Therefore, the time-varying formation control problem is transformed into time-varying formation tracking problem.

To make the definition more comprehensible, we take a time-invariant formation of square as an example, which is shown in Fig.2. Consider a multi-UAV system containing four UAVs, which means $I=\{1,2,3,4\}$. Since the formation is invariant, $h_i(t)$ is equal to h_i . Choose the formation center as the virtual leader. If equation (2) in Definition 2.1 is satisfied, and $\|h_i\|=\|h_j\|$, $\forall i,j\in I$ as showed in Fig.2, the square formation is achieved. Let $\xi_i(t)=[x_i^T(t),v_i^T(t)]^T$, and $\xi_0(t)=[x_0^T(t),v_0^T(t)]^T$. It is worth noting that $h_i(t)$ represents the relative offset vector of $[x_i(t),v_i(t)]^T$ with respect to $[x_i(0),v_i(0)]^T$ if Definition 2.1 is satisfied. Therefore, we can achieve any desired formation by choosing the appropriate $h_i(t)$.

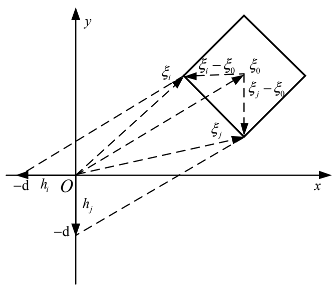


Fig. 2. Illustration example of square formation.

Next, the paper will focus on the time-varying formation tracking control law design based on NFTSM.

3 NONSINGULAR FAST TERMINAL SLIDING MODE CONTROL LAW DESIGN

In this section, we will firstly present the expression of the NFTSM, after which the control law based on NFTSM is proposed.

In this paper, it is assumed that only local neighbor information can be used for the control law design, such that the formation control is fully distributed. Consequently, for the ith UAV, we define local neighborhood tracking position and velocity errors respectively as

$$e_{ix}(t)=\sum_{v_j\in N_i} a_{ij}\left[x_i(t)-h_{ix}(t)-(x_j(t)-h_{jx}(t))\right] \\ +b_i(x_i(t)-x_0(t)-h_{ix}(t)) \tag{4}$$

and

$$e_{iv}(t)=\sum_{v_j\in N_i} a_{ij}\left[v_i(t)-h_{iv}(t)-(v_j(t)-h_{jv}(t))\right] \\ +b_i(v_i(t)-v_0(t)-h_{iv}(t)). \tag{5}$$

Taking both convergence rate and singularity problem into consideration, we construct nonsingular fast sliding surface [9][12] for the ith UAV as

$$s_i=e_{ix}+k_1\,\mathrm{sig}(e_{ix})^{\eta_1}+k_2\,\mathrm{sig}(e_{iv})^{\eta_2} \tag{6}$$

where $\mathrm{sig}(\cdot)^{\gamma}=|\cdot|^{\gamma}\,\mathrm{sign}(\cdot)$; $k_1,k_2>0$; $\eta_2<\eta_1$, $1<\eta_2<2$.

In order to further improve the convergence speed, a fast reaching law is chosen as

$$\dot{s}_i=-c'\,\mathrm{sgn}(s_i)-c''s_i \tag{7}$$

where $c',c''>0$.

Comparing (7) with the differentiation of equation (6), we can get the control law as follows

$$u_i=\frac{1}{(\sum_{v_j\in N_i} a_{ij}+b_i)}\Big[\sum_{v_j\in N_i} a_{ij}(\dot{h}_{iv}-\dot{h}_{jv}+u_j)+b_i(\dot{v}_0+\dot{h}_{iv}) \\ -\frac{1}{k_2\eta_2}(|e_{iv}|^{2-\eta_2}(1+k_1\eta_1|e_{ix}|^{\eta_1-1})+c_1\,\mathrm{sgn}(s_i)+c_2 s_i\Big] \tag{8}$$

Where $c_1,c_2>0$.

Remark 3.1. In control law (8), since $1<\eta_2<2$, the value of the item $|e_{iv}|^{2-\eta_2}$ stays small when $|e_{iv}|$ is sufficiently small, resulting in no peak value in the control input curve as $|e_{iv}|\to 0$. Thus, the control law is free from singular problem.

Remark 3.2. Compared to the conventional TSM surface defined as

$$s_i=e_{iv}+\lambda|e_{ix}|^{p/q}\text{ , } q>p>0,\ p \text{ and } q \text{ are odd,} \tag{9}$$

the item $k_1\,\mathrm{sig}(e_{ix})^{\eta_1}$ in the NFTSM surface (6) can accelerate the convergence speed when the states e_{ix} and e_{iv} move along the sliding surface. Moreover, by utilizing the fast reaching law (7) , we can further improve the speed of convergence.

Remark 3.3. The values of a_{ij} and b_i represent the confident degree on the formation received from the neighbors or the leader. For simplicity, choose $a_{ij}=1$ or 0 and $b_i=1$ or 0 in this paper.

4 CLOSE-LOOP SYSTEM STABILITY ANALYSIS

In this section, we will focus on the finite-time stability analysis of the close-loop multi-UAV system under NFTSM control.

Assumption 4.1. The augmented graph $\bar{G}$ contains a spanning tree with the root node being the virtual leader.

Assumption 4.2. The external disturbance is bounded by $\bar{d}$, that is, $|d_i|<\bar{d}$, $\bar{d}>0$.

Define $e_i=[e_{ix},e_{iv}]^T$ and global neighborhood formation tracking error $e=[e_1^T,e_2^T,\dots e_N^T(t)]^T$, then we can get

$$e=[(L+B)\otimes I_2]\delta \tag{10}$$

where $\delta=\left[\delta_1^T,\delta_2^T,\dots\delta_N^T\right]^T$; L denotes the Laplace matrix between the UAVs; $B=diag\{b_1,b_2\dots,b_N\}$.

Lemma 4.1.[14] Assume assumption 4.1 is satisfied, then $L+B$ is nonsingular. Besides, $\delta=0$ if and only if $e=0$.

Remark 4.1. From Lemma 4.1, the condition (2) in definition 2.1 is equal to

$$\lim_{t\to\infty} e_i(t)=\mathbf{0},\ i=1,2,\dots,N. \tag{11}$$

Thus, we can achieve the specified time-varying formation through adjusting $e_i(t)$ to $\mathbf{0}$.

Lemma 4.2. [15] Consider a continuous system $\dot{x} = f(x)$ with $f(0) = 0$. Suppose there exist real numbers $c > 0$, $\beta > 0$, $0 < \alpha < 1$, a positive-definite function $V : D \rightarrow R$, and a neighborhood $U \subset D$ of the origin such that $\dot{V} + cV + \beta V^{\alpha} \leq 0$. Then, the origin is a finite-time stable equilibrium. In addition, the settle time T satisfies $T \leq [1/c(1-\alpha)]\ln[(cV(0)^{1-\alpha} + \beta) / \beta]$.

Lemma 4.3. [15] Suppose $a_i > 0, i = 1,2,\ldots,n$, and $0 < p < 2$, then $(\sum_{i=1}^{n} a_i^2)^p \leq (\sum_{i=1}^{n} a_i^p)^2$.

Theorem 4.1. Suppose Assumption 4.1 and 4.2 hold. Then time-varying formation for the multi-UAV system (1) can be achieved in finite-time under the NFTSM control law (8), if the condition $c_1 > 3N\bar{d}$ is satisfied.

Proof. Define $e_x = [e_{1x}, e_{2x}, \ldots e_{Nx}]^T$, $e_v = [e_{1v}, e_{2v}, \ldots e_{Nv}]^T$ and $U = [u_1, u_2, \ldots u_N]^T$. Then system (1) can be written as

$$\begin{cases} \dot{e}_x = e_v \\ \dot{e}_v = (L+B)(U + D - \dot{h}_v) - B\dot{v}_0 \end{cases} \tag{12}$$

where $h_v = [h_{1v}, h_{2v}, \ldots, h_{Nv}]^T$, $D = [d_1, d_v, \ldots, d_N]^T$.

Let $|e_x|^{\eta_1 - 1} = diag\{|e_{1x}|^{\eta_1 - 1}, \ldots, |e_{Nx}|^{\eta_1 - 1}\}$, $|e_v|^{\eta_2 - 1} = diag\{|e_{1v}|^{\eta_2 - 1}, \ldots, |e_{Nv}|^{\eta_2 - 1}\}$, $\text{sig}(e_v)^{2-\eta_2} = [\text{sig}(e_{1v})^{2-\eta_2}, \ldots, \text{sig}(e_{Nv})^{2-\eta_2}]^T$, $S = [s_1, s_2, \ldots s_N]^T$ and $\text{sgn}(S) = [\text{sgn}(s_1), \text{sgn}(s_2), \ldots \text{sgn}(s_N)]^T$. The control law (8) can be transformed into matrix form as follows

$$\begin{aligned} U = (L+B)^{-1}\big[&(L+B)\dot{h}_v + B\dot{v}_0 - k_2^{-1}\eta_2^{-1} \\ &(I + k_1\eta_1 |e_x|^{\eta_1 - 1})|e_{iv}|^{2-\eta_2} + c_1\,\text{sgn}(S) + c_2 S\big]. \end{aligned} \tag{13}$$

Consider the following Lyapunov function

$$V = \frac{1}{2}S^T S. \tag{14}$$

Differentiating V with respect to time, we obtain

$$\dot{V} = S^T \dot{S}, \tag{15}$$

where

$$\begin{aligned} \dot{S} &= (I + k_1\eta_1 |e_x|^{\eta_1 - 1})e_v + k_2\eta_2 |e_v|^{\eta_2 - 1}\dot{e}_v \\ &= k_2\eta_2 |e_v|^{\eta_2 - 1}\big[(L+B)(U + D - \dot{h}_v) - B\dot{v}_0\big] \\ &\quad + (I + k_1\eta_1 |e_x|^{\eta_1 - 1})e_v, \end{aligned} \tag{16}$$

Substituting for U from (13), yields

$$\dot{S} = -k_2\eta_2 |e_v|^{\eta_2 - 1}\big[(c_1\,\text{sgn}(S) + c_2 S) - (L+B)D\big]. \tag{17}$$

Therefore, we can get the derivation of the Lyapunov function

$$\begin{aligned} \dot{V} &= -S^T k_2\eta_2 |e_v|^{\eta_2 - 1}\big[(c_1\,\text{sgn}(S) + c_2 S) - (L+B)D\big] \\ &\leq -k_2\eta_2 |e_v|^{\eta_2 - 1}\sum_{i=1}^{N}((c_1 - 3N\bar{d})|s_i| + c_2 |s_i|^2), \end{aligned} \tag{18}$$

By applying Lemma 4.3, we can further obtain

$$\dot{V} \leq -\alpha_1 V^{1/2} - \alpha_2 V, \tag{19}$$

where $\alpha_1 = \sqrt{2}k_2\eta_2 |e_v|^{\eta_2 - 1} N(c_1 - 3N\bar{d})$, and $\alpha_2 = 2k_2\eta_2 |e_v|^{\eta_2 - 1} Nc_2$. If $c_1 > 3N\bar{d}$ is satisfied, then according to Lemma 4.2, the sliding mode will be reached in finite time in the case of any $e_v \neq 0$. From [16], $e_v = 0$ is not an attractor in the reaching phase, so that the finite-time reachability of NFTSM (6) is guaranteed. Furthermore, the neighborhood formation tracking error e will converge to zero along the sliding surface in finite time. Thus, from Lemma 4.1, the desired time-varying formation for the multi-UAV system (1) can be achieved in finite-time.

5 Simulations

In this section, numerical simulations are given to illustrate the effectiveness of theoretical results obtained in the previous sections.

Consider a multi-UAV system with six UAVs where the dynamics of each UAV are described by (1). In the case where $m = 2$, $x_i(t)$, $v_i(t)$, $x_0(t)$, $v_0(t)$, $u_i(t)$, $h_i(t)$ can be respectively written as $x_i(t) = [x_{iX}(t), x_{iY}(t)]^T$, $v_i(t) = [v_{iX}(t), v_{iY}(t)]^T$, $x_0(t) = [x_{0X}(t), x_{0Y}(t)]^T$, $v_0(t) = [v_{0X}(t), v_{0Y}(t)]^T$, $u_i(t) = [u_{iX}(t), u_{iY}(t)]^T$, and $h_i(t) = [h_{ixX}(t), h_{ivX}(t), h_{ixY}(t), h_{ivY}(t)]^T$, $i = 1,2,\ldots,6$.

To verify the effectiveness of the proposed control law, choose the interaction digraph as showed in Fig. 3, which contains a spanning tree.

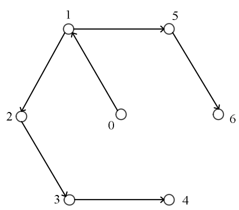
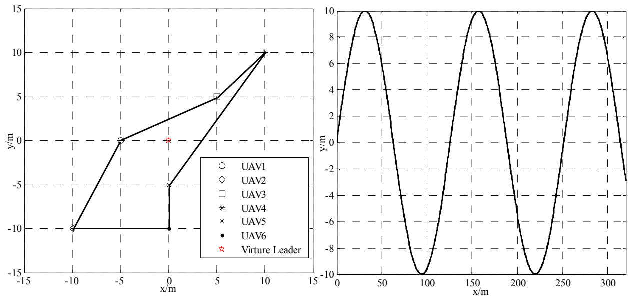


Fig. 3. interaction digraph containning a spanning tree.

The six UAVs are supposed to keep a parallel hexagon formation with time-varying edges while keeping rotation around the virtual leader which is also the formation center. Thus, the time-varying formation can be specified by

$$h_i(t) = \begin{bmatrix} 0.2t\cos(0.1t + \frac{\pi}{3}(i-1)) \\ 0.2[\cos(0.1t + \frac{\pi}{3}(i-1)) - 0.1t\sin(0.1t + \frac{\pi}{3}(i-1)] \\ 0.2t\sin(0.1t + \frac{\pi}{3}(i-1)) \\ 0.2[\sin(0.1t + \frac{\pi}{3}(i-1)) + 0.1t\cos(0.1t + \frac{\pi}{3}(i-1))] \end{bmatrix}$$

The trajectory of the virtual leader is expressed by $x_0(t) = [4t, 10\sin(0.2t)]^T$. The external disturbance is assumed to be expressed by $d_i(t) = 0.1\sin(0.2t)\text{m/s}^2$. In procotol (8), choose parameters $c_1 = 2 > 3N\bar{d}$, $c_2 = 0.05$, $k_1 = k_2 = 3$, and $\eta_1 = 2$, $\eta_2 = 5/3$ which satisfy $\eta_2 < \eta_1$, $1 < \eta_2 < 2$. To reduce chattering in the simulation, replace sign funtion sgn(·) with saturation function sat(·) in the control law (8). The simulation results are presented by Figs. 4-7. Fig. 4 (a) shows the initial positions of the six UAVs and the virtual leader, while Fig. 4 (b) shows the motion trajectory of the virtual leader; Fig. 5 depicts the positions of the six UAVs and the virtual leader at $t = 30, 40, 50$ s . Fig. 6 displays the formation position tracking errors $\overline{x}_i(t)$ on X-axis and Y-axis. Fig. 7 shows the changing curve of the control input $u_i(t) = [u_{iX}(t), u_{iY}(t)]^T$.



(a) Initial states (b) the trajectory of the virtual leader

Fig. 4. Initial states and the trajectory of the virtual leader.

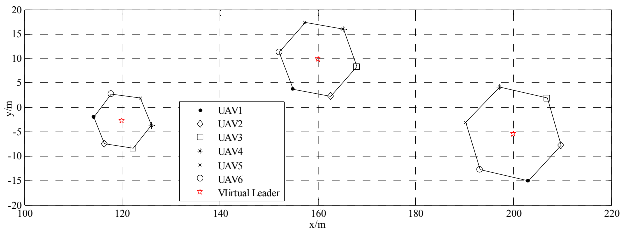
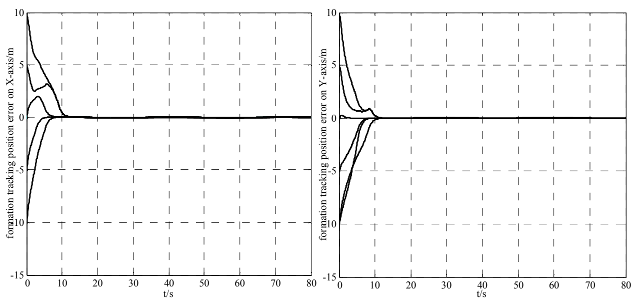


Fig. 5. Position snapshots of the six UAVs and the virtual leader at $t = 30, 40, 50$ s.

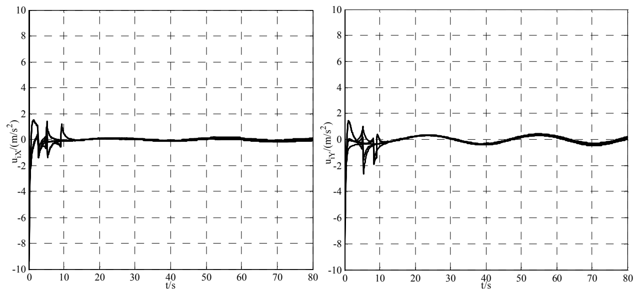
From Figs. 4-5, the following phenomena can be observed: (i) the six UAVs successfully keep a parallel hexagon formation; (ii) the edge of the parallel hexagon is getting longer as time goes on as expected; (iii) the parallel hexagon keeps rotation around the virtual leader; (iv) the virtual leader moves along a sine curve, and keeps lying in the center of the formation, so that the formation moves along the same sine curve.



(a) error on X-axis (b) error on Y-axis

Fig. 6. Formation position tracking errors on X-axis and Y-axis.

From Fig. 6, we can see that the formation position tracking errors converge around 0 after about t=10s although there exits time-varying input disturbance. From Fig. 7, it can be observed that there exists no peak value in the steady state process, that is, the proposed control law (8) is free from singularity. Besides, in the transient process, the control inputs have a little vibrations due to the initial simulation condition and the existence of the input disturbance. However, they also stay in an admissible range. Therefore, these demonstrate the desired time-varying formation can be achieved in finite time by the NFTSM-based control law (8).



(a) input u_{iX} (b) input u_{iY}

Fig. 7. Formation position tracking errors on X-axis and Y-axis.

6 CONCLUSION

Compared with time-invariant formations, time-varying formation is more practical in many applications. To enhance the robustness and obtain finite-time stability of the multi-UAV system, we design a NFTSM approach to achieve time-varying formation tracking as well as to avoid singularity problem and improve convergence rate. Based on graph theory, the finite time stability of the close-loop system with the proposed control law is proven by applying Lyapunov stability theory. Numerical simulation results illustrate the effectiveness of the obtained theoretical results. Future research directions are to establish an adaptive NFTSM control scheme or to extend the results in this paper to the case where there exist time delays.

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