Neural Network based Distributed Adaptive Time-varying Formation Control for Multi-UAV Systems with Varying Time Delays

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***Abstract*—This paper investigates the time-varying formation control problem for multiple unmanned aerial vehicle (multi-UAV) systems with unknown uncertainties and varying time delays. Firstly, a radial basis function neural network (RBFNN) is adopted to estimate the lumped model uncertainties online for compensation. Then, a novel RBFNN-based fully distributed adaptive control scheme consisting of control law to stabilize the system and adaptive law to adjust RBFNN weights is developed to tackle the time-varying formation tracking problem in the presence of varying time delay. The uniformly ultimately boundedness (UUB) of the formation tracking errors is theoretically analyzed through Lyapunov approach. Comparative simulation results demonstrate the effectiveness of the control and adaptive laws proposed in this paper.**

Keywords—formation, neural network, time delays, multi-UAV

I. Introduction

Since outstanding advantages of formation flight for multiple unmanned aerial vehicle (multi-UAV) systems have been witnessed, including wide area sensing coverage, low cost, good flexibility performance, strong robustness to system failure, and so on, numerous efforts have been made in this field, recently. Generally, the primary research concern is to shape and stabilize the expected formations for multi-UAV systems by means of effective control methods. As a matter of fact, three typical frameworks have been widely investigated and applied for formation control, including leader-follower based approach [1], behavior based method [2], and virtual structure based strategy [3]. However, all these three frameworks have their own weaknesses [4].

With the development of consensus theory for multi-agent systems, increasing researchers have extended consensus protocols to deal with formation control problem and numerous results have been derived. Ref. [5] studied finite-time control problem for a group of nonholonomic mobile robots using a distributed finite-time observer. Ref. [6] considered the output-feedback formation problem for multi-UAVs by introducing a state observer. Ref. [7] dealt with finite time decentralized formation tracking problem for multiple autonomous vehicles through sliding mode estimator method. However, it should be pointed out that formations in the results above are time-invariant, which often cannot satisfy practical requirements on various situations where we need to cover changing environment, avoid moving obstacles and perform other complex tasks. Therefore, it motivates some researches on time-varying formation control. Ref. [8] studied time-varying formation tracking problem for multiple manipulators in finite time. Ref. [9] focused on time-varying formation tracking analysis and design problems for second-order multi-agent systems with switching interaction graphs, and the obtained results were applied to solve the target enclosing problem of a UAV swarm system.

For multi-UAVs formation control design, one important issue is how to work with the uncertainties of the UAV dynamics, which may result from parametric uncertainties, modeling errors, external disturbances, and so on. To deal with it, neural network (NN), which can be regarded as a universal approximator, is usually applied in formation control to estimate the unknown uncertainties. An adaptive nonsingular terminal sliding mode formation control by means of output recurrent fuzzy wavelet neural networks for a group of networked heterogeneous Mecanum-wheeled omni-directional robots was presented in [10]. A novel neural-network based approach for tracking control with switching formation in nonomniscient constrained space for multi-agent systems was proposed in [11]. A distributed adaptive attitude synchronization control strategy of spacecraft formation flying based on modified fast terminal sliding mode and NN in the presence of unknown external disturbances was designed in [12].

In addition, time delay, which can severely deteriorate the formation tracking performance and even make the system broken down in real engineering, is another important issue that should be stressed on. Aiming to investigate formation control problem of a multi-UAV system with nonuniform time-delays and jointly connected topologies, Ref. [13] proposed a consensus-based distributed formation control protocol which required only the local neighbor-to-neighbor information between UAVs. Ref. [14] investigated a leader-following formation control problem for multi-agent systems with non-uniform time-varying communication delays. Ref. [15] designed a consensus based algorithm to solve formation control problems for multiple unmanned underwater vehicles under time-varying delays.

Nevertheless, to the best of our knowledge, few investigations in the existing literature, including the aforementioned works, have simultaneously considered the problems of unknown uncertainties and varying time delays in time-varying formation control, even in the consensus control for multi-agent systems. In [16], a coordinated tracking control problem was considered in multiple Euler-Lagrange systems with nonlinear uncertainties, external disturbances, and communication delays, which was solved by designing distributed observers such that all the followers could keep access to the state information of the leader. However, the control law was not distributed, which may greatly limit the applications in reality. An adaptive output feedback control scheme was proposed to solve the output consensus regulation of multi-agent systems with parameter uncertainties and time-invariant communication time delays in [17]. It should be pointed out that the observer-based control scheme may suffer from the limited learning capability of the utilized observers, which will result in a huge challenge in the design stage.

Focusing on the above problems, in this paper, we propose a novel radial basis function neural network (RBFNN) based distributed adaptive time-varying formation control scheme for multi-UAV systems with varying time delays. Firstly, a RBFNN is adopted to estimate the lumped model uncertainties for compensation. Secondly, a novel RBFNN-based fully distributed adaptive control scheme is developed to tackle the time-varying formation tracking problem in the presence of varying time delays. Thirdly, the uniformly ultimately boundedness (UUB) of the formation tracking errors is obtained through Lyapunov approach. Finally, to intuitively show the effects of the RBFNN, a counterpart without RBFNN is chosen as the comparison in numerical simulation, and the results verify the robustness and superiority of our proposed control scheme.

Compared with the previous relevant results, the main contributions of this paper can be summarized as follows. Firstly, model uncertainties and varying time delays are simultaneously considered in the time-varying formation problem in this paper, and the stability analysis of our proposed control scheme is theoretically explored through Lyapunov approach. Secondly, differing from the work in [16], a fully distributed control scheme which only requires the time-delayed neighbor-to-neighbor information between UAVs and the leader is developed for time-varying formation, which will greatly broaden the application scenario in real engineering.

The rest of the paper is organized as follows. The formation control problem description along with some preliminaries and notations is given in Section II. The design control law and the stability analysis are studied in Section III. Numerical simulation results are presented in Section IV, after which the conclusions are drawn.

II. Problem Description

A. Modeling an UAV

Generally, the dynamics of a UAV can be classified into the trajectory dynamics and the attitude dynamics. Since the dynamics of attitude are much faster than those of trajectory, the formation controller can be established by an inner-loop controller and an outer-loop controller, where the former is employed to stabilize the attitude and the latter is to drive the UAV towards the desired position [18]. The two-loop control structure schematic diagram for a UAV is depicted in Fig. 1 [9]. Since the formation tracking in this paper is mainly concerned with the positions and velocities, the paper mainly focuses on the outer-loop controller design, while the inner-loop can be stabilized by a feasible controller, such as PD controller. In this case, the UAV can be simply regarded as a point-mass system, and the outer-loop dynamics of the *i*th UAV can be approximately described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) + f_i(x_i, v_i), \end{cases} \tag{1}$$

where $x_i(t) \in R^m$ and $v_i(t) \in R^m$ are the position and velocity of the i th UAV, m is the dimension of the states; $u_i(t) \in R^m$ represents the control input vector of the i th UAV, respectively; $f_i(x_i, v_i)$ stands for the lumped uncertainty term brought by modeling errors or external disturbances.

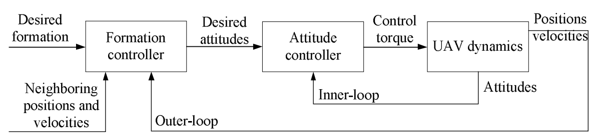

Fig. 1. The two-loop control structure scheme for an UAV.

B. Modeling a Multi-UAV system

In this paper, a multi-UAV system comprising N UAVs can be regarded as a multi-agent system where a UAV shares information with other UAVs via communication architecture. Generally, we use a graph denoted by $G = (V, E, A)$ to describe the information exchanges among UAVs. Denote a single UAV as υ_i, then $V = \{\upsilon_1, \upsilon_2, \dots, \upsilon_N\}$ is the set of UAVs, and $E \subseteq V \times V$ represents the set of edges, where E is defined such that if $(\upsilon_j, \upsilon_i) \in E, j \neq i$, there is an edge from UAV j to the UAV *i*, which means that UAV j can deliver information to UAV *i*. In addition, $A = [a_{ij}] \in R^{N \times N}$ is the associated adjacency matrix with $a_{ij} \geq 0$. We set where $a_{ij} > 0,\ j \neq i$ if and only if $(\upsilon_j, \upsilon_i) \in E$; otherwise $a_{ij} = 0$. In this case, UAV *j* is said to be the neighbor of UAV *i* if and only if $a_{ij} > 0$, and $N_i = \{\upsilon_j \in V : (\upsilon_j, \upsilon_i) \in E\}$ represents the neighbor set of the i th UAV. A graph is an undirected graph if and only if $a_{ij} = a_{ji}$. Define $D = diag\{d_1, d_2, \cdots, d_N\} \in R^{N \times N}$ as the in-degree matrix, where $d_i = \sum_{\upsilon_j \in N_i} a_{ij}$. Then, the Laplacian matrix of graph G is defined as $L = D - A$. A direct path from UAV i to j is a sequence of successive edges in the form of $\{(\upsilon_i,\upsilon_k),(\upsilon_k,\upsilon_l),\cdots,(\upsilon_m,\upsilon_j)\}$. Furthermore, an undirected graph is called connected if there is a path between any two UAV of graph G. In the case of leader-following, another graph $\bar{G}$ with a virtual leader (labled as UAV 0) and N UAVs should be considered. The adjacency matrix element associated with the edge from the ith UAV to the virtual leader is denoted by b_i, with $b_i > 0$ if and only if the ith UAV can receive information from the leader. For more details about graph theory, readers can refer to [19].

In addition, define $x(t)=[x_1^T(t),...,x_N^T(t)]^T$, $v(t)=[v_1^T(t),...,v_N^T(t)]^T$, $u(t)=[u_1^T(t),...,u_N^T(t)]^T$ and $f(x,v)=[f_1^T(x_1,v_1), f_2^T(x_2,v_2),...,f_N^T(x_N,v_N)]^T$. Then the multi-UAV systems in which the dynamics of each UAV is depicted as (1) can be written in the following compact form:

$$\begin{cases}\dot{x}(t)=v(t)\\ \dot{v}(t)=u(t)+f(x,v)\end{cases} \tag{2}$$

Throughout this paper, we suppose the following assumption naturally holds.

Assumption 1: The interaction network graph $\bar{G}$ is connected.

C. Formation Tracking Problem Description

In this paper, we can choose any predefined point or specified UAV as the virtual leader with desired dynamics. Denote the position and velocity of the virtual leader as $x_0(t)\in R^{\mathrm{m}}$ and $v_0(t)\in R^{\mathrm{m}}$ with

$$\begin{cases}\dot{x}_0(t)=v_0(t)\\ \dot{v}_0(t)=f_0(t).\end{cases} \tag{3}$$

In addition, the expected time-varying formation is specified by a command position vector $h_x(t)=[h_{1x}^T(t),h_{2x}^T(t),\ldots,h_{Nx}^T(t)]^T$, where $h_{ix}(t)\in R^{\mathrm{m}}$ is a piecewise continuously differentiable vector. Besides, define $h_v=\left[h_{1v}^T,h_{2v}^T,\ldots,h_{Nv}^T\right]^T$ with $h_{iv}(t)=\dot{h}_{ix}(t)$. It should be pointed out that $h_x(t)$ is only used to represent the relative offset vector of the desired positon of the ith UAV with respect to $x_0(t)$ rather than providing reference trajectory for each UAV to follow [20].

Furthermore, we define the formation tracking error as $\delta(t)=[\delta_x^T(t),\delta_v^T(t)]^T$, where $\delta_x(t)=[\delta_{1x}^T(t),...,\delta_{Nx}^T(t)]^T$ and $\delta_v(t)=[\delta_{1v}^T(t),...,\delta_{Nv}^T(t)]^T$ with $\delta_{ix}(t)=x_i(t)-x_0(t)-h_{ix}(t)$ and $\delta_{iv}(t)=v_i(t)-v_0(t)-h_{iv}(t)$ are position and velocity formation tracking errors, respectively. For simplicity, let $m=1$ without loss of generality, since the analysis and the results can be easily extended to a higher dimensional case by using the Kronecker product.

In this paper, we consider not only the varying transfer delay $\tau_{ij}(t)$ for the ith UAV to get the states of the jth UAV, but also the self-delay $\tau_{ii}(t)$ caused by measurement or computation. Note that $\tau_{ij}(t)$ and $\tau_{ii}(t)$ are particularly considered as uniform time delay $\tau_i(t)$, which is generally assumed in the multi-agent consensus control [21].

Assumption 2: $0\le\tau_i(t)\le\tau_m$, $i=1,2,\ldots,N$ for $t\ge 0$, where τ_m is a positive constant.

Assumption 3: The derivate of the velocity of the virtual leader is bounded, that is, there exists a positive constant v_m such that $\left\|\underline{f}_0(t)\right\|\le v_m$, where $\underline{f}_0=[f_0^T,f_0^T,\ldots,f_0^T]^T$.

Assumption 4: The derivate of h_v is bounded, that is, there exists a positive constant h_m such that $\left\|\dot{h}_v\right\|\le h_m$.

The control objective of this paper is to design an adaptive control scheme such that the formation tracking error $\delta(t)$ can be rendered small in the presence of varying time delays and uncertainties, which leads to a successful time-varying formation for multi-UAV system (2).

D. Notations

Throughout this paper, the following notations are used. I represents the identity matrix with appropriate dimension; $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ denotes the Euclidian 2-norm of a vector; $\|\cdot\|_F$ is the Frobenius norm of a matrix; $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ is the maximum and minimum singular value of a matrix, respectively; matrix $P>0$ means P is positive definite.

III. Proposed Control Method

In this section, we will firstly introduce the RBFNN to approximate the unknown uncertainties. Then, both a distributed adaptive time-varying formation tracking control law design and stability analysis for multi-UAV system (2) with varying time delays and unknown uncertainties are investigated.

A. Radial Basis Function Neural Network Approximator

Since the accurate model information cannot be always obtained in practice which may lead to the failure of formation, we need to estimate it for compensation. In this paper, we adopt an adaptive RBFNN approach.

Assume that the uncertain term $f_i(x_i,v_i)$ can be expressed on a prescribed compact set $\Omega\in R^m$ by

$$f_i(x_i,v_i)=W_i^{*T}\phi_i(x_i,v_i)+\varepsilon_i \tag{4}$$

where $\phi_i\in R^{l_i}$ is a suitable set of l_i Gaussian functions, $W_i^*\in R^{l_i}$ is the ideal neural network weight vector, and $\varepsilon_i\in R$ is the RBFNN approximation error.

To compensate for the unknown uncertainties, select the approximation of f_i as

$$\hat{f}_i = \hat{W}_i^T \phi_i, \tag{5}$$

where $\hat{W}_i \in R^{l_i}$ is the current actual values of the RBFNN weights for the ith UAV.

Define $W^* = diag(W_1^*, W_2^*, \ldots, W_N^*)$, $\hat{W} = diag(\hat{W}_1, \hat{W}_2, \ldots, \hat{W}_N)$, $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N]^T$, $\phi = [\phi_1^T, \phi_2^T, \ldots, \phi_N^T]^T$, then the uncertain term f can be written as the following form

$$f = W^{*T}\phi + \varepsilon, \tag{6}$$

and its approximation is

$$\hat{f} = \hat{W}^T \phi. \tag{7}$$

Besides, the error of the RBFNN weights is defined as $\tilde{W} = W^* - \hat{W}$.

Remark 1 [19]*:* According to Stone-Weierstrass approximation theorem, it is obvious to see that there exist positive numbers ϕ_M , W_M and ε_M , such that $\|\phi\| \le \phi_M$, $\|W^*\| \le W_M$, and $\|\varepsilon\| \le \varepsilon_M$.

B. Control Law Design and Stability Analysis

In this paper, it is supposed that only local neighbor information with time delay can be used for the control law design, such that the formation control is fully distributed. Consequently, for the ith UAV, we define local neighborhood tracking position and velocity errors respectively as

$$\begin{aligned} e_{ix}(t) = &\sum_{\upsilon_j \in N_i} a_{ij}\left[x_i(t) - h_{ix}(t) - (x_j(t) - h_{jx}(t))\right] \\ &+ b_i(x_i(t) - x_0(t) - h_{ix}(t)) \end{aligned} \tag{8}$$

and

$$\begin{aligned} e_{iv}(t) = &\sum_{\upsilon_j \in N_i} a_{ij}\left[v_i(t) - h_{iv}(t) - (v_j(t) - h_{jv}(t))\right] \\ &+ b_i(v_i(t) - v_0(t) - h_{iv}(t)). \end{aligned} \tag{9}$$

Let $e_x = \left[e_{1x}^T, e_{2x}^T, \ldots, e_{Nx}^T\right]^T$, $e_v = \left[e_{1v}^T, e_{2v}^T, \ldots, e_{Nv}^T\right]^T$. Then the system (2) can be written as follows

$$\begin{cases} \dot{e}_x = e_v \\ \dot{e}_v = (L + L_B)(u + f - \underline{f_0} - \dot{h}_v) \end{cases} \tag{10}$$

where $\dot{h}_v = \left[\dot{h}_{1v}^T, \dot{h}_{2v}^T, \ldots, \dot{h}_{Nv}^T\right]^T$, $L = D - A$ is the Laplacian matrix, and $L_B = \mathrm{diag}\{b_i\}$.

To approximate the lumped unknown uncertainty in (1), a two-input-one-output RBFNN is chosen for each UAV, where the inputs are the time-delayed states $x_i(t-\tau_i)$ and $v_i(t-\tau_i)$, while the output is $\hat{f}_i = \hat{W}_i^T \phi_i$ which is the approximation of f_i . Hence, the control input and adaptive law for the ith UAV can be designed as follows, respectively

$$u_i(t) = -k_1 e_{ix\tau} - k_2 e_{iv\tau} - \hat{W}_i^T \phi_i + \dot{h}_{iv\tau} \tag{11}$$

$$\dot{\hat{W}}_i = J_i \phi_i (\beta e_{ix\tau} + \alpha e_{iv\tau}) - \kappa J_i \hat{W}_i \tag{12}$$

where control gains $k_1, k_2 > 0,$, $\dot{h}_{iv\tau} = \dot{h}_{iv}(t-\tau_i)$, $J_i = p_i I \in R^{l_i \times l_i}$, $i = 1, 2, \ldots, N$ is positive definite diagonal matrix with $p_i > 0$, α, β and κ are positive constants. Besides,

$$\begin{aligned} e_{ix\tau} = e_{ix}(t-\tau_i) = &b_i\left[x_i(t-\tau_i) - x_0(t-\tau_i) - h_{ix}(t-\tau_i)\right] \\ &+ \sum_{j \in N_i} a_{ij}\left[x_i(t-\tau_i) - h_{ix}(t-\tau_i) - (x_j(t-\tau_i) - h_{jx}(t-\tau_i))\right], \end{aligned}$$

and

$$\begin{aligned} e_{iv\tau} = e_{iv}(t-\tau_i) = &b_i\left[v_i(t-\tau_i) - v_0(t-\tau_i) - h_{iv}(t-\tau_i)\right] \\ &+ \sum_{j \in N_i} a_{ij}\left[v_i(t-\tau_i) - h_{iv}(t-\tau_i) - (v_j(t-\tau_i) - h_{jv}(t-\tau_i))\right]. \end{aligned}$$

To express collectively,

$$u = -k_1 e_{x\tau} - k_2 e_{v\tau} - \hat{W}^T \phi + \dot{h}_{v\tau} \tag{13}$$

$$\dot{\hat{W}} = J\phi(\beta e_{x\tau}^T + \alpha e_{v\tau}^T) - \kappa J \hat{W} \tag{14}$$

where $e_{x\tau} = \left[e_{1x\tau}^T, e_{2x\tau}^T, \ldots, e_{Nx\tau}^T\right]^T$, $e_{v\tau} = \left[e_{1v\tau}^T, e_{2v\tau}^T, \ldots, e_{Nv\tau}^T\right]^T$, $\dot{h}_{v\tau} = \left[\dot{h}_{1v\tau}^T, \dot{h}_{2v\tau}^T, \ldots, \dot{h}_{Nv\tau}^T\right]^T$.

Remark 2: From (11)-(14), it is obvious that the control law and adaptive law only require the time-delayed neighbor states and formation information, which is consistent with reality. Meanwhile, only a part of UAVs can get the time-delayed states of the leader. In this respect, the proposed control scheme is fully distributed and possesses higher superiorities of less computation and stronger reliability than that in [22].

Remark 3: It should be pointed out that the parameters k_1, k_2, α, β can be chosen to be different for each UAV in the control law (11) and adaptive law (12). Here, we set them as the same for simplification.

Then, by applying (6), (7), (10), (11) and (13), the following equation yields

$$\dot{e}_v = (L + L_B)(-k_1 e_{x\tau} - k_2 e_{v\tau} + \tilde{W}^T \phi + \varepsilon - \underline{f_0} + \dot{h}_{v\tau} - \dot{h}_v) \tag{15}$$

Next we will present the main result of this paper, before which the following three lemmas are needed.

Lemma 1 [23]: Under Assumption 1, the matrix $L + L_B$ is positive definite.

Lemma 2 [24]: For any real differentiable vector function $y(t) \in R^n$, any differentiable scalar function $\tau(t) \in [0, \tau_0]$ where τ_0 is a positive constant, and any constant matrix $0 < U = U^T \in R^{n \times n}$, we have the following inequality:

$$\begin{aligned}&\tau_0^{-1}[y(t)-y(t-\tau)]^T U[y(t)-y(t-\tau)]\\ &\le \int_{t-\tau}^{t} \dot{y}^T(\xi)U\dot{y}(\xi)d\xi,\ t>0\\ &\le \int_{t-\tau_0}^{t} \dot{y}^T(\xi)U\dot{y}(\xi)d\xi,\ t>0\end{aligned} \tag{16}$$

Lemma 3 [19]: Under Assumption 1, $L+L_B$ is positive definite. Thus, $\|\delta_x\| \le \|e_x\|/\underline{\sigma}(L+L_B)$ and $\|\delta_v\| \le \|e_v\|/\ \underline{\sigma}(L+L_B)$.

Following the idea of [24], here we state the main result of this study.

Theorem 1: Suppose Assumptions 1-4 hold. Then under control law (13) and adaptive law (14), time-varying formation for the multi-UAV system (2) with uncertainties and varying time delays can be achieved, if the positive parameters k_1, k_2, α, γ, β, c_1 and c_2 with $\alpha = 4c_2\tau_m^2 k_2$, $\gamma = 4c_2\tau_m^2 k_1 k_2$, $\beta = 4c_2\tau_m^2 k_1$ are chosen, such that symmetric matrix $M>0$, where

$$M=\begin{bmatrix} \begin{matrix}k_1\beta\\ -2c_2\tau_m^2k_1^2\end{matrix} & 0 & 0 & 0 & 0\\ 0 & \begin{matrix}k_2\alpha - c_1\tau_m^2 - \beta\bar{\sigma}(G^{-1})\\ -2c_2\tau_m^2k_2^2\end{matrix} & 0 & 0 & 0\\ 0 & 0 & c_1-2c_2\tau_m^2k_1^2 & -2c_2\tau_m^2k_1k_2 & -\frac{\beta}{2}\phi_M\\ 0 & 0 & * & c_2(\underline{\sigma}(H^{-1})-2\tau_m^2k_2^2) & -\frac{\alpha}{2}\phi_M\\ 0 & 0 & * & * & \kappa-2c_2\tau_m^2\phi_M^2 \end{bmatrix} \tag{17}$$

and the matrix $G=L+L_B$, $H=(L+L_B)^T(L+L_B)$.

Proof: Construct the following Lyapunov-Krasovskii candidate function $V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)$ with

$$V_1(t)=\frac{1}{2}\alpha e_v^T G^{-1}e_v + \frac{1}{2}\gamma e_x^T e_x + \beta e_x^T G^{-1} e_v \tag{18}$$

$$V_2 = c_1\tau_m \int_{t-\tau_m}^{t} (\xi - t + \tau_m)\dot{e}_x^T(\xi)\dot{e}_x(\xi)d\xi \tag{19}$$

$$V_3 = c_2\tau_m \int_{t-\tau_m}^{t} (\xi - t + \tau_m)\dot{e}_v^T(\xi)H^{-1}\dot{e}_v(\xi)d\xi \tag{20}$$

$$V_4(t) = tr\left\{\tilde{W}^T J^{-1}\tilde{W}\right\} \tag{21}$$

Taking the derivative of V_1 with respect to *t*, we obtain

$$\begin{aligned}\dot{V}_1+\dot{V}_4 &= (e_v^T\alpha + e_x^T\beta)G^{-1}\dot{e}_v + \gamma e_x^T e_v + e_v^T\beta G^{-1}e_v + tr\left\{\tilde{W}^T J^{-1}\dot{\tilde{W}}\right\}\\ &= (e_v^T\alpha + e_x^T\beta)(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v) + \\ &\quad (e_v^T\alpha + e_x^T\beta)B + \gamma e_x^T e_v + e_v^T\beta G^{-1}e_v + tr\left\{\tilde{W}^T\kappa\hat{W}\right\}\\ &\quad + tr\left\{\tilde{W}^T\phi\left[(e_v^T\alpha + e_x^T\beta) - (\beta e_{x\tau}^T + \alpha_{v\tau}^T)\right]\right\}\\ &= (e_v^T\alpha + e_x^T\beta)(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v) + \kappa tr\left\|\tilde{W}\right\|_F W_M\\ &\quad + (e_v^T\alpha + e_x^T\beta)B + \gamma e_x^T e_v + e_v^T\beta G^{-1}e_v - \kappa\left\|\tilde{W}\right\|_F^2\\ &\quad + tr\left\{\tilde{W}^T\phi\left[(e_v^T\alpha + e_x^T\beta) - (\beta e_x^T + \beta\theta_x^T + \alpha e_v^T + \alpha\theta_v^T)\right]\right\}\end{aligned} \tag{22}$$

where $\theta_x = e_{x\tau} - e_x$, and $\theta_v = e_{v\tau} - e_v$, $B = \varepsilon - \underline{f_0} + \dot{h}_{v\tau} - \dot{h}_v$.

Applying Lemma 2, the derivatives of V_2 and V_3 are shown as

$$\begin{aligned}\dot{V}_2 &= c_1\tau_m^2\dot{e}_x^T(t)\dot{e}_x(t) - c_1\tau_m\int_{t-\tau_m}^{t}\dot{e}_x^T(\xi)\dot{e}_x(\xi)d\xi\\ &\le -c_1\theta_x^T\theta_x + c_1\tau_m^2 e_v^T(t)e_v(t)\\ &\le -c_1\|\theta_x\|^2 + c_1\tau_m^2\|e_v\|^2\end{aligned} \tag{23}$$

and

$$\begin{aligned}\dot{V}_3 &= c_2\tau_m^2\dot{e}_v^T(t)H^{-1}\dot{e}_v(t) - c_2\tau_m\int_{t-\tau_m}^{t}\dot{e}_v^T(\xi)H^{-1}\dot{e}_v(\xi)d\xi\\ &\le -c_2\theta_v^T H^{-1}\theta_v + c_2\tau_m^2\dot{e}_v^T(t)H^{-1}\dot{e}_v(t)\\ &\le -c_2\underline{\sigma}(H^{-1})\|\theta_v\|^2 + c_2\tau_m^2(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v + \tilde{W}^T\phi\\ &\quad +B)^T(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v + \tilde{W}^T\phi + B)\\ &\le -c_2\underline{\sigma}(H^{-1})\|\theta_v\|^2 + 2c_2\tau_m^2\left\|\tilde{W}\right\|_F^2\phi_M^2 + 2c_2\tau_m^2(-k_1\theta_x - k_1e_x\\ &\quad -k_2\theta_v - k_2e_v + B)^T(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v + B)\\ &\le -c_2\underline{\sigma}(H^{-1})\|\theta_v\|^2 + c_2\tau_m^2(2k_1^2\|\theta_x\|^2 + 4k_1^2e_x^T\theta_x\\ &\quad +4k_1k_2\theta_v^T\theta_x + 4k_1k_2e_v^T\theta_x + 4k_1k_2e_v^Te_x + 2k_1^2\|e_x\|^2\\ &\quad +4k_1k_2\theta_v^Te_x + 2k_2^2\|\theta_v\|^2 + 4k_2^2e_v^T\theta_v + 2k_2^2\|e_v\|^2\\ &\quad +2B_M^2 + 2\left\|\tilde{W}\right\|_F^2\phi_M^2 + 4B^T(-k_1\theta_x - k_1e_x - k_2\theta_v - k_2e_v))\end{aligned} \tag{24}$$

where $B_M = 2h_m + v_m + \varepsilon_m$. Combining equations (22)-(24), we have

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4\\ &\le -z^T Mz + z^T s + \Xi = -V_z(z)\end{aligned} \tag{25}$$

where the matrix M is shown as (17), and $z=\left[\|e_x\|,\|e_v\|,\|\theta_x\|,\|\theta_v\|,\left\|\tilde{W}\right\|_F\right]^T$, $s=[0\quad 0\quad 4k_1B_M\ 4k_2B_M\quad W_M\kappa]^T$, $\Xi = 2c_2\tau_m^2B_M^2$.

$V_z(z)$ is positive definite if the matrix M is positive definite and

$$\|z\| \ge \frac{-\|s\| + \sqrt{\|s\|^2 + 4\underline{\sigma}(M)\Xi}}{2\underline{\sigma}(M)} \tag{26}$$

According to [25],we can draw the conclusion that $z(t)$ is UUB. Furthermore, it can be obtained that both e_x and e_v are bounded stable. Then following Lemma 3, the formation tracking error $\delta(t)$ are UUB. Thus, we can achieve the desired formation under the control law (13) and adaptive law (14). This completes the proof.

Remark 4: In Theorem 1, k_1, k_2 α β and κ are related to the control and adaptive laws, γ, c_1 and c_2 are parameters to be selected. τ_m is the upper bound of the time delays. Except τ_m, all of these parameters are tunable to guarantee the solvability of the matrix *M*. Furthermore, it is obvious that the diagonal elements of the matrix *M* are positive with τ_m being a certain value. Therefore, the solution of the matrix *M* in Theorem 1 exists.

IV. Simulations

To illustrate the theoretical results obtained in the previous section, comparative simulations are conducted.

Consider a multi-UAV system described by (2) with four UAVs. In addition, the virtual leader is set as the formation center. Trivially choose the interaction graph shown as Fig. 2, which is a connected graph.

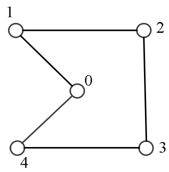

Fig. 2. The connected interaction graph.

The four UAVs are supposed to keep a periodic time-varying square formation in the horizontal X-Y plane and at the same time to keep rotating around the varying virtual leader $x_0(t)=[5t,5t]^T$. Thus, $m=2$, and in this case, $x_i(t)$, $v_i(t)$, $x_0(t)$, $v_0(t)$, $u_i(t)$, $f_i(x_i,v_i)$, $h_i(t)$, can be respectively written as $x_i(t)=[x_{iX}(t),x_{iY}(t)]^T$, $v_i(t)=[v_{iX}(t),v_{iY}(t)]^T$, $x_0(t)=[x_{0X}(t),x_{0Y}(t)]^T$, $v_0(t)=[v_{0X}(t),v_{0Y}(t)]^T$, $u_i(t)=[u_{iX}(t), u_{iY}(t)]^T$, $f_i(x_i,v_i)=[f_{iX},f_{iY}]^T$ and $h_{ix}(t)=[h_{ixX}(t),h_{ixY}(t)]^T$, $i=1,2,3,4$. Besides, the time-varying formation is specified by

$$h_{ix}(t)=\begin{bmatrix}10\cos(0.1t+(i-1)\pi/2))\\ 10\sin(0.1t+(i-1)\pi/2)\end{bmatrix},\ i=1,2,3,4. \tag{27}$$

The control gains in (11) and parameters in (12) are shown in Table 1. The number of nodes for each RBFNN is set as l_i=5, while the initial weights are randomly chosen. Besides, choose $c_1=0.1$ and $c_2=0.54$, so that the matrix M in (17) is positive definite. Furthermore, the different varying time delays are set as $\tau_1=\tau_4$=0.03+0.02sin(0.1t) and $\tau_2=\tau_3$=0.03+0.01sin(0.1t). Besides, let $f_1=$ [sin(0.2v_{1X}), sin(0.1v_{1Y})]^T, $f_2=[\cos(0.2v_{2X}),\ \cos(0.2v_{2Y})]^T$, $f_3=[\sin(0.1v_{3X}^2),\ \sin(0.1v_{3Y}^2)]^T$, $f_4=[\cos(0.1v_{4X}^2),\ \cos(0.1v_{4Y}^2)]^T$. Furthermore, to show the superiority of the adopted RBFNN based scheme, a counterpart without RBFNN is chosen as the comparison in numerical simulation.

TABLE I. CONTROL PARAMETES

Parameters	k_1	k_2	β	α	κ_i	p_i
ith UAV	1.2	3	0.0012	0.0030	0.001221	2000

The simulation results are presented by Figs. 3-5. Fig. 3 shows the positions of the four UAVs at $t=0,10,\ 20,\ 30,\ 40,\ 50$ s . Fig. 4 depicts the formation position tracking errors $\delta_{ix}(t)$ on X-axis and Y-axis for each UAV under the proposed control scheme versus its comparison, respectively. From Figs. 3-4 the following phenomena can be observed: (i) the four UAVs successfully keep a square formation while keeping rotating around the dynamic virtual leader; (ii) the virtual leader moves along a straight line, and keeps lying in the center of the formation, so that the formation moves along the same straight line; (iii) under the proposed control scheme, the formation position tracking errors keep around 0 after about t=10s although there exist varying time delays in the communication network. However, for the comparison case, the formation position tracking errors cannot converge to 0 due to the unknown uncertainties, which means the uncertainties can be compensated pretty well by the adopted RBFNN. As a result, the desired time-varying formation can be achieved by the proposed control law under unknown uncertainties and varying time delays.

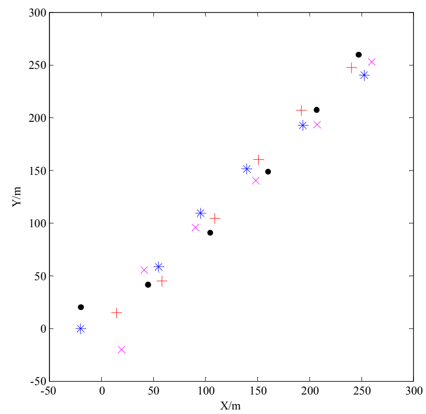

Fig. 3. Position snapshots of the four UAVs $t=0,\ 10,\ 20,\ 30,\ 40,\ 50$ s .

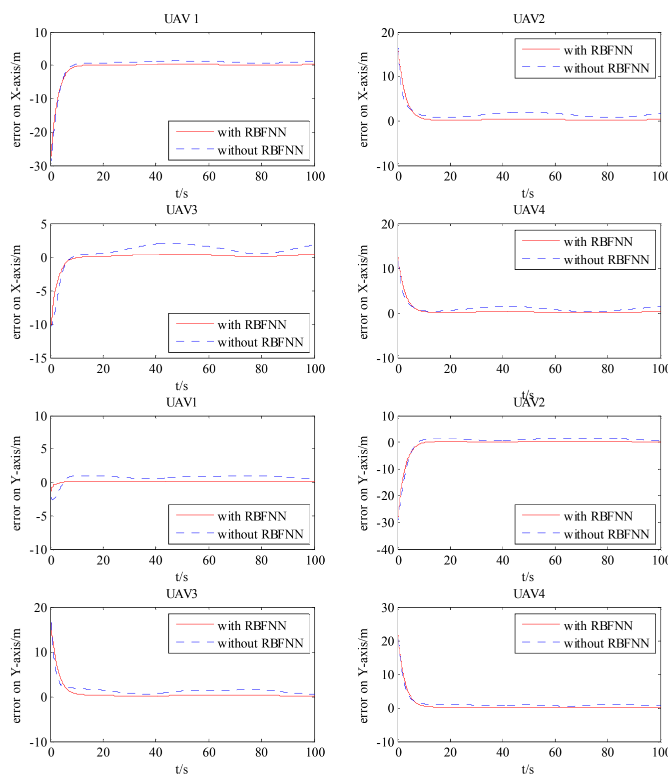

Fig. 4. Formation position tracking errors on X-axis and Y-axis for each UAV under the control law wth RBFNN and without RBFNN.

Fig. 5 presents the RBFNN approximation errors for four UAVs. Form Fig. 5, it is obvious to see that although there exist small vibrations in the first 5 seconds due to the random setting of the initial weights in the network nodes, the approximation errors of the RBFNNs can still quickly converge, which shows the strong learning ability of the proposed RBFNNs.

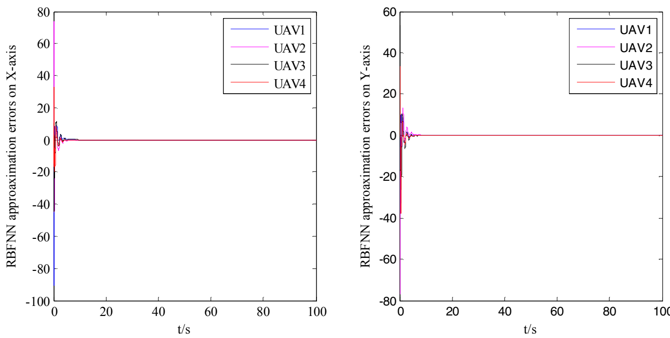

Fig. 5. RBFNN approximation errors on X-axis and Y-axis for four UAVs.

V. Conlusions

This paper investigates the time-varying formation control problem for multi-UAV systems. By adopting the RBFNN, a novel RBFNN-based fully distributed adaptive control scheme which simultaneously deals with unknown uncertainties and varying time delays is proposed. The stability analysis is theoretically explored through Lyapunov function approach. Comparative simulation results verify the effectiveness and superiority of the developed control scheme in this paper. In our future work, we will further consider the attitude dynamics of the UAVs, and try to formulate a novel RBFNN-based integrated two-loop control scheme for the multi-UAV system.

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