Adaptive Neural Network Time-varying Formation Tracking Control for Multi-agent Systems via Minimal Learning Parameter Approach

Tianyi Xiong[1,2], Zhiqiang Pu[1,2], Jianqiang Yi[1,2] and Zezhi Sui[1,2]

[1]*School of Artificial Intelligence*
University of Chinese Academy of Sciences
Beijing 100049, China

[2]*Institute of Automation*
Chinese Academy of Sciences
Beijing 100190, China

email:{xiongtianyi2016, zhiqiang.pu, jianqiang.yi, suizezhi2015} @ia.ac.cn

***Abstract*—This paper investigates the time-varying formation tracking control problem for multi-agent systems with consideration of model uncertainties. For each dimension of an agent, a radial basis function neural network (RBFNN) is first adopted to approximate the model uncertainties online. Taking the square of the norm of the neural network weight vector as a newly developed adaptive parameter, a novel RBFNN-based adaptive control law with minimal learning parameter (MLP) approach is then constructed to tackle the time-varying formation tracking problem. The uniformly ultimately boundedness (UUB) of formation tracking errors is guaranteed through Lyapunov analysis. Compared with other traditional RBFNN-based formation tracking control laws for multi-agent systems, very few parameters need to be adapted online in our proposed one, which can greatly lessen the computational burden. Finally, comparative simulation results demonstrate the effectiveness and superiority of the proposed adaptive control law.**

***Keywords*—Formation control, minimal learning parameter, multi-agent system, neural network**

I. Introduction

To date, cooperative formation control for multi-agent systems has achieved abundant accomplishments in various fields. Generally, three typical frameworks, namely leader-follower based strategy [1], behavior based strategy [2], and virtual structure based strategy [3], have been widely investigated and applied for formation control. However, all these three frameworks have their own weaknesses [4]. Since consensus control [5], [6] is fully distributed and only requires the local neighbor-to-neighbor information, it possesses strong robustness and high flexibility. Thus, consensus-based formation control has been a focus, and numerous exciting results have been derived.

Ref [7] extended consensus theory to formation control for second-order multi-agents, and pointed out that many existing leader-follower based, behavior based and virtual structure based formation control approaches could be unified in the framework of consensus based approaches. To achieve finite-time formation control, a novel finite-time formation control framework for first-order multi-agent system with a large population of agents was constructed in [8], which could greatly reduce the information sharing and easily realize a variety of complex formations. On the basis of a distributed finite-time observer for each follower to estimate the leader's states, finite-time formation control problem for a group of nonholonomic mobile robots was solved in [9]. Aiming at dealing with time-delay problem, a leader-following formation control for second-order multi-agent systems with nonuniform time-varying communication delays under directed topologies was studied in [10]. Considering velocity constraints, a leader-follower formation control problem of nonholonomic vehicles of unicycle-type with velocity constraints was investigated in [11], in which the velocity constraints of each vehicle were described by saturated angular velocity and bounded linear velocity lying between two positive constants.

However, it should be pointed out that formations in the majority of the existing results are time-invariant, which often cannot satisfy practical requirements on various situations where we need to cover changing environment, avoid moving obstacles and perform other complex tasks. Therefore, it motivates some researches on time-varying formation control. Time-varying formation problems for linear multi-agent systems under switching directed topologies were deeply studied in [12], in which necessary and sufficient conditions of asymptotic stability for the multi-agent systems were given. By using adaptive output-feedback approach, fully distributed time-varying formation control problems for general linear swarm systems with fixed and switching topologies were investigated in [13]. Besides, the formation may also need to track the trajectory generated by the virtual/real leader to perform a task, so tracking problem arises, where a group of agents keep the desired time-varying formation while tracking the trajectory of the leader. Ref. [14] studied time-varying formation tracking problem for multiple manipulators in finite time. Ref. [15] focused on time-varying formation tracking

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problems for second-order multi-agent systems with switching graphs, and the obtained results were applied to solve the target enclosing problem for a multiquadrotor unmanned aerial vehicle system.

For multi-agent system formation control design, one important issue is how to deal with inevitable uncertainties of the agent dynamics, which may result from parametric uncertainties, modeling errors, and so on. To deal with it, neural network (NN), as a universal approximator, can be applied in formation control to estimate the model uncertainties online. A novel neural-network based approach for switching formation tracking control in nonomniscient constrained space for multi-agent systems was proposed in [16]. A distributed adaptive attitude synchronization control strategy of spacecraft formation flying based on modified fast terminal sliding mode and NN in the presence of unknown external disturbances was designed in [17]. NN was also adopted in [18] to simultaneously handle the problems of matched and mismatched heterogeneous nonlinearities, such that the proposed protocol could achieve time-varying formation tracking for second-order nonlinear multi-agent systems with multiple leaders. Nevertheless, in all the aforementioned results, too many parameters needed to be adapted online in the adaptive control laws due to the (actual large number of agents as well as) numerous nodes set in NNs, which was time-consuming and burdensome in practice. To the best of our knowledge, few results in the existing literature, including the aforementioned works, have considered and effectively solved this problem in the formation control, even in the consensus control for multi-agent systems. To minimize the number of adaptive parameters, in 2003, Yang et al. [19] firstly proposed a solution called minimal learning parameter (MLP) approach, which took the square of the norm of NN weight matrix as an adaptive parameter and became an effective way. Following this idea, an adaptive T-S fuzzy control algorithm with MLP for a class of MIMO system was developed in [20]. Xu et al. [21] and Bu et al. [22] used MLP approach in the hypersonic flight control design. However, these previous results are not easy to be directly extended to tackling the consensus-based formation tracking control problem with MLP approach for multi-agent systems as the complexities of the stability analysis and control law design will greatly increase in formation tracking control objective.

Motivated by the above observations, in this paper, we propose an adaptive time-varying formation tracking control law for multi-agent systems using MLP approach. Compared with the previous relevant results, the outstanding features of the adaptive control law proposed in this paper can be summarized as follows. First, time-varying formation tracking can be realized under the proposed control law. Second, the precise model information is not necessary due to the employment of radial basis function neural network (RBFNN) in the control design. Furthermore, the burdensome computation of the adaptive control law can be lightened to a great extent, which leads to an easier realization of the control law in real engineering.

The rest of the paper is organized as follows. The formation control problem description along with some preliminaries and notations is given in Section II. The problem description is introduced in section III. Control law design and the stability analysis are studied in Section IV. Numerical simulation results are presented in Section V, after which the conclusions are drawn.

II. Preliminaries

A. Basic Concepts on Graph Theory

Consider a multi-agent system comprising N agents. Generally, we use a graph denoted by $G=(V,E,A)$ to describe the information exchanges among the agents. Denote a single agent as node υ_i. Then $V=\{\upsilon_1,\upsilon_2,\ldots,\upsilon_N\}$ is the set of the agents, and $E\subseteq V\times V$ represents the set of edges, where E is defined such that if $(\upsilon_j,\upsilon_i)\in E, j\neq i$, there is an edge from agent j to agent i, which means that agent j can deliver information to agent i. In addition, $A=[a_{ij}]\in R^{N\times N}$ is the associated adjacency matrix with $a_{ij}\geq 0$. We set $a_{ij}>0,\ j\neq i$ if and only if $(\upsilon_j,\upsilon_i)\in E$; otherwise $a_{ij}=0$. In this case, agent j is said to be the neighbor of agent i if and only if $a_{ij}>0$, and $N_i=\{\upsilon_j\in V:\ (\upsilon_j,\upsilon_i)\in E\}$ represents the neighbor set of agent i. A graph is an undirected graph if and only if $a_{ij}=a_{ji}$. Define $D=\mathrm{diag}\{\rho_1,\rho_2,\cdots,\rho_N\}\in R^{N\times N}$ as the in-degree matrix, where $\rho_i=\sum_{\upsilon_j\in N_i}a_{ij}$. Then, the Laplacian matrix of graph G is defined as $L=D-A$. A direct path from agent i to agent j is a sequence of successive edges in the form of $\{(\upsilon_i,\upsilon_k),(\upsilon_k,\upsilon_l),\cdots,(\upsilon_m,\upsilon_j)\}$. Furthermore, an undirected graph is called connected if there is a path between any two agents in the graph.

In the case of formation tracking, another graph $\bar{G}$ with a virtual leader (labeled as agent 0) and N agents should be considered. The adjacency matrix element associated with the edge from agent i to the virtual leader is denoted by b_i, with $b_i>0$ if and only if agent i can receive information from the leader.

In this paper, we suppose the following assumption naturally holds.

Assumption 1*:* The graph $\bar{G}$ among the agents and the leader is connected.

B. Notations

Throughout this paper, the following notations are used. I_n represents n dimensional identity matrix; 1_n stands for n dimensional vector with all elements being 1; $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ denotes the Euclidian 2-norm of a vector; $\|\cdot\|_F$ is the Frobenius norm of a matrix; $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ are the maximum and minimum singular values of a matrix, respectively; $\otimes$ denotes Kronecker product; matrix $P>0$ means P is positive definite.

III. PROBLEM DESCRIPTION

Consider N agents with distinct dynamics. The dynamics of agent i can be described as follows.

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = f_i(x_i) + u_i + d_i, \end{cases} \tag{1}$$

where $x_i = \left[x_{i,1}^T, x_{i,2}^T\right]^T \in R^{2m}$ is the state vector of agent i with $x_{i,1} \in R^m$ and $x_{i,2} \in R^m$, m is the dimension of the states $x_{i,1}$ and $x_{i,2}$. $u_i \in R^m$ denotes the control input vector of agent i. $f_i(\bullet) \in R^m$ is the model uncertainty vector which is locally Lipschitz in R^{2m} with $f_i(0) = 0$. And $d_i \in R^m$ is the unknown external disturbance vector.

The dynamics of the leader (generally nonautonomous) is described as follows

$$\begin{cases} \dot{x}_{0,1} = x_{0,2} \\ \dot{x}_{0,2} = f_0(t, x_0), \end{cases} \tag{2}$$

where $x_0 = \left[x_{0,1}^T, x_{0,2}^T\right]^T \in R^{2m}$ is the state vector of the leader with $x_{0,1} \in R^m$ and $x_{0,2} \in R^m$. $f_0(t, x_0)$ is piecewise continuous in t and locally Lipschitz in x_0 with $f_0(t,0) = 0$, and it is assumed to be unknown to all the other agents.

In addition, the expected time-varying formation is specified by a command vector $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_N^T(t)]^T$, where $h_i(t) = \left[h_{i,1}^T(t), h_{i,2}^T(t)\right]^T \in R^{2m}$ is a piecewise continuously differentiable vector. Besides, let $h_1(t) = \left[h_{1,1}^T(t), h_{2,1}^T(t), \ldots, h_{N,1}^T(t)\right]^T$ and $h_2(t) = \left[h_{1,2}^T(t), h_{2,2}^T(t), \ldots, h_{N,2}^T(t)\right]^T$ with $h_{i,2}(t) = \dot{h}_{i,1}(t)$.

We define the formation tracking error as $\delta(t) = [\delta_1^T(t), \delta_2^T(t)]^T$, where $\delta_1(t) = [\delta_{1,1}^T(t), ..., \delta_{N,1}^T(t)]^T$ and $\delta_2(t) = [\delta_{1,2}^T(t), ..., \delta_{N,2}^T(t)]^T$ with $\delta_{i,1}(t) = x_{i,1}(t) - x_{0,1}(t) - h_{i,1}(t)$ and $\delta_{i,2}(t) = x_{i,2}(t) - x_{0,2}(t) - h_{i,2}(t)$. Next, the following assumptions are given.

Assumption 2: There exists a positive constant d_M such that the overall disturbance vector d is also bounded by $\|d\| \le d_M$, where $d = [d_1^T, d_2^T \ldots, d_N^T]^T \in R^{Nm}$ and d_M is unknown.

Assumption 3: There exists a positive constant f_M such that $\|\underline{f}\| \le f_M$, where $\underline{f} = [f_0^T, f_0^T, \ldots, f_0^T]^T \in R^{Nm}$.

The control objective of this paper is to design a consensus based adaptive formation tracking control law such that the norm of the formation tracking error $\delta(t)$ can converge to a small neighborhood of zero which leads to a successful time-varying formation tracking for the multi-agent system (1).

IV. FORMATION CONTROL DESIGN AND ANALYSIS

In this section, we will firstly employ RBFNNs to compensate for the model uncertainties. Then, both the design and stability analysis of the MLP-based adaptive neural network time-varying formation tracking control law and stability analysis for multi-agent system (1) are investigated.

A. Radial Basis Function Neural Network Approximator

Assume that the uncertain term $f_i(x_i) \in R^m$ can be expressed on a prescribed compact set $\Omega \in R^{2m}$ by

$$f_i(x_i) = W_i^{*T} \phi_i(x_i) 1_m + \varepsilon_i \tag{3}$$

where $\phi_i = \mathrm{diag}\left\{\phi_{i,1}, \phi_{i,2}, \ldots, \phi_{i,m}\right\} \in R^{ml_i \times m}$ is a matrix with $\phi_{i,q} = \left[\phi_{i,q,1}, \ldots, \phi_{i,q,l_i}\right]^T \in R^{l_i}$, $q = 1, \ldots, m$ being a suitable set of l_i Gaussian functions. $W_i^* = \mathrm{diag}\left\{W_{i,1}^*, W_{i,2}^*, \ldots, W_{i,m}^*\right\} \in R^{ml_i \times m}$ is the ideal neural network weight matrix with $W_{i,q}^* = \left[W_{i,q,1}^*, W_{i,q,2}^*, \ldots, W_{i,q,l_i}^*\right]^T \in R^{l_i}$. And $\varepsilon_i \in R^m$ is the RBFNN approximation error vector.

To compensate for the unknown uncertainties, select the approximation of f_i as

$$\hat{f}_i = \hat{W}_i^T \phi_i 1_m, \tag{4}$$

where $\hat{W}_i = \mathrm{diag}\left\{\left[\hat{W}_{i,1}, \hat{W}_{i,2}, \ldots, \hat{W}_{i,m}\right]\right\} \in R^{ml_i \times m}$ with $\hat{W}_{i,q} = \left[\hat{W}_{i,q,1}, \hat{W}_{i,q,2}, \ldots, \hat{W}_{i,q,l_i}\right]^T \in R^{l_i}$ is the current actual values of the RBFNN weights for agent i.

Define $W^* = \mathrm{diag}(W_1^*, W_2^*, \ldots, W_N^*)$, $\hat{W} = \mathrm{diag}(\hat{W}_1, \hat{W}_2, \ldots, \hat{W}_N)$, $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T]^T$ and $\phi = \mathrm{diag}\left(\phi_1, \phi_2, \ \ldots, \phi_N\right)$. Then the uncertain term $f = [f_1^T(t), f_2^T(t), \ldots, \ f_N^T(t)]^T$ and its approximation $\hat{f}$ can be written as the follows

$$f = W^{*T} \phi 1_{mN} + \varepsilon, \tag{5}$$

$$\hat{f} = \hat{W}^T \phi 1_{mN}. \tag{6}$$

Besides, the error of the RBFNN weights is defined as $\tilde{W} = W^* - \hat{W}$.

Remark 1 [23]: According to Stone-Weierstrass approximation theorem, there exist positive numbers ϕ_M, W_M and ε_M, such that $\|\phi\|_F \le \phi_M$, $\|W^*\|_F \le W_M$, and $\|\varepsilon\| \le \varepsilon_M$.

B. Adaptive Control Law Design and Stability Analysis

To present the adaptive control law design and stability analysis, first we define local neighbourhood formation tracking errors for agent i as

$$e_{i,1}(t)=\sum_{\upsilon_j\in N_i} a_{ij}\left[x_{i,1}-h_{i,1}-(x_{j,1}-h_{j,1})\right] \\ +b_i(x_{i,1}-x_{0,1}-h_{i,1}), \tag{7}$$

and

$$e_{i,2}=\sum_{\upsilon_j\in N_i} a_{ij}\left[x_{i,2}-h_{i,2}-(x_{j,2}-h_{j,2})\right] \\ +b_i(x_{i,2}-x_{0,2}-h_{j,2}). \tag{8}$$

Let $u=[u_1^T(t),u_2^T(t),\ldots,u_N^T(t)]^T$, and $d=[d_1^T(t),d_2^T(t),\ldots,d_N^T(t)]^T$. Then a straightforward derivation yields

$$\begin{cases}\dot{e}_1=e_2\\ \dot{e}_2=\left[(L+B)\otimes I_m\right](u+f+d-h_2-\underline{f}_0)\end{cases} \tag{9}$$

where $e_1=[e_{1,1}^T(t),e_{2,1}^T(t),\ldots,e_{N,1}^T(t)]^T$ and $e_2=[e_{1,2}^T(t),e_{2,2}^T(t),\ldots,e_{N,2}^T(t)]^T$ are global formation tracking errors, respectively.

Define auxiliary error r_i for agent *i* as

$$r_i=\lambda e_{i,1}+e_{i,2}, \tag{10}$$

where $\lambda>0$ is a constant. Then on the error surface $r_i=0$, $e_i=[e_{i,1}^T,e_{i,2}^T]^T\to 0$ as $t\to\infty$. Therefore, the control objective in this paper can be transformed to keep the auxiliary error r_i for agent *i* staying on error surface $r_i=0$, or near the error surface.

The following two lemmas illustrate that uniformly ultimately boundedness (UUB) of r_i implies UUB of δ_i , $i=1,2,\ldots,N$.

Lemma 1 [23], [24]**.** Under Assumption 1, the matrix $L+B$ is positive definite, and $\|\delta_1\|\le\|e_1\|/\underline{\sigma}\left[(L+B)\otimes I_m\right]$, $\|\delta_2\|\le\|e_2\|/\underline{\sigma}\left[(L+B)\otimes I_m\right]$.

Lemma 2 [23]**.** For each agent, suppose

$$\|r_i(t)\|\le\xi_i,\ \forall t\ge t_0;\ \|r_i(t)\|\le\zeta_i,\ \forall t\ge T_i$$

for some bounds $\xi_i>0$, $\zeta_i>0$, and time $T_i>t_0$. Then there exist bounds $\psi_i>0$, $\varsigma_i>0$ and time $\tau_i>t_0$, such that

$$\|e_i(t)\|\le\psi_i,\ \forall t\ge t_0;\ \|e_i(t)\|\le\varsigma_i,\ \forall t\ge\tau_i.$$

Let $r=[r_1^T,r_2^T,\ldots,r_N^T]^T$. Then $r=\lambda e_1+e_2$, and the dynamics of auxiliary error r is

$$\dot{r}=\lambda e_2+\left[(L+B)\otimes I_m\right](u+f+d-\dot{h}_2-\underline{f}_0) \tag{11}$$

Here, we propose the MLP based formation tracking control law. Let $\theta_{i,q}^*=\|W_{i,q}^*\|^2$ and $\theta_i^*=\mathrm{diag}(\theta_{i,1}^*,\theta_{i,2}^*,\ldots,\theta_{i,m}^*)$, where $\theta_{i,q}^*$ is a positive constant. $\hat{\theta}_i$ is the approximation of θ_i^*. Hence, for agent *i*, the time-varying formation control law is designed as follows

$$u_i=-kr_i-\frac{1}{2}\hat{\theta}_i\phi_i^T\phi_i r_i-\lambda e_{i,2}+\dot{h}_{i,2} \tag{12}$$

and the NN adaptive tuning law is designed to be

$$\dot{\hat{\theta}}_i=\frac{1}{2}\phi_i^T\phi_i R_i F_i-\alpha_i\hat{\theta}_i F_i \tag{13}$$

where α_i and F_i are positive constants. $R_i\in R^{m\times m}$ is a diagonal matrix with the elements of the principal diagonal being the same as those of the matrix $r_i^T r_i$. To express collectively, the control and adaptive laws can be written as

$$u=-kr-\frac{1}{2}\hat{\theta}\phi^T\phi r-\lambda e_2+\dot{h}_2 \tag{14}$$

$$\dot{\hat{\theta}}=\frac{1}{2}\phi^T\phi RF-\alpha\hat{\theta}F \tag{15}$$

where $\hat{\theta}=\mathrm{diag}(\hat{\theta}_1,\ldots,\hat{\theta}_N)$, $\phi=\mathrm{diag}(\phi_1,\ldots,\phi_N)$, $R=\mathrm{diag}(R_1,\ldots,R_N)$, $\alpha=\mathrm{diag}(\alpha_1\otimes I_m,\ldots,\alpha_N\otimes I_m)$, and $F=\mathrm{diag}(F_1\otimes \mathrm{I}_m,\ldots,F_N\otimes \mathrm{I}_m)$.

Next, we will present the main results of this paper.

Theorem 1. Considering the multi-agent system (1) with *N* agents and the leader agent (2). Suppose Assumptions 1-3 hold. Then under the control law (12) and adaptive law (13) for each agent, time-varying formation tracking control for the multi-agent system (1) can be achieved, if the following condition is satisfied,

$$\begin{cases}k>\mu\\ 4\lambda(k-\mu)>(1+\lambda^2\mu)^2,\end{cases} \tag{16}$$

where $\mu=\bar{\sigma}\left\{\left[I_N-(L+B)^{-1}\right]\otimes I_m\right\}$.

Proof: Consider the Lyapunov function candidate

$$V=V_1+V_2+V_3 \tag{17}$$

where $V_1=\frac{1}{2}r^T\left[(L+B)\otimes I_m\right]^{-1}r$, $V_2=\frac{1}{4}tr\{\tilde{\theta}^T F^{-1}\tilde{\theta}\}$ and $V_3=\frac{1}{2}e_1^T e_1$.

Firstly, we compute the derivative of V_1.

$$\begin{aligned}\dot{V}_1&=\lambda r^T\left[(L+B)\otimes I_m\right]^{-1}e_2\\ &\quad+r^T(u+f+d-\dot{h}_2-\underline{f}_0)\\ &=r^T(-kr+W^{*T}\phi 1_{mN}+\varepsilon-\frac{1}{2}\hat{\theta}\phi^T\phi r+d-\underline{f}_0)\\ &\quad-\lambda r^T e_2+\lambda r^T\left[(L+B)\otimes I_m\right]^{-1}e_2.\end{aligned} \tag{18}$$

By applying Young's inequality, we can obtain $r_{i,q}W_{i,q}^{*T}\phi_{i,q}\le\frac{1}{2}r_{i,q}^2\theta_{i,q}^*\phi_{i,q}^T\phi_{i,q}+\frac{1}{2}$, where $r_{i,q}$ is an arbitrary element in the vector r_i. Furthermore, it is easy to deduce that $r^T W^{*T} \phi 1_{mN} \le \frac{1}{2} r^T \theta^* \phi^T \phi r + \frac{mN}{2}$. Let $\tilde{\theta} = \theta^* - \hat{\theta}$, we get

$$\begin{aligned} \dot{V}_1 \le & -kr^T r + \frac{1}{2} r^T \tilde{\theta} \phi^T \phi r + \frac{mN}{2} + r^T (\varepsilon + d - \underline{f}_0) \\ & -\lambda r^T \left\{ \left[I_N - (L+B)^{-1} \right] \otimes I_m \right\} e_2. \end{aligned} \tag{19}$$

Noticing that $y^T z = tr\{zy^T\}$ if $y, z \in R^n$, we get $\dot{V}_1 + \dot{V}_2$ as follows

$$\begin{aligned} \dot{V}_1 + \dot{V}_2 \le & -kr^T r + \frac{1}{2} tr(\phi^T \phi r r^T \tilde{\theta}) + \frac{mN}{2} \\ & + r^T (\varepsilon + d - \underline{f}_0) + \frac{1}{2} tr\{\dot{\tilde{\theta}}^T F^{-1} \tilde{\theta}\} \\ & -\lambda r^T \left\{ \left[I_N - (L+B)^{-1} \right] \otimes I_m \right\} e_2. \\ = & kr^T r + \frac{1}{2} tr(\phi^T \phi R \tilde{\theta}) + \frac{mN}{2} \\ & + r^T (\varepsilon + d - \underline{f}_0) + \frac{1}{2} tr\{\dot{\tilde{\theta}}^T F^{-1} \tilde{\theta}\} \\ & -\lambda r^T \left\{ \left[I_N - (L+B)^{-1} \right] \otimes I_m \right\} e_2. \end{aligned} \tag{20}$$

Considering $\dot{\tilde{\theta}} = \dot{\theta}^* - \dot{\hat{\theta}} = -\dot{\hat{\theta}}$, and substituting the adaptive law (15) into (20) gives

$$\begin{aligned} \dot{V}_1 + \dot{V}_2 = & -kr^T r - \alpha tr(\tilde{\theta}^T \tilde{\theta}) + \alpha tr(\theta^{*T} \tilde{\theta}) \\ & -\lambda r^T \left\{ \left[I_N - (L+B)^{-1} \right] \otimes I_m \right\} (r - \lambda e_1) \\ & + r^T (\varepsilon + d - \underline{f}_0) + \frac{mN}{2} \\ \le & -(k-\mu)\|r\|^2 - \alpha \|\tilde{\theta}\|_F^2 + \frac{mN}{2} + T_M \|r\| \\ & + \alpha \|\tilde{\theta}\|_F \|\theta^*\|_F + \lambda^2 \mu \|e_1\| \|r\|, \end{aligned} \tag{21}$$

where $\mu = \bar{\sigma}\left\{ \left[I_N - (L+B)^{-1} \right] \otimes I_m \right\}$, $T_M = \varepsilon_M + d_M + f_M$.

The derivative of V_3 is

$$\dot{V}_3 = e_1^T e_2 = e_1^T (r - \lambda e_1) \le -\lambda \|e_1\|^2 + \|e_1\| \|r\|. \tag{22}$$

Therefore,

$$\begin{aligned} \dot{V} \le & -(k-\mu)\|r\|^2 - \alpha \|\tilde{\theta}\|_F^2 - \lambda \|e_1\|^2 + \alpha \|\tilde{\theta}\|_F \|\theta^*\|_F \\ & + (1+\lambda^2 \mu)\|e_1\| \|r\| + \frac{mN}{2} + T_M \|r\| \\ = & -z^T M z + \omega^T z + \frac{mN}{2} = -V_z(z), \end{aligned} \tag{23}$$

where $z = \begin{bmatrix} \|r\| & \|\tilde{\theta}\|_F & \|e_1\| \end{bmatrix}^T$, $\omega = \begin{bmatrix} T_M & \alpha \|\theta^*\|_F & 0 \end{bmatrix}^T$,

$$M = \begin{bmatrix} (k-\mu) & 0 & -(1+\lambda^2 \mu)/2 \\ 0 & \alpha & 0 \\ -(1+\lambda^2 \mu)/2 & 0 & \lambda \end{bmatrix}.$$

If condition (16) is satisfied, the matrix M is positive definite. In addition,

$$\dot{V} \le -V_z(z), \ \forall \|z\| \ge s \tag{24}$$

with $V_z(z)$ being a continuous positive function, where

$$s = \frac{-\|\omega\| + \sqrt{\|\omega\|^2 + 2\underline{\sigma}(M) mN}}{2\underline{\sigma}(M)}.$$

According to (17), we can get

$$\underline{\sigma}(\Omega)\|z\|^2 \le V \le \bar{\sigma}(\Psi)\|z\|^2 \tag{25}$$

where $\Omega = \text{diag}\{\frac{1}{2\bar{\sigma}\left[(L+B)\otimes I_m\right]}, \frac{1}{4\bar{\sigma}(F)}, \frac{1}{2}\}$, $\Psi = \text{diag}\{\frac{1}{2\underline{\sigma}\left[(L+B)\otimes I_m\right]}, \frac{1}{4\underline{\sigma}(F)}, \frac{1}{2}\}$. Then according to [25], we can draw the conclusion that for any initial value $z(t_0)$, there exists a time T_0 such that

$$\|z(t)\| \le \sqrt{\frac{\bar{\sigma}(\Psi)}{\underline{\sigma}(\Omega)}}\ s, \ \forall t \ge t + T_0 \tag{26}$$

Inequality (26) shows that $z(t)$ is UUB. Then following Lemmas 1 and 2, the formation tracking error $\delta(t)$ is UUB. Thus, we can achieve the desired formation under the control law (12) and adaptive law (13). This completes the proof.

Remark 2. Different from the traditional radial basis function neural network (TRBFNN) based design, the MLP approach takes the square of the norm of the RBFNN weight matrix $\hat{\theta}_{i,q}$ as an adaptive parameter. Thus, in the proposed adaptive control law, for each agent, if m RBFNNs with l_i nodes in the hidden layer are applied, then only m parameters need to be adapted online, while ml_i parameters need in the TRBFNN based control law. Therefore, the burden of computation can be reduced to a large extent.

Remark 3. By choosing parameters k and λ appropriately, s in (26) can reach to a small neighborhood of zero, such that the ultimate bound of $z(t)$ can be small and the effects of the external disturbances can be suppressed.

V. SIMULATIONS

To illustrate the theoretical results obtained in the previous sections, a comparative numerical simulation is conducted.

Consider a multi-agent system with six agents where the dynamics of each agent are described by (1). In the case where $m = 2$, x_i can be written as $x_i = [x_{i,1X}, x_{i,1Y}, x_{i,2X}, \ x_{i,2Y}]^T$, in the meanwhile f_i and d_i are set as

$$f_1 = [3\sin(0.2x_{1,2X}), 2\cos(0.2x_{1,2Y})]^T,$$
$$f_2 = [3\cos(0.2x_{2,2X}), 2\cos(0.2x_{2,2Y})]^T,$$

$$f_3 = [3\sin(0.1(x_{3,2X})^2), 2\cos(0.1(x_{3,2Y})^2)]^T,$$
$$f_4 = [3\sin(0.1(x_{4,2X})^2), 2\cos(0.1(x_{4,2Y})^2)]^T,$$
$$f_5 = [3\sin(0.1(x_{5,2X})^2), 2\sin(0.1(x_{5,2Y})^2)]^T,$$
$$f_6 = [3\sin(0.1(x_{6,2X})^2), 2\cos(0.1(x_{6,2Y})^2)]^T,$$

$$d_1 = [0.5(\sin x_{1,1X} + \cos x_{1,2X} + 1),\ 0.1(\sin(x_{1,2Y})^2 + 2]^T,$$
$$d_2 = [0.5(\cos x_{2,1X} + \cos(x_{2,2X})^2 + 1),\ 0.1\cos(x_{2,2Y})^2 + 2]^T,$$
$$d_3 = [0.5\sin(x_{3,1X} x_{3,2X}) + 0.5,\ 0.1\cos(x_{3,1Y} x_{3,2Y}) + 2]^T,$$
$$d_4 = [0.5\sin((x_{4,1X} x_{4,2X})^2 + 0.5,\ 0.1\cos((x_{4,1Y} x_{4,2Y})^2) + 2]^T,$$
$$d_5 = [0.5\sin(x_{5,1X}) + 0.5,\ 0.2\cos(x_{5,1Y}) + 2]^T,$$
$$d_6 = [0.5\cos(x_{6,1X}) + 0.5,\ 0.1\cos(x_{6,1Y}) + 2]^T.$$

Besides, the leader is set as the formation center. Trivially choose the interaction graph $\bar{G}_0$ shown as Fig. 1, which is a connected graph.

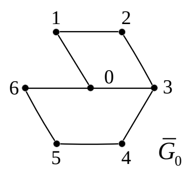


Fig. 1. The connected interaction graph.

The six agents are supposed to keep a periodic time-varying square formation in the horizontal X-Y plane and at the same time to keep rotating around the varying leader with $x_0(t) = [5t, 5t, 5, 5]^T$. Besides, the time-varying formation is specified by

$$h_i(t) = \begin{bmatrix} 0.2t\cos(0.1t + (i-1)\pi/3) \\ 0.2[\cos(0.1t + (i-1)\pi/3) - 0.1t\sin(0.1t + (i-1)\pi/3)] \\ 0.2t\sin(0.1t + (i-1)\pi/3) \\ 0.2[\sin(0.1t + (i-1)\pi/3) + 0.1t\cos(0.1t + (i-1)\pi/3)] \end{bmatrix}$$

, $i = 1, 2, \ldots, 6$.

The initial state of each agent are shown in Table I. Choose the control parameters in (14)-(15) as shown in Table II, which satisfies the condition (16) in Theorem 1.

TABLE I. THE INITIAL POSITIONS AND VELOCITIES OF SIX AGENTS

Initial states	agent1	agent2	agent3	agent4	agent5	agent6
$x_{i,1}(0)$	$[-20,10]^T$	$[20,-20]^T$	$[-20,20]^T$	$[15,15]^T$	$[-10,20]^T$	$[15,15]^T$
$x_{i,2}(0)$	$[0,0]^T$	$[0,0]^T$	$[0,0]^T$	$[0,0]^T$	$[0,0]^T$	$[0,0]^T$

TABLE II. VALUES OF THE CONTROL PARAMETERS

k	λ	F	α
9	2	$\mathrm{diag}\{0.3, 0.35, 0.2, 0.4, 0.55, 0.4\} \otimes I_2$	$0.0001 I_{12}$

In addition, the number of nodes of each NN is set as $l_i = 5$, the initial value of $\hat{\theta}$ is chosen as $\hat{\theta}(0) = \mathrm{diag}\{0, 0, 5, 0, 0, 1, 0, 0, 0, 0, 0, 0\}$.

To better show the effectiveness and superiorities of the proposed control law with MLP approach, two different control laws are taken as comparisons. The first one is a counterpart but without RBFNN, which adopts the following control law for agent i

$$u_i' = -kr_i - \lambda e_{i,2} + \dot{h}_{i,2}. \tag{27}$$

The second one utilizes the similar control scheme but using the TRBFNN without MLP, of which the control and NN adaptive laws for agent i are designed as follows

$$u_i'' = -kr_i - \hat{W}_i^T \phi_i 1_m - \lambda e_{i,2} + \dot{h}_{i,2} \tag{28}$$

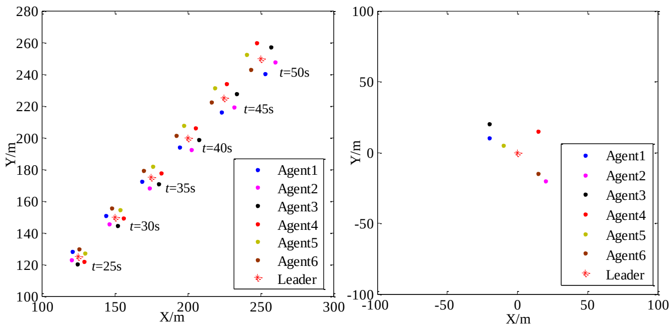
$$\dot{\hat{W}}_i = \left(c_i \otimes I_{ml_i}\right) \phi_i \tilde{r}_i - \beta_i \left(c_i \otimes I_{ml_i}\right) \hat{W}_i, \tag{29}$$

where c_i and β_i are positive constants. $\tilde{r}_i \in R^{m \times m}$ is a diagonal matrix with the ith principal diagonal elements $\tilde{r}_{i,q}$ satisfying $\tilde{r}_{i,q} = r_{i,q}$, and $r_{i,q}$ is the qth element of r_i, $q = 1, \ldots, m$.

In (27) and (28), we set the parameters k and λ be the same as the ones in TABLE II. Moreover, in (29), c_i and β_i are set as $\beta_i = 0.001$ and $c = \mathrm{diag}\{c_1, c_2, \ldots, c_6\} = \mathrm{diag}\{0.3, 0.35, 0.2, 0.4, 0.55, 0.4\}$. Besides, the number of nodes of each RBENN is also set as $l_i = 5$ for the second comparison case.

The simulation results are presented by Fig. 2-4. Fig. 2 shows the positions of the six agents and the leader at $t = 0,\ 25,\ 30,\ 35,\ 40,\ 45,\ 50\mathrm{s}$. Fig. 3 and Fig. 4 depict the formation tracking error $\delta_1(t)$ on X-axis and Y-axis for each agent under the proposed control law with MLP versus its comparisons, respectively. From Fig. 2, the following phenomena can be observed: (i) the six agents successfully keep a regular hexagon formation with varying edges while keeping rotating around the dynamic leader; (ii) the leader moves along a straight line, and keeps lying in the center of the formation, so that the formation moves along the same straight line while tracking the leader.

From Fig. 3-4, it can be obtained that under the proposed control law, the formation position tracking errors keep around 0 after about t=5s although there exist uncertainties and disturbances. However, for the first comparison case without adopting RBFNN, the formation position tracking errors on X-axis and Y-axis cannot converge near 0 due to the uncertainties and disturbances, especially formation tracking errors of the agents 4 and 5 on X-axis are over 0.4, which may lead to failure of formation. Therefore, the uncertainties can be compensated pretty well by the proposed control law with MLP.



(a). at $t = 25,\ 30,\ 35, 40,\ 45,\ 50$ s (b) at $t = 0$ s

Fig. 2. Position snapshots of the six agents and the leader at $t = 0,\ 25,\ 30,\ 35,\ 40,\ 45,\ 50$ s .

For the second comparison case with TRBFNN, we can get that under the same parameters, the formation tracking errors perform similar. However, in the proposed adaptive law (15), only 2 parameters for each agent need to be adapted, whereas 10 parameters need in the adaptive law (27), which means that the burdensome computation of the control law can be lightened to a great extent and it will be easy to realize the control law in real engineering. To sum up, the desired time-varying formation can be achieved by the proposed control law under unknown uncertainties and disturbances.

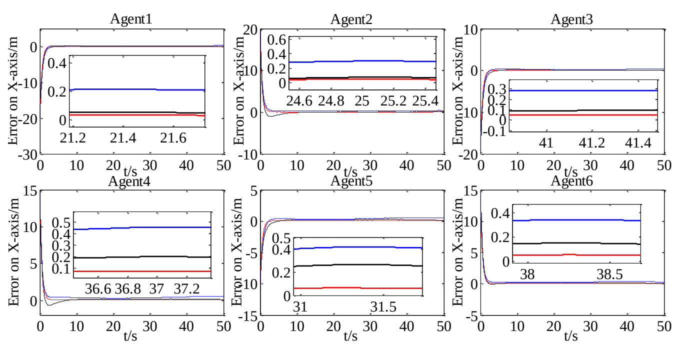


Fig. 3. Formation position tracking error on X-axis for each agent under the control laws with MLP (the red line) and its two comparison control laws (the first one is the blue line and the second is the black line).

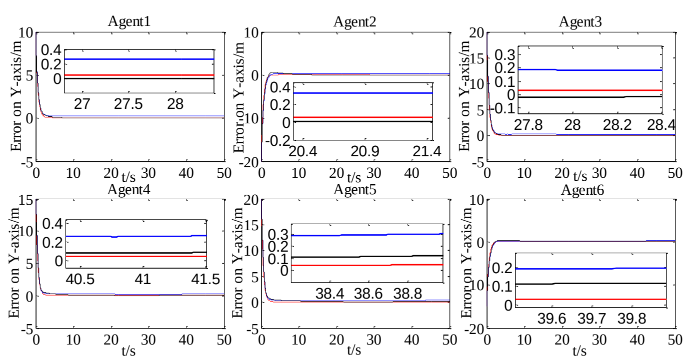


Fig. 4. Formation position tracking error on Y-axis for each agent under the control laws with MLP (the red line) and its two comparison control laws (the first one is the blue line and the second is the black line).

VI. Conlusions

In comparison with time-invariant formation, time-varying formation tracking is obviously more practical in many applications. For the purpose of achieving expected time-varying formation tracking under model uncertainties and disturbances, a novel RBFNN-based adaptive control scheme with MLP is proposed. By adopting the RBFNNs, the uncertainties are well compensated. Through MLP approach, the burdensome computation can be lightened to a great extent. The stability analysis is theoretically explored through Lyapunov function approach. Comparative simulation results verify the effectiveness and superiority of the developed adaptive control law in this paper. In our future work, we will combine MLP with other robust adaptive control methods to further improve the robustness of the multi-agent control systems or extend this work to the case with directed graph.

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