Type-2 Fuzzy Comprehension Evaluation for Tourist Attractive Competency

Hong Mo, Kefu Yan, Xuanming Zhao, Yaqiong Zeng, Xiao Wang, *Member, IEEE*, and Fei-Yue Wang, *Fellow, IEEE*

Abstract—In general, the evaluation for tourist attractive competency is always by means of natural language, owing to the difficulty for people to use conventional mathematics method to describe the evaluation modeling and their evaluation results. In this paper, discrete and partially connected type-2 fuzzy sets, fuzzy comprehension evaluation, are synthesized, and then discrete type-2 fuzzy comprehension valuation and partially connected type-2 fuzzy comprehension evaluation are presented to provide the analysis of evaluation and represent the results of different conditions. Finally, the method of evaluation for tourist attractive competency is given, and the results illustrate that the method can describe the otherness of evaluated data.

Index Terms—Fuzzy sets, interval type-2 fuzzy sets (IT2 FSs), T2 FSs, tourist attractive competency, type-2 fuzzy comprehension evaluation.

I. INTRODUCTION

TODAY, problems of social systems are related to human behavior [1]–[3], such as evaluation for tourist attraction competence; the competence of employees is described in natural language, so how to deal with natural language is the key to solve such problems.

Because tourist's attractive competency is always evaluated by the means of natural language, it is mainly expressed as perceptual information which is difficult to be described by conventional mathematics models. Computing with words [4] and linguistic dynamic systems [5]–[7] are effective methods to deal with perceptual information, where words are always represented as fuzzy sets [8] or type-2 fuzzy sets (T2 FSs) [9]. Fuzzy logic has been used to deal with many practical problems [10], [11]. T2 FSs can provide more freedom to describe membership information and, hence, can address the problem of linguistic ambiguity and data noise better than type-1 fuzzy sets. Also, many other applications using T2 FSs have demonstrated definite improvement over using fuzzy sets [12]–[14]. However, in the last 20 years, the research mainly focused on interval T2 FSs (IT2 FSs) [15]–[18] that eventually transformed into the upper membership function and lower membership function of their corresponding footprint of uncertainty that is actual two type-1 fuzzy sets. This is due to the difficulty of representation for general T2 FSs. To a certain extent, the application of T2 FSs and logic is actually two type-1 fuzzy sets. Recently, Wang and Mo [19] and Wang *et al.* [20] presented new methods to described general T2 FSs by the partition of CoS, which can help the direct use of T2 FSs and logic.

In 1973, McCleland presented the concept of competency. Since then, the study of competency has been the hot topic for the problems related to management. Many related works focus on building competency modeling and discuss the relationship between the internal management function of organization and competency management system. Fuzzy comprehension evaluation is an effective way to analyze the problems of multifactor evaluation [10], but it is limited to single-person decision. For multipersons decision making or group decision, this needs type-2 fuzzy comprehension evaluation.

In this paper, by synthesizing T2 FSs, fuzzy comprehension evaluation, type-2 fuzzy comprehension is presented to appraise the tourist attractive competency. If the data are not very big, discrete type-2 fuzzy comprehension is used, otherwise, partially connected type-2 fuzzy comprehension is considered.

The rest of this paper is organized as follows. In Section II, we introduce some related terminologies. Section III is the description of discrete and partially connected T2 FSs, and in Section IV, discrete and partially connected type-2 fuzzy comprehension evaluation are presented. Section V discusses type-2 fuzzy comprehension evaluation for tourist attractive competency. Section VI concludes this paper.

II. PRELIMINARIES

A. Fuzzy Sets

In this paper, X is the universe; $A, B, C, \ldots$ are the conventional sets; $\widetilde{A}, \widetilde{B}, \widetilde{C}, \ldots$ are the fuzzy sets on X; $\breve{A}, \breve{B}, \breve{C}, \ldots$ are the vectors or matrixes; $\omega, \omega_1, \omega_2, \ldots$ are the T2 FSs on X; and $K, M, N, \ldots$ are the nature numbers.

Let $\widetilde{A}$ be a fuzzy set on X, defined as

$$\mu_{\widetilde{A}} : X \to [0, 1] \tag{1}$$

where $\mu_{\widetilde{A}}$ is called the membership function of $\widetilde{A}$, and for every $x \in X$, there is a function value $\mu_{\widetilde{A}}(x) \in [0, 1]$, which is called membership grade of $\widetilde{A}$ at x, written as $\widetilde{A}(x)$.

A fuzzy set $\widetilde{A}$ on X can be represented as

$$\widetilde{A} = \int_{x \in X} \frac{\widetilde{A}(u)}{u}. \tag{2}$$

If $X = \{x_1, x_2, \ldots, x_N\}$ is discrete, then $\widetilde{A}$ can be written as follows:

$$\widetilde{A} = \sum_{i=1}^{N} \frac{\widetilde{A}(x_i)}{x_i}. \tag{3}$$

For a fuzzy sets $\widetilde{A}$ on X, the set $\{x | \mu_{\widetilde{A}}(x) > 0\}$ is called the support of $\widetilde{A}$, written as $supp(\widetilde{A})$. $\overline{\{x | \mu_{\widetilde{A}}(x) > 0\}}$ is called the closure of the support of $\widetilde{A}$, written as $CoS(\widetilde{A})$, where $\overline{A}$ is the closure set of A.

If $X \subseteq [0, 1]$, then $\widetilde{A}$ is called an unit fuzzy set, and L is used to substitute X.

B. Type-2 Fuzzy Sets

Let $C(2^I)$ be the set of all the nonempty close subsets of I. A T2FS ω on X is defined as

$$\omega = \left\{ (x, u, z) | x \in X, u \in L_x \subseteq C(2^I), z = \mu_\omega^2(x, \mu) \right\} \tag{4}$$

where x (or u, z) is the primary (or secondary, third) variable, $L_x \subseteq [0.1]$ is the primary membership grade, defined by the following multivalue mapping:

$$\mu_\omega^1 : X \to C(2^I) \tag{5}$$

i.e., for every $x \in X$, there exist $L_x \in C(2^I)$, such that $\mu_\omega^1(x) = L_x$ and $0 \leq \mu_\omega^2(x, u) \leq 1$, μ_ω^2 is the secondary membership function, defined as

$$\mu_\omega^2 : \bigcup_{x \in X} x \times L_x \to I. \tag{6}$$

Let X be a discrete set, if L_x is connected, and μ_ω^2 is continuous on every $x \times L_x$, then ω is a partially connected T2 FS on X, written as

$$\omega = \sum_{x \in X} \int_{u \in L_x} \frac{\mu_\omega^2(x, u)}{(x, u)}. \tag{7}$$

If X and L_x are all discrete, then ω is a discrete T2 FS on X, written as

$$\omega = \sum_{x \in X} \sum_{u \in L_x} \frac{\mu_\omega^2(x, u)}{(x, u)}. \tag{8}$$

If for every $x \in X$ and $u \in L_x$, there is $\mu_\omega^2(x, u) = 1$, then ω is an IT2 FSs, written as

$$\omega = \sum_{x \in X} \frac{\frac{1}{[a_x, b_x]}}{x}. \tag{9}$$

C. Fuzzy Comprehensive Evaluation

Let the universe $X = \{x_1, x_2, \ldots, x_M\}$ be the set of all evaluated objects. Fuzzy comprehensive evaluation generally contains the following six steps.

Step 1: Determination of the factor set.

The factor set A contains K factors, written as

$$A = \{a_1, a_2, \ldots, a_K\} \tag{10}$$

where $a_k, k = 1, 2, \ldots, K$ is one of the factor being evaluated, and K is a nature number.

Step 2: Determination of the evaluation set V.

For every factor $a_k, k = 1, 2, \ldots, K$, the assess value $\widetilde{V}$ is expressed by some basis words $\{\widetilde{V}_1, \widetilde{V}_2, \ldots, \widetilde{V}_{M_1}\}$, that is

$$\widetilde{V} = \{\widetilde{V}_1, \widetilde{V}_2, \ldots, \widetilde{V}_{M_1}\} \tag{11}$$

where $\widetilde{V}_1, \widetilde{V}_2, \ldots, \widetilde{V}_{M_1}$ are the fuzzy sets on X.

Step 3: Determination of the weight of every factor.

The factor of different element set may play different role in the comprehensive evaluation, then the weight vector is represented as

$$\check{W} = \{w_1, w_2, \ldots, w_K\} \tag{12}$$

where w_k is the weight for the factor $a_k, (k = 1, 2, \ldots, K)$, and $\sum_{k=1}^{K} w_k = 1$.

Step 4: Determination of the fuzzy comprehension evaluation matrix $\check{R}$ for the evaluated element $x_m (m = 1, 2, \ldots, M)$

$$\check{R}^m = \begin{pmatrix} r_{11}^m & r_{12}^m & r_{13}^m & \cdots & r_{1M_1}^m \\ r_{21}^m & r_{22}^m & r_{23}^m & \cdots & r_{2M_1}^m \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{K1}^m & r_{K2}^m & r_{K3}^m & \cdots & r_{KM_1}^m \end{pmatrix} \tag{13}$$

where r_{kz}^m is the membership grade of the factor a_k for the element x_m is evaluated to be the grade $\widetilde{V}_z$.

Step 5: Computation for comprehension evaluation vector. Let

$$\check{A} = \check{W} \circ \check{R}^m. \tag{14}$$

Step 6: Comprehension evaluation by the fuzzy algorithm.

III. DISCRETE AND PARTIALLY CONNECTED TYPE-2 FUZZY SETS

Let ω be a T2 FSs on $X = \{x_1, x_2, \ldots, x_M\}$, then its primary membership grade L_x and secondary membership

function $\mu_\omega^2(x, u)$ can be written as

$$
L_x = \begin{cases} \{u_{11}, u_{12}, \ldots, u_{1N_1}\}, & x = x_1 \\ \vdots & \vdots \\ \{u_{21}, u_{22}, \ldots, u_{2N_2}\}, & x = x_2 \\ \vdots & \vdots \\ \{u_{M1}, u_{M2}, \ldots, u_{MN_M}\}, & x = x_M \end{cases}
$$

$$
\widetilde{\mu}_\omega^2(x, u) = \begin{cases} c_{11}, & x = x_1, \ u = u_{11} \\ c_{12}, & x = x_1, \ u = u_{12} \\ \vdots & \vdots \\ c_{1N_1}, & x = x_1, \ u = u_{1N_1} \\ c_{21}, & x = x_2, \ u = u_{21} \\ c_{22}, & x = x_2, \ u = u_{22} \\ \vdots & \vdots \\ c_{2N_2}, & x = x_2, \ u = u_{2N_2} \\ \vdots & \vdots \\ c_{M1}, & x = x_M, \ u = u_{M1} \\ c_{M2}, & x = x_M, \ u = u_{M2} \\ \vdots & \vdots \\ c_{MN_M}, & x = x_M, \ u = u_{MN_M}. \end{cases}
$$

Then, a discrete T2 FS can be written as follows:

$$
\omega = \sum_{m=1}^{M} \sum_{t=1}^{N_m} \frac{c_{mt}}{(x_m, u_{mt})} \tag{15}
$$

or

$$
\omega = \sum_{m=1}^{M} \sum_{t=1}^{N_m} \frac{\frac{c_{mt}}{u_{mt}}}{x_m}. \tag{16}
$$

v is a partially connected T2 FS on $X = \{x_1, x_2, \ldots, x_M\}$, and its primary membership grade and secondary membership function can be defined as follows:

$$
L_x = \begin{cases} [r_1, s_1], & x = x_1 \\ [r_2, s_2], & x = x_2 \\ \vdots & \vdots \\ [r_M, s_M], & x = x_M \end{cases}
$$

$$
\widetilde{\mu}_v^2(x, u) = \begin{cases} f_1(u), & x = x_1, \ u \in [r_1, s_1] \\ f_2(u), & x = x_2, \ u \in [r_2, s_2] \\ \vdots & \vdots \\ f_M(u), & x = x_M, \ u \in [r_M, s_M]. \end{cases}
$$

If for every $m \in \{1, 2, \ldots, M\}$, there is $f_m(u) = d_m$, then v is called a general IT2 FSs, which can be written as follows:

$$
v = \sum_{m=1}^{M} \frac{\frac{d_m}{[r_m, s_m]}}{x_m}. \tag{17}
$$

Example 1: Let ω_1 be a discrete T2 FS on $\{x_1, x_2, x_3\}$, then its primary grade is defined as

$$
L_x = \begin{cases} \{0.1, 0.3, 0.5\}, & x = x_1 \\ \{0.2, 0.7\}, & x = x_2 \\ \{0.3, 0.6\}, & x = x_3 \end{cases}
$$

$$
\widetilde{\mu}_{\omega_1}^2(x, u) = \begin{cases} 0.2, & x = x_1, \ u = 0.1 \\ 0.1, & x = x_1, \ u = 0.3 \\ 0.4, & x = x_1, \ u = 0.5 \\ 0.1, & x = x_2, \ u = 0.2 \\ 0.5, & x = x_2, \ u = 0.7 \\ 0.6, & x = x_3, \ u = 0.3 \\ 0.4, & x = x_3, \ u = 0.6. \end{cases}
$$

Then ω_1 can be represented as

$$
\omega_1 = \frac{0.2}{(x_1, 0.1)} + \frac{0.1}{(x_1, 0.3)} + \frac{0.4}{(x_1, 0.5)}
$$
$$
+ \frac{0.1}{(x_2, 0.2)} + \frac{0.5}{(x_2, 0.7)} + \frac{0.6}{(x_3, 0.3)} + \frac{0.4}{(x_3, 0.6)} \tag{18}
$$

or

$$
\omega_1 = \frac{\frac{0.2}{0.1}}{x_1} + \frac{\frac{0.1}{0.3}}{x_1} + \frac{\frac{0.4}{0.5}}{x_1} + \frac{\frac{0.1}{0.2}}{x_2} + \frac{\frac{0.5}{0.7}}{x_2} + \frac{\frac{0.6}{0.3}}{x_3} + \frac{\frac{0.4}{0.6}}{x_3}. \tag{19}
$$

Let v_1 be a partially connected T2 FS on $\{x_1, x_2, x_3\}$, then its primary grade is defined as

$$
L_x = \begin{cases} [0.2, 0.5], & x = x_1 \\ [0.4, 0.6], & x = x_2 \\ [0.7, 0.9], & x = x_3. \end{cases}
$$

The secondary membership function is defined as follows:

$$
\widetilde{\mu}_{v_1}^2(x, u) = \begin{cases} 0.35, & x = x_1, \ u \in [0.2, 0.5] \\ 0.52, & x = x_2, \ u \in [0.4, 0.6] \\ 0.68, & x = x_3, \ u \in [0.7, 0.9] \end{cases}.
$$

Then v_1 can be represented as

$$
v_1 = \frac{\frac{0.35}{[0.2, 0.5]}}{x_1} + \frac{\frac{0.52}{[0.4, 0.6]}}{x_2} + \frac{\frac{0.68}{[0.7, 0.9]}}{x_3}. \tag{20}
$$

IV. TYPE-2 FUZZY COMPREHENSION EVALUATION

Let the universe $X = \{x_1, x_2, \ldots, x_M\}$, and N experts are invited to provide the evaluation scores. For type-2 fuzzy comprehension evaluation, discrete and partially connected T2 FSs are used to substitute the corresponding fuzzy sets, and the procedure for evaluation also includes six steps.

A. Discrete Type-2 Fuzzy Comprehension Evaluation

The evaluation set and the evaluation results can be represented as some discrete IT2 FSs. Three steps (i.e., steps 2, 4, and 5) of fuzzy comprehension evaluation are modified as follows.

Step 4*: Determination of the fuzzy comprehension evaluation matrix.

TABLE I
EVALUATION BY p_n FOR ALL OBJECTS BEING EVALUATED

	a_1				a_2				...	a_K			
	$\widetilde{V}_1$	$\widetilde{V}_2$	$\widetilde{V}_3$	$\widetilde{V}_4$	$\widetilde{V}_1$	$\widetilde{V}_2$	$\widetilde{V}_3$	$\widetilde{V}_4$	...	$\widetilde{V}_1$	$\widetilde{V}_2$	$\widetilde{V}_3$	$\widetilde{V}_4$
$x_1\|p_n$	r_{11}^{1n}	r_{12}^{1n}	r_{13}^{1n}	r_{14}^{1n}	r_{21}^{1n}	r_{22}^{1n}	r_{23}^{1n}	r_{24}^{1n}	...	r_{K1}^{1n}	r_{K2}^{1n}	r_{K3}^{1n}	r_{K4}^{1n}
$x_2\|p_n$	r_{11}^{2n}	r_{12}^{2n}	r_{13}^{2n}	r_{14}^{2n}	r_{21}^{2n}	r_{22}^{2n}	r_{23}^{2n}	r_{24}^{2n}	...	r_{K1}^{n2}	r_{K2}^{n2}	r_{K3}^{n2}	r_{K4}^{n2}
$x_3\|p_n$	r_{11}^{3n}	r_{12}^{3n}	r_{13}^{3n}	r_{14}^{3n}	r_{21}^{3n}	r_{22}^{3n}	r_{23}^{3n}	r_{24}^{3n}	...	r_{K1}^{3n}	r_{K2}^{3n}	r_{K3}^{3n}	r_{K4}^{3n}
$\vdots$	$\vdots$				$\vdots$				$\vdots$	$\vdots$			
$x_M\|p_n$	r_{11}^{Mn}	r_{12}^{Mn}	r_{13}^{Mn}	r_{14}^{Mn}	r_{21}^{Mn}	r_{22}^{Mn}	r_{23}^{Mn}	r_{24}^{Mn}	...	r_{K1}^{Mn}	r_{K2}^{Mn}	r_{K3}^{Mn}	r_{K4}^{Mn}

R^{mn} is provided by the nth expert $p_n, n \in \{1, 2, \ldots, N\}$ for the evaluated element x_m, where $n = 1, 2, \ldots, N, m = 1, 2, \ldots, M$

$$\check{R}^{mn} = \begin{pmatrix} r_{11}^{mn} & r_{12}^{mn} & r_{13}^{mn} & \cdots & r_{1M_1}^{mn} \\ r_{21}^{mn} & r_{22}^{mn} & r_{23}^{mn} & \cdots & r_{2M_1}^{mn} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{K1}^{mn} & r_{K2}^{mn} & r_{K3}^{mn} & \cdots & r_{KM_1}^{mn} \end{pmatrix} \quad (21)$$

where r_{kz}^{mn} is the membership grade for the factor $a_k (k = 1, 2, \ldots, K)$ of the element $x_m (m = 1, 2, \ldots, M)$ to the grade $\widetilde{V}_z$ by the nth expert. The evaluation result by all the experts is the primary membership grade $L_{x_m}^{kz} = \{r_{kz}^{m1}, r_{kz}^{m2}, \ldots, r_{kz}^{mN}\}$, then the corresponding evaluation matrixes are achieved.

Thus, the evaluation values for every factor $a_k (k = 1, 2, \ldots, K)$ of every element $x_m (m = 1, 2, \ldots, M)$ to every evaluation grade $\widetilde{V}_1, \widetilde{V}_2, \widetilde{V}_3, \widetilde{V}_4$ by the same evaluation assessment are shown in Table I.

Step 5*: Computation for comprehension evaluation vector. Let

$$\check{A}^{mn} = \check{W} \circ \check{R}^{mn} \quad (22)$$

where $\check{A}^{mn} = \{a_1^{mn}, a_2^{mn}, \ldots, a_{M_1}^{mn}\}, m \in \{1, 2, \ldots, M\}, n \in \{1, 2, \ldots, N\}$, and for any $z \in \{1, 2, \ldots, M_1\}$, there is

$$a_z^{mn} = (w_1 \times r_{1z}^{mn}) + (w_2 \times r_{2z}^{mn}) + \cdots + (w_K \times r_{Kz}^{mn}). \quad (23)$$

Step 6*: Comprehension evaluation by the type-2 fuzzy algorithm.

B. Partially Connected Type-2 Fuzzy Comprehension Evaluation

The evaluation set and the evaluation results can be represented as some partially connected T2 FSs. Corresponding three steps (i.e., steps 2, 4, and 5) of fuzzy comprehension evaluation are modified as follows.

Step 4**: Determination of the fuzzy comprehension evaluation matrix.

For the factor $a_k (k = 1, 2, \ldots, K)$ related to the element $x_m (m = 1, 2, \ldots, M)$, the evaluation result to the grade $\widetilde{V}_z$ by all the experts is the primary membership grade $L_{x_m}^{kz} = [r_{kz}^m, s_{kz}^m]$, where $m = 1, 2, \ldots, M; k = 1, 2, \ldots, K$, and

the corresponding two evaluation matrixes are represented as follows:

$$\check{R}^m = \begin{pmatrix} r_{11}^m & r_{12}^m & r_{13}^m & \cdots & r_{1M_1}^m \\ r_{21}^m & r_{22}^m & r_{23}^m & \cdots & r_{2M_1}^m \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{K1}^m & r_{K2}^m & r_{K3}^m & \cdots & r_{KM_1}^m \end{pmatrix} \quad (24)$$

$$\check{S}^m = \begin{pmatrix} s_{11}^m & s_{12}^m & s_{13}^m & \cdots & s_{1M_1}^m \\ s_{21}^m & s_{22}^m & s_{23}^m & \cdots & s_{2M_1}^m \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ s_{K1}^m & s_{K2}^m & s_{K3}^m & \cdots & s_{KM_1}^m \end{pmatrix}. \quad (25)$$

Step 5**: Computation for comprehension evaluation vector. Let

$$\check{A}^m = \check{W} \circ \check{R}^m \quad (26)$$
$$\check{B}^m = \check{W} \circ \check{S}^m \quad (27)$$
$$\check{D}^* = \check{W} \circ \check{D} \quad (28)$$

where

$$\check{A}^m = \{a_1^m, a_2^m, \ldots, a_{M_1}^m\} \quad (29)$$
$$\check{B}^m = \{b_1^m, b_2^m, \ldots, b_{M_1}^m\} \quad (30)$$
$$\check{D} = \{d_1, d_2, \ldots, d_M\} \quad (31)$$
$$\check{D}^* = \{d_1^*, d_2^*, \ldots, d_M^*\} \quad (32)$$

where $m \in \{1, 2, \ldots, M\}$, and for any $z \in \{1, 2, \ldots, M_2\}$, there is

$$a_z^m = (w_1 \times r_{1z}^m) + (w_2 \times r_{2z}^m) + \cdots + (w_K \times r_{Kz}^m) \quad (33)$$
$$b_z^m = (w_1 \times s_{1z}^m) + (w_2 \times s_{2z}^m) + \cdots + (w_K \times s_{Kz}^m) \quad (34)$$
$$d^* = (w_1 \times d_1) + (w_2 \times d_2) + \cdots + (w_K \times d_m). \quad (35)$$

Step 6**: Comprehension evaluation by the type-2 fuzzy algorithm.

V. TYPE-2 FUZZY COMPREHENSION EVALUATION FOR TOURIST ATTRACTIVE COMPETENCY

In this section, type-2 fuzzy comprehension evaluation is used to assess the tourist attractive competency. If the number of evaluation experts or the voters is not very large, discrete type-2 fuzzy comprehension evaluation is used, otherwise,

TABLE II
EVALUATION FOR ALL OBJECTS BY THREE ASSESSMENTS

	a_1				a_2				a_3			
	pr	av	gd	ex	pr	av	gd	ex	pr	av	gd	ex
$x_1\|p_1$	0	0	0.35	0.65	0	0	0.15	0.85	0	0	0.33	0.67
$x_1\|p_2$	0	0	0.23	0.77	0	0	0.25	0.75	0	0	0.35	0.65
$x_1\|p_3$	0	0	0.34	0.66	0	0	0.18	0.82	0	0	0.34	0.66
$x_2\|p_1$	0	0.23	0.77	0	0	0	0.13	0.87	0	0.45	0.55	0
$x_2\|p_2$	0	0.35	0.65	0	0	0	0.15	0.85	0	0.13	0.87	0
$x_2\|p_3$	0	0.24	0.76	0	0	0	0.12	0.88	0	0.42	0.58	0
$x_3\|p_1$	0	0	0.11	0.89	0	0.35	0.65	0	0	0.24	0.76	0
$x_3\|p_2$	0	0	0.15	0.85	0	0.23	0.77	0	0	0.28	0.72	0
$x_3\|p_3$	0	0	0.25	0.75	0	0.32	0.68	0	0	0.23	0.77	0
$x_4\|p_1$	0	0	0.12	0.88	0	0	0.22	0.78	0	0	0.26	0.74
$x_4\|p_2$	0	0	0.14	0.86	0	0	0.22	0.78	0	0	0.43	0.57
$x_4\|p_3$	0	0	0.16	0.84	0	0	0.21	0.79	0	0	0.34	0.66
$x_5\|p_1$	0.91	0.09	0	0	0.72	0.28	0	0	0	0.22	0.78	0
$x_5\|p_2$	0.85	0.15	0	0	0.75	0.25	0	0	0	0.26	0.74	0
$x_5\|p_3$	0.84	0.16	0	0	0.78	0.22	0	0	0	0.24	0.76	0

the partially connected type-2 fuzzy comprehension evaluation can be adopted. The following example tells us how to use the upper two methods.

Example 3: Five scenic spots x_1, x_2, x_3, x_4, x_5 are evaluated by three evaluation values p_1, p_2, p_3. The set of evaluated factors is {a_1 = tourism safety, a_2 = tourism traffic, a_3 = tourism quality}, and the weight vector is $(0.4, 0.3, 0.3)$, and the evaluation set is {excellent(ex), good(gd), average(av), poor(pr)}, and their corresponding membership functions are as follows:

$$\mu_{ex}(y) = \begin{cases} 0.1x - 9, & 90 \le y \le 100 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{gd}(y) = \begin{cases} 0.1x - 8, & 80 \le y \le 90 \\ 10 - 0.1x, & 90 \le y \le 100 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{av}(y) = \begin{cases} 0.1x - 7, & 70 \le y \le 80 \\ 9 - 0.1x, & 80 \le y \le 90 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{pr}(y) = \begin{cases} 1, & 0 \le y \le 70 \\ 8 - 0.1x, & 70 \le y \le 80 \\ 0, & \text{otherwise.} \end{cases}$$

The corresponding figure is similar to Fig. 1.

By the method of discrete type-2 fuzzy comprehension evaluation, the evaluation results pr for the scene spots by
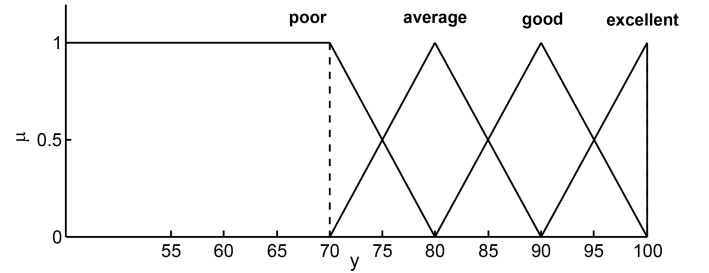

Fig. 1. Membership functions of evaluation sets.

the data from Table II can be written as follows:

$$pr = \frac{\frac{1}{0}}{x_1} + \frac{\frac{1}{0}}{x_2} + \frac{\frac{1}{0}}{x_3} + \frac{\frac{1}{0}}{x_4} + \frac{\frac{1}{0.58} + \frac{1}{0.565} + \frac{1}{0.57}}{x_5}$$

$$av = \frac{\frac{1}{0}}{x_1} + \frac{\frac{1}{0.227} + \frac{1}{0.179} + \frac{1}{0.222}}{x_2}$$
$$+ \frac{\frac{1}{0.177} + \frac{1}{0.153} + \frac{1}{0.165}}{x_3} + \frac{\frac{1}{0}}{x_4} + \frac{\frac{1}{0.186} + \frac{1}{0.213} + \frac{1}{0.202}}{x_5}$$

$$gd = \frac{\frac{1}{0.284} + \frac{1}{0.272} + \frac{1}{0.292}}{x_1} + \frac{\frac{1}{0.512} + \frac{1}{0.566} + \frac{1}{0.514}}{x_2}$$
$$+ \frac{\frac{1}{0.4677} + \frac{1}{0.507} + \frac{1}{0.535}}{x_3} + \frac{\frac{1}{0.192} + \frac{1}{0.251} + \frac{1}{0.229}}{x_4}$$
$$+ \frac{\frac{1}{0.234} + \frac{1}{0.222} + \frac{1}{0.228}}{x_5} \tag{36}$$

TABLE III
EVALUATION FOR ALL OBJECTS BY 500 QUESTIONNAIRES

	a_1				a_2				a_3			
	pr	av	gd	ex	pr	av	gd	ex	pr	av	gd	ex
x_1	0	0	$\frac{0.43}{[0.89,0.95]}$	$\frac{0.57}{[0.95,0.99]}$	0	0	$\frac{0.15}{[0.88,0.95]}$	$\frac{0.85}{[0.95,0.97]}$	0	0	$\frac{0.33}{[0.9,0.95]}$	$\frac{0.67}{[0.95,0.98]}$
x_2	0	$\frac{0.22}{[0.8,0.85]}$	$\frac{0.78}{[0.85,0.88]}$	0	0	$\frac{0.13}{[0.81,0.85]}$	$\frac{0.87}{[0.85,0.9]}$	0	0	$\frac{0.45}{[0.82,0.85]}$	$\frac{0.55}{[0.85,0.89]}$	0
x_3	0	$\frac{0.89}{[0.77,0.85]}$	$\frac{0.11}{[0.85,0.87]}$	0	0	$\frac{0.65}{[0.78,0.85]}$	$\frac{0.35}{[0.85,0.9]}$	0	0	$\frac{0.76}{[0.75,0.85]}$	$\frac{0.24}{[0.85,0.88]}$	0
x_4	0	0	$\frac{0.12}{[0.89,0.95]}$	$\frac{0.88}{[0.95,0.98]}$	0	0	$\frac{0.22}{[0.91,0.95]}$	$\frac{0.78}{[0.95,0.98]}$	0	0	$\frac{0.26}{[0.88,0.95]}$	$\frac{0.74}{[0.95,0.98]}$
x_5	$\frac{0.91}{[0.66,0.75]}$	$\frac{0.09}{[0.75,0.81]}$	0	0	$\frac{0.72}{[0.69,0.75]}$	$\frac{0.28}{[0.75,0.78]}$	0	0	$\frac{0.78}{[0.71,0.75]}$	$\frac{0.22}{[0.75,0.8]}$	0	0

$$ex = \frac{\frac{1}{0.716} + \frac{1}{0.728} + \frac{1}{0.708}}{x_1} + \frac{\frac{1}{0.261} + \frac{1}{0.255} + \frac{1}{0.264}}{x_2}$$
$$+ \frac{\frac{1}{0.356} + \frac{1}{0.34} + \frac{1}{0.3}}{x_3} + \frac{\frac{1}{0.808} + \frac{1}{0.749} + \frac{1}{0.771}}{x_4} + \frac{\frac{1}{0}}{x_5}. \tag{37}$$

From (32) and (33), by the primary membership grade of every element, it is easy to give the evaluation results for all elements as follows: x_5 is poor; x_2 and x_3 are good; and x_1 and x_4 are excellent. At the same time, x_2 is better than x_3, and x_4 is more excellent than x_1.

In the example, if the evaluation for the five scenes spots x_1, x_2, x_3, x_4, x_5 are by the means of questionnaires being sent to 500 tourists, and the set of factors, the evaluation set, and weight vector are the same as earlier. The evaluated results being normalization are shown in Table III.

By the method of partially connected type-2 fuzzy comprehension evaluation, the evaluation results for the scene spots by the data from Table III can be described as follows:

$$pr = \frac{\frac{0}{0}}{x_1} + \frac{\frac{0}{0}}{x_2} + \frac{\frac{0}{0}}{x_3} + \frac{\frac{0}{0}}{x_4} + \frac{\frac{0.814}{[0.684,0.75]}}{x_5}$$
$$av = \frac{\frac{0}{0}}{x_1} + \frac{\frac{0.262}{[0.809,0.85]}}{x_2} + \frac{\frac{0.799}{[0.767,0.85]}}{x_3} + \frac{\frac{0}{0}}{x_4} + \frac{\frac{0.186}{[0.75,0.789]}}{x_5} \tag{38}$$
$$gd = \frac{\frac{0.316}{[0.89,0.95]}}{x_1} + \frac{\frac{0.738}{[0.85,0.889]}}{x_2} + \frac{\frac{0.221}{[0.85,0.882]}}{x_3} + \frac{\frac{0.192}{[0.893,0.95]}}{x_4} + \frac{\frac{0}{0}}{x_5}$$
$$ex = \frac{\frac{0.684}{[0.95,0.981]}}{x_1} + \frac{\frac{0}{0}}{x_2} + \frac{\frac{0}{0}}{x_3} + \frac{\frac{0.808}{[0.95,0.98]}}{x_4} + \frac{\frac{0}{0}}{x_5}. \tag{39}$$

By (34) and (35), the evaluation grade for five scene spots is as follows: x_5 is poor, x_3 is average, x_2 is good, x_1 and x_4 are excellent, and x_4 is more excellent than x_1.

VI. CONCLUSION

In this paper, the type-2 fuzzy comprehension evaluation is presented. At the same time, discrete T2 FSs, partially connected T2 FSs and their representations are discussed. Then fuzzy comprehension based on discrete T2 FSs and partially connected T2 FSs are provided, respectively, and the corresponding computation steps are also introduced.

In our future work, the next step is to extend this paper from the following aspects: 1) study dynamic evolution of the general T2 FS comprehension evaluation and 2) explore the application of T2 FS comprehension evaluation to parallel blockchain [21] and semantic social network [22].

REFERENCES

[1] F.-Y. Wang, "Computational social systems in a new period: A fast transition into the third axial age," *IEEE Trans. Computat. Social Syst.*, vol. 4, no. 3, pp. 52–53, Sep. 2017.

[2] F.-Y. Wang, "Computational dissemination: Toward precision and smart impacts for computational social systems," *IEEE Trans. Computat. Social Syst.*, vol. 4, no. 4, pp. 193–195, Dec. 2017.

[3] Y. Yuan, F.-Y. Wang, and D. Zeng, "Competitive analysis of bidding behavior on sponsored search advertising markets," *IEEE Trans. Computat. Social Syst.*, vol. 4, no. 3, pp. 179–190, Sep. 2017.

[4] L. A. Zadeh, "Fuzzy logic = Computing with words," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 2, pp. 103–111, May 1996.

[5] F.-Y. Wang, "Modeling, analysis and synthesis of linguistic dynamic systems: A computational theory," in *Proc. IEEE Int. Workshop Archit. Semiotic Modelling Situation Control Large Complex Syst.*, Monterey, CA, USA, Aug. 1995, pp. 173–178.

[6] F.-Y. Wang, "On the abstraction of conventional dynamic systems: From numerical analysis to linguistic analysis," *Inf. Sci.*, vol. 171, pp. 233–259, Mar. 2005.

[7] H. Mo and F. Wang, "Linguistic dynamic systems based on computing with words and their stabilities," *Sci. China F, Inf. Sci.*, vol. 52, no. 5, pp. 780–796, 2009.

[8] L. A. Zadeh, "Fuzzy set," *Inf. Control*, vol. 18, no. 3, pp. 338–353, 1965.

[9] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—I," *Inf. Sci.*, vol. 8, no. 3, pp. 199–249, 1975.

[10] Y. Li, Z. Sun, L. Han, and N. Mei, "Fuzzy comprehensive evaluation method for energy management systems based on an Internet of Things," *IEEE Access*, vol. 5, pp. 21312–21322, 2017.

[11] J. M. Mendel, "Uncertainty, fuzzy logic, and signal processing," *Signal Process.*, vol. 80, no. 6, pp. 913–933, 2000.

[12] H. Mo, X. Hao, H. Zheng, Z. Liu, and D. Wen, "Linguistic dynamic analysis of traffic flow based on social media—A case study," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 9, pp. 2668–2676, Sep. 2016.

[13] H. A. Hagras, "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 4, pp. 524–539, Aug. 2004.

[14] C. W. Tao, J. S. Taur, C.-W. Chang, and Y.-H. Chang, "Simplified type-2 fuzzy sliding controller for wing rock system," *Fuzzy Sets Syst.*, vol. 207, no. 8, pp. 111–129, 2012.

[15] J. R. Aguero and A. Vargas, "Calculating functions of interval type-2 fuzzy numbers for fault current analysis," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 1, pp. 31–40, Feb. 2007.

[16] H. Mo, F.-Y. Wang, M. Zhou, R. Li, and Z. Xiao, "Footprint of uncertainty for type-2 fuzzy sets," *Inf. Sci.*, vol. 272, pp. 96–110, Jul. 2014.

[17] H. Mo and F.-Y. Wang, *Language Dynamic System and Type-2 Fuzzy Logic*. Beijing, China: Science and Technology of China Press, 2013.

[18] H. Mo, J. Wang, X. Li, and Z. Wu, "Linguistic dynamic modeling and analysis of psychological health state using interval type-2 fuzzy sets," *IEEE/CAA J. Automatica Sinica*, vol. 2, no. 4, pp. 366–373, Oct. 2015.

[19] F.-Y. Wang and H. Mo, "Some fundamental issues on type-2 fuzzy sets," (in Chinese), *Acta Automatica Sinica*, vol. 43, no. 7, pp. 1114–1141, 2017.

[20] F.-Y. Wang, H. Mo, L. Zhao, and R. M. Li, *Type-2 Fuzzy Sets and Logic*, (in Chinese). Beijing, China: Tsinghua Univ. Press, 2018.

[21] F.-Y. Wang, Y. Yuan, C. Rong, and J. J. Zhang, "Parallel blockchain: An architecture for CPSS-based smart societies," *IEEE Trans. Comput. Social Syst.*, vol. 5, no. 2, pp. 303–310, Jun. 2018.

[22] M. Nakatsuji, Q. Zhang, X. Lu, B. Makni, and J. A. Hendler, "Semantic social network analysis by cross-domain tensor factorization," *IEEE Trans. Comput. Social Syst.*, vol. 4, no. 4, pp. 207–217, Dec. 2017.

Hong Mo received the M.S. degree in fundamental mathematics from Guangxi University, Nanning, China, in 2001, and the Ph.D. degree in control theory and control engineering from the University of Chinese Academy of Sciences, Beijing, China, in 2004.

She is currently a Professor in the College of Electric and Information Engineering, Changsha University of Science and Technology, Changsha, China. She has authored more than 60 papers published in academic journals and conferences and her two academic monographs were published in 2013 and 2018, respectively. Her current research interests include type-2 fuzzy sets, linguistic dynamic systems, and fuzzy theory.

Dr. Mo is currently the member of IEEE ITSS Social Transportation Systems Technical Committee and the Chinese Association of Automation (CAA) Technical Committee of Parallel Intelligence. She is currently the Associate Editor of *International Journal of Fuzzy Systems* and *ACTA Automatica Sinica*. She is currently the Vice Director of CAA Technical Committee of Granular Computing and Multiscale Analysis.

Kefu Yan received the B.Eng. degree from Nanjing Agricultural University, Nanjing, China, in 2015. He is currently working toward the M.S. degree in the College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China.

His current research interests include type-2 fuzzy sets, unmanned vehicle, and intelligent transportation systems.

Xuanming Zhao received the B.Eng. degree from Nankai University, Tianjin, China, in 2016. He is currently working toward the M.S. degree in the College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China.

His current research interests include type-2 fuzzy sets, unmanned vehicle, and intelligent transportation systems.

Yaqiong Zeng received the B.Eng. degree from the Hunan Institute of Engineering, Xiangtan, China, in 2017. She is currently working toward the M.S. degree in the College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China.

Her current research interests include fuzzy sets and intelligent transportation systems.

Xiao Wang (M'16) received the B.S. degree in network engineering from the Dalian University of Technology, Dalian, China, in 2011, and the Ph.D. degree in social computing from the University of Chinese Academy of Sciences, Beijing, China, in 2016.

She is currently an Assistant Researcher in the State Key Laboratory of Management and Control Systems for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing. Her current research interests include social transportation, cyber movement organization, artificial intelligence, and social network analysis.

Fei-Yue Wang (S'87–M'89–SM'94–F'03) received the Ph.D. degree in computer and systems engineering from the Rensselaer Polytechnic Institute, Troy, NY, USA, in 1990.

In 1990, he joined The University of Arizona, Tucson, AZ, USA, where he became a Professor and the Director of the Robotics and Automation Laboratory and the Program in Advanced Research for Complex Systems. In 1999, he founded the Intelligent Control and Systems Engineering Center, Institute of Automation, Chinese Academy of Sciences (CAS), Beijing, China, under the support of the Outstanding Overseas Chinese Talents Program from the State Planning Council and 100 Talent Program from CAS. In 2002, he joined the Lab of Complex Systems and Intelligence Science, CAS, as the Director, where he was the Vice President for Research, Education, and Academic Exchanges with the Institute of Automation from 2006 to 2010. In 2011, he was the State Specially Appointed Expert and the Director of the State Key Laboratory for Management and Control of Complex Systems, Beijing, China. His current research interests include methods and applications for parallel systems, social computing, parallel intelligence, and knowledge automation.

Dr. Wang was elected as Fellow of INCOSE, IFAC, ASME, and AAAS. In 2007, he was a recipient of the National Prize in Natural Sciences of China, the Outstanding Scientist by ACM for his research contributions in intelligent control and social computing, IEEE ITS Outstanding Application, and Research Awards in 2009, 2011, and 2015, and the IEEE SMC Norbert Wiener Award in 2014. He has been the General or Program Chair for more than 30 IEEE, INFORMS, ACM, and ASME conferences. He was the Chair of IFAC TC on Economic and Social Systems from 2008 to 2011. He was the Founding Editor-in-Chief of the *International Journal of Intelligent Control and Systems* from 1995 to 2000 and the *IEEE ITS Magazine* from 2006 to 2007. He was the Editor-in-Chief of IEEE INTELLIGENT SYSTEMS from 2009 to 2012 and the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS from 2009 to 2016. He is currently the President-Elect of the IEEE Council on RFID. He is currently the Editor-in-Chief of the IEEE TRANSACTIONS ON COMPUTATIONAL SOCIAL SYSTEMS, the Founding Editor-in-Chief of the IEEE/CAA JOURNAL OF AUTOMATICA SINICA, and the *Chinese Journal of Command and Control*. He was the President of the IEEE Intelligent Transportation Systems (ITS) Society from 2005 to 2007, the Chinese Association for Science and Technology, USA, in 2005, and the American Zhu Kezhen Education Foundation from 2007 to 2008. He was the Vice President of the ACM China Council from 2010 to 2011. Since 2008, he has been the Vice President and the Secretary General of the Chinese Association of Automation.