

Modeling of mutual inductance between planar inductors on the same plane

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Abstract—This paper presents a compact model of mutual inductance between two planar inductors on the same plane, which is essential to design and optimize multiple output isolated power supply or multi-channel digital isolators. The proposed model considers the influence of shapes, geometric parameters and relative lateral distances of the inductors, and it is derived by solving Neumann's integral. Different planar inductors are simulated by 3-D electromagnetic(EM) software and fabricated on a printed circuit board (PCB) or silicon wafer. The proposed model shows good agreement with simulation and measurement results.

Keywords—mutual inductance; planar inductor; multiple output isolated power supply; multi-channel digital isolator

I. INTRODUCTION

Transformers which provide isolation and transmission of energy and signal are widely used in multiple output isolated power supply and multi-channel digital isolators [1-3]. Crosstalk between inductors on the same plane of multiple transformers will weaken the performance of each transformer, and it is a serious problem in design. Crosstalk is caused by magnetic coupling, which can be expressed by mutual inductance M. The evaluation of M is very important to predict crosstalk and improve device performance. Various approaches have been reported to calculate M, such as the numerical integration of elliptical [4], Bessel function [5] and Neumann's integral function [6]. All of these approaches are used to calculate M between vertical inductors. Thus, it is desirable to have a methodology to calculate M between inductors on the same plane.

In this study, a compact model for predicting M between planar inductors on the same plane is presented. Different shapes, geometric parameters and relative lateral distances have influence on M. Different planar inductors are simulated by 3-D EM software and fabricated on PCB board or silicon wafer. The proposed model shows good agreement with 3-D EM simulation and test results, and the maximum error is only about 8%.

II. ANALYTICAL MODEL OF MUTUAL INDUCTANCE

For circular inductor-a and inductor-b, their physical configuration and geometric parameters are described in Fig.1(a). Geometric parameters include outer diameter d_{out} , turn number N, track width w and track separation s and lateral distance d. To estimate M between circular inductor-a and inductor-b, each turn of the inductors is approximated as constant current carrying filament [7] as shown in Fig.1(b). It

needs to find all the possible combinations of M_{ij} between individual filaments. Then M_{ij} is calculated by Neumann's equation as shown in (1). Total M is calculated by adding all the combinations of M_{ij} . If inductor-a and inductor-b are square, octagon or other shapes, they are required to be equivalent to circular inductor with the same geometric parameters. Then the M between equivalent circular inductors is calculated by using the above method, and finally it is corrected to obtain the final M between inductor-a and inductor-b. Equation (2) is M between inductor-a and inductor-b which considers the influence of shapes, geometric parameters and relative lateral distances.

$$\begin{aligned} M_{ij} &= \frac{\mu_0}{4\pi} \oint_{c_i} \oint_{c_j} \frac{1}{R_{ij}} d\vec{l}_1 d\vec{l}_2 \\ &= \frac{\mu_0}{4\pi} a_i b_j \oint_0^{2\pi} \oint_0^{2\pi} \frac{\cos(\beta - \alpha)}{R_{ij}} d\beta d\alpha \end{aligned} \quad (1)$$

where:

$$\begin{aligned} R_{ij} &= \sqrt{(a_i + b_j + d_{ij} + b_j \cos\beta - a_i \cos\alpha)^2 + (b_i \sin\beta - a_i \sin\alpha)^2} \\ a_i &= \frac{d_{out_a}}{2} - (i-1)(w_a + s_a) - \frac{w_a}{2} \\ b_j &= \frac{d_{out_b}}{2} - (j-1)(w_b + s_b) - \frac{w_b}{2} \\ d_{ij} &= d + (i-1)(w_a + s_a) + (j-1)(w_b + s_b) + \frac{w_a}{2} + \frac{w_b}{2} \end{aligned}$$

Where d_{out_a}/d_{out_b} , N_a/N_b , w_a/w_b , s_a/s_b are outer diameter, turn number, track width, and track separation of inductor-a and inductor-b respectively.

$$M = k_1 k_2 \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} M_{ij} \quad (2)$$

Where k_1 and k_2 are correction factor, which are equal to area ratio of the inductor-a and inductor-b to the equivalent circular inductors, respectively. k_1 and k_2 are summarized in table1. When the shape of inductor is modified octagon as shown in Fig.2, k_1 and k_2 can be corrected as (3), where C_1 and C_2 are shown in Fig.2.

$$k_{1,2} = k_0 \frac{4}{\pi} = \frac{d_{out} d_{out} - 2C_1 C_2}{d_{out} d_{out}} \frac{4}{\pi} \quad (3)$$

TABLE I. K VERSUS SHAPE

shape	circular	square	octagon	Modified-octagon
k_1	1	$4/\pi$	1.05	$4k_0/\pi$
k_2	1	$4/\pi$	1.05	$4k_0/\pi$

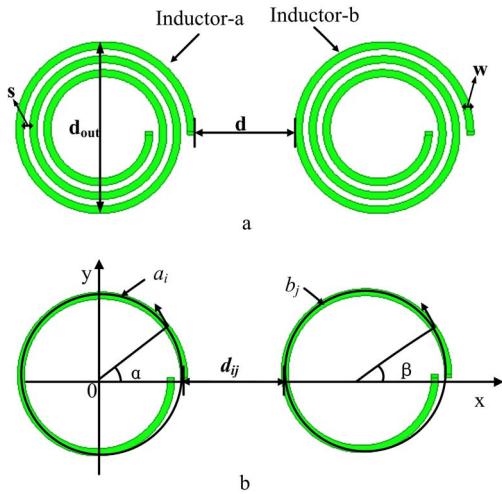


Fig.1. The circular inductors on the same plane(a) Physical configuration (b) Each turn is approximated as constant current carrying filament.

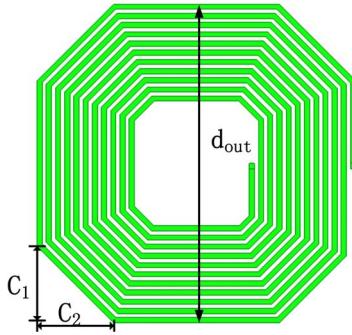


Fig.2. Physical configuration of the modified octagon inductor

III. RESULTS AND DISCUSSION

To validate the proposed model, several inductors are simulated by 3-D EM software and fabricated on PCB board and silicon wafer. Inductor-a and Inductor-b have same geometric parameters which are described in table 2. Agilent N5242A vector network analyzer (VNA) is used to measure 2-port s-parameters which are converted to Z-parameters. M of simulation and measurement are calculated by $\text{Im}(Z_{21})/2\pi f$ at 20MHz.

Fig.3 shows M of proposed model calculated, simulated and measured with different shapes, geometric parameters and lateral distances. Fig.4 shows the comparison of model with simulation and measurement. In Fig.3(a)-3(c) and Fig.4(a)-4(c), inductors are fabricated on PCB board and the shapes are circular, square and octagon respectively. Two outer diameters 136mil and 160mil are considered. In Fig.3(d) and Fig.4(d), inductors are fabricated on silicon wafer and the shape is modified octagon. It can be seen that M is decided by the shapes, geometric parameters and relative lateral distances of the inductors. It increases with

the increase of outer diameter and turn number and the decrease of relative lateral distance. The calculated value of the model is in good agreement with simulated and measured results. The maximum error is only about 8%.

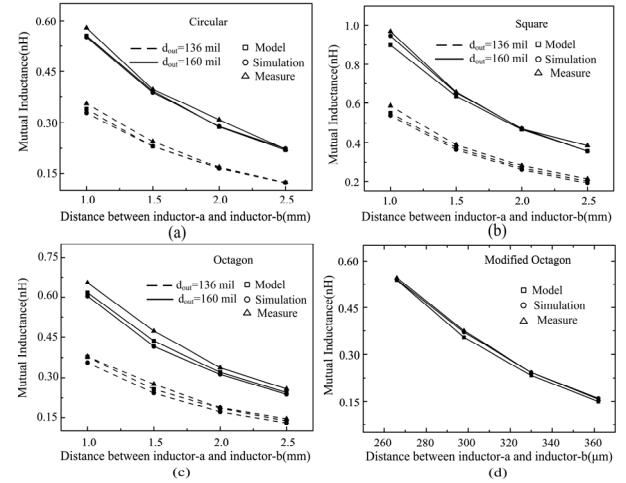


Fig.3. M of proposed model calculated, simulated and measured with different shapes, geometric parameters and distance.(a)Circular, (b)Square, (c) Octagon, (d) Modified octagon

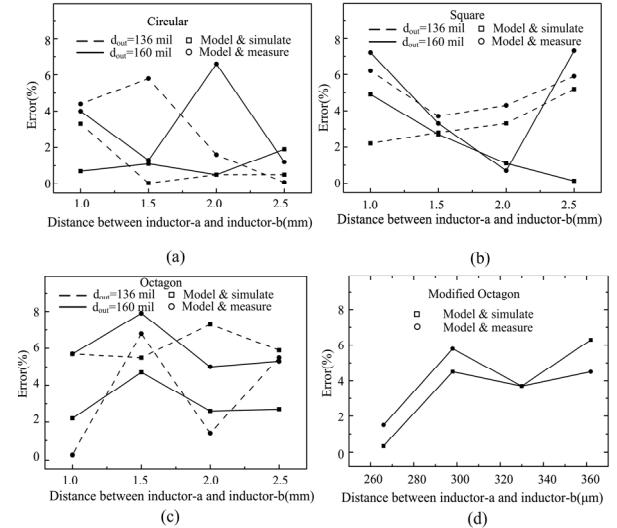


Fig.4. Comparison of model, simulation and measurement.(a) Circular, (b)Square, (c) Octagon, (d) Modified octagon

TABLE II. GEOMETRIC PARAMETERS OF INDUCTOR-A/B

shape	type	d_{out}	w	s	N	d
cir	PCB	136/160mil	8mil	8mil	3	1/1.5/2/2.5mm
squ	PCB	136/160mil	8mil	8mil	3	1/1.5/2/2.5mm
oct	PCB	136/160mil	8mil	8mil	3	1/1.5/2/2.5mm
mod-oct	chip	554um	9um	7um	11	266um
mod-oct	chip	522um	9um	7um	10	298um
mod-oct	chip	490um	9um	7um	9	330um
mod-oct	chip	458um	9um	7um	8	362um

IV. CONCLUSION

The mutual inductance M between two inductors on the same plane causes crosstalk, which affects device performance. A compact model to predict M and its

mathematical derivation are presented. The model considers the influence of shapes, geometric parameters and relative lateral distances. The calculated value of model is in good agreement with simulated and measured value with a maximum error of 8%. The proposed model can provide theoretical guidance for reducing crosstalk and improving device performance.

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