

# An expression for oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator

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**Abstract**—Oscillation amplitude is a key factor for NMOS/PMOS complementary cross-coupled LC-tank oscillator, and its proper estimation is beneficial to design oscillator and select process. An ordinary expression for oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator is derived by solving Shichman-Hodge equation using Fourier Transform. It mainly depends on power supply voltage and threshold voltage. The effects of power supply voltages, transistor's sizes and inductor values are discussed. The calculation and Cadence simulation of oscillation amplitude show excellent agreement with error less than 5%.

**Keywords**—NMOS/PMOS complementary cross-coupled; LC-tank oscillator; oscillation amplitude

## I. INTRODUCTION

Cross-coupled LC-tank oscillator is widely used in fully integrated electronic device such as digital isolator<sup>[1,2]</sup>, isolated power supply<sup>[3]</sup> and transceivers<sup>[4]</sup> et al. The common structures are NMOS cross-coupled and NMOS/PMOS complementary cross-coupled. NMOS cross-coupled LC-tank oscillator needs to use center tapped inductor which usually implemented by two cascaded inductors. NMOS/PMOS complementary cross-coupled LC-tank oscillator uses only one inductor that significantly saves chip area. Therefore, NMOS/PMOS complementary cross-coupled LC-tank oscillator is very popular with strict area applications.

Oscillation amplitude will directly affect the withstand voltage of MOS transistors and electronic device performance, such as signal transmission quality of digital isolator<sup>[2]</sup>, output voltage of isolated power supply<sup>[3]</sup> and phase noise of transceiver<sup>[5]</sup>. Therefore its proper estimation is a key factor in circuit design and process selection. Owing to the fact that a cross-coupled LC-tank oscillator is a nonlinear circuit, it is difficult to accurately predict oscillation amplitude. In [6], oscillation amplitude of source-tailed NMOS/PMOS complementary cross-coupled LC-tank oscillator is analyzed, which depends on tail current. But it is not suitable for the oscillators without tail current source.

In this paper, an ordinary expression for oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator without tail current source is derived by solving Shichman-Hodge equation using Fourier Transform. Cadence simulation has been carried out to assert the accuracy of the expression. The simulated and calculated oscillation amplitude show excellent agreement with error less than 5%.

## II. OSCILLATION AMPLITUDE

The schematic of NMOS/PMOS complementary cross-coupled LC-tank oscillator is shown in Fig.1.

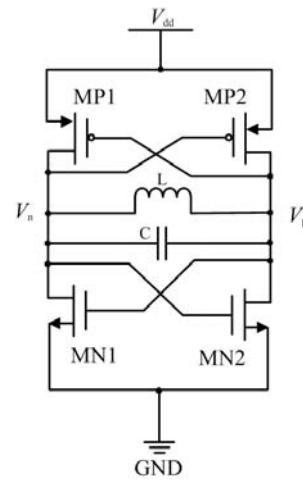


Fig. 1. Schematic of NMOS/PMOS complementary cross-coupled LC-tank oscillator

Since the two outputs of the oscillator are 180° out of phase, the output voltages of Fig.1 can be expanded in a Fourier series as the following

$$V_n(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)$$

$$V_p(t) = A_0 + \sum_{n=1}^{\infty} (-1)^n A_n \cos(n\omega t) + \sum_{n=1}^{\infty} (-1)^n B_n \sin(n\omega t) \quad (1)$$

Where  $A_0$ ,  $A_n$  and  $B_n$  are the Fourier series coefficients.

Owing to the fact that MOSFETS are acting like switches, for simplicity, we can approximately consider that during the first half of each period  $0 \leq t \leq T/2$ , MN1, MP2 are on and work in the triode region<sup>[7]</sup>, their current are  $i_{s1}$ , MN2, MP1 are off and their current are zero. During the second half of each period  $T/2 \leq t \leq T$ , MN1, MP2 are off and their current are zero, MN2, MP1 are on and work in the triode region<sup>[7]</sup>, their current are  $i_{s2}$ .  $i_{s1}$  of MN1 can be obtained from the following Shichman-Hodge equation

$$i_{s1} = k \left[ (V_p - V_m) \times V_n - \frac{1}{2} V_n^2 \right] \quad (2)$$

$$k = \mu_n C_{ox} \frac{W}{L}$$

where  $\mu_n$  is inversion layer mobility,  $C_{ox}$  is gate oxide capacitance per unit area,  $V_{in}$  is threshold voltage, and  $W$  and  $L$  are the width and length of MN1, respectively.

Since the drain current  $i_{s1}$  is periodic, it can be represented by the Fourier series as the following

$$i_{s1} = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \quad (3)$$

In (3),  $a_n$  and  $b_n$  can be given by

$$a_n = \frac{\omega}{\pi} \left[ \int_0^{\pi} i_{s1} \cos(n\omega t) dt + \int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} 0 \cos(n\omega t) dt \right]$$

$$b_n = \frac{\omega}{\pi} \left[ \int_0^{\pi} i_{s1} \sin(n\omega t) dt + \int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} 0 \sin(n\omega t) dt \right] \quad (4)$$

For the sake of simplicity, assuming that only the amplitude of first harmonic is considered, second and higher order harmonic can be ignored. Equation (1) is simplified to the following equation

$$V_n(t) = A_0 + A_1 \cos(\omega t) + B_1 \sin(\omega t)$$

$$V_p(t) = A_0 - A_1 \cos(\omega t) - B_1 \sin(\omega t) \quad (5)$$

Using (2)-(5) for solving  $A_1$  and  $B_1$

$$A_1 = \frac{\pi}{4} (A_0 + V_{tn}) \quad (6)$$

$$B_1 = \sqrt{|A_0(A_0 - 2V_{tn})|} \quad (7)$$

$A_0$  can be approximated as followings

$$A_0 = V_{dd} / 2 \quad (8)$$

Where  $V_{dd}$  is power supply voltage.

Oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator in differential output can be approximated as (9), which depends on power supply voltage and threshold voltage.

$$h = 2\sqrt{A_1^2 + B_1^2} = 2\sqrt{\frac{\pi^2}{16} \left( \frac{V_{dd}}{2} + V_{tn} \right)^2 + \left| \frac{V_{dd}^2}{4} - V_{dd}V_{tn} \right|} \quad (9)$$

### III. RESULTS AND DISCUSSION

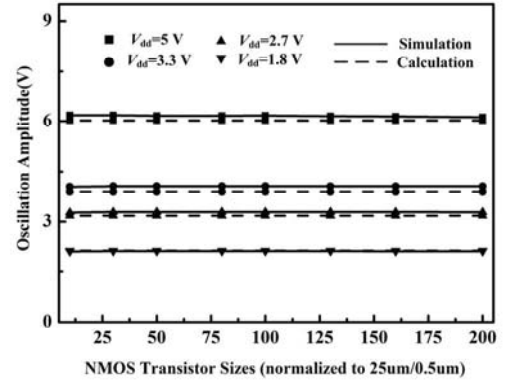
In order to check the accuracy of (9), several NMOS/PMOS complementary cross-coupled LC-tank oscillators are simulated in Cadence. In these oscillators, inductance values and transistor sizes are varied over a relatively large range, PMOS size is always 1.2 times larger than NMOS size for having the same transconductance, parasitic capacitance of transistors is used as capacitance of LC-tank, power supply voltage  $V_{dd}$  are 5 V, 3.3 V, 2.7 V and 1.8 V respectively, threshold voltage  $V_{tn}$  is fixed as 0.72 V.

In Fig. 2, the inductance value is fixed as 45nH, NMOS transistor size changes from 10 to 200 which is normalized to the minimum size 25um/0.5um. Fig.2(a) shows calculated

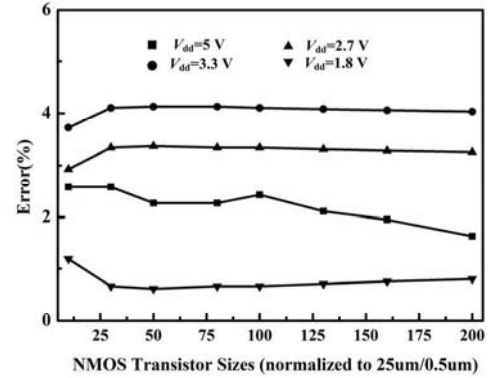
and simulated oscillation amplitude at different transistor sizes. Fig.2(b) shows the comparison between expression calculation and simulation results.

In Fig. 3, NMOS transistor size is fixed as 100x25um/0.5um, inductance value changes from 15nH to 95nH. Fig.3(a) shows calculated and simulated oscillation amplitude at different inductance values. Fig.3(b) shows the comparison between expression calculation and simulation results.

In these figures, it can be found transistor sizes and inductance values have little effect on the oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator. Equation (9) predict the oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank oscillator quite accurately. The error in all cases is less than 5%.

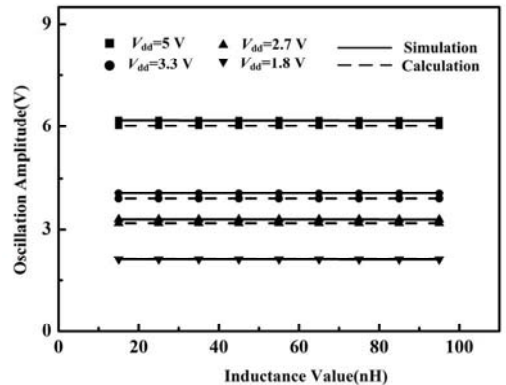


(a)



(b)

Fig. 2. (a) Calculated and simulated oscillation amplitude at different transistor sizes, (b) Comparison with simulation results



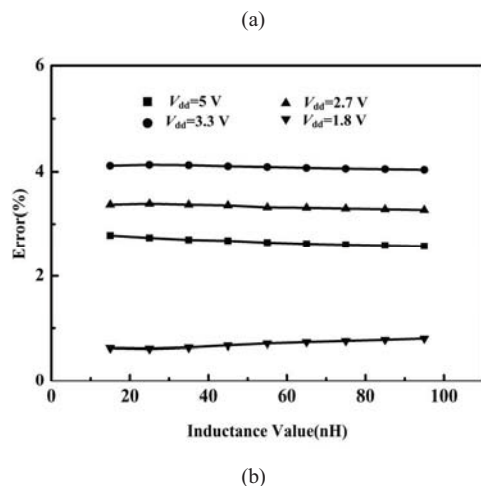


Fig. 3. (a) Calculated and simulated oscillation amplitude at different inductance values, (b) Comparison with simulation results

#### IV. CONCLUSION

In this paper, an ordinary expression for oscillation amplitude of NMOS/PMOS complementary cross-coupled LC-tank is derived, which depends on power supply voltage and threshold voltage. Oscillation amplitude at different power supply voltages, transistor's sizes and inductor values are simulated by Cadence and compared with the derived expression. The proposed expression can successfully predict

oscillation amplitude and be applicable to the design of NMOS/PMOS complementary cross-coupled LC-tank oscillator. This method is also applicable to derive the oscillation amplitude of NMOS or PMOS cross-coupled LC-tank oscillator, but the expression and precision may be different.

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