Optimizing Sum-Capacity Through Power Allocation for SLNR-precoding-based Cognitive Networks

Kai Zhao, Haixia Zhang, and Dongfeng Yuan
Wireless Mobile Communication and Transmission Lab.
School of Information Science and Engineering
Shandong University, Jinan, China, 250100
E-mail: sduzhaokai@mail.sdu.edu.cn, {haixia.zhang,dfyuan}@sdu.edu.cn

***Abstract*—Cognitive radio (CR) has great potential to improve the spectral efficiency of future wireless networks. This paper focuses on maximizing the sum-capacity of cognitive wireless networks based on signal to leakage noise ratio (SLNR) precoding and hybrid opportunistic spectrum access scheme. We propose a globally optimal power allocation scheme based on a combination of the Branch and Bound framework (B&B) and convex relaxation technique to maximize the sum capacity of all secondary users (SUs). Simulation results indicate that, with the proposed power allocation scheme, the sum capacity of the secondary network can be improved compared to conventional SLNR-precoding-based power allocation schemes.**

***Index Terms*—Power allocation, cognitive radio (CR), signal to leakage noise ratio (SLNR) precoding.**

I. Introduction

New applications operating on the mobile internet network demand for higher speed wireless communications. However, the spectrum authorized by Federal Communications Commission (FCC) is limited, and then, utilizing the spectrum efficiently has been an important and challenging task for wireless communications. Cognitive radio (CR) is proposed and taken as a acknowledged potential technology that will be used in the future standard to improve the spectrum efficiency [1], [2]. To further improve the spectrum efficiency, a hybrid opportunistic spectrum access scheme is proposed in reference [3].

Multiple antenna technique provides some improvements for the spectrum efficiency due to the space utilization. In multiple antenna system, especially the multi-user system, precoding is applied to combat fading and suppress interference from other antennas and other users [4], [5]. Thus, the spectrum efficiency can be ensured from a certain point of view. Considering the characteristic of cognitive radio, the traditional precoding can be expanded to CR system [6], [7]. Additionally, in the precoding design, the transmit signals and the norms of the precoding matrix are generally all normalized, which means that equal power is allocated at the transmitter. Therefore, there still exists improvement space for the precoding-based system through power allocation. Some schemes that jointly considered precoding and power allocation were proposed in multi-user MIMO downlink [8],[9], cognitive radio networks [10] and cooperative communications scenario [11].

In reference [12], Sadek et al. proposed a new precoding scheme with the help of a new concept signal-to-leakage-and-noise ratio (SLNR) and got the designed precoding in multiuser MIMO downlink. Furthermore, some improved schemes such as jointly considering the dirty paper coding (DPC) [13] and SLNR precoding research in MIMO-OFDM system [14] enriched the study of SLNR precoding. We find that, from a certain perspective, the SLNR precoding is suit to be used in the CR system especially. To be specific, the objective of the SLNR precoding is to increase the power ratio between the intending signal and the leakage signal plus noise. This is rather helpful in CR networks, where the secondary users need to reduce the interference to the primary user to satisfy the interference temperature constraint. Therefore, we focus on the SLNR-precoding-based CR systems and investigate the further improvements of spectrum efficiency through power allocation.

In this paper, we concentrate on multi-carrier cognitive wireless networks with the objective of improving the spectrum efficiency through jointly considering the effects of spectrum access mode, interference suppression and power controlling. To be specific, for the SLNR-precoding-based cognitive wireless networks where hybrid opportunistic spectrum access is applied, we propose a globally optimal power allocation scheme for the SUs to maximize their sum capacity. We formulate the power allocation problem as a non-convex optimization problem, which is hard to be solved by the traditional convex optimization method. To overcome this, we design an algorithm based on branch and bound (B&B) framework together with convex relaxation technique, and acquire the globally optimal solution. Simulation results indicate that the proposed power allocation can achieve some sum-capacity improvements compared to three existing power allocations according to the SLNR precoding.

The rest of the paper is organized as follows. In Sec-

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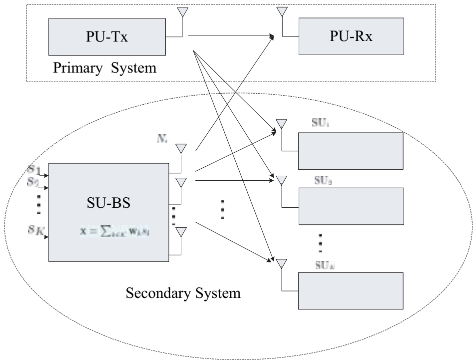

Fig. 1: System Model

tion II, we introduce the system model and in Section III, we formulate the problem of sum capacity maximized through power allocation. In Section IV, we describe the proposed globally optimal power allocation scheme. Simulation results are presented in section V. Finally, we draw some conclusions in Section VI.

Notations: $\mathbf{V}^{\mathrm{H}}$, $\|\mathbf{V}\|$ and $\mathbf{V}^{-1}$ represent the conjugate transpose, the norm and inverse transformation of matrix $\mathbf{V}$, respectively. Matrix $\mathbf{I}$ denotes the identity matrix. $|v|$ denotes the absolute value of scalar v.

II. System Model

In this paper, we consider a cognitive radio networks shown in Fig. 1. A single antenna primary user transmitter communicates with a receiver equipped with a single antenna in the licensed frequency, which is divided into N sub-channels, denoted by $\mathcal{N}$. Each sub-channel is taken as a flat fading channel with constant channel coefficients in a time block.

The secondary user system is modeled as a multi-user multiple-input single-output (MISO) system and accesses the sub-channels following the hybrid opportunistic spectrum access (H-OSA) scheme referred in [3], based on which the SUs access the idle sub-channels with no constraints and the sub-channels where the PUs exist under the interference constraint. A secondary base station with N_t antennas serves a set of K single antenna secondary receivers denoted as $\mathcal{K}$.

We focus on the downlink communication where SLNR-based precoding is processed on each subcarrier at the secondary base station. Then, the transmitted signal $\mathbf{x}_j$ before power allocation on subcarrier $j \in \mathcal{N}$ writes

$$\mathbf{x}_j = \sum_{k\in\mathcal{K}} \mathbf{w}_{k,j} s_{k,j},\ \forall j \in \mathcal{N}, \tag{1}$$

where $\mathbf{w}_{i,j} \in \mathbb{C}^{N_t\times 1}$ is the SLNR precoding vector, which will be discussed later in Section III and $s_{i,j}$ is the transmitted symbol of secondary user $i, i \in \mathcal{K}$ on sub-channel j. Without loss of generality, both the precoding vectors and the transmitted signals are normalized. Then, the total transmitted power is taken as $P_t = K \times N$.

The full channel state information including the secondary user system and the licensed user can be perceived and obtained at the secondary transmitter. Considering both the large scale loss and small scale loss simultaneously, we formulate the wireless channel on each subcarrier as a flat fading Rayleigh model. The parameters $\rho_{i,j}^{(s,s)}, \rho_{i,j}^{(l,s)}$ and $\rho_j^{(s,l)}$ are taken to denote the large scale loss caused by path loss effect according to the channels from the secondary base station to the secondary user i, from primary transmitter to the secondary user i, and from secondary base station to primary receiver on the subcarrier j. In addition, we denote the interference channel vector from the secondary base station to the primary receiver as $\mathbf{g}_j \in \mathbb{C}^{1\times N_t}$, the channel component from the primary user to the secondary receiver i as $h_{i,j}^{l,s} \in \mathbb{C}^{1\times 1}$ and the channel vector from the secondary transmitter to the secondary user i as $\mathbf{h}_{i,j} \in \mathbb{C}^{1\times N_t}$ to measure the small scale loss caused by multipath effect on the subcarrier j. Every element of the channel vectors referred above is an independent complex gaussian variable with zero-mean together with unit-variance. Then, the path-loss factors can be denoted as

$$\rho_{i,j}^{(s,s)} = (d_{i,j}^{s,s})^{-\alpha}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{2}$$

$$\rho_{i,j}^{(l,s)} = (d_{i,j}^{l,s})^{-\alpha}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{3}$$

$$\rho_j^{(s,l)} = (d_j^{s,l})^{-\alpha}, \forall j \in \mathcal{N}, \tag{4}$$

and the channel vectors write as

$$\hat{\mathbf{h}}_{i,j} = \sqrt{\rho_{i,j}^{(s,s)}}\mathbf{h}_{i,j}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{5}$$

$$\hat{h}_{i,j}^{l,s} = \sqrt{\rho_{i,j}^{(l,s)}} h_{i,j}^{l,s}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{6}$$

$$\hat{\mathbf{g}}_j = \sqrt{\rho_j^{(s,l)}}\mathbf{g}_j, \forall j \in \mathcal{N}. \tag{7}$$

where α is the pass loss exponent while d is the corresponding distance.

Then, we denote the interference from the licensed transmitter to the secondary user i on the sub-carrier j by $\mu_{i,j}$ and the power of $\mu_{i,j}$ can be quantized as follows

$$I_{i,j}^{(l,s)} = \theta_j P_{tj}^{(l)} \rho_{i,j}^{(l,s)} |h_{i,j}^{l,s}|^2, \tag{8}$$

where $P_{tj}^{(l)}$ is the primary transmitter power on the subcarrier j and the parameter θ_j denotes whether the primary user occupies the subcarrier j. This parameter is set one when the primary user occupies the subcarrier j and otherwise, the parameter is set zero.

After the power allocation, the signal at the receiver i on the subcarrier j can be shown as

$$y_{i,j} = \hat{\mathbf{h}}_{i,j}\sqrt{p_{i,j}}\mathbf{w}_{i,j}s_{i,j} + \hat{\mathbf{h}}_{i,j}\sum_{k\in\mathcal{K}/i}\sqrt{p_{k,j}}\mathbf{w}_{k,j}s_{k,j} + \mu_{i,j} + n_{i,j}, \tag{9}$$

where $p_{i,j}$ is the power allocated to the user i on the subcarrier j. The term $n_{i,j}$ denotes the additive noise at the receiver i on the subcarrier j with zero-mean and variance $\sigma_{i,j}^2$.

In addition, the power of the interference from the secondary system to the primary receiver is denoted as

$$I_j^{(s,l)} = \sum_{k\in\mathcal{K}} |\hat{\mathbf{g}}_j \mathbf{w}_{k,j}|^2 p_{k,j}. \tag{10}$$

III. Problem Formulation

With the help of the notation leakage, the SLNR precoding has been proposed in the reference [12]. In our communication model, the SLNR defined when equal power is allocated to each user can be denoted as (11)

$$SLNR_{i,j} = \frac{|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{i,j}|^2}{\sigma_{i,j}^2 + I_{i,j}^{(l,s)} + |\hat{\mathbf{g}}_j \mathbf{w}_{i,j}|^2 + \sum_{k\in\mathcal{K}/i} |\hat{\mathbf{h}}_{k,j}\mathbf{w}_{i,j}|^2}, \tag{11}$$

By maximizing (11), the normalized precoding vector, expressed as $\mathbf{w}_{i,j}$, can be got by the method in [12], i.e., the normalized eigenvector corresponding to the maximum eigenvalue of the matrix defined in (12),

$$((\sigma_{i,j}^2 + I_{i,j}^{(l,s)})\mathbf{I} + \hat{\mathbf{g}}_j^{\mathrm{H}}\hat{\mathbf{g}}_j + \sum_{k\in\mathcal{K}/i} \hat{\mathbf{h}}_{k,j}^{\mathrm{H}}\hat{\mathbf{h}}_{k,j})^{-1}\hat{\mathbf{h}}_{i,j}^{\mathrm{H}}\hat{\mathbf{h}}_{i,j} \tag{12}$$

For a fixed subcarrier, the existence state of the primary user leads to the different precoding vectors.

Our objective is to search for the power allocation scheme based on the precoding vector acquired with equal power allocation that can maximize the sum capacity of all the secondary users on the whole bandwidth. On the subcarrier j with bandwidth B_s, in a certain time block, the SINR for user i after the power allocation writes as

$$SINR_{i,j} = \frac{p_{i,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{i,j}|^2}{\sigma_{i,j}^2 + I_{i,j}^{(l,s)} + \sum_{k\in\mathcal{K}/i} p_{k,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{k,j}|^2}, \tag{13}$$

and the corresponding capacity can be expressed as

$$C_{i,j} = B_s \log(1 + SINR_{i,j}), \tag{14}$$

where all the channel vectors are constant and known. [1]

We use the set notation $\mathcal{N}_1$ to cover all the subcarriers where the primary user occupies and the notation $\mathcal{N}_0$ to include the subcarriers idle in a certain time block. Then, with $\mathbf{P} = [(p_{i,j})_{i\in\mathcal{K},j\in\mathcal{N}}]$ denoted as the notation to represent the power allocation scheme, the sum capacity of all the users on all the subcarriers can be expressed as

$$C(\mathbf{P}) = \sum_{j\in\mathcal{N}}\sum_{i\in\mathcal{K}} C_{i,j} = \sum_{j_1\in\mathcal{N}_1}\sum_{i\in\mathcal{K}} C_{i,j_1} + \sum_{j_0\in\mathcal{N}_0}\sum_{i\in\mathcal{K}} C_{i,j_0}, \tag{15}$$

and the optimization problem can be formulated as

$$\text{Given}: \mathbf{h}_{i,j}, \mathbf{w}_{i,j}, \mathbf{g}_j, \sigma_{i,j}^2, h_{i,j}^{l,s}, P_t, P_{tj}^{(l)}, Q_j, B_s \tag{16}$$

$$\rho_{i,j}^{(s,s)}, \rho_{i,j}^{(l,s)}, \rho_j^{(s,l)}, \theta_j, \epsilon, \forall i\in\mathcal{K}, \forall j\in\mathcal{N} \tag{17}$$

$$\text{Find}: p_{i,j}, \forall i\in\mathcal{K}, \forall j\in\mathcal{N} \tag{18}$$

$$\text{Maximize}: C(\mathbf{P}) \tag{19}$$

$$\text{Subject to}: \sum_{j\in\mathcal{N}}\sum_{i\in\mathcal{K}} p_{i,j} \le P_t \tag{20}$$

$$p_{i,j} \ge \epsilon, \forall i\in\mathcal{K}, \forall j\in\mathcal{N} \tag{21}$$

$$I_j^{s,l} \le Q_j, \forall j\in\mathcal{N}_1, \tag{22}$$

[1] The capacity in (14) is defined by the natural log function and with the unit nats per second per hertz (nps/Hz). This makes the descriptions about the derivation of the log curve in the IV-B more convenient and makes no difference on the pursuit of the optimal power allocation compared with the capacity defined with the unit bits per second per hertz (bps/Hz).

where a positive number ϵ small enough compared to zero is used to replace zero in the constraint (21) to make this constraint compact. In addition, the constraint (22) denotes that the interference to the primary receiver should be kept below the interference temperature constraint Q_j defined in advance when the primary user exists on the subcarrier j.

IV. Globally Optimal Power Allocation Scheme

The formulated problem is a nonlinear nonconvex problem and can not be solved by the traditional convex optimization, In our paper, with the help of the branch and bound (B&B) framework and the convex relaxation technique, we design a globally optimal power allocation algorithm to achieve the maximized sum capacity of all the secondary users. The readers can be referred to [15] and [16] for more details about branch and bound framework and relaxation technique. The following introduction of the designed algorithm will be started by the structure of the main algorithm and then continued with the key steps in the main algorithm, including the convex relaxation processing, bounds determining, together with the variable space partitioning.

A. Structure of the Designed Algorithm

The designed globally optimal power allocation scheme is on the basis of the branch and bound framework. Through the convex relaxation technique, the formulated problem defined in (16)-(22) can be transferred to a linear programming, which can be solved by the traditional convex optimization and lead to a upper bound of the primitive problem. On the other hand, this transformation will bring some relaxation errors which will tend to be smaller as the variable space decreases and this leads to smaller differences between relaxed optimal and actual optimal. These two aspects make the branch and bound framework take a role and converge. The detailed description of the convex relaxation technique about this problem will be described in the next subsection.

The objective of the designed algorithm is to obtain a solution f^* close to the primitive optimal f according to $C(\mathbf{P})$ defined in (19) at any precision. The precision can be measured by the parameter ε and this relationship between f^* together with f can be denoted as

$$f^* \ge f(1-\varepsilon). \tag{23}$$

Under the branch and bound framework, both the globally upper bound ($\mathrm{UB}_{\mathrm{glb}}$) and the globally lower bound ($\mathrm{LB}_{\mathrm{glb}}$) of f can be acquired. In our designed algorithm, we use the relatively stricter criterion

$$\mathrm{LB}_{\mathrm{glb}} \ge (1-\varepsilon)\mathrm{UB}_{\mathrm{glb}}. \tag{24}$$

to replace the inequality defined in (23) as the flag to terminate the algorithm and take the corresponding $\mathrm{LB}_{\mathrm{glb}}$ as the obtained f^* when (24) is satisfied.

We define the initial variable space as $\mathcal{S}_1$, which represents the original search space for the globally optimal power allocation (including all the possible power allocation schemes for different users on each subcarrier). In the initial variable space, the whole variable space is taken as one variable set, denoted as $\mathcal{S}_1^1$, and, as the branch and bound framework continues, after each iterative step, certain variable set in the variable space at the last iterative step will be partitioned into two new generated variable sets. We denote the variable space at the iterative step t as

$$\mathcal{S}_t = \{\mathcal{S}_t^s,\ s = 1, 2, \cdots t\}, \tag{25}$$

which consists of t variable sets denoted as $\mathcal{S}_t^1 \cdots \mathcal{S}_t^t$. At this iterative step t, through the relaxed linear programming, according to every variable set $\mathcal{S}_t^s$ in the variable space $\mathcal{S}_t$, both the upper bound $\mathrm{UB}(\mathcal{S}_t^s)$ and the lower bound $\mathrm{LB}(\mathcal{S}_t^s)$ can be acquired. The convex relaxation technique and bound determining will be introduced in detail in the IV-B.

A key procession of the globally optimal algorithm is to determine the globally upper bound $\mathrm{UB}_{\mathrm{glb}}$ and the globally lower bound $\mathrm{LB}_{\mathrm{glb}}$. At the iterative step t, all the variable sets corresponding to the iterative step t lead to a complete but narrower relaxed space to search for the upper bound of the primitive optimal compared with the variable sets at the iterative step $(t-1)$, which can be well understood after the subsection IV-B. Due to this, we take the maximum upper bound according to the variable sets at this variable space $\mathcal{S}_t$ as $\mathrm{UB}_{\mathrm{glb}}$, which is larger than the maximum of the primitive objective function and non-increased as the iterative algorithm based on branch and bound framework continues, i.e. ,

$$\mathrm{UB}_{\mathrm{glb}} = \max\{\mathrm{UB}(\mathcal{S}_t^s),\ s = 1, 2, \cdots t\}. \tag{26}$$

In addition, due to the method to determine the lower bound for each variable set, which will be introduced in the subsection IV-B, we choose the maximum lower bound in all the computed variable sets since the algorithm begins. So, the $\mathrm{LB}_{\mathrm{glb}}$ is non-decreased as the algorithm continues, i.e. ,

$$\mathrm{LB}_{\mathrm{glb}} = \max\{\mathrm{LB}(\mathcal{S}_u^s),\ u = 1, 2, \cdots t, s = 1, 2, \cdots u\}. \tag{27}$$

Then, the convergence of the algorithm can be ensured.

After each iterative step, the criterion in (24) will be checked to decide whether the algorithm should be continued. If the criterion in (24) is satisfied, the $\mathrm{LB}_{\mathrm{glb}}$ is taken as the maximum $C(\mathbf{P})$ and the corresponding power allocation is used as the optimal power scheme. Otherwise, certain variable set at this iterative step should be chosen and partitioned to continue the branch and bound framework, which will be introduced at length in IV-C.

B. Convex relaxation and bounds determining

The capacity expression for the user i on the subcarrier j defined in (14) can be expressed as

$$\begin{aligned} C_{i,j} =& B_s \log(\sigma_{i,j}^2 + I_{i,j}^{(l,s)} + \sum_{k\in\mathcal{K}} p_{k,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{k,j}|^2) \\ & \underbrace{-B_s \log(\sigma_{i,j}^2 + I_{i,j}^{(l,s)} + \sum_{k\in\mathcal{K}/i} p_{k,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{k,j}|^2)}_{(\star)}. \end{aligned} \tag{28}$$

In (28), the term $(\star)$ makes the optimization problem a nonconvex problem and hard be solved by the traditional convex optimization. Therefore, we construct some new variables and with the help of these variables, reform the primitive nonconvex problem into a linear programming problem.

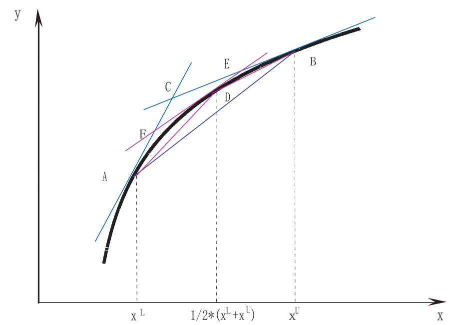

Fig. 2: Illustration of the Linear Relaxation

To be specific, we take an illustration to express the main idea of convex relaxation. As described in Fig. 2, a standard log curve $\log x$ between x^L and x^U can be restricted and expressed by the triangle region ABC which is constructed by the tangents at the $(x^L, \log x^L)$ together with $(x^U, \log x^U)$, and the secant linked them. This relaxation operation brings some relaxation errors. After the variable space is partitioned, the primitive log curve can be further represented by the triangle regions BDE and ADF, which will lead to smaller relaxation errors compared to the primitive relaxation. Because the relaxed triangle regions to obtain optimal are enlarged and include the primitive $\log$ curve, the maximum acquired by the relaxed triangle regions is not lower than the maximum of the primitive combination of $\log$ curves. So, we use the optimal value got by the relaxed linear programming as the upper bound of primitive $\log$ function. In addition, the upper bound of the primitive function acquired by the relaxed linear programming is non-increased after the variable space is partitioned because of the smaller relaxation errors.

In allusion to our formulated problem, we use several new variables corresponding to the relaxed linear regions to substitute the $\log$ function in (28) and, then, the optimization objective is transferred into a linear problem. To be specific, firstly, we construct the new variables as

$$a_{i,j} = \sigma_{i,j}^2 + I_{i,j}^{(l,s)} + \sum_{k\in\mathcal{K}} p_{k,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{k,j}|^2 \tag{29}$$

$$x_{i,j} = \sigma_{i,j}^2 + I_{i,j}^{(l,s)} + \sum_{k\in\mathcal{K}/i} p_{k,j}|\hat{\mathbf{h}}_{i,j}\mathbf{w}_{k,j}|^2. \tag{30}$$

The variables defined in (29)-(30) can be expressed by the variables $p_{i,j}$, which are named core variables, and then, the variables defined in (29)-(30) can be bounded through bounding the core variables, i.e.,

$$p_{i,j} \le p_{i,j}^U, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} \tag{31}$$

$$p_{i,j} \ge p_{i,j}^L, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}. \tag{32}$$

Using the relaxation technique introduced above, we use the variables $b_{i,j}$ and $y_{i,j}$ to substitute the corresponding $\log(a_{i,j})$ together with $\log(x_{i,j})$ successively. Then, the primitive optimization objective $C(\mathbf{P})$ defined in (19) can be written as

$$\widehat{C}(\mathbf{P}) = B_s \sum_{j\in\mathcal{N}} \sum_{i\in\mathcal{K}} (b_{i,j} - y_{i,j}), \tag{33}$$

and the new variables should be constrained by the secant constraints,

$$b_{i,j} \geq \log a_{i,j}^L + \frac{\log a_{i,j}^L - \log a_{i,j}^U}{a_{i,j}^L - a_{i,j}^U}(a_{i,j} - a_{i,j}^L), \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{34}$$

$$y_{i,j} \geq \log x_{i,j}^L + \frac{\log x_{i,j}^L - \log x_{i,j}^U}{x_{i,j}^L - x_{i,j}^U}(x_{i,j} - x_{i,j}^L), \forall i \in \mathcal{K}, \forall j \in \mathcal{N}, \tag{35}$$

together with the corresponding tangent constraints,

$$b_{i,j} \leq \log a_{i,j}^L + \frac{a_{i,j} - a_{i,j}^L}{a_{i,j}^L}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} \tag{36}$$

$$y_{i,j} \leq \log x_{i,j}^L + \frac{x_{i,j} - x_{i,j}^L}{x_{i,j}^L}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} \tag{37}$$

$$b_{i,j} \leq \log a_{i,j}^U + \frac{a_{i,j} - a_{i,j}^U}{a_{i,j}^U}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} \tag{38}$$

$$y_{i,j} \leq \log x_{i,j}^U + \frac{x_{i,j} - x_{i,j}^U}{x_{i,j}^U}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N}. \tag{39}$$

Then, for each variable set in every variable space, the optimization defined in (16)-(22) can be transformed as

$$\begin{aligned}
\text{Given :} & \mathbf{h}_{i,j}, \mathbf{w}_{i,j}, \mathbf{g}_j, \sigma_{i,j}^2, h_{i,j}^{l,s}, P_t, P_{tj}^{(l)}, Q_j, B_s, & (40)\\
& p_{i,j}^L, p_{i,j}^U, P_{tj}^{(l)}, \rho_{i,j}^{(s,s)}, \rho_{i,j}^{(l,s)}, \rho_j^{(s,l)}, \theta_j, & (41)\\
& a_{i,j}^L, a_{i,j}^U, x_{i,j}^L, x_{i,j}^U, \epsilon, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} & (42)\\
\text{Find :} & p_{i,j}, \forall i \in \mathcal{K}, \forall j \in \mathcal{N} & (43)\\
\text{Maximize :} & \widehat{C}(\mathbf{P}) & (44)\\
\text{Subject to :} & (20), (22), (31), (32), (34) - (39). & (45)
\end{aligned}$$

Through solving the new constructed optimization problem defined in (40)-(45), we can acquire the upper bound according to every variable set and the corresponding power allocation. Afterwards, we substitute the acquired power allocation to the primitive objective function $C(\mathbf{P})$ defined in (19) and take the result as the lower bound according to this variable set. Due to the process above to obtain the lower bound for each variable set, the $\mathrm{LB}_{\mathrm{glb}}$ will be chosen as the maximum lower bound in all the variable sets at all the iteration steps.

C. Variable Space Determining and Partitioning

In the initial variable space $\mathcal{S}_1$, all power variables according to each user on each subcarrier can be achieved in the scope from ϵ to $P_t - (KN-1)\epsilon$ and they construct the sole variable set in the initial variable space. According to each variable set at every iterative step, both the corresponding upper bound and lower bound would be acquired. After each iterative step, based on the solved upper bounds and lower bounds, a certain variable set at this iterative step will be chosen and partitioned to construct the variable space at the next step.

At the iterative step t, for certain $\mathcal{S}_t^s \in \mathcal{S}_t$, we denote the scope of the variable $p_{i,j}$ as $[p_{i,j}^L, p_{i,j}^U]$ and substitute all the $p_{i,j}^L$ to $a_{i,j}$ together with $x_{i,j}$ to acquire the $a_{i,j}^L$ together with $x_{i,j}^L$. In addition, we obtain $a_{i,j}^U$ together with $x_{i,j}^U$ through designing a linear programming, which is constructed to solve the upper bounds of $a_{i,j}$ together with $x_{i,j}$ under both the scope constraint of related power variables and the total power constraint. Then, for this variable set, through the optimization problem defined in (40)-(45), both the upper bound $\mathrm{UB}(\mathcal{S}_t^s)$ and the lower bound $\mathrm{LB}(\mathcal{S}_t^s)$ can be acquired. The other variable sets in $\mathcal{S}_t$ can be processed in the same way.

Because every $a_{i,j}$ together with $x_{i,j}$ can be represented and expressed by the power variables, we partition the power variable scope to continue the branch and bound algorithm if (24) can not be fulfilled. The objective of partitioning is to decrease the globally upper bound and we can find the smaller variable scope brings the smaller relaxation errors in the last subsection. Therefore, we choose the power variable with the largest scope in the variable set corresponding to the $\mathrm{UB}_{\mathrm{glb}}$ at this iterative step t for further partitioning, i.e.,

$$s^* = \arg \max_{s \in \{1,2,\cdots t\}} \mathrm{UB}(\mathcal{S}_t^s), \tag{46}$$

$$i^*, j^* = \arg \max_{i \in \mathcal{K}, j \in \mathcal{N}} (p_{i,j}^{U(s^*)} - p_{i,j}^{L(s^*)}), \tag{47}$$

and then, divide the power variable scope $[p_{i^*,j^*}^{L(s^*)}, p_{i^*,j^*}^{U(s^*)}]$ from the middle value $(p_{i^*,j^*}^{L(s^*)} + p_{i^*,j^*}^{U(s^*)})/2$ and keep the scopes of other power variables unchanged to generate two new variable sets. We combine the two new variable sets and the other variable sets excluding the chosen variable set at iterative step t to construct the new variable space at iterative step $(t+1)$.

V. Simulation Results

In this section, we simulate the performance of the proposed power allocation. The relative distance of the components in simulated communication topology, including a pair of PU and a three user SU MISO system, is represented by the coordinates. We assume the distance between SU base station and SU1 as the normalized distance. Correspondingly, the relative locations of SU base station, SU_1, SU_2, SU_3, PU transmitter and PU receiver are respectively positioned at the coordinates (0,0), (1,0), (0,1), (1,1), (0,2) and (2,2). The frequency band is divided into four unit-bandwidth sub-carriers and the path loss exponent is set as $\alpha = 4$. In our simulation, we assume the sub-carriers $1, 3$ are used by the primary user and the sub-carriers $2, 4$ are idle. The additive noise is set the same variance σ^2 for each user on every subcarrier, and the sum capacity curves are plotted versus the received SNR of SU1 with normalized distance in the communication topology, i.e., $10 \times \log(1/\sigma^2)$. The power of the primary transmitter on each subcarrier is predefined same and with the value 4. The minimum power ϵ is set 10^{-4} and the termination flag ε is set 0.05, which means that the sum capacity solved by the algorithm proposed is greater than or equal to 95% of the actual optimal. The proposed power allocation is evaluated comparing with three existing power allocations based on SLNR precoding in the multi-user MIMO system, including: i) the equal power allocation, ii) the proportional power allocation scheme in [8] based on the reciprocal of the $\mathrm{SLNR}_{i,j}$ value when the equal power is allocated to each user, and iii) the proportional power allocation scheme in [9] based on the reciprocal of the trace of the matrix $\mathbf{h}_{i,j}^{\mathrm{H}}\mathbf{h}_{i,j}^{\mathrm{H}}$. Fig. 3 (a) and Fig. 3 (c) are acquired through averaging over 500 channel realizations while Fig. 3 (b) is got by 100 channel realizations. The convex optimizations referred in the simulation are solved by the CVX Matlab package [17].

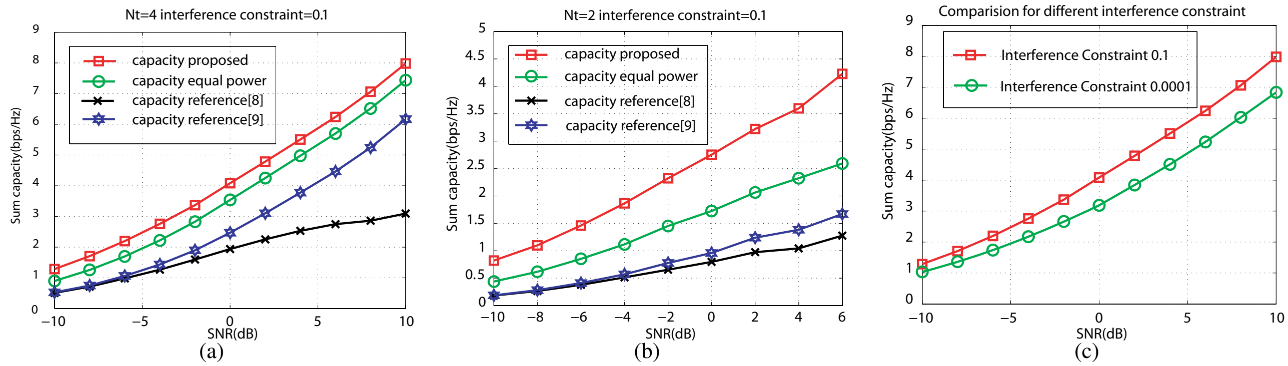

Fig. 3: Performance of the proposed power allocation. (a) Sum-capacity comparation in the case of $N_t = 4$ (b) Sum-capacity comparation in the case of $N_t = 2$ (c) Sum-capacity comparation with different interference constraints.

From the Fig. 3 (a) and the Fig. 3 (b), equipped with four and two transmit antennas at the secondary transmitter respectively, we can see that the proposed power allocation can provide some sum-capacity performance improvements compared with the three existing power allocations under the interference constraint on each subcarrier 0.1. In the simulations, both the equal power allocation and the proportional power allocations in [8], [9] are designed without considering the interference constraint set by the licensed user. If this factor is taken into consideration, part of the sub-carriers can not be accessed by the SUs and the sum-capacity will be worse than the corresponding simulation curves. In addition, larger transmit antenna number can lead to better sum-capacity performance because that more transmit antennas can support more space dimensions for the signal transmission.

The sum-capacity based on the hybrid opportunistic spectrum access mode is affected by the tolerance extent of primary user. As the interference temperature constraint predefined becomes lower, stricter restrictions for the secondary system should be satisfied. Then, the sum-capacity performance will be decreased and the simulations in the Fig. 3 (c) verify this judgement in the case of four transmit antenna with the interference constraint on each subcarrier 0.1 and 10^{-4}.

VI. Conclusion

In this paper, we have investigated a sum-capacity maximization power allocation scheme for the SLNR-precoding-based cognitive wireless network where hybrid opportunistic spectrum access is applied. The power allocation problem is formulated as a nonconvex optimization and with the help of branch and bound framework (B&B) together with the convex relaxation technique, we proposed the globally optimal power allocation algorithm which can maximize sum-capacity of all the secondary users. Simulation results showed that the proposed power allocation could provide some improvements on sum-capacity performance compared with the three existing SLNR-precoding-based power allocations.

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