

Optimized Relaying Method for Wireless Multi-antennas Cooperative Networks

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Abstract—An optimized relaying method for multi-antennas cooperative networks, estimate-and-forward (EF) strategy for MIMO relay, is proposed and analyzed in this paper. According to our theoretical analysis, it performs like amplify-and-forward (AF) for the low signal noise ratio (SNR) region and behaves like detect-and-forward (DF) for the high SNR region. For the relay networks with a large number of antennas and/or high order constellations, two approximate methods are proposed to reduce the complexity of the signal estimation at the relay. The first one uses a list sphere decoder to generate an estimate list and to obtain the approximate minimum mean squared error (MMSE) estimate based on the reduced list. The proposed list EF retains the advantages of the exact EF relay strategy in large MIMO relay networks at a negligible performance loss. The second algorithm computes the estimate of the symbols using Gaussian approximation to approximate the summation to be integral. We find the proposed EF scheme performs better than both AF and DF across all SNRs without switching algorithms for different SNRs.

I. INTRODUCTION

Next generation wireless communication systems need to accommodate high speed multimedia and internet-related services. To this end, multiple-input multiple-output (MIMO) technology based on multiple-antenna transmitter and receiver terminals has been shown to bring about enormous improvements in data rates and system's reliability [1]. However, even with MIMO technology, the demands and expectations on wireless communication systems still cannot be satisfied [2]–[5]. One way to address these problems is cooperation between wireless nodes that enables the relays to forward messages from the source to the destination. The destination receives multiple versions of the message. It is then able to obtain a more reliable estimate of the transmitted signal by combining the multiple received signals.

Relay strategies have been studied in [6], [7], for amplify-and-forward (AF) relaying, the relays are allowed to amplify their received signals subject to their power constraints and to retransmit the messages to the destination. This simple processing benefits cooperative wireless networks with full spatial diversity at high signal-to-noise ratios (SNRs). How-

ever, because the noise component is also amplified in AF, the bit error rate (BER) performance could be degraded. For decode/detect-and-forward (DF) relaying, the relays first decode/detect the received signals and then forward the signals to the destination. However, when the channel link suffers from deep fading (low SNRs), the decoding could produce errors which will be propagated to the destination.

One relay strategy called estimate-and-forward (EF) was proposed in [8], which was shown to be a powerful approach for uncoded single antenna relay networks. It uses the unconstrained minimum mean squared error (MMSE) estimation at the relays instead of amplifying/detecting the received signals. EF strategy leads to the optimized relay function for all SNRs. Due to the advantages of EF, several recent papers [9], [10] have investigated MMSE relaying. However, the EF strategy for the MIMO relay networks especially with high order constellations and/or a large number of antennas has not been investigated in the literature.

In this paper, we propose an EF strategy for MIMO relay networks, which performs better than both AF and DF relay strategies across all SNRs. The relay function at the relay is analyzed and shown to approach AF in low SNR region and to perform like DF in high SNR region. For large MIMO relay networks with high order constellations and/or a large number of antennas, a list EF is proposed to reduce the complexity of computing the MMSE estimation at the relay. The list EF uses a list sphere decoder (such as [11], [12]) to constrain the candidates list for the MMSE estimation. To reduce the complexity, we also propose using Gaussian approximation to replace the summation in MMSE estimation with a Gaussian integral, which reduces the multi-dimensional MMSE estimation to an one-dimensional MMSE solution. Note that in this paper, we focus on the signal processing algorithm design at the relay. We do not consider joint signal transmission design between the source and the relay.

The rest of this paper is organized as follows. Section II introduces the system model. And Section III describes the relay strategies for MIMO relay networks including AF, DF



Fig. 1. MIMO relay channel

and EF. Section IV presents the proposed list EF and Gaussian approximation EF. Simulation results and discussions are given in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The MIMO relay system model is illustrated in Fig 1, where the source has M_s transmit antennas, the relay has N_r receive antennas and M_r transmit antennas, and the destination has N_d receive antennas. The source transmits signal to the relay which generates

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}_1, \quad (1)$$

where $\mathbf{H} = [h_{ij}] \in \mathcal{C}^{N_r \times M_s}$ denotes the MIMO channel between the source and the relay and the elements of \mathbf{H} are independent identically distributed (i.i.d.) complex Gaussian ($H_{ij} \sim \mathcal{CN}(0, 1)$); $\mathbf{n}_1 = [n_{11}, n_{12}, \dots, n_{1N_r}]^T$ and $n_{1i} \sim \mathcal{CN}(0, \sigma_1^2)$ is an additive white Gaussian noise (AWGN) with mean zero and variance σ_1^2 ; the transmitted signal is denoted by $\mathbf{x} = [x_1, x_2, \dots, x_{M_s}]^T$, assuming i.i.d. elements in \mathbf{x} . We also assume each element in the transmitted symbols is chosen from the constellation \mathcal{Q} , and the average transmitted power is $\mathcal{E}[\|\mathbf{x}\|^2] = P_s$, where P_s is the source average power. The relay generates the transmitted signal to the destination with the relay function $\mathcal{G}(\mathbf{r})$ using received signal \mathbf{r} . Assuming P_r is the relay average power, the transmitted signal $\mathcal{G}(\mathbf{r})$ should satisfy its power constraint $\mathcal{E}[\|\mathcal{G}(\mathbf{r})\|^2] = P_r$. At the destination, the received signal can be written as

$$\mathbf{y} = \mathbf{G}\mathcal{G}(\mathbf{r}) + \mathbf{n}_2, \quad (2)$$

where $\mathbf{G} = [g_{ij}] \in \mathcal{C}^{N_d \times M_r}$ denotes the MIMO channel between the relay and the destination and the elements of \mathbf{G} are also i.i.d. complex Gaussian ($g_{ij} \sim \mathcal{CN}(0, 1)$); and $\mathbf{n}_2 = [n_{21}, n_{22}, \dots, n_{2N_d}]^T$ ($n_{2i} \sim \mathcal{CN}(0, \sigma_2^2)$).

Throughout this paper, we assume that there is only one relay in the network. For the case of multiple relays, each relay will perform the same operation as we do not consider the relay cooperation. For simplicity, we only consider the case that the destination receives the signal only from the relay. The destination cannot receive signal from the source directly, e.g., destination is far away from the source. But the algorithms proposed in this paper can be readily extended to the case with line of sight (LOS).

III. RELAY STRATEGIES

The performance of MIMO relay networks depends on different relay strategies. Therefore, the relay functions of common memoryless forwarding strategies for MIMO relay systems are discussed in this section.

A. Amplify-and-Forward

1) *Pure AF*: AF relaying is the most common relay strategy in relay networks, where a linear relay function is used. Furthermore, to satisfy the average power constraint at the relay, the relay function is equal to

$$\mathcal{G}_{AF}(\mathbf{r}) = \sqrt{\frac{P_r}{\mathcal{E}[\|\mathbf{H}\|^2]P_s + N_r\sigma_1^2}}\mathbf{r}. \quad (3)$$

Thus, the received signal at the destination is

$$\mathbf{y} = \mathbf{G}\sqrt{\frac{P_r}{\mathcal{E}[\|\mathbf{H}\|^2]P_s + N_r\sigma_1^2}}\mathbf{r} + \mathbf{n}_2 = \mathbf{H}'\mathbf{x} + \mathbf{n}', \quad (4)$$

where

$$\mathbf{H}' = \sqrt{\frac{P_r}{\mathcal{E}[\|\mathbf{H}\|^2]P_s + N_r\sigma_1^2}}\mathbf{G}\mathbf{H} \quad (5)$$

$$\mathbf{n}' = \sqrt{\frac{P_r}{\mathcal{E}[\|\mathbf{H}\|^2]P_s + N_r\sigma_1^2}}\mathbf{n}_1 + \mathbf{n}_2. \quad (6)$$

2) *LMMSE AF*: Instead of directly amplifying the received signal at the relay, the linear MMSE (LMMSE) AF relay strategy firstly estimate the received signal by the LMMSE estimator [1], which is

$$\hat{\mathbf{x}}_{LMMSE} = (\mathbf{H}^H\mathbf{H} + N_r\sigma_1^2\mathbf{I})^{-1}\mathbf{H}^H\mathbf{r}. \quad (7)$$

The estimated signal is then scaled by a scaling factor

$$\alpha_{LMMSE} = \sqrt{\frac{P_r}{\mathcal{E}[\|\hat{\mathbf{x}}_{LMMSE}\|^2]}}. \quad (8)$$

Then, the relay function for the LMMSE AF relay strategy is

$$\mathcal{G}_{LMMSE}(\mathbf{r}) = \alpha_{LMMSE}\hat{\mathbf{x}}_{LMMSE}. \quad (9)$$

Thus, the received signal at the destination will be

$$\mathbf{y} = \mathbf{G}\mathcal{G}_{LMMSE}(\mathbf{r}) + \mathbf{n}_2, \quad (10)$$

B. Detect-and-Forward

In DF, the relay detects all the transmitted symbols, and then forwards them to the destination. Without loss of generality, by using ML detection at the relay, the detected signal could be given as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathcal{Q}^{M_s}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2, \quad (11)$$

where \mathcal{Q}^{M_s} is the set of the transmitted vector symbols. This detection can be accomplished by using the sphere decoder [12] in order to reduce the detection complexity. The relay function for DF is given as

$$\mathcal{G}_{DF}(\mathbf{r}) = \sqrt{\frac{P_r}{\mathcal{E}[\|\hat{\mathbf{x}}\|^2]}}\hat{\mathbf{x}} = \sqrt{\frac{P_r}{P_s}}\hat{\mathbf{x}}. \quad (12)$$

Therefore, at the destination, the received signal is

$$\mathbf{y} = \mathbf{G}\sqrt{\frac{P_r}{P_s}}\hat{\mathbf{x}} + \mathbf{n}_2 \quad (13)$$

$$\begin{aligned}
\mathcal{E}(\|\hat{x}\|^2) &= \int_{-\infty}^{\infty} \left(\frac{\exp\left(\frac{hr}{\sigma_1^2}\right) - \exp\left(-\frac{hr}{\sigma_1^2}\right)}{\exp\left(\frac{hr}{\sigma_1^2}\right) + \exp\left(-\frac{hr}{\sigma_1^2}\right)} \right)^2 \times f(r) dr \\
&\approx \begin{cases} \frac{\exp\left(-\frac{h^2}{2\sigma_1^2}\right)}{2\sqrt{2\pi\sigma_1^2}} \int_{-\infty}^{\infty} \frac{\exp\left(\frac{2hr}{\sigma_1^2}\right) - \exp\left(-\frac{2hr}{\sigma_1^2}\right)}{2} \exp\left(-\frac{r^2}{2\sigma_1^2}\right) dr & : \text{when } \sigma_1^2 \rightarrow \infty \\ \int_{-\infty}^{\infty} 1^2 \times \frac{1}{2\sqrt{2\pi\sigma_1^2}} \left(\exp\left(-\frac{(r-h)^2}{2\sigma_1^2}\right) + \exp\left(-\frac{(r+h)^2}{2\sigma_1^2}\right) \right) dr & : \text{when } \sigma_1^2 \rightarrow 0 \end{cases} \\
&= \begin{cases} \frac{h^2}{\sigma_1^2} & : \text{when } \sigma_1^2 \rightarrow \infty \\ 1 & : \text{when } \sigma_1^2 \rightarrow 0 \end{cases}
\end{aligned} \tag{22}$$

C. Estimate-and-Forward

Instead of AF and DF, the soft information is transmitted at the relay in EF relay networks. The relay forwards the unconstrained MMSE estimate to the destination. The MMSE estimation of the transmitted symbols \mathbf{x} at the relay is

$$\begin{aligned}
\hat{x} &= \mathcal{E}(\mathbf{x}|\mathbf{r}, \mathbf{H}) = \sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} P(\mathbf{x}|\mathbf{r}, \mathbf{H}) \\
&= \frac{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} f(\mathbf{r}|\mathbf{x}, \mathbf{H}) P(\mathbf{x})}{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} f(\mathbf{r}|\mathbf{x}, \mathbf{H}) P(\mathbf{x})},
\end{aligned} \tag{14}$$

where $f(\mathbf{r}|\mathbf{x}, \mathbf{H})$ is the probability distribution function (PDF) of \mathbf{r} conditioned by \mathbf{x} and \mathbf{H} . Under the assumption of equal priori probabilities for the transmitted symbols, \hat{x} is given as

$$\hat{x} = \frac{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} f(\mathbf{r}|\mathbf{x}, \mathbf{H})}{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} f(\mathbf{r}|\mathbf{x}, \mathbf{H})}. \tag{15}$$

Because \mathbf{n}_1 is an AWGN, the PDF $f(\mathbf{r}|\mathbf{x}, \mathbf{H})$ is

$$f(\mathbf{r}|\mathbf{x}, \mathbf{H}) = \frac{1}{(\pi\sigma_1^2)^{M_s}} \exp\left(-\frac{\|\mathbf{r} - \mathbf{Hx}\|^2}{\sigma_1^2}\right). \tag{16}$$

Then,

$$\hat{x} = \frac{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} \exp\left(-\frac{\|\mathbf{r} - \mathbf{Hx}\|^2}{\sigma_1^2}\right)}{\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \exp\left(-\frac{\|\mathbf{r} - \mathbf{Hx}\|^2}{\sigma_1^2}\right)}. \tag{17}$$

In order to obtain the relay function and satisfy the relay power constraint, we need to calculate $\mathcal{E}[\|\hat{x}\|^2]$, and then the scaling parameter for the relay function should be

$$\beta = \sqrt{\frac{P_r}{\mathcal{E}(\|\hat{x}\|^2)}} = \sqrt{\frac{P_r}{\int_{-\infty}^{\infty} \|\hat{x}(\mathbf{r})\|^2 f(\mathbf{r}) d\mathbf{r}}}, \tag{18}$$

where

$$\begin{aligned}
f(\mathbf{r}) &= \sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} f(\mathbf{r}|\mathbf{x}, \mathbf{H}) P(\mathbf{x}) \\
&= \sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \frac{1}{(\pi\sigma_1^2)^{M_s}} \exp\left(-\frac{\|\mathbf{r} - \mathbf{Hx}\|^2}{\sigma_1^2}\right) \frac{1}{|\mathcal{Q}|^{M_s}}.
\end{aligned} \tag{19}$$

The relay then sends $\beta\hat{x}$ to the destination.

In the following, we give an example to illustrate the EF strategy.

Example (BPSK and $M_s = N_r = M_r = N_d = 1$): In this case, we consider a real-valued system as

$$\hat{x} = \mathcal{E}(x|r, h) = \tanh\left(\frac{hr}{\sigma_1^2}\right), \tag{20}$$

where

$$\tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}. \tag{21}$$

Thus, $\mathcal{E}(\|\hat{x}\|^2)$ can be shown as Eq. (22). The EF relay function for this system is

$$g_{EF}(r) = \sqrt{\frac{P_r}{\mathcal{E}(\|\hat{x}\|^2)}} \hat{x}. \tag{23}$$

From Eq. (21), we can find that when the SNR at the relay is very low,

$$\begin{aligned}
\lim_{\sigma_1^2 \rightarrow \infty} \hat{x} &= \lim_{\sigma_1^2 \rightarrow \infty} \frac{\exp\left(\frac{hr}{\sigma_1^2}\right) - \exp\left(-\frac{hr}{\sigma_1^2}\right)}{\exp\left(\frac{hr}{\sigma_1^2}\right) + \exp\left(-\frac{hr}{\sigma_1^2}\right)} \\
&= \lim_{\sigma_1^2 \rightarrow \infty} \frac{\left(1 + \frac{hr}{\sigma_1^2}\right) - \left(1 - \frac{hr}{\sigma_1^2}\right)}{\left(1 + \frac{hr}{\sigma_1^2}\right) + \left(1 - \frac{hr}{\sigma_1^2}\right)} \\
&= \lim_{\sigma_1^2 \rightarrow \infty} \frac{hr}{\sigma_1^2}.
\end{aligned} \tag{24}$$

This becomes an AF alike relaying with amplify factor $\frac{h}{\sigma_1^2}$.

When the SNR at the relay is very high,

$$\begin{aligned}
\lim_{\sigma_1^2 \rightarrow 0} \hat{x} &= \lim_{\sigma_1^2 \rightarrow 0} \frac{\exp\left(\frac{hr}{\sigma_1^2}\right) - \exp\left(-\frac{hr}{\sigma_1^2}\right)}{\exp\left(\frac{hr}{\sigma_1^2}\right) + \exp\left(-\frac{hr}{\sigma_1^2}\right)} \\
&= \begin{cases} -1 & : \frac{hr}{\sigma_1^2} < 0 \\ 1 & : \frac{hr}{\sigma_1^2} > 0 \end{cases} = \text{sign}\left(\frac{hr}{\sigma_1^2}\right).
\end{aligned} \tag{25}$$

Thus, the MMSE relaying becomes DF relaying.

D. Relationship between AF, DF and EF

In this section, the relationship between AF, DF and EF is discussed. We assume that the constellation is symmetric, i.e., $x \in \mathcal{Q} \Leftrightarrow -x \in \mathcal{Q}$.

Low SNR case: When the receive SNR at the relay is very low, by using Eq. (17), the MMSE estimate can be derived as

$$\begin{aligned} \lim_{\sigma_1^2 \rightarrow \infty} \hat{\mathbf{x}} &= \frac{1}{\sigma_1^2 |\mathcal{Q}|^{M_s}} \left(\left(\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} \mathbf{x}^H \right) \mathbf{H}^H \mathbf{r} + \left(\sum_{\mathbf{x} \in \mathcal{Q}^{M_s}} \mathbf{x} \mathbf{x}^T \right) \mathbf{H}^T \mathbf{r}^* \right) \\ &\stackrel{(a)}{=} \frac{1}{\sigma_1^2 |\mathcal{Q}|} \left(\sum_{x \in \mathcal{Q}} |x|^2 \mathbf{H}^H \mathbf{r} + \sum_{x \in \mathcal{Q}} x^2 \mathbf{H}^T \mathbf{r}^* \right) \end{aligned} \quad (26)$$

$$\lim_{\sigma_1^2 \rightarrow 0} \hat{\mathbf{x}} = \lim_{\sigma_1^2 \rightarrow 0} \frac{\mathbf{x}_{ML} + \sum_{\mathbf{x} \in \mathcal{Q}^{M_s} - \mathbf{x}_{ML}} \mathbf{x} \exp \left(-\frac{\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 - \|\mathbf{r} - \mathbf{H}\mathbf{x}_{ML}\|^2}{\sigma_1^2} \right)}{1 + \sum_{\mathbf{x} \in \mathcal{Q}^{M_s} - \mathbf{x}_{ML}} \exp \left(-\frac{\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 - \|\mathbf{r} - \mathbf{H}\mathbf{x}_{ML}\|^2}{\sigma_1^2} \right)} = \mathbf{x}_{ML}. \quad (30)$$

Eq. (26), where we have used the fact that the constellation is symmetric and thus $\sum x f(x) = 0$ if $f(x)$ is an even function in x ; we have also used the fact that $\lim_{\sigma_1^2 \rightarrow \infty} n_1/\sigma^2 = 0$ with probability one and the approximation $e^x \approx 1+x$; in (a) we have used the fact that

$$\sum_{x_1, x_2 \in \mathcal{Q}} x_1 x_2^* = 0, \text{ and } \sum_{x_1, x_2 \in \mathcal{Q}} x_1 x_2 = 0.$$

From (26), we can see that EF becomes an AF alike strategy in the low SNR region. But it is slightly different from the AF strategy in (3). However, this could be further simplified for difference constellation methods. When working on the real-valued constellations, such as BPSK, Eq. (26) could be rewritten as

$$\lim_{\sigma_1^2 \rightarrow \infty} \hat{\mathbf{x}} = \frac{2 \sum_{x \in \mathcal{Q}} x^2}{\sigma_1^2 |\mathcal{Q}|} \Re(\mathbf{H}^H \mathbf{r}). \quad (27)$$

When $M_s = N_r = 1$, (27) becomes (24).

When working on complex-valued systems, such as M -QAM, we have $\sum_{x \in \mathcal{Q}} x^2 = 0$ according to the symmetric characteristic. Eq. (26) becomes

$$\lim_{\sigma_1^2 \rightarrow \infty} \hat{\mathbf{x}} = \frac{\sum_{x \in \mathcal{Q}} |x|^2}{\sigma_1^2 |\mathcal{Q}|} \mathbf{H}^H \mathbf{r}. \quad (28)$$

Therefore, EF becomes a matched filter AF when the SNR is very low.

High SNR case: Assuming the ML detection is

$$\mathbf{x}_{ML} = \arg \min_{\mathbf{x} \in \mathcal{Q}^{M_s}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2. \quad (29)$$

When the SNR is very high, the MMSE estimate could be derived by Eq. (30), which becomes DF relaying. We have used the fact that

$$\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \neq \|\mathbf{r} - \mathbf{H}\mathbf{x}_{ML}\|^2, \quad \forall \mathbf{x} \neq \mathbf{x}_{ML},$$

with probability one. In other words, $\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 - \|\mathbf{r} - \mathbf{H}\mathbf{x}_{ML}\|^2$ is lower bounded by $\epsilon > 0$ with probability one as SNR goes to infinity. Therefore, we have

$$\lim_{\sigma_1^2 \rightarrow 0} \exp \left(-\frac{\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 - \|\mathbf{r} - \mathbf{H}\mathbf{x}_{ML}\|^2}{\sigma_1^2} \right) = 0.$$

IV. APPROXIMATE ESTIMATE-AND-FORWARD

From Eq. (17), if \mathcal{Q}^{M_s} is small, it is easy to compute $\hat{\mathbf{x}}$. However, when \mathcal{Q}^{M_s} is large, it is hard to compute $\hat{\mathbf{x}}$ in (17) directly. In this section, we propose two approximate EF methods to solve this problem.

A. List Estimate-and-Forward

Before introducing the list EF, we firstly analyze the element $\Psi = \exp \left(-\frac{\|\mathbf{r} - \mathbf{H}\tilde{\mathbf{x}}\|^2}{\sigma_1^2} \right)$ in Eq. (17), when $\tilde{\mathbf{x}} \neq \hat{\mathbf{x}}$. As we know \mathbf{n}_1 is an AWGN. Thus $\frac{\|\mathbf{r} - \mathbf{H}\tilde{\mathbf{x}}\|^2}{\sigma_1^2} \sim \chi^2(M_s)$, which is a chi-square distribution $f(\cdot)$, where M_s is the degree of freedom. Therefore, the PDF of Ψ ($0 \leq \psi \leq 1$) can be derived as

$$\begin{aligned} f_\Psi(\psi) &= \frac{1}{\psi} f(-\ln \psi) \\ &= \frac{1}{\psi} \times \frac{(-\ln \psi)^{M_s/2-1} \exp(-\frac{\ln \psi}{2})}{2^{M_s/2} \Gamma(M_s/2)}. \end{aligned} \quad (31)$$

The probability of $\psi \geq a$ (a is a larger number) could be given by

$$F_\Psi(\psi \geq a) = \int_a^1 f_\Psi(\psi) d\psi. \quad (32)$$

For example, when $M_s = 2$, $F_\Psi(\psi \geq a) = \frac{1-\sqrt{a}}{\Gamma(1)}$. If a is a number close to 1, then $F_\Psi(\psi \geq a)$ would be a very small value. This demonstrates that the probability of large ψ is very small. That is the main motivation of the list EF.

Most of the terms in the sum of Eq. (17) are typically very small and contribute very little to the final result especially in high SNR. It is therefore intuitive to find a subset of \mathcal{Q}^{M_s} . Because $\exp \left(-\frac{\|\mathbf{r} - \mathbf{H}\tilde{\mathbf{x}}\|^2}{\sigma_1^2} \right)$ has small possibility to be a large number, there is no need to compute all the elements for $\mathbf{x} \in \mathcal{Q}^{M_s}$. The subset \mathcal{L} of \mathcal{Q}^{M_s} could be derived by the sphere decoder to generate a list of \mathbf{x} satisfying $\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \leq d^2$. We call this method to be list EF. The constraint radius d^2 could be defined to be $d^2 = -\sigma_1^2 \ln a$ ($0 < a < 1$). Furthermore, with different a , the generated list has different size, which

$$\hat{x}_i = E(x_i | \mathbf{r}, \mathbf{H}) = \frac{\sum_{x_i \in \mathcal{Q}} x_i \sum_{\mathbf{x}_{-i} \in \mathcal{Q}^{M_s-1}} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}{\sum_{x_i \in \mathcal{Q}} \sum_{\mathbf{x}_{-i} \in \mathcal{Q}^{M_s-1}} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}. \quad (36)$$

$$\hat{x}_i \approx \frac{\sum_{x_i \in \mathcal{Q}} x_i \int \exp\left(-\frac{\|\mathbf{x}_{-i}\|^2}{\sigma_x^2}\right) \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}_{-i}\mathbf{x}_{-i}-\mathbf{h}_i x_i\|^2}{\sigma_1^2}\right) d\mathbf{x}_{-i}}{\sum_{x_i \in \mathcal{Q}} \int \exp\left(-\frac{\|\mathbf{x}_{-i}\|^2}{\sigma_x^2}\right) \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}_{-i}\mathbf{x}_{-i}-\mathbf{h}_i x_i\|^2}{\sigma_1^2}\right) d\mathbf{x}_{-i}}. \quad (37)$$

$$\int \exp\left[-\frac{1}{2}\mathbf{x}^H \mathbf{A}\mathbf{x} + \mathbf{c}^H \mathbf{x}\right] d\mathbf{x} = \sqrt{\det(2\pi\mathbf{A}^{-1})} \exp\left[\frac{1}{2}\mathbf{c}^H \mathbf{A}^{-H} \mathbf{c}\right]. \quad (38)$$

$$\hat{x}_i = \frac{\sum_{x_i \in \mathcal{Q}} x_i \exp\left(-\frac{\|\mathbf{h}_i x_i\|^2}{2\sigma_1^2} - \frac{\mathbf{C} - (\mathbf{A}^{-1}\mathbf{B})^H(\mathbf{A}^{-1}\mathbf{B})}{2\sigma_x^2\sigma_1^2}\right)}{\sum_{x_i \in \mathcal{Q}} \exp\left(-\frac{\|\mathbf{h}_i x_i\|^2}{2\sigma_1^2} - \frac{\mathbf{C} - (\mathbf{A}^{-1}\mathbf{B})^H(\mathbf{A}^{-1}\mathbf{B})}{2\sigma_x^2\sigma_1^2}\right)}. \quad (39)$$

affects the computational accuracy and complexity. The list is denoted by \mathcal{L} , and we can then approximate (17) to be

$$\begin{aligned} \hat{\mathbf{x}} &= E(\mathbf{x} | \mathbf{r}, \mathbf{H}) \\ &\approx \frac{\sum_{\mathbf{x} \in \mathcal{L}} \mathbf{x} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}{\sum_{\mathbf{x} \in \mathcal{L}} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}. \end{aligned} \quad (33)$$

Thus, the summation of this small list of \mathbf{x} can be easily derived. For clarity, in this paper, we use the size of the list $N_{\mathcal{L}}$ to obtain the trade-offs of the computational accuracy and complexity. When $N_{\mathcal{L}}$ gets smaller, the computation of $\hat{\mathbf{x}}$ becomes easier. Because of

$$\lim_{\sigma^2 \rightarrow 0} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right) = 0, \quad (34)$$

the list EF can achieve the exact computation of MMSE estimate even with a very small set for the high SNR region.

The sphere decoder [11] was proposed to reduce the computational complexity of MIMO detection. It only searches the symbol vector in the hypersphere constrained by the searching radius d . In this paper, the sphere decoder is used to generate the list for the EF relaying scheme in MIMO relay systems.

After obtaining the $\hat{\mathbf{x}}$ by Eq. (33). The relay function could be given as

$$\mathcal{G}_{EF}(\mathbf{r}) = \beta \frac{\sum_{\mathbf{x} \in \mathcal{L}} \mathbf{x} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}{\sum_{\mathbf{x} \in \mathcal{L}} \exp\left(-\frac{\|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2}{\sigma_1^2}\right)}, \quad (35)$$

where β can be derived by Eq. (18). Thus, the MMSE estimation at the relay would be retransmitted to the destination.

B. Gaussian Approximation

In order to reduce the complexity of the MMSE estimation at the relay, another approximate method is proposed in this subsection. Instead of computing \mathbf{x} in Eq. (17) vector by vector, we propose to compute $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{M_s}]^T$ entry by entry for large systems. The summation is hard to

compute directly because of the large set \mathcal{Q}^{M_s} . However, it can be approximated by converting the summation to an integration, especially for high order constellation and large antenna systems. The conventional computation of MMSE estimation needs to compute the summations of all the $|\mathcal{Q}|^{M_s}$ elements. While, only $|\mathcal{Q}|$ elements in the constellation are summed by using the proposed integral approximation as shown in Eq. (36), where \mathbf{x}_{-i} denotes the vector containing all other entries except x_i .

In a MIMO system with large constellation, it is common to use a Gaussian approximation [13], which makes it easy to compute the integrals. Thus, assuming \mathbf{x}_{-i} to be a Gaussian distribution with mean zero and variance σ_x^2 derived from the constellation, the finite sum in Eq. (36) could be replaced by an integration as shown in Eq. (37), where the integral can be derived by using the vector integration.

By using the integration of Gaussian vector shown in Eq. (38) and decomposition and combination of the vectors, the integration in Eq. (37) could be derived in a closed-form, thus x_i could be rewritten as Eq. (39), where

$$\mathbf{A} = \sigma_1^2 \mathbf{I} + \sigma_x^2 \mathbf{H}_{-i}^H \mathbf{H}_{-i}, \quad (40)$$

$$\mathbf{B} = \sigma_x^2 (\mathbf{r} - \mathbf{h}_i x_i)^H \mathbf{H}_{-i}, \quad (41)$$

$$\mathbf{C} = \sigma_x^2 (\mathbf{r}^H \mathbf{r} - \mathbf{r}^H \mathbf{h}_i x_i - (\mathbf{h}_i x_i)^H \mathbf{r}). \quad (42)$$

Similarly, all the estimation of the symbol \hat{x}_i in the $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{M_s}]^T$ could be derived by (39). After deriving $\hat{\mathbf{x}}$,

$$\mathcal{G}_{EF}(\mathbf{r}) = \beta \hat{\mathbf{x}}, \quad (43)$$

where β is the scaling factor satisfying the transmit power constraint in the relay, which can also be derived according to (18).

V. SIMULATION RESULTS

In this section, we compare the performance of different strategies (the proposed list EF, DF, AF, LMMSE EF, and matched filter AF) with $P_s = P_r = M_s = M_r$ and $\sigma_1^2 = \sigma_2^2$ for MIMO relay networks. The performance is evaluated by

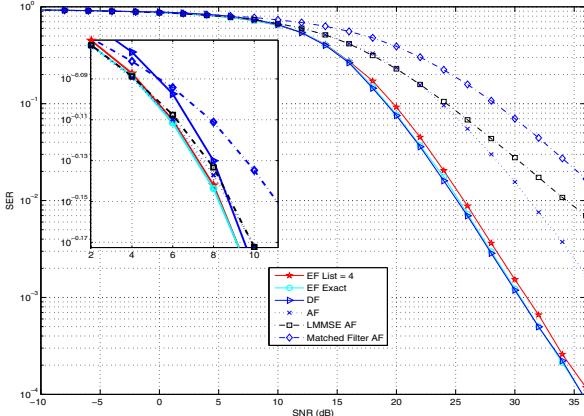


Fig. 2. SER of relay strategies in MIMO relay system, $M_s = N_r = M_r = N_d = 2$

the symbol error rate (SER). At the destination, the sphere decoder could be used to detect the transmitted signal based on the received signal from the relay.

The SER performance of different relay strategies is given for 2×2 16-QAM relay network in Fig. 2. The list EF with $N_{\mathcal{L}} = 4$ and the exact EF relay function are shown. The LMMSE AF and the matched filter AF (Eq. (28)) are also given to compare with the proposed EF. From Section III, the exact EF needs to compute all the $16^2 = 256$ elements in \mathcal{Q}^{M_s} . The list EF significantly reduces the computational complexity. It only needs 4 elements for the EF List = 4 case. In the low SNR region, all the list EF cases achieve similar performance with the AF, LMMSE AF and the matched filter AF, which agrees with the result in Eq. (28). It can also be seen that the EF obtains performance gain than all the AF cases. For example, at an $SER = 10^{-2}$, it obtains 6.5 dB and 9 dB gains than AF and LMMSE AF, respectively. Further, with larger list size $N_{\mathcal{L}}$, the list EF performs much closer to the exact EF case. It is worth to mention that even with $N_{\mathcal{L}} = 4$ the list EF approaches the performance of the exact EF and DF in the high SNR region with negligible performance loss.

The advantage of the proposed scheme is that we do not need to switch algorithms as SNR changes. We can simply use an unified algorithm for all cases.

VI. CONCLUSIONS

This paper proposed an estimate-and-forward (EF) relay strategy for MIMO relay networks. The characteristic of the EF relay function was analyzed and shown. The EF relay performs like AF in the low SNR region and approaches to DF in the high SNR region. For the MIMO relay networks with a large number of antennas and/or high order constellations, both the proposed list EF and the Gaussian approximation EF can make the computation of the MMSE estimate feasible. For the list EF, we proposed to use the sphere decoder for generating the reduced computation list. It achieves the exact EF relay strategy in large MIMO relay networks at a negligible

performance loss. The simulation results demonstrated that the proposed EF performs better than AF and DF for all the SNRs in large MIMO relay systems.

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