

Relative Torque Contribution Based Model Simplification for Robotic Dynamics Identification

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Abstract—It has been proved that minimizing the condition number of the observation matrix, which is calculated from the robot dynamic model and the associated exciting trajectories, is very effective for improving the identification accuracy of robotic dynamics. A relative simple dynamic model is beneficial for reduction of the associated condition number, and hence, several model simplification methods have been proposed in the literature. However, the existed methods cannot be used to efficiently process model structural errors, which will inevitably cause inaccurate estimation of the dynamics. Therefore, a novel model simplification method based on relative contribution of the undetermined parameters, is proposed to overcome the deficiency. Firstly, exciting trajectories for model simplification are designed by using finite Fourier series and optimized by using the condition number criteria. Then, the optimized exciting trajectory is implemented on the robot, and joint torques and motion data are recorded, which are used to calculate relative contribution of the undetermined parameters to joint torques. The model can be simplified repeatedly by neglecting the parameter that contributes least until the condition number is small enough. Finally, the performance of the proposed method is demonstrated by the identification and validation experiments conducted on a lower limb rehabilitation robot.

Index Terms—Identification, Model Simplification, Robotic Dynamics.

I. INTRODUCTION

A relatively accurate dynamic model is of great importance for model based control [1] or recognition of human motion intention in the application of rehabilitation robots [2] [3]. Structural errors caused by modeling methods, and parameter estimation errors mainly caused by measurement noise or by inappropriate design of the exciting trajectories, are two of the main factors that affect the accuracy of the estimated model. On one hand, in order to reduce structural errors, the dynamics should be modeled accurately to consider all related factors. On the other hand, the identification experiment should be carefully designed to improve the parameter estimation accuracy, among which design of the optimized exciting trajectories (OETs) and estimation of the undetermined parameters are two of the most important issues. The most popular method for optimization of the exciting trajectories is that, exciting trajectories are parameterized by using the finite Fourier series [4], and minimization of the condition number which is calculated from the dynamic

model and the associated exciting trajectories, is used as the objective function [5].

Since the condition number represents an upper limit for input/output error transmissibility [5], it should be designed as small as possible to reduce the effect of measurement noise. Two factors, including the complexity of the dynamic model and the suitability of the exciting trajectories used in the identification, can affect the condition number. If the dynamic model is very complicated or the exciting trajectories failed to fully excite the dynamics, the condition number will become large. In the literature, the second factor is usually considered by optimization of the exciting trajectories.

However, how to reduce the complexity of the dynamic model has not been well addressed. In [6] [7], model simplification methods based on mechanical structure have been proposed, which are effective for elimination of parameters related directly to mechanical structure. Whereas, if the friction related parameters, which are not only related to the mechanical structure but also to lubrication, roughness of the surface, etc.[8], should be considered, the methods of [6] [7] will become invalid. Data-driven model reduction methods have been proposed in [9] [10]. However, the data used in the parameter simplification methods of [9] [10], were obtained from the designed OET and not measured from sensors during actual implementation of the OET, and the dynamic parameters were simplified by considering the singular values of the associated observation matrix. Therefore, the structural errors of the designed dynamic model cannot be processed by the methods of [9] [10], which is the deficiency. In [11], a model simplification method based on the contribution of the parameters to joint torques is proposed, where a user defined contribution threshold was given and the parameters whose contribution below the threshold were neglected together. The method of [11] make it possible to set the desired accuracy of the simplified model, as is claimed by the authors. However, since contributions of the parameters are closely related to the selected OET, it is difficult to decide which one should be eliminated when the contributions of several parameters are close to each other, and meanwhile, elimination of several parameters together at a time will possibly cause over elimination, which will lead to too large structural errors.

Therefore, in this paper, a novel simplification method is proposed for parameter reduction of the robot dynamic model. The method of this paper is also data-driven, however,

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the data used in the simplification method are measured from the associated sensors during actual implementation of the associated OET. Therefore, the model structural errors can be handled to a certain extent. Meanwhile, relative torque contributions, which are different from the torque contributions in [11], are used to eliminate the parameter that has the least contribution. By this method, the dynamic model can be simplified step by step. In each step, only one parameter is eliminated, and hence, over elimination can be avoided.

The remainder of this paper is organized as follows: Section II describes the method of this paper; Section III gives the experiments and result; this paper is concluded in Section IV.

II. METHOD

A. The Dynamic Model and Design of the OET

The dynamic model of a serial robot with n joints can be described by:

$$\boldsymbol{\tau} = \boldsymbol{\Phi}(\boldsymbol{\Theta}, \dot{\boldsymbol{\Theta}}, \ddot{\boldsymbol{\Theta}}) \mathbf{P} \quad (1)$$

where $\boldsymbol{\Phi}$, a $n \times l$ regression matrix, is the function of joint angles, angular velocities and accelerations. \mathbf{P} and $\boldsymbol{\tau}$ are respectively the parameter vector and joint torque vector, which are given respectively by:

$$\mathbf{P} = (p_1, p_2, \dots, p_l)^T \quad (2)$$

and

$$\boldsymbol{\tau} = (\tau_1, \dots, \tau_n)^T \quad (3)$$

The elements of \mathbf{P} are the undetermined parameters to be identified, including inertial and friction related parameters. The model defined by (1) - (3) is linear, and referred as the preliminary dynamic model (PDM) in the following text to distinguish it from the simplified model obtained in the model simplification process.

The finite Fourier series are used to parameterize exciting trajectories, and minimization of the condition number is used as the objective function for the optimization of exciting trajectories. The associated optimization problem for exciting trajectories can be solved (namely OETs for the PDM (OET_{PDM}) can be obtained) by using the method given in [2].

B. Design of MSTs

The simplification method designed by this paper consists of three steps: design of the model simplification trajectories (MSTs), data acquisition and model simplification.

Data measured from associated sensors during implementation of the MSTs, are used for simplifying the dynamic model in the proposed simplification method. In order to implement the simplification successfully, the MSTs should satisfy the following three conditions: 1) they can be implemented, specifically the joint angles, angular velocities and

accelerations should be designed in the suitable ranges which can be implemented by the robot; 2) the undetermined parameters can be well excited by the MSTs; 3) the observation matrix computed from the MSTs should be full rank, such that the parameters can be accurately estimated.

As these conditions can be satisfied easily by the OET_{PDM} obtained in the above subsection, it is used as the first MST. Other MSTs used in the subsequent model simplification process can also be designed by using the method same to that for the OET_{PDM}.

C. Model Simplification

During the simplification process, the MST corresponding to current dynamic model is implemented by the robot first. Then the data obtained during the implementation, including joint angles, angular velocities and accelerations, and joint torques, are used to calculate contribution of each undetermined parameter. Finally the undetermined parameters can be simplified by neglecting those parameters with less contribution. The overdetermined equation calculated from the current dynamic model and the measured data, can be described by:

$$\boldsymbol{\Gamma} = \mathbf{W}_c \mathbf{P}_c \quad (4)$$

where the vector, \mathbf{P}_c , represents current parameter set; the elements of the current observation matrix \mathbf{W}_c , are calculated from the current dynamic model and the measured data; $\boldsymbol{\Gamma}$ is the torque vector, which consists of the measured joint torques. Then, \mathbf{P}_c can be calculated by the least square estimation method, as follows:

$$\mathbf{P}_c = (\mathbf{W}_c^T \mathbf{W}_c)^{-1} \mathbf{W}_c^T \boldsymbol{\Gamma} \quad (5)$$

For the k th equation of (4), the torque contributed by the j th parameter of current parameter set can be described by:

$$\tau_{k,j} = w_{k,j}^c p_j^c \quad (6)$$

where $k = 1, 2, \dots, K_\tau$; K_τ is the number of the equations of (4); $j = 1, 2, \dots, N_p$; N_p is the number of current undetermined parameters; $w_{k,j}^c$ is the (k, j) th element of \mathbf{W}_c ; p_j^c is the j th element of \mathbf{P}_c . Then, the relative contribution of the j th parameter to joint torques for the whole MST can be defined by:

$$\gamma_j = \frac{\tau_j}{\sum_{s=1}^{N_p} \tau_s} \quad (7)$$

where τ_j and τ_s are defined respectively by:

$$\tau_j = \sqrt{\sum_{k=1}^{K_\tau} (\tau_{k,j})^2} \quad (8)$$

and

$$\tau_s = \sqrt{\sum_{k=1}^{K_\tau} (\tau_{k,s})^2}. \quad (9)$$

The dynamic model can be simplified by neglecting the parameter that corresponds to the least relative contribution and a new dynamic model can be obtained. Then, this model

can be used to optimize the exciting trajectories again and a new OET can be obtained. The observation matrix can be computed from the new OET and current dynamic model. If the condition number of the observation matrix is small enough, the simplification is finished; otherwise, the new OET can be used as a new MST to repeat the simplification again. Therefore, the simplification method of this paper is a recursive one, which is referred as RSA (recursive simplification algorithm) in the following text, and can be summarized as follows:

- 1) Optimize exciting trajectories, which are parameterized by using the FFS method, for the PDM and the first MST can be obtained.
- 2) The MST is implemented by the robot and the motion data and joint torques are recorded and preprocessed.
- 3) Calculate the relative contribution of each dynamic parameter by (7).
- 4) Neglect the parameter that has the smallest relative contribution and a new dynamic model can be obtained.
- 5) Optimize exciting trajectories for the new dynamic model and a new OET can be obtained.
- 6) Calculate the observation matrix by using the current dynamic model and the associated OET;
- 7) Calculate the condition number of the observation matrix; if the condition number is small enough (e.g. less than 100, as is suggested by Schr  er [12] and to be discussed in the following context), the simplification is finished, or else the OET obtained in 5) can be used as a new MST and repeat 2).

III. EXPERIMENTS AND DISCUSSION

A. The Experiment Platform and Data Acquisition

The right leg mechanism of *iLeg* [2] [13], which is given in Fig. 1, is used as the experiment platform in this paper. It can be taken as a 2 DOF planar robot which consists of two active joints: the hip and knee joints, and two links: the thigh and crus links. The schematic plot of the leg mechanism is given in Fig. 2, and the associated dynamic model has 12 undetermined parameters, the detail of which can be seen in [2]. Since the coupling factors between the hip and knee joints, have been considered in the joint frictions, the dynamic model is more complicated than the usual 2 DOF robot model which has 10 undetermined parameters. Meanwhile, ranges of the joint angles, angular velocities and accelerations, are relatively small. Therefore, the condition number calculated from the preliminary dynamic model and the associated OET-s is large. A recursive optimization algorithm (ROA) has been proposed in [2], by which the obtained optimized solutions of the optimization problem for the exciting trajectories were very close to the actual global optima, and the condition number of the associated observation matrix was reduced effectively while the model structural accuracy being kept. However, since the dynamic model is not changed actually

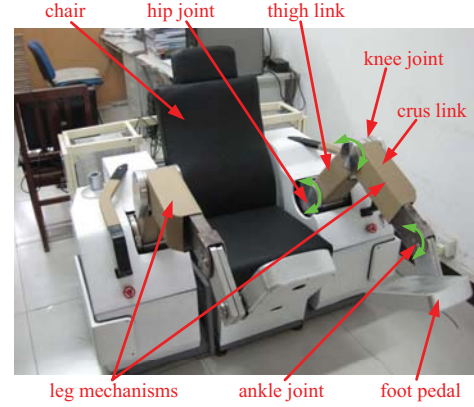


Fig. 1. The prototype of *iLeg*[2].

by using the ROA method, the global optima are not changed, and hence, the condition number cannot be further reduced by using the ROA method alone.

In the experiments of this paper, the joint angles and torques were measured respectively by the position and torque sensors mounted on the hip and knee joints of the leg mechanism. In order to obtain the data necessary for model simplification, each MST was implemented for 12 complete periods by the leg mechanism, and joint angles and torques were recorded at every 30 ms. The data measured in the first and last periods were neglected in order to decrease the effect of measurement noise. The torques were filtered by moving average filters, which were carried out by smooth functions of Matlab. The spans of the smooth functions for the torques were set to 9. Let K points of the trajectory are recorded, the angular velocities and accelerations can be obtained respectively by:

$$\dot{\theta}_{i,k} = \frac{\theta_{i,k+1} - \theta_{i,k-1}}{2\Delta t}, \forall i = 1, 2; k = 2, 3, \dots, K-1 \quad (10a)$$

$$\ddot{\theta}_{i,k} = \frac{\dot{\theta}_{i,k+1} - \dot{\theta}_{i,k-1}}{2\Delta t}, \forall i = 1, 2; k = 3, 4, \dots, K-2 \quad (10b)$$

where Δt is the time interval between two adjacent recorded points, θ_1 and θ_2 are respectively the hip and knee joint angles. The angular velocities were used directly in the parameter estimation; meanwhile, the angular accelerations obtained by (10b) were preprocessed by using the smooth functions whose spans were set to 5.

B. Model Simplification Experiment

The model simplification method given in Section II was used in this experiment. The first MST, namely the OET_{PDM} of the leg mechanism is given in Fig. 2. A series of monotonously decreasing condition numbers were obtained during the simplification process, which is given in Fig. 3. The final simplified dynamic model (SDM) obtained in the

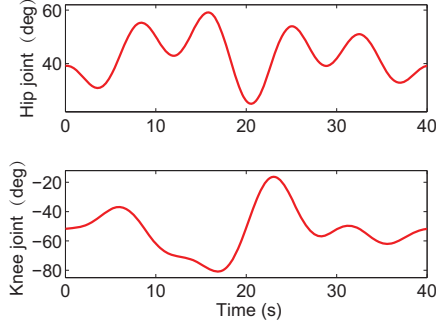


Fig. 2. The joint angles for OET_{PDM}.

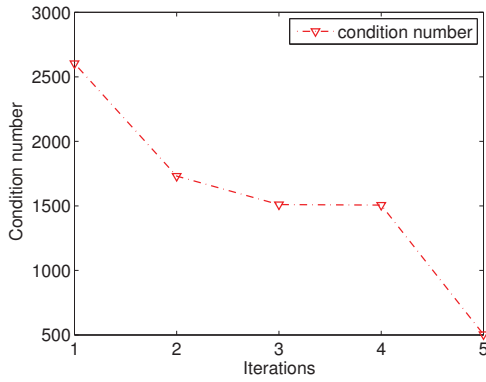


Fig. 3. Variation tendency of the condition number during the model simplification process.

experiment is given by:

$$\tau = \Phi_s \mathbf{P}_s \quad (11)$$

where \mathbf{P}_s was derived by neglecting parameters p_1 , p_3 , p_{10} , and p_{12} , in turn; the elements of Φ_s are corresponding to the simplified parameter set. On one hand, since p_{10} and p_{12} are friction related parameters, the result of this experiment denotes that, the friction model can be further simplified to improve model accuracy. On the other hand, the inertia parameters, p_3 and p_1 , were neglected, which can be explained by the small joint accelerations and shows the unusual dynamic characteristics of the robot under strong motion constraints.

The OET for the SDM (OET_{SDM}) is also obtained when the simplification is finished. The condition number of the observation matrix for the OET_{SDM} is 503.4. It should be noted that, since the simplification method of this paper is directly related to actual data, the stop condition should be designed carefully for specific case. In this paper, when the parameter p_{12} is neglected, it becomes difficult or very slow to further decrease the condition number by neglecting parameters that have less contribution. Since torque estimation was accurate enough in the experiment, which can be seen

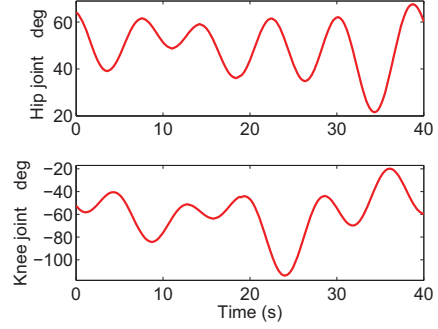


Fig. 4. The joint angles for the OET_{SDM}.

in the following subsections, the simplification process was stopped; or else, it should be further implemented. The joint angles of *iLeg* for the OET_{SDM}, are given in Fig. 4.

C. Parameter Estimation Experiment

In this experiment, the OET_{SDM} obtained in the above subsection was implemented first, during which the motion data and joint torques were measured and recorded. Then, these data were preprocessed by the method described in Section III-A. The undetermined parameters of the SDM were estimated by using the LSE method, as follows:

$$\mathbf{P}_s = (\mathbf{W}_s^T \mathbf{W}_s)^{-1} \mathbf{W}_s^T \mathbf{\Gamma} \quad (12)$$

where \mathbf{W}_s was derived from Φ_s . The estimation experiment was implemented for five times and the results were statistically analyzed. The estimated values of the undetermined parameters are given in Table I, where the average value, \bar{X} , and the relative standard deviation, $\% \sigma$, are defined respectively by:

$$\bar{X} = \frac{1}{5} \sum_{i=1}^5 X_i. \quad (13)$$

and

$$\% \sigma = \frac{\sqrt{\sum_{i=1}^5 (X_i - \bar{X})^2}}{5\bar{X}} \times 100\%. \quad (14)$$

Then the obtained dynamic model was used to reconstruct the joint torques, as is shown in Fig. 5. The reconstructed torque errors are given in Table II, where the root-mean-square error of the reconstructed torques is defined by:

$$\tau_{i,rms} = \sqrt{\frac{1}{K_c} \sum_{k=1}^{K_c} (\tau_{e,i,k} - \tau_{m,i,k})^2}, \forall i = 1, 2, \quad (15)$$

where K_c is the number of points used in the calculation; $\tau_{e,i,k}$ and $\tau_{m,i,k}$ are respectively the k th estimated and measured torques for the i th joint (the hip and knee joints are taken as the first and second joints respectively). β_{are} is

TABLE I
THE UNDETERMINED PARAMETERS ESTIMATED BY LSE.

parameter	unit	value					\bar{X}	% σ
		1	2	3	4	5		
p_2	Nm	13.8238	14.0202	13.7847	13.8903	13.8968	13.8831	0.5783
p_4	kg·m ²	0.9722	0.9721	0.9725	0.9722	0.9722	0.9722	0.0144
p_5	kg·m ²	0.0120	0.0122	0.0119	0.0120	0.0117	0.0120	1.4498
p_6	Nm	-2.3991	-2.2644	-2.4859	-2.4584	-2.4552	-2.4126	3.2857
p_7	Nm	0.5071	0.4829	0.4715	0.4430	0.4632	0.4735	4.4917
p_8	kg·m ²	43.9145	40.8640	45.6607	38.2915	41.7885	42.1038	6.0180
p_9	kg·m ²	38.6113	31.2062	38.5468	28.1294	33.0645	33.9116	12.1614
p_{11}	Nm	0.2772	0.2668	0.2750	0.2751	0.2739	0.2736	1.3063

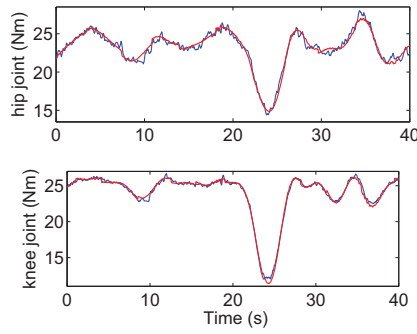


Fig. 5. The measured and reconstructed torques for the OET_{SDM}, which are respectively represented by the blue and red lines.

TABLE II
THE RECONSTRUCTED TORQUE ERRORS FOR THE OET.

$\tau_{rmse}(\text{Nm})$		$\beta_{are}(\%)$
hip	knee	
0.4089	0.1841	1.01

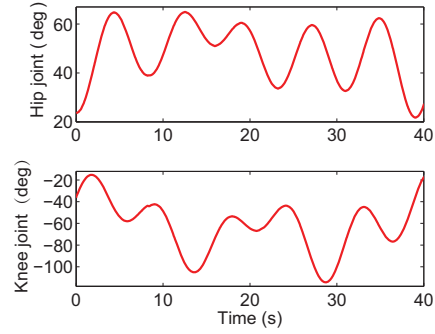


Fig. 6. The joint angles for the OET_{ODM}.

TABLE III
THE ESTIMATED TORQUE ERRORS FOR THE VALIDATION TRAJECTORY.

$\tau_{rmse}(\text{Nm})$		PDM	OPM	SDM
	hip	0.4487	0.4182	0.37
	knee	0.7666	0.3672	0.288
$\beta_{are}(\%)$		2.95	1.85	1.56

the average relative error of the reconstructed or estimated joint torques, which is defined by:

$$\beta_{are} = \frac{\sqrt{\frac{1}{2K_c} \sum_{k=1, \dots, K_c} \sum_{i=1,2} (\tau_e^{i,k} - \tau_m^{i,k})^2}}{\sqrt{\frac{1}{2K_c} \sum_{k=1, \dots, K_c} \sum_{i=1,2} (\tau_m^{i,k})^2}}. \quad (16)$$

It can be seen that, the estimation errors are very small, which shows that the dynamics of the leg mechanism can be well described by the SDM.

D. Validation Experiment and Discussion

In order to illustrate the performance of the proposed model simplification method, two models different from the SDM were used to describe the dynamics of the leg mechanism: the PDM, which has 12 parameters, and the optimized dynamic model (ODM) designed by using the ROA method of [2].

The undetermined parameters of PDM and ODM were identified first. The optimized exciting trajectory for ODM (OET_{ODM}) is given in Fig. 6 ((OET_{PDM}) was given already in Fig. 2). Then, a circular trajectory often used in the lower limb rehabilitation training, which is different from the OETs for the PDM, SDM, and ODM, was used as the validation trajectory. The validation trajectory is given in Fig. 7(a). The validation trajectory was implemented on the robot and the joint angles and torques were recorded at the same time at every 30 ms. Finally, the obtained dynamic models, the PDM, ODM, and SDM, were used to estimate the joint torques based on the recorded data from the validation trajectory. The measured and estimated joint torques are given in Fig. 7. A more quantitative interpretation of the figures is given in Table III.

It is shown that, the estimation errors for the PDM are bigger than those for the ODM and SDM. Therefore, the

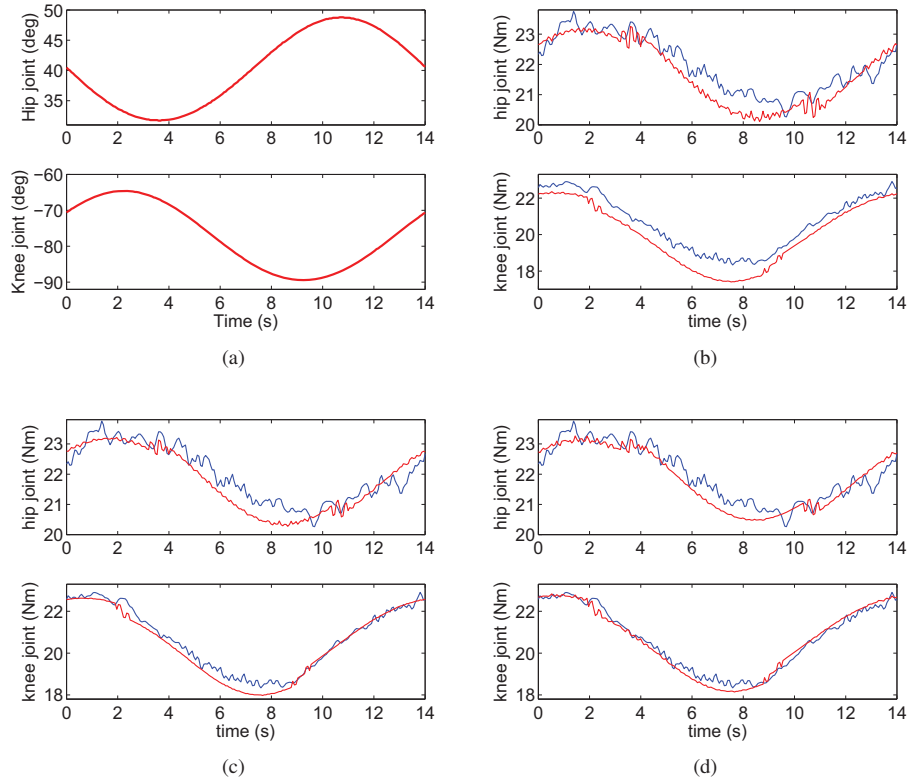


Fig. 7. The validation trajectory (a) and joint torques for the validation trajectory for the (b) PDM, (c) ODM, and (d) SDM. The measured and estimated torques are represented respectively by the blue and red lines.

ROA method of [2] and the RSA method of this paper are both effective for improving estimation accuracy. Meanwhile, the estimation errors for the SDM are slightly smaller than those for the ODM, which shows that the RSA method can obtain similar estimation accuracy to the ROA method.

The ROA method provides an efficient method for searching the global optima. However, since the actual dynamic model is not changed in the ROA method, when the global optima is reached, the condition number cannot be further reduced by using the ROA method alone. In this case, the RSA can be used. The RSA method is a tradeoff between the structural accuracy and the immunity to measurement noise. In order to obtain an accurate torque estimation, it is expected that, the model structure should be relatively accurate and the noise immunity should be stronger, which however are two contradictory factors. In other words, if the model structure is very accurate, the dynamic model will become relatively complicated, and the condition number calculated from the dynamic model and the associated OET will become relatively large. The RSA method can be used to balance in the two factors.

The advantage of the ROA method is that, the structural accuracy can be kept while improving the noise immunity.

Therefore, if the dynamic model is not too complicated, it is suggested to use the ROA method alone; or else, the RSA method can be used together with the ROA method to use the benefits of two methods to obtain satisfied estimation accuracy.

IV. CONCLUSION

In order to obtain an accurate dynamic model for robots, two aspects of methods can be used: i) improving structural accuracy of the dynamic model; ii) raising noise immunity of the identification experiment. However, these two factors are usually contradictory to each other. In one of our previous papers [2], a ROA method was proposed to reduce the condition number, which has been proven effective to improve the noise immunity. Whereas, since the dynamic model is not changed in fact in the ROA method, when global optima of the optimization problem for exciting trajectories are reached, the condition number cannot be further reduced by using the ROA method alone. It is not satisfying especially when the condition number is relatively large brought out by the complicated dynamic model.

The RSA method proposed in this paper can be used to overcome the deficiency. Based on contribution of the dynamic parameters to joint torques, the method can be

used to simplify the dynamic model, and hence, to further reduce the condition number. The RSA method is a tradeoff between the structural accuracy and noise immunity. The details of the method is given and the identification and validation experiments were carried out, from which it can be seen that, the RSA method is effective for improving the estimation accuracy. Future works will be focused on further improvement of the algorithm efficiency and its application in practice.

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