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Distributed event-triggered synchronization for robotic teleoperation systems with randomly occurring gain fluctuations

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Abstract
This article investigates the synchronization problem of robotic teleoperation systems by developing a novel event-triggered networking strategy. Furthermore, the randomly occurring controller gain fluctuations are considered for the robotic teleoperation systems, such that more robustness can be obtained in the controller implementation. Based on model transformation, sufficient synchronization conditions are derived by employing the Lyapunov–Krasovskii method. Then, distributed controllers are designed for achieving the mean square synchronization. Finally, the effectiveness and feasibility of our proposed control strategy are demonstrated by an illustrative example.

Keywords
Asynchronous event-triggered scheme, event-triggered synchronization, robotic teleoperation systems, randomly occurring gain fluctuations, Lyapunov–Krasovskii method

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Introduction

In the past decade, robotic teleoperation systems (RTSs) have received increasing attention due to the urgent demand in both civil and military applications, such as remote surgery,[1] rescuing in hostile zones,[2] exploration in unknown environments,[3] and so on.[4–6] RTSs are typically composed of a master system (MS) with the human operator and the corresponding slave systems (SSs), where the information can be exchanged via the communication network. A basic function of RTS is achieving the synchronization, which means that the SS can track the motion of the MS controlled by the human operator. To deal with this issue, many effective control approaches have been reported (see, e.g.,refs [7–9] and the references therein). It should be pointed out that the communication network has played a key role for successful synchronization. However, it is well known that the communication networks are with certain constraints, such as limited transmission speed, stability, network bandwidth, and so on.[10–13] As a result, these constraints should be addressed for the analysis and design of RTSs.

Recently, many researchers have intensively investigated the so-called event-triggered networked control systems and related results have been proposed following this research line. Different from traditional control strategy, the event-triggered strategy can obtain benefits in

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increasing the robustness and reliability of signal transmission and decreasing the network load.[14–16] Among many event-triggered control strategies, discrete-time event-triggered approach is designed,[17,18] where the computing and networking resources can be utilized more effectively without relying on continuous-time monitoring the event-triggered conditions. Although the usefulness of event-triggered control approaches has been proved in many open literature, so far, there are few results on the event-triggered synchronization problem of RTSs, which is a challenging issue.

On another research frontline, it is noteworthy that the controller parameter perturbations in the real-world implementation could decrease the closed-loop performance.[19-22] These facts give rise to the so-called nonfragile control problem. As a result, some effective control method with regard to gain fluctuations have been developed.[23–25] In addition, further studies have shown that the gain fluctuations often happen in a random way,[25,26] such that considering the randomly occurring gain fluctuations in design can lead to less conservatism. For the synchronization problem of RTSs in the realistic applications, the randomly occurring gain fluctuations of the synchronization controllers should be addressed to be more practical. Unfortunately, this problem still remains open.

The above considerations motivate us for the investigation on the event-triggered synchronization design of RTSs with distributed information exchanges. In comparison with the existing results, the main contributions of this article can be summarized as twofold: (i) A novel asynchronous event-triggered networking strategy is developed for the communication of the MS and multi-SSs, where discrete-time measurement is introduced and event-triggered transmission is adopted. (ii) The developed synchronization controllers are in distributed configuration with nonfragile properties. The random model obeying the Bernoulli distribution is used for characterizing the randomly occurring controller gain fluctuations.

The remainder of this arranged is arranged as follows. Some preliminaries on the algebraic graph basics and the RTS dynamics are first introduced. Then, the event-triggered synchronization controllers are designed, and the corresponding sufficient synchronization conditions are established in the mean square sense. Furthermore, we give the simulation results to demonstrate the effectiveness of our developed methods. In the end, the conclusion and discussions are given.

Notation. The notations are standard throughout this article. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n dimensional Euclidean space and the space of $m\times n$ real matrices, respectively. $P>0$ means P is positive definite. $\mathrm{col}_N\{x_i\}$ presents $[x_1^T, x_2^T, \ldots, x_N^T]^T$. $A\otimes B$ stands for the Kronecker product. $\mathbb{E}\{\cdot\}$ denotes mathematical expectation; $\Pr\{\alpha\}$ represents the probability of an event α.

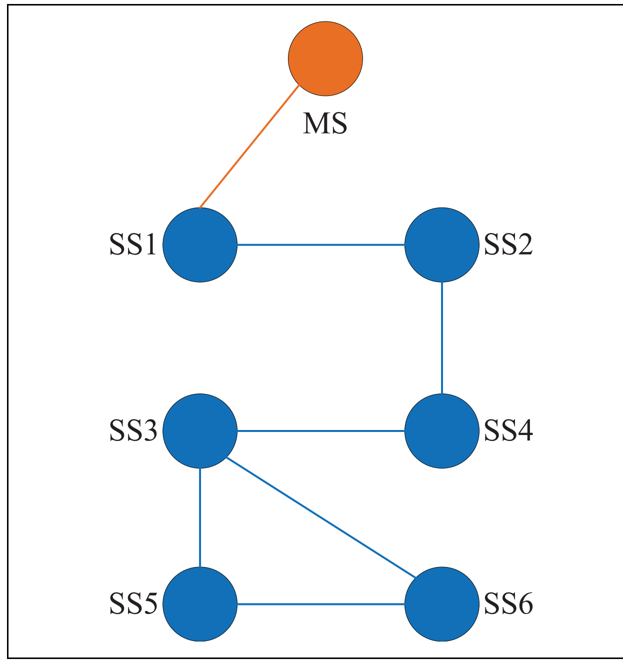

Figure 1. An illustration of the communication graph of 1 MS and 6 SSs, where the lines between the nodes denote the communication connections of the RTS, respectively ($b_1, a_{12}, a_{21}, a_{24}, a_{42}, a_{34}, a_{43}, a_{35}, a_{53}, a_{36}, a_{63}, a_{56}, a_{65} > 0$). MS: master system; SS: slave system; RTS: robotic teleoperation systems.

Preliminaries and problem formulation

Algebraic graph basics

For representing the teleoperation communication, the undirected graph $\mathcal{G} = [\mathcal{V}, \mathcal{E}, \mathcal{A}]$ with N nodes is utilized, which is depicted in Figure 1. $\mathcal{V} = \{v_i\}_N$ denotes the set of nodes, $\mathcal{E}$ denotes the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix with

$$\begin{cases} a_{ij} > 0, \ (v_i, v_j) \in \mathcal{E}, \\ a_{ij} = 0, \ \text{Otherwise} \end{cases} \tag{1}$$

$\mathcal{G}$ is called connected if any two nodes can be connected via a path. $L = (l_{ij})_{N\times N}$ denotes the Laplacian matrix with

$$\begin{cases} l_{ij} = -a_{ij}, i \neq j \\ l_{ii} = \sum_{j=1, i\neq j}^{N} a_{ij}. \end{cases} \tag{2}$$

$B = \mathrm{diag}\{b_1, b_2, \ldots, b_N\}$ denotes the MS adjacency matrix with

$$\begin{cases} b_i > 0, \ \text{MS is connected with SS}i \\ b_i = 0, \ \text{Otherwise.} \end{cases} \tag{3}$$

Robotic teleoperation systems

For the sake of simplicity, consider the following RTS with 1 MS and N SSs with one degree-of-freedom[27]:

$$J\ddot{\theta}_i(t) + b\dot{\theta}_i(t) = \tau_i(t), \; i = 0, 1, 2, \ldots, N \tag{4}$$

where J denotes the inertia, θ_i denotes the rotate angle, b denotes the viscous fiction coefficient, and $\tau_i(t)$ denotes the control torque.

System (4) can be rewritten in state-space formulation as follows:

$$\dot{x}_i(t) = \mathcal{A}x_i(t) + \mathcal{C}\tau_i(t), \; i = 0, 1, 2, \ldots, N \tag{5}$$

where $x_i(t) = [\theta_i(t), \dot{\theta}_i(t)]^T$ and

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ 0 & -b/J \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 0 \\ 1/J \end{bmatrix}$$

Then, by applying the same multi-agent framework in the work by Rakkiyappan et al.,[28] system (5) can be further described in a general framework:

$$\begin{cases} \text{MS} : \dot{x}_0(t) = \mathcal{A}x_0(t) \\ \text{SSs} : \dot{x}_i(t) = \mathcal{A}x_i(t) + \mathcal{C}\tau_i(t), \; i = 1, 2, \ldots, N \end{cases} \tag{6}$$

Hence, it can be verified that the synchronization is achieved if $x_i(t) \to x_0(t)$, $t \to \infty$.

The aim of this article is to design event-triggered controllers for RTS (equation (4)) such that the synchronization teleoperation can be achieved.

Event-triggered synchronization controller

Assume that the controller and the actuator are event driven. For the time-driven sampler, the sampling period is set as h. In order to trigger the transmissions, the following function is introduced:

$$\begin{aligned} t^i_{\sigma+1}h := t^i_\sigma h + \min_{l_i \geq 1}\{l_i h | \Upsilon_i^T(t^i_\sigma h + l_i h)\Phi_1 \Upsilon_i(t^i_\sigma h + l_i h) \\ \geq \varepsilon \gamma_i^T(t^i_\sigma h + l_i h)\Phi_2 \gamma_i(t^i_\sigma h + l_i h)\} \end{aligned} \tag{7}$$

$$\begin{aligned} \Upsilon_i(t^i_\sigma h + l_i h) &:= x_i(t^i_\sigma h + l_i h) - x_i(t^i_\sigma h), \gamma_i(t^i_\sigma h + l_i h) \\ &:= \sum_{j \in \mathcal{N}_i} a_{ij}[x_i(t^i_\sigma h) - x_j(t_{\sigma^*} ih)] \\ &\quad + b_i\Big([x_i(t^i_\sigma h) - x_0(t_{\sigma^*} ih)]\Big) \\ \sigma^* &\triangleq \arg\min_\kappa\{t^i_\sigma + l_i - t^j_\kappa | t^i_\sigma + l_i > t^j_\kappa, \; \kappa \in \mathbb{N}\} \end{aligned} \tag{8}$$

where $0 < \varepsilon < 1$, $\Phi_1, \Phi_2 > 0$, and $t^i_\sigma h$ are the latest σth transmission.

Remark 1. In the proposed asynchronous event-triggered networking strategy, once the triggering function is satisfied, the neighboring SSs and the MS would exchange the information to the local SS.

Consequently, the synchronization controllers for multi-SSs are designed by

$$\begin{aligned} \tau_i(t) = -K\Bigg\{ & \sum_{j \in \mathcal{N}_i} a_{ij}[x_i(t^i_\sigma h) - x_j(t^i_{\sigma^*} h)] \\ & + b_i[x_i(t^i_\sigma h) - x_0(t^i_{\sigma^*} h)]\Bigg\}, \; t \in [t^i_\sigma h, t^i_{\sigma+1} h) \end{aligned} \tag{9}$$

where K denotes the controller gain.

Due to the randomly occurring controller gain fluctuations, the synchronization inputs are rewritten as

$$\begin{aligned} \tau_i(t) = -(K + \beta(t)\Delta K(t))\Bigg\{ & \sum_{j \in \mathcal{N}_i} a_{ij}[x_i(t^i_\sigma h) - x_j(t^i_{\sigma^*} h)] \\ & + b_i[x_i(t^i_\sigma h) - x_0(t^i_{\sigma^*} h)]\Bigg\}, \; t \in [t^i_\sigma h, t^i_{\sigma+1} h) \end{aligned} \tag{10}$$

where $\Delta K(t)$ is the controller gain fluctuation satisfying $\Delta K = H\Psi(t)E$, $\Psi^T(t)\Psi(t) \leq I$; $\beta(t)$ is a Bernoulli-distributed variable defined as

$$\beta(t) = \begin{cases} 1, & \text{controller gain fluctuation happens} \\ 0, & \text{controller gain fluctuation does not happen} \end{cases} \tag{11}$$

Moreover, assume that

$$\Pr\{\beta(t) = 1\} = \bar{\beta} \tag{12}$$

$$\Pr\{\bar{\beta}\,(t) = 0\} = 1 - \bar{\beta}\,, \; \bar{\beta}\, \in 0, 1] \tag{13}$$

Remark 2. As Bernoulli distribution has been well utilized to represent the randomly occurring phenomenon in control systems, the Bernoulli distribution is adopted to describe the randomly occurring controller gain fluctuation. Moreover, in order to use the convex optimization method for solving the controller design problem in terms of linear matrix inequalities, $\Delta K = H\Psi(t)E$ is given for matrix inequality transformation.

Define $\tilde{x}_i(t) := x_i(t) - x_0(t)$. The closed-loop dynamics of the RTS can be derived as

$$\begin{aligned} \dot{\tilde{x}}_i(t) = \mathcal{A}\tilde{x}_i(t) - \mathcal{C}(K + \beta(t)\Delta K)\Bigg\{ & \sum_{j \in \mathcal{N}_i} a_{ij}[\tilde{x}_i(t^i_\sigma h) \\ & - \tilde{x}_j(t^i_{\sigma^*} h)] + b_i\tilde{x}_i(t^i_\sigma h)\Bigg\}, \; t \in [t^i_\sigma h, t^i_{\sigma+1} h) \end{aligned} \tag{14}$$

The following lemma is needed for later use.

Lemma 1.[29] Given $L^T = L$, real matrices H, E and $F(t)$ satisfying $F^T(t)F(t) \leq I$. $L + HFE + E^T F^T H^T < 0$, if and only if there exists $\varepsilon > 0$, such that $L + \varepsilon^{-1}HH^T + \varepsilon E^T E < 0$, or equivalently

$$\begin{bmatrix} L & H & \varepsilon E^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{15}$$

Main results

In this section, sufficient mean square synchronization criteria will be established by the linear matrix inequality technic.

Theorem 1. With the event-triggered control input (equation (10)) and connected $\mathcal{G}$, the synchronization of RTS (equation (4)) can be achieved if there exist positive symmetric matrices $P > 0$, $Q > 0$, $\Phi_1 > 0$, and $\Phi_2 > 0$, such that $\Pi < 0$, where

$$\Pi := \begin{bmatrix} \Pi_1 & \Pi_2 \\ \star & \Pi_3 \end{bmatrix} \tag{16}$$

$$\Pi_1 := \begin{bmatrix} \Pi_{11} & G \otimes P\mathcal{C}K - \varepsilon G^T G \otimes \Phi_2 & G \otimes P\mathcal{C}K \\ \star & -I \otimes Q + \varepsilon G^T G \otimes \Phi_2 & 0 \\ \star & \star & -I \otimes \Phi_1 \end{bmatrix} \tag{17}$$

$$\Pi_{11} := 2I \otimes P\mathcal{A} - 2G \otimes P\mathcal{C}K + \varepsilon G^T G \otimes \Phi_2 \tag{18}$$

$$\Pi_2 := \begin{bmatrix} \Pi_{21} & \bar{\beta} G \otimes P\mathcal{C}H & -\varepsilon (I \otimes E)^T \\ hG^T \otimes K^T \mathcal{C}^T & 0 & \varepsilon (I \otimes E)^T \\ hG^T \otimes K^T \mathcal{C}^T & 0 & \varepsilon (I \otimes E)^T \end{bmatrix} \tag{19}$$

$$\Pi_{21} := hI \otimes \mathcal{A}^T - hG^T \otimes K^T \mathcal{C}^T \tag{20}$$

$$\Pi_3 := \begin{bmatrix} -I \otimes Q^{-1} & \Pi_{31} & \varepsilon (I \otimes E)^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{bmatrix} \tag{21}$$

$$\Pi_{31} := \left(\sqrt{\bar{\beta}(1-\bar{\beta})} + \bar{\beta}\right) G \otimes \mathcal{C}H \tag{22}$$

$$G := L + B \tag{23}$$

Proof. For $t \in \left[lh, (l+1)h\right)$ one has

$$\begin{aligned} \dot{\tilde{x}}(t) = (I \otimes \mathcal{A})\tilde{x}(t) - [(L+B) \otimes (\mathcal{C}K \\ + \beta(t)\mathcal{C}\Delta K)(\tilde{x}(t - \mathrm{d}(t)) - e(lh)) \end{aligned} \tag{24}$$

where $\tilde{x}(t) = \mathrm{col}_N\{\tilde{x}_i(t)\}$, $d(t) := t - lh$ and $e(lh) := \tilde{x}(lh) - \tilde{x}(t_\sigma h)$

Choose the following Lyapunov–Krasovskii function:

$$\begin{aligned} V(t) = \tilde{x}^T(t)(I \otimes P)\tilde{x}(t) \\ + h \int_{-h}^{0} \int_{t+s}^{t} \dot{\tilde{x}}^T(\eta)(I \otimes Q)\dot{\tilde{x}}(\eta)\, \mathrm{d}\eta\, \mathrm{d}s. \end{aligned} \tag{25}$$

Define the infinitesimal operator $\mathcal{L}$ of $V(t)$ as

$$\mathcal{L}V(t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \{\mathbb{E}\{V(t+\Delta)|t\} - V(t)\} \tag{26}$$

It yields that

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V(t)\} &= \mathbb{E}\Big\{\dot{\tilde{x}}^T(t)(I \otimes P)\tilde{x}(t) + \tilde{x}^T(t)(I \otimes P)\dot{\tilde{x}}(t) + h^2 \dot{\tilde{x}}^T(t)(I \otimes Q)\dot{\tilde{x}}(t) - h\int_{t-h}^{t} \dot{\tilde{x}}^T(s) \times (I \otimes Q)\dot{\tilde{x}}(s)\, \mathrm{d}s\Big\} \\ &= \mathbb{E}\Big\{2\tilde{x}^T(t)(I \otimes P)[(I \otimes \mathcal{A})\tilde{x}(t) - [(L+B) \otimes (\mathcal{C}K + \beta(t)\mathcal{C}\Delta K)(\tilde{x}(t - d(t)) - e(lh))] \\ &\quad + h^2 \dot{\tilde{x}}^T(t)(I \otimes Q)\dot{\tilde{x}}(t) - h\int_{t-h}^{t} \dot{\tilde{x}}^T(s)(I \otimes Q)\dot{\tilde{x}}(s)\, \mathrm{d}s\Big\} \\ &= \mathbb{E}\Big\{2\tilde{x}^T(t)(I \otimes P\mathcal{A})\tilde{x}(t) - 2\tilde{x}^T(t)[(L+B) \otimes (P\mathcal{C}K + \bar{\beta}P\mathcal{C}\Delta K](\tilde{x}(t - \mathrm{d}(t))) - e(lh))] \\ &\quad - h\int_{t-h}^{t} \dot{\tilde{x}}^T(s)(I \otimes Q)\dot{\tilde{x}}(s)\, \mathrm{d}s + h^2 \mathbb{E}\Big\{\dot{\tilde{x}}^T(t)(I \otimes Q)\dot{\tilde{x}}(t)\Big\} \\ &\leq \mathbb{E}\Big\{2\tilde{x}^T(t)(I \otimes P\mathcal{A})\tilde{x}(t) - 2\tilde{x}^T(t)[(L+B) \otimes (P\mathcal{C}K + \bar{\beta}P\Delta K)](\tilde{x}(t - \mathrm{d}(t))) - e(lh))] \\ &\quad - \int_{t-d(t)}^{t} \dot{\tilde{x}}^T(s)\, \mathrm{d}s(I \otimes Q)\int_{t-d(t)}^{t} \dot{\tilde{x}}(s)\, \mathrm{d}s + h^2 \mathbb{E}\Big\{\dot{\tilde{x}}^T(t)(I \otimes Q)\dot{\tilde{x}}(t)\Big\} \end{aligned} \tag{27}$$

Note that $\tilde{x}(t - d(t)) = \tilde{x}(t) - \int_{t-\mathrm{d}(t)}^{t} \dot{\tilde{x}}^T(s)\, \mathrm{d}s$, it can be derived that

$$\mathbb{E}\Big\{\dot{\tilde{x}}^T(t)(I \otimes Q)\dot{\tilde{x}}(t)\Big\} = \vartheta^T(t)\tilde{\Pi}\vartheta(t) \tag{28}$$

where

$$\vartheta^T(t) := [\tilde{x}^T(t), \int_{t-\mathrm{d}(t)}^{t} \dot{\tilde{x}}^T(s)\, \mathrm{d}s, \tilde{e}^T(lh)]^T \tag{29}$$

$$\tilde{\Pi} := [\,(I\otimes\mathcal{A}) - \tilde{X} \quad \tilde{X} \quad \tilde{X}\,](I\otimes Q)\begin{bmatrix}(I\otimes\mathcal{A}^T) - \tilde{X}^T \\ \tilde{X}^T \\ \tilde{X}^T\end{bmatrix} \tag{30}$$

$$\begin{aligned}\tilde{X} := (L+B)\otimes\mathcal{C}K + \sqrt{\bar{\beta}(1-\bar{\beta})}(L+B)\otimes\mathcal{C}\Delta K \\ + \bar{\beta}(L+B)\otimes\mathcal{C}\Delta K\end{aligned} \tag{31}$$

Moreover, one obtains from equation (7) that

$$\begin{aligned}&-\tilde{e}^T(lh)(I\otimes\Phi_1)\tilde{e}(lh) + \varepsilon\left(\tilde{x}^T(t) - \int_{t-\mathrm{d}(t)}^{t}\dot{\tilde{x}}^T(s)\,\mathrm{d}s\right)\\ &\times[(L+B)^T(L+B)\otimes\Phi_2]\times\left[\left(\tilde{x}(t) - \int_{t-\mathrm{d}(t)}^{t}\dot{\tilde{x}}^T(s)\,\mathrm{d}s\right)\geq 0\end{aligned} \tag{32}$$

Hence, by Schur complement, one has if $\bar{\Pi} < 0$, then $\Pi < 0$ holds, where

$$\bar{\Pi} := \begin{bmatrix}\bar{\Pi}_1 & \bar{\Pi}_2 \\ \star & \bar{\Pi}_3\end{bmatrix}, \tag{33}$$

$$\bar{\Pi}_1 := \begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12} \\ \star & -(I\otimes Q) + \varepsilon G^T G\otimes\Phi_2\end{bmatrix} \tag{34}$$

$$\begin{aligned}\bar{\Pi}_{11} := 2(I\otimes P\mathcal{A}) - 2G\otimes(P\mathcal{C}K + \bar{\beta}P\mathcal{C}\Delta K)\\ + \varepsilon G^T G\otimes\Phi_2\end{aligned} \tag{35}$$

$$\bar{\Pi}_{12} := G\otimes(P\mathcal{C}K + \bar{\beta}P\mathcal{C}\Delta K) - \varepsilon G^T G\otimes\Phi_2 \tag{36}$$

$$\bar{\Pi}_2 := \begin{bmatrix}G\otimes(P\mathcal{C}K + \bar{\beta}P\mathcal{C}\Delta K) & \bar{\Pi}_{21} \\ 0 & \bar{\Pi}_{22}\end{bmatrix} \tag{37}$$

$$\begin{aligned}\bar{\Pi}_{21} := (I\otimes\mathcal{A})^T - [G\otimes\mathcal{C}K + \sqrt{\bar{\beta}(1-\bar{\beta})}G\\ \otimes\mathcal{C}\Delta K + \bar{\beta}G\otimes\mathcal{C}\Delta K]^T\end{aligned} \tag{38}$$

$$\bar{\Pi}_{22} := [G\otimes\mathcal{C}K + \sqrt{\bar{\beta}(1-\bar{\beta})}G\otimes\mathcal{C}\Delta K + \bar{\beta}G\otimes\mathcal{C}\Delta K]^T, \tag{39}$$

$$\bar{\Pi}_3 := \begin{bmatrix}\bar{\Pi}_{31} \\ \bar{\Pi}_{32} \\ \bar{\Pi}_{32} \\ -(I\otimes Q)^{-1}\end{bmatrix} \tag{40}$$

$$\begin{aligned}\bar{\Pi}_{31} := h(I\otimes\mathcal{A})^T - h[G\otimes\mathcal{C}K + \sqrt{\bar{\beta}(1-\bar{\beta})}G\\ \otimes\mathcal{C}\Delta K + \bar{\beta}G\otimes\mathcal{C}\Delta K]^T\end{aligned} \tag{41}$$

$$\bar{\Pi}_{32} := h[G\otimes\mathcal{C}K + \sqrt{\bar{\beta}(1-\bar{\beta})}G\otimes\mathcal{C}\Delta K + \bar{\beta}G\otimes\mathcal{C}\Delta K]^T \tag{42}$$

By $\Delta K = H\Psi(t)E$, it follows that

$$\begin{aligned}\bar{\Pi} = &\begin{bmatrix}\bar{\Pi}_a & \bar{\Pi}_b \\ \star & \bar{\Pi}_c\end{bmatrix} + \begin{bmatrix}\bar{\beta}G\otimes(P\mathcal{C}H) \\ 0 \\ 0 \\ \sqrt{\bar{\beta}(1-\bar{\beta})} + \bar{\beta}\big)G\otimes(\mathcal{C}H)\end{bmatrix}(I\otimes\Psi(t))\\ &\times[\,-(I\otimes E) \;\; (I\otimes E) \;\; (I\otimes E) \;\; 0\,] + \left\{\begin{bmatrix}\bar{\beta}G\otimes(P\mathcal{C}H) \\ 0 \\ 0 \\ \sqrt{\bar{\beta}(1-\bar{\beta})} + \bar{\beta})G\otimes(\mathcal{C}H)\end{bmatrix}(I\otimes\Psi(t))\times[\,-(I\otimes E) \;\; (I\otimes E) \;\; (I\otimes E) \;\; 0\,]\right\}^T\end{aligned} \tag{43}$$

where

$$\bar{\Pi}_a := \begin{bmatrix}\bar{\Pi}_{a1} & G\otimes P\mathcal{C}K - \varepsilon G^T G\otimes\Phi_2 \\ \star & -(I\otimes Q) + \varepsilon G^T G\otimes\Phi_2\end{bmatrix} \tag{44}$$

$$\bar{\Pi}_{a1} := 2(I\otimes P\mathcal{A}) - 2G\otimes P\mathcal{C}K + \varepsilon G^T G\otimes\Phi_2 \tag{45}$$

$$\bar{\Pi}_b := \begin{bmatrix}G\otimes P\mathcal{C}K & h(I\otimes\mathcal{A}^T) - hG^T\otimes K^T\mathcal{C}^T \\ 0 & hG^T\otimes K^T\mathcal{C}^T\end{bmatrix} \tag{46}$$

$$\bar{\Pi}_c := \begin{bmatrix}-(I\otimes\Phi_1) & hG^T\otimes K^T\mathcal{C}^T \\ \star & -(I\otimes Q)^{-1}\end{bmatrix} \tag{47}$$

Consequently, it can be obtained by Lemma 1 that $\Pi < 0$ holds and completes the proof.

Remark 3. It is noteworthy that the developed results of event-triggering networking strategy can be extended to the so-called leader–follower consensus problem of multiagent systems.[28,30]

By Theorem 1, the following Theorem is given to deal with the synchronization controller design problem.

Theorem 2. With the event-triggered control input (equation (10)) and connected $\mathcal{G}$, the synchronization of RTS (equation (4)) can be achieved if there exist positive symmetric matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{\Phi}_1 > 0$, $\tilde{\Phi}_2 > 0$ and matrix V, such that $\breve{\Pi} < 0$, where

$$\breve{\Pi} := \begin{bmatrix} \breve{\Pi}_1 & \breve{\Pi}_2 \\ \star & \breve{\Pi}_3 \end{bmatrix}, \tag{48}$$

$$\breve{\Pi}_1 := \begin{bmatrix} \breve{\Pi}_{11} & G \otimes \mathcal{C}V - \varepsilon G^T G \otimes \tilde{\Phi}_2 & G \otimes \mathcal{C}V \\ \star & \breve{\Pi}_{12} & 0 \\ \star & \star & -I \otimes \tilde{\Phi}_1 \end{bmatrix}, \tag{49}$$

$$\breve{\Pi}_{11} := 2I \otimes \mathcal{A}\tilde{P} - 2G \otimes \mathcal{C}V + \varepsilon G^T G \otimes \tilde{\Phi}_2, \tag{50}$$

$$\breve{\Pi}_{12} := I \otimes \tilde{Q} - 2I \otimes \tilde{P} + \varepsilon G^T G \otimes \tilde{\Phi}_2, \tag{51}$$

$$\breve{\Pi}_2 := \begin{bmatrix} \breve{\Pi}_{21} & \bar{\beta} G \otimes (\mathcal{C}H) & -\varepsilon (I \otimes \tilde{P}E)^T \\ hG^T \otimes V^T \mathcal{C}^T & 0 & \varepsilon (I \otimes \tilde{P}E)^T \\ hG^T \otimes V^T \mathcal{C}^T & 0 & \varepsilon (I \otimes \tilde{P}E)^T \end{bmatrix} \tag{52}$$

$$\breve{\Pi}_{21} := h(I \otimes \tilde{P}\mathcal{A}^T) - hG^T \otimes V^T \mathcal{C}^T \tag{53}$$

$$\breve{\Pi}_3 := \begin{bmatrix} -I \otimes \tilde{Q} & \breve{\Pi}_{31} & \varepsilon (I \otimes E)^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{bmatrix} \tag{54}$$

$$\breve{\Pi}_{31} := \left(\sqrt{\bar{\beta}(1 - \bar{\beta})} + \bar{\beta} \right) G \otimes (\mathcal{C}H) \tag{55}$$

The synchronization controller gains can be obtained by $K = V\tilde{P}^{-1}$.

Proof. Let $\tilde{P} = P^{-1}$, $\tilde{Q} = Q^{-1}$, $V = K\tilde{P}$, $\tilde{\Phi}_1 = \tilde{P}\Phi_1\tilde{P}$ and $\tilde{\Phi}_2 = \tilde{P}\Phi_2\tilde{P}$.

Noting that $-\tilde{P}\tilde{Q}^{-1}\tilde{P} < \tilde{Q} - 2\tilde{P}$ and multiplying both sides of $\Pi < 0$ by $\mathrm{diag}\{I \otimes \tilde{P}, I \otimes \tilde{P}, I \otimes \tilde{P}, I, I, I\}$, the proof can be completed.

Remark 4. The established conditions in Theorems 1 and 2 are in the form of strict linear matrix inequalities, which can be easily solved by Matlab or YALMIP. These positive symmetric matrices should be chosen appropriately to ensure that the conditions have feasible solutions. For practical applications, once the state-space equations can be obtained, then by solving the given conditions, the controller gain can be determined accordingly.

Illustrative example

In this section, simulation results are provided to show the effectiveness of our derived results.

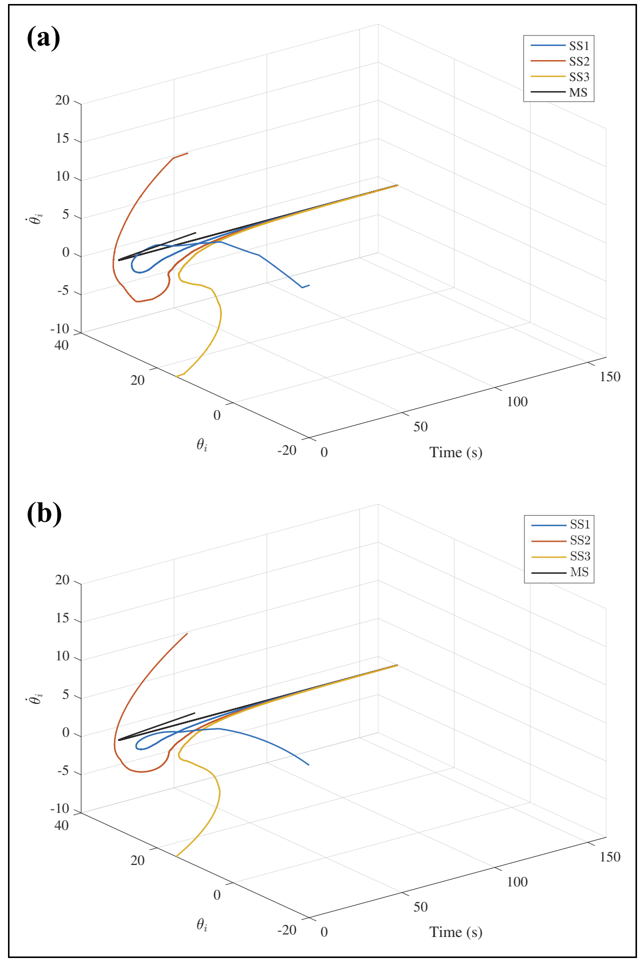

Figure 2. Comparison results of our proposed event-triggered scheme and the common time-triggered scheme. (a) The dynamics of the MS and SSs with event-triggered scheme and (b) The dynamics of the MS and SSs with time-triggered scheme.

Consider the RTS (equation (4)) with the following parameters as $J = 10$ kgm^2 and $b = 4$ Nms/rad.

Moreover, the Laplacian matrix of the communication topology is assumed to be

$$L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The controller gain fluctuations are set as follows:

$$H = [\,0.5 \quad 0.5\,]$$
$$E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$\Psi(t)$ being Perlin noise (persistence: 0.25, octaves: 6) and $\bar{\beta} = 0.5$.

Choose the sampling period $h = 0.2$ s, the parameter $\varepsilon = 0.02$ and the parameter $\varepsilon = 1$. With the above parameters, the synchronization controller gain can be solved by Theorem 2 as $K = [0.2700, 0.3677]$.

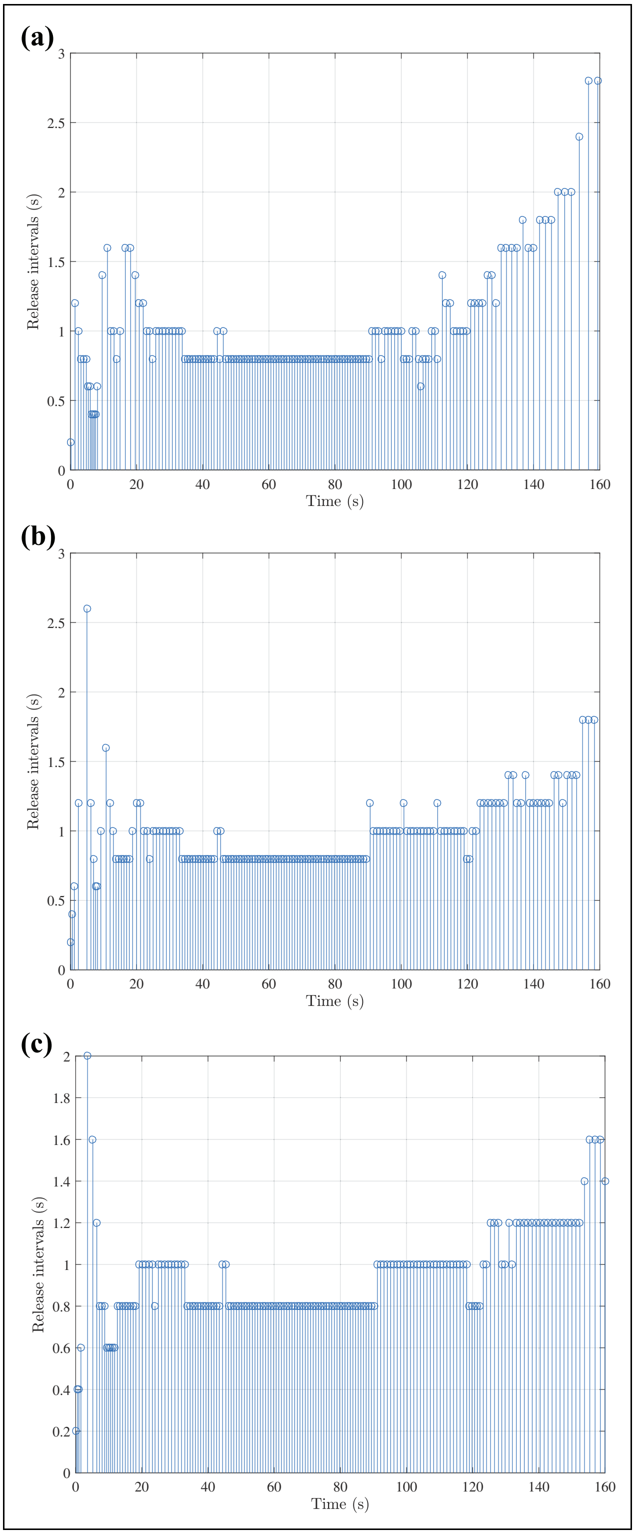

Figure 3. Broadcasting instants and release intervals of the SSs: (a) SS1, (b) SS2, and (c) SS3.

By setting random initial conditions, the closed-loop dynamics of the RTS (equation (4)) with event-triggered scheme are depicted in Figure 2(a). It can be seen that all the SSs can track the rotate angle and the rotate angle velocity of the MS effectively, which means that the synchronization can be well achieved with our designed controllers and demonstrates our theoretical results. In addition, the closed-loop dynamics of the traditional time-triggered case with the same sampling period $h = 0.2$ s is also presented in Figure 2(b). It can be found that in the time-triggered case, the synchronization can be achieved a bit faster than the event-triggered case since there are more information exchanges of the RTS. However, by the corresponding broadcasting instants and release intervals shown in Figure 3, the event-triggered case can effectively reduce the information exchanges with less communications, such that the communication burden of the network can be considerably reduced with advantage. In practical applications, this trade-off of the synchronization time and the communication burden can be considered according to the design requirement by adjusting the event-triggered networking function.

Conclusion and discussions

In this article, the asynchronous event-triggered synchronization problem of robotic teleoperation systems is studied with gain fluctuations. In particular, a Bernoulli distributed variable is introduced to describe the randomly occurring gain fluctuations. By model transformation and applying the stochastic analysis, sufficient conditions are established based on Lyapunov–Krasovskii functionals such that the synchronization can be achieved in the mean square sense. Then, the related synchronization controllers are designed with the help of linear matrix inequalities. In the end, the effectiveness of our proposed control method is validated via the provided numerical simulations. In comparison with common time-triggered schemes, our proposed event-triggered synchronization scheme can effectively reduce the information exchanges among the RTS. Our future work encompasses investigating the cases with force reflection, which is more applicable for implementations.

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