Received: 24 April 2017 | Accepted: 2 March 2018
DOI: 10.1002/oca.2421

RESEARCH ARTICLE

WILEY

A novel triggering condition of event-triggered control based on heuristic dynamic programming for discrete-time systems

Ziyang Wang[1] | Qinglai Wei[2] | Derong Liu[3]

[1]School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China

[2]The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China

[3]School of Automation, Guangdong University of Technology, Guangzhou, China

Correspondence
Qinglai Wei, State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China.
Email: qinglai.wei@ia.ac.cn

Funding information
National Natural Science Foundation of China, Grant/Award Number: 61233001, 61722312, 61374105, and 61533017

Summary

In this paper, an event-triggered heuristic dynamic programming algorithm for discrete-time nonlinear systems with a novel triggering condition is studied. Different from traditional heuristic dynamic programming algorithms, the control law in this algorithm will only be updated when the triggering condition is satisfied to reduce the computational burden. Three neural networks are employed, which are model network, action network, and critic network. Model functions, control laws, and value functions are estimated using neural networks, respectively. The main contribution of this algorithm is the novel triggering condition with simpler form and fewer assumptions. Additionally, a proof of the stability for discrete-time systems using Lyapunov technique is given. Finally, two simulations are shown to verify the effectiveness of the developed algorithm.

KEYWORDS

adaptive dynamic design, adaptive dynamic programming, approximate dynamic programming, event-triggered control, optimal control

1 | INTRODUCTION

Adaptive dynamic programming (ADP) refers to a family of practical actor-critic methods for finding optimal solutions in real time,[1] and it is a self-learning method.[2-8] In 1977, adaptive critic design was first proposed by Werbos,[9] which takes the advantages of neural networks (NNs). Then, several names emerged, eg, approximate dynamic programming and asymptotic dynamic programming. Iterative methods are widely used in ADP to obtain solutions of the Bellman equation indirectly.[10-12] Adaptive dynamic programming can be divided into many categories, such as heuristic dynamic programming (HDP)[13-15] dual heuristic dynamic programming (DHP),[16] action dependent DHP (also called Q-learning[17]), globalized DHP,[18] and so on. Adaptive dynamic programming has been widely used in real-world applications.[19-22]

Event-triggered control is an effective method to increase the efficiency because the amount of calculation is reduced and the performance is maintained.[23,24] Generally, conditions are required in event-triggered control, thus the controller only works or updates when the conditions are satisfied.[25,26] Many researchers started to pay attention to event-triggered control methods in recent years. For discrete-time Markov jump systems, the event generator was proposed to select the sampled states in the works of Song et al[27] and Shen et al.[28] An online event-triggered algorithm was developed using NNs in the work of Vamvoudakis.[29] Decentralized control for wireless sensor/actuator networks based on event-triggered control was studied in the work of Mazo.[30] In the works of Eqtami et al[26] and Sahoo et al,[31] event-triggered control methods for discrete-time systems were given. Analysis of event-triggered control methods for linear systems was presented in the work of Heemels et al.[32] In 2012, an event-triggered control algorithm was studied for multiagent systems in the work of Dimarogonas et al.[33] Besides, the event-triggered scheme was used for tracking control systems in the works of Tallapragada and Chopra[34] and Liu et al.[35]

With extensive research on ADP, many event-triggered ADP algorithms have been generated. Zhong and He[36] proposed an event-triggered control method based on ADP algorithms with an observer, which only used input and output data. In the work of Dong et al,[37] a novel event-triggered method with stability analysis was studied for nonlinear discrete-time systems based on HDP algorithm.

In this paper, a novel event-triggered HDP method is studied to reduce the computational burden and computing time. The difficulty of solving the event-triggered optimal control problems using HDP algorithm is that many hypotheses need to be established to stabilize the system. Compared with the existing work,[37] main contributions of this paper include the following.

1. A novel triggering condition of event-triggered HDP algorithm is studied, which needs fewer assumptions to stabilize the system. Hence, this event-triggered HDP method under the new condition will be more practical for applications.
2. The stability of the system is guaranteed with the new method by Lyapunov technique[38] in 2 situations, ie, the event is triggered or not. Additionally, we will show how to implement the algorithm using 3 NNs. The proof is given to show that the system states and the estimation errors of the NN weights are uniformly ultimately bounded (UUB).

The rest of this paper is organized as follows. The formulation of the optimization problems for event-triggered nonlinear discrete-time systems is presented in Section 2. A new triggering condition is studied in Section 3, and the stability of the system is also considered under the new condition. In Section 4, the implementation of the event-triggered HDP algorithm is given. In Section 5, 2 simulations and analysis are presented to show the effectiveness of the method. The conclusion is given in Section 6.

2 | PROBLEM FORMULATION

Consider a nonlinear control system in discrete-time domain, which is formulated as

$$x(k+1) = f(x(k), u(k)), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$. Assume that $f\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz continuous. The state $x(k) = 0$ is the unique equilibrium point of the system under $u(k) = 0$, ie, $f(0,0) = 0$. The control law $u(k)$ is updated only when the triggering condition is met. Hence, when the triggering condition is dissatisfied, the system will work under the control input updated last time until the next triggering. A positive integer sequence $\{k_i\}_{i=0}^{\infty}$ is defined as triggering instants. Then, the control input $u(k)$ can be expressed as

$$u(k) = u(k_i), \quad k_i \leqslant k < k_{i+1}. \tag{2}$$

The triggering error is described as

$$e(k) = x(k_i) - x(k), \quad k_i \leqslant k < k_{i+1}, \tag{3}$$

where $x(k_i)$ is the state at the ith triggering instant and $x(k)$ is the real-time state. A function $v(x(k)) = u(k)$ is used to represent the relationship between the control law and states. According to (3), the event-triggered control law $u(k_i)$ can be rewritten as $u(k_i) = v(e(k) + x(k))$. (1) can be rewritten as

$$x(k+1) = f(x(k), u(k_i)). \tag{4}$$

Define $U(x(k), u(k))$ as a continuous positive definite function called utility function given by

$$U(x(k), u(k)) = x^{\mathsf{T}}(k)Qx(k) + u^{\mathsf{T}}(k)Ru(k), \tag{5}$$

where $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are positive definite symmetric matrices with appropriate dimensions. Then, the performance index function of system (1) can be represented as

$$J(x(k)) = \sum_{j=k}^{\infty} U\left(x(j), u(j)\right). \tag{6}$$

Our objective is to minimize the performance index by designing the feedback control input $u(k)$. On the basis of Bellman optimality principle, the optimal cost function $J^*(x(k))$ at the instant k satisfies the discrete-time Hamilton-Jacobi-Bellman equation

$$\begin{aligned} J^*(x(k)) &= \min_{u(k)} \{U(x(k), u(k)) + J^*(x(k+1))\} \\ &= \min_{u(k_i)} \{U(x(k), u(k_i)) + J^*(f(x(k), u(k_i)))\}. \end{aligned} \tag{7}$$

Then, the optimal control input $u^*(k_i)$ can be obtained as

$$u^*(k_i) = \arg\min_{u(k_i)} \{U(x(k), u(k_i)) + J^*(f(x(k), u(k_i)))\}. \tag{8}$$

It is hard to solve nonanalytical equations like (7) and (8). Thus, an event-triggered HDP algorithm with a novel triggering condition is studied in the next section to solve the problem.

3 | TRIGGERING CONDITION AND STABILITY ANALYSIS

We define a threshold e_T as the triggering condition with a positive constant $C \in \left(0, \frac{\sqrt{2}}{2}\right]$ to be designed as

$$e_T = \sqrt{\frac{1-2C^2}{2C^2}} \|x(k)\|. \tag{9}$$

When the threshold e_T is less than the triggering error $\|e(k)\|$, the control law in system (4) will be updated and the triggering error will return to zero. Thus, the triggering error $e(k)$ is always less than or equal to e_T in the stable operation of the system, ie,

$$\|e(k)\| \leqslant e_T. \tag{10}$$

Definition 1. (See the work of Jiang and Wang[39])
A continuous function $\beta(t)$ is a κ-function if it is rigorously increasing and $\beta(0) = 0$; $\beta(t)$ is a κ_∞-function if it is a κ-function, and also, $\beta(t) \to \infty$ as $t \to \infty$.

Lemma 1. (See the work of Dong et al[37])
A function $V : \mathbb{R}^n \to \mathbb{R}$ is called an input-to-state stability Lyapunov function if the following inequalities:

$$\alpha_1(\|x\|) \leqslant V(x(k)) \leqslant \alpha_2(\|x\|) \tag{11}$$

$$V(f(x(k), v(e(k)+x(k)))) - V(x(k)) \leqslant -\alpha_3(\|x(k)\|) + \sigma(\|e(k)\|), \tag{12}$$

hold, where σ is a κ-function, and α_1, α_2, and α_3 are κ_∞-functions.

Assumption 1. (See the work of Eqtami et al[26])
There must be a positive constant $C \in \left(0, \frac{\sqrt{2}}{2}\right]$, which makes the following equation hold:

$$\|f(x(k), v(e(k)+x(k)))\| \leqslant C\|x(k)\| + C\|e(k)\|. \tag{13}$$

Theorem 1. *Under the Assumption 1, the discrete-time event-triggered system (4) is asymptotically stable under the triggering condition*

$$\|e(k)\| \leqslant \sqrt{\frac{1-2C^2}{2C^2}} \|x(k)\|. \tag{14}$$

Proof. Define a Lyapunov function in the following form:

$$V(x(k)) = x^{\mathsf{T}}(k)Qx(k) + u^{\mathsf{T}}(k)Ru(k). \tag{15}$$

Then, we will consider whether the Lyapunov function is a nonincreasing function under 2 situations that the triggering condition is dissatisfied and the triggering condition is satisfied.

I. The triggering condition is dissatisfied

We define a series of functions as follows:

$$\alpha_1(\|x\|) = x^{\mathsf{T}}(k)Q_l x(k) + u^{\mathsf{T}}(k)Ru(k), \tag{16}$$

$$\alpha_2(\|x\|) = x^{\mathsf{T}}(k)Q_g x(k) + u^{\mathsf{T}}(k)Ru(k), \tag{17}$$

$$\Delta V = V(f(x(k), v(e(k) + x(k)))) - V(x(k)), \tag{18}$$

$$\alpha_3(\|x(k)\|) = \|q\|^2 \left(1 - 2C^2\right) \|x(k)\|^2, \tag{19}$$

$$\sigma(\|e(k)\|) = 2C^2\|q\|^2\|e(k)\|^2. \tag{20}$$

Q_l and Q_g in (16) and (17) can be obtained by (11). The matrix q in (19) and (20) is determined from $x^{\mathsf{T}}Qx = x^{\mathsf{T}}qq^{\mathsf{T}}x = \|x^{\mathsf{T}}q\|^2$. According to (15) and (18), the left side of (12) can be rewritten as

$$\begin{aligned}\Delta V = {} & f(x(k), v(x(k_i)))^{\mathsf{T}}Qf(x(k), v(x(k_i))) + v(x(k_i))^{\mathsf{T}}Rv(x(k_i)) \\ & - \left(x^{\mathsf{T}}(k)Qx(k) + v(x(k_i))^{\mathsf{T}}Rv(x(k_i))\right).\end{aligned} \tag{21}$$

Since the triggering condition is dissatisfied, the control law will not be updated, ie, $u(k) = v(x(k_i))$, in the period. Thus, (21) can be simplified as

$$\Delta V = f(x(k), v(x(k_i)))^{\mathsf{T}}Qf(x(k), v(x(k_i))) - x^{\mathsf{T}}(k)Qx(k). \tag{22}$$

Substituting (13) to (22), we can get

$$\Delta V \leqslant \|q\|^2 \left((C\,\|x(k)\| + C\,\|e(k)\|)^2 - \|x(k)\|^2\right). \tag{23}$$

According to the Cauchy-Schwarz inequality, (23) becomes

$$\begin{aligned}\Delta V & \leqslant \|q\|^2 \left(2C^2\|x(k)\|^2 + 2C^2\|e(k)\|^2 - \|x(k)\|^2\right) \\ & \leqslant \|q\|^2(2C^2 - 1)\|x(k)\|^2 + 2C^2\|q\|^2\|e(k)\|^2 \\ & \leqslant -\alpha_3\left(\|x(k)\|\right) + \sigma\left(\|e(k)\|\right).\end{aligned} \tag{24}$$

Thus, the condition (12) holds, and the function V is an input-to-state stability Lyapunov function. Additionally, substituting the triggering condition (14) into (24), then we will get

$$V(f(x(k), v(e(k) + x(k)))) - V(x(k)) \leqslant 0. \tag{25}$$

Hence, the function V is guaranteed to be nonincreasing when triggering condition is dissatisfied.

II. The triggering condition is satisfied

When the triggering condition is satisfied, the control law will be updated, and the system will work under the updated control law. According to the work of Wei et al,[6] the Lyapunov function $V(x(k))$ is nonincreasing.

Combining the aforementioned results, we can get that, no matter whether the triggering condition is satisfied or not, the Lyapunov function $V(x(k))$ is nonincreasing based on a proper constant C. Therefore, the system (4) is asymptotically stable, and the proof is completed. □

Remark 1. It can be seen that the discrete-time event-triggered system (4) is asymptotically stable with triggering condition (14) under Assumption 1. Compared with the existing work,[37] the new triggering condition needs fewer assumptions to stabilize the discrete-time systems.

4 | NEURAL NETWORK IMPLEMENTATION

There are 3 main parts in the studied event-triggered HDP algorithm. The model network approximates the system. The critic network approximates the value function by iteration, and the action network approximates the control law. The weights of action network update only when the triggering error is greater than the threshold.

First, some notations are given. $W(k)$ represents the weight matrix between the hidden layer and output layer, and $Y(k)$ represents the weight matrix between the input layer and hidden layer. Define $\sigma(\gamma(k)) = (1 - e^{-\gamma(k)})/(1 + e^{-\gamma(k)})$ as the activation function. η is the learning rate.

I. Model Network

Approximate the system (4) with a 3-layer NN, which is denoted as

$$\hat{x}(k+1) = W_m^{\mathsf{T}} \sigma \left(Y_m^{\mathsf{T}} \left[x(k); u(k) \right] \right). \tag{26}$$

Our target is to minimize the error in the following form by regulating the weight matrix:

$$D_m(k) = \frac{1}{2} d_m^{\mathsf{T}}(k) d_m(k), \tag{27}$$

where d_m is the estimation error defined as

$$d_m(k) = \hat{x}(k+1) - x(k+1). \tag{28}$$

The update algorithm of the weight matrix of model networks can be expressed as

$$\begin{aligned} \hat{W}_m(k+1) &= \hat{W}_m(k) - \eta_m \left(\frac{\partial D_m(k)}{\partial W_m(k)} \right) \\ &= \hat{W}_m(k) - \eta_m \left(\frac{\partial D_m(k)}{\partial x_m(k+1)} \frac{\partial x_m(k+1)}{\partial W_m(k)} \right). \end{aligned} \tag{29}$$

II. Critic Network

The critic network is used to approximate the iterative value function $J(x(k))$, which is denoted as

$$\hat{J}(x(k)) = W_c^{\mathsf{T}}(k) \sigma \left(Y_c^{\mathsf{T}}(k) x(k) \right). \tag{30}$$

The target is to minimize the error function in the following form by regulating the weight matrix:

$$d_c(k) = J(x(k)) - \left[J\left(\hat{x}(k+1)\right) + U(k) \right], \tag{31}$$

$$D_c(k) = \frac{1}{2} d_c^{\mathsf{T}}(k) d_c(k). \tag{32}$$

The update algorithm of hidden-to-output weight matrix of the critic network is expressed as

$$\begin{aligned} \hat{W}_c(k+1) &= \hat{W}_c(k) - \eta_c \left(\frac{\partial D_c(k)}{\partial W_c(k)} \right) \\ &= \hat{W}_c(k) - \eta_c \left(\frac{\partial D_c(k)}{\partial J(k)} \frac{\partial J(k)}{\partial W_c(k)} \right). \end{aligned} \tag{33}$$

III. Action network

The action network is used to approximate the iterative control input $u(k)$, which can be expressed as

$$\hat{u}(k) = W_a^{\mathsf{T}}(k) \sigma \left(Y_a^{\mathsf{T}}(k) x(k) \right). \tag{34}$$

Our target is to minimize the error in the following form by regulating the weight matrix:

$$d_a(k) = J\left(\hat{x}(k+1)\right) + U(k) - U_z, \tag{35}$$

$$D_a(k) = \frac{1}{2} d_a^{\mathsf{T}}(k) d_a(k), \tag{36}$$

where U_z can be set to zero according to the work of Si and Wang.[40] The weights update algorithm for the action network can be expressed as

$$\begin{aligned} \hat{W}_a(k+1) &= \hat{W}_a(k) - \eta_a \left(\frac{\partial D_a(k)}{\partial W_a(k)} \right) \\ &= \hat{W}_a(k) - \eta_a \left(\frac{\partial D_a(k)}{\partial J(k+1)} \frac{\partial J(k+1)}{\partial \hat{x}(k+1))} \frac{\partial \hat{x}(k+1))}{\partial u(k)} \frac{\partial u(k)}{\partial W_a(k)} + \frac{\partial D_a(k)}{\partial U(k)} \frac{\partial U(k)}{\partial u(k)} \frac{\partial u(k)}{\partial W_a(k)} \right). \end{aligned} \tag{37}$$

Define the weight approximation error as

$$\tilde{w}(k) = \hat{w}(k) - w^*(k). \tag{38}$$

Then, the stability analysis is given as follows.

Theorem 2. *If the event-triggered systems update the weights of NNs through the update laws (33) and (37), no matter whether the triggering condition is satisfied or not, the closed-loop state $x(k)$ and the network weight approximation errors $\tilde{w}_c$ and $\tilde{w}_a$ are UUB under the conditions that*

$$\frac{1}{\sqrt{2}} < \alpha < 1, \tag{39}$$

$$\eta_c < \frac{1}{\alpha^2 \|\sigma_c(k)\|^2}, \tag{40}$$

$$\eta_a < \frac{1}{\|\sigma_a(k)\|^2}. \tag{41}$$

Proof. I. The triggering condition is satisfied

Define a Lyapunov function as

$$L = \frac{1}{\eta_c}\mathrm{tr}\left[\tilde{w}_c^{\mathsf{T}}\tilde{w}_c\right] + \frac{1}{\gamma\eta_a}\mathrm{tr}\left[\tilde{w}_a^{\mathsf{T}}\tilde{w}_a\right] + \frac{1}{2}\|\delta(k-1)\|^2\left\|\tilde{w}_c^{\mathsf{T}}(k-1)\right\|^2, \tag{42}$$

where $\gamma > 4/(\alpha^2 - \frac{1}{2})$. According to the work of Liu et al,[41] we can get that the first difference function of (42) is negative under the conditions (39) to (41). Thus, the closed-loop state $x(k)$ and the network weight approximation errors $\tilde{w}_c$ and $\tilde{w}_a$ are UUB when the triggering condition is saitisfied.

II. The triggering condition is dissatisfied

Define a Lyapunov function in the following form:

$$L = x^{\mathsf{T}}(k)x(k) + \mathrm{tr}\left\{\tilde{w}_c^{\mathsf{T}}\tilde{w}_c\right\} + \mathrm{tr}\left\{\tilde{w}_a^{\mathsf{T}}\tilde{w}_a\right\}. \tag{43}$$

The first-order difference is calculated as

$$\begin{aligned}\Delta L = {} & x^{\mathsf{T}}(k+1)x(k+1) - x^{\mathsf{T}}(k)x(k) \\ & + \mathrm{tr}\left\{\tilde{w}_c^{\mathsf{T}}(k+1)\tilde{w}_c(k+1)\right\} - \mathrm{tr}\left\{\tilde{w}_c^{\mathsf{T}}(k)\tilde{w}_c(k)\right\} \\ & + \mathrm{tr}\left\{\tilde{w}_a^{\mathsf{T}}(k+1)\tilde{w}_a(k+1)\right\} - \mathrm{tr}\left\{\tilde{w}_a^{\mathsf{T}}(k)\tilde{w}_a(k)\right\}.\end{aligned} \tag{44}$$

When the triggering condition is dissatisfied, the 2 networks stop working, and the network weights remain constant. Therefore, the first difference function (44) becomes

$$\Delta L = x^{\mathsf{T}}(k+1)x(k+1) - x^{\mathsf{T}}(k)x(k). \tag{45}$$

According to (25), the first-order difference function $\Delta L < 0$ holds. Thus, the closed-loop system state $x(k)$ and the network weight approximation errors $\tilde{w}_c$ and $\tilde{w}_a$ are UUB when the triggering condition is dissatisfied. □

5 | EXAMPLES

Two examples are used to validate the studied algorithm.

Example 1. First, apply the method to the mass-spring-damper system.[37] The state-space model is given as follows:

$$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = -\frac{b}{m} - \frac{k_{s1}}{m} + \frac{F_{\mathrm{ext}}}{m},\end{cases} \tag{46}$$

where $m = 1$ kg is the mass of the body and $k_{s1} = 9$ N/m is the linear spring constant. The drag force is $b = 3$ N·s/m. The F_{ext} represents the force from outside, ie, the control law $u(k)$. Choose the sampling period as $T = 0.01$ s, and the discrete-time state-space function is

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.0099x_2(k) + 0.9996x_1(k) \\ -0.0887x_1(k) + 0.97x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0099 \end{bmatrix} u(k). \tag{47}$$

Define the performance index function as

$$J = \sum_{k=0}^{\infty}\left(x^{\mathsf{T}}(k)Qx(k) + u^{\mathsf{T}}(k)Ru(k)\right), \tag{48}$$

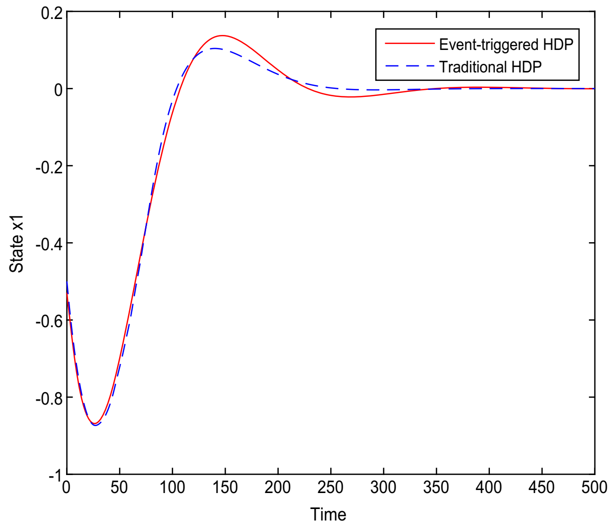

FIGURE 1 The trajectories of the current angle x_1. HDP, heuristic dynamic programming [Colour figure can be viewed at wileyonlinelibrary.com]

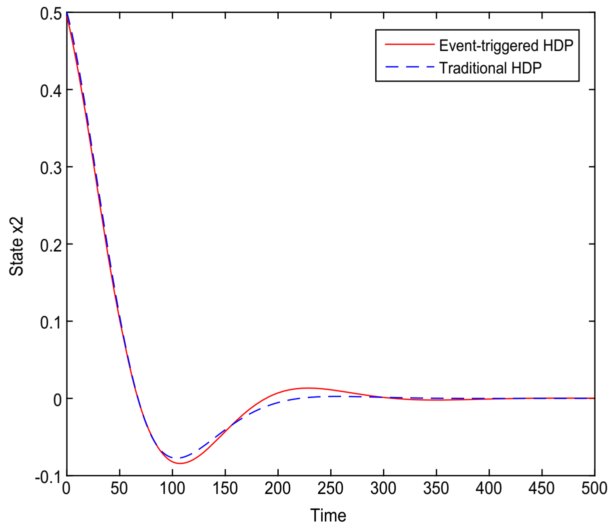

FIGURE 2 The trajectories of the angular velocity x_2. HDP, heuristic dynamic programming [Colour figure can be viewed at wileyonlinelibrary.com]

where $Q = I_1, R = I_2$, and I is the identity matrix with appropriate dimensions. We use both traditional HDP algorithm and event-triggered HDP algorithm to optimize the control law for comparison. Two 3-layer 2-12-1 back propagation NNs are chosen for the critic network and the action network, respectively. Choose the initial state as $x_0 = [-0.5, 0.5]$ and the constant $C = 0.7$.

In Figures 1 and 2, the trajectories of 2 states using 2 algorithms are presented, respectively. Figure 3 shows the control law in the event-triggered HDP algorithm. Apparently, the control law is updated only when the triggering condition is satisfied. Figure 4 represents the summary of the performance index function. The triggering error $||e(k)||$ and the triggering threshold e_T are shown in Figure 5. It can be seen that the threshold is always greater than the triggering error. We also record the number of the control law updates in Figure 6. The traditional HDP algorithm is implemented by 500 updates, while the event-triggered HDP algorithm is completed by only 115 updates. Thus, it can be seen that the event-triggered method gets a competitive result with less calculation.

Example 2. In this part, we consider a nonlinear discrete-time control system in (49)

$$x(k+1) = F(x_k, u_k) = x_k + \sin(0.1x_k^2 + u_k). \tag{49}$$

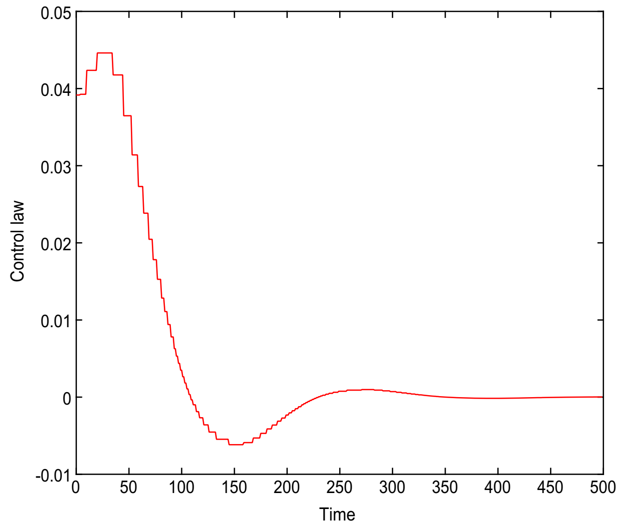

FIGURE 3 The trajectories of event-triggered heuristic dynamic programming control law [Colour figure can be viewed at wileyonlinelibrary.com]

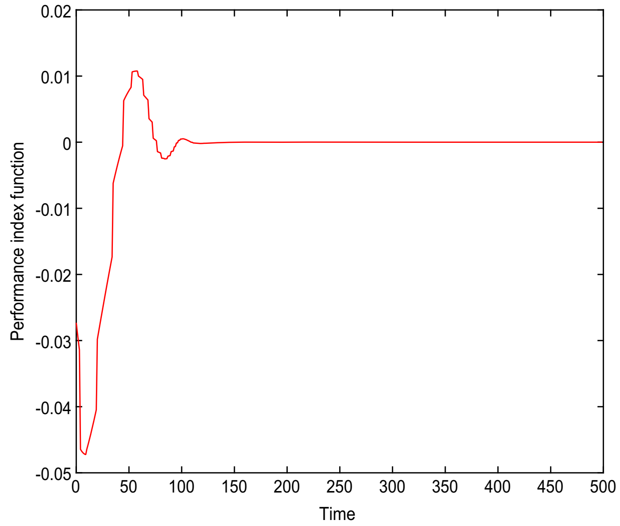

FIGURE 4 The trajectories of performance index [Colour figure can be viewed at wileyonlinelibrary.com]

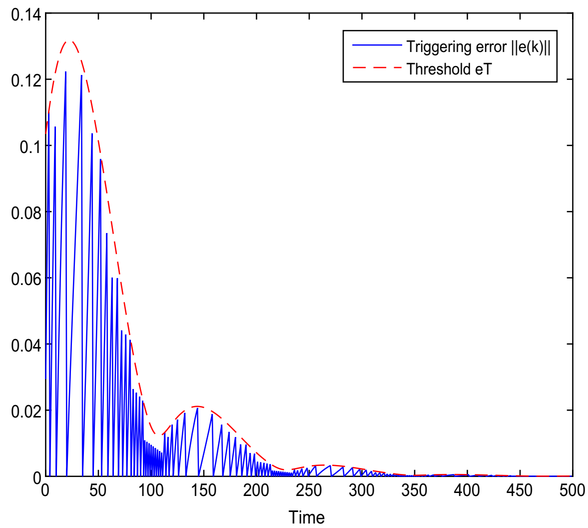

FIGURE 5 Triggering error and threshold [Colour figure can be viewed at wileyonlinelibrary.com]

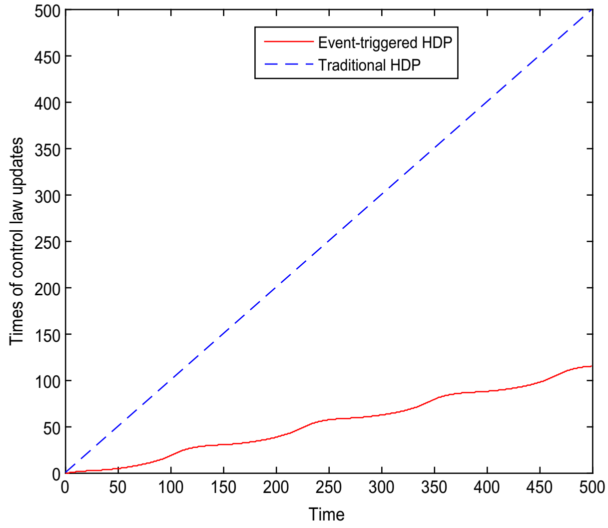

FIGURE 6 Comparison of training times. HDP, heuristic dynamic programming [Colour figure can be viewed at wileyonlinelibrary.com]

Choose the initial state as $x_0 = 0.5$ and the constant $C = 0.55$. The form of the performance index function is the same as (48), where $Q = 0.04I$ and $R = 0.01I$. We use both traditional HDP algorithm and event-triggered HDP algorithm to control the system and obtain the optimal performance index function. Two 3-layer 1-8-1 back propagation NNs are chosen for the critic network and the action network, respectively. For each iteration, the action NN and the critic NN are trained for 100 times. Both traditional HDP algorithm and event-triggered HDP algorithm are operated at the same time for comparison.

In Figure 7, the trajectories of the state in 2 algorithms are presented to show the effectiveness. The control law in event-triggered HDP algorithm is shown in Figure 8. Apparently, the control law is updated only when the event is triggered. The triggering error $\|e(k)\|$ and the triggering threshold e_T are presented in Figure 9. It can be seen that the triggering error will be reset if it is greater than the threshold. Figure 10 represents the summary of the performance index function. We also record the number of training times in Figure 11. The traditional HDP algorithm is implemented by 50 times of training, while the event-triggered HDP algorithm by 10 times.

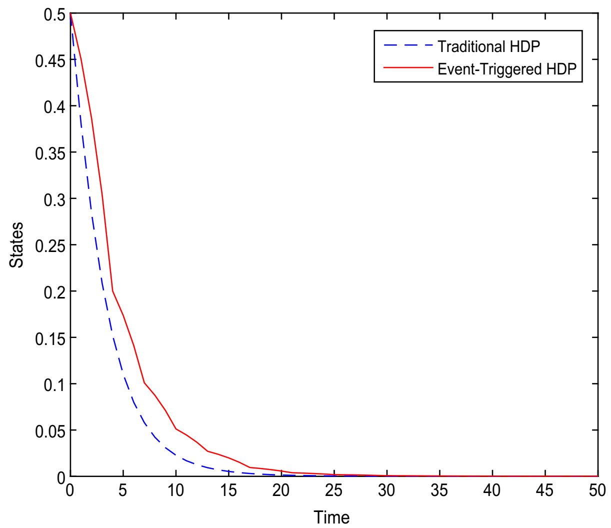

FIGURE 7 The trajectories of states. HDP, heuristic dynamic programming [Colour figure can be viewed at wileyonlinelibrary.com]

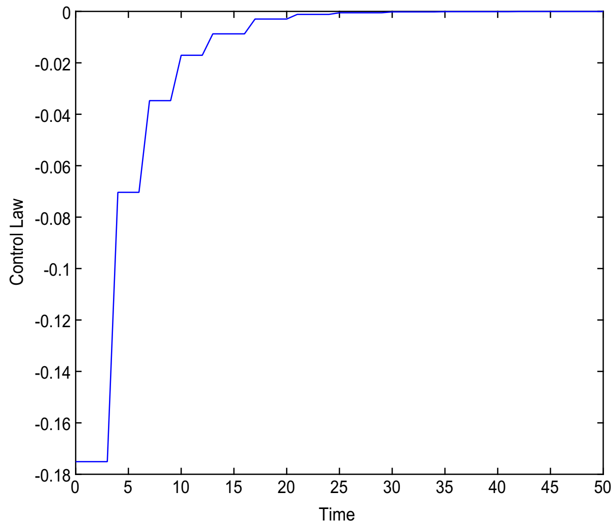

FIGURE 8 The trajectories of event-triggered heuristic dynamic programming control law [Colour figure can be viewed at wileyonlinelibrary.com]

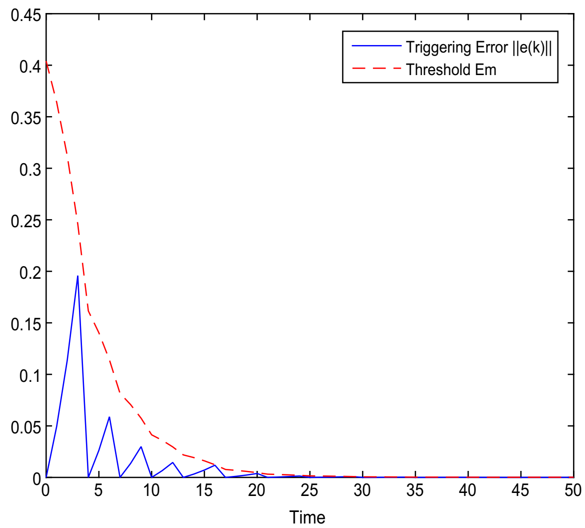

FIGURE 9 Triggering error and threshold [Colour figure can be viewed at wileyonlinelibrary.com]

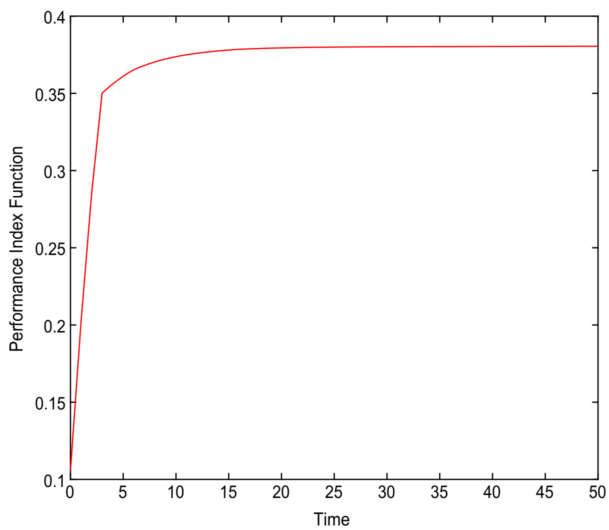

FIGURE 10 The trajectories of performance index [Colour figure can be viewed at wileyonlinelibrary.com]

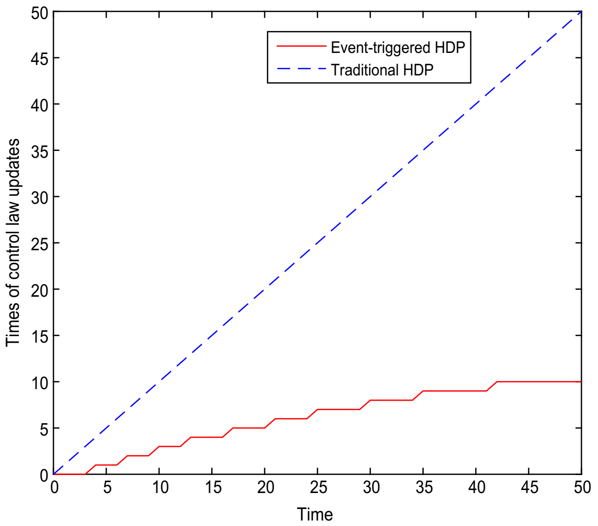

FIGURE 11 Comparison of training times. HDP, heuristic dynamic programming [Colour figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSION

In this paper, a new triggering condition has been proposed based on event-triggered HDP algorithm for discrete-time systems. Under this triggering condition, the stability of the system is proved. Simulations are presented to show the effectiveness that the event-triggered HDP algorithm could reduce more than three-quarters of the computation. The selection of constant C will be studied in future work.

ACKNOWLEDGEMENT

This work was supported in part by the National Natural Science Foundation of China under grants 61233001, 61722312, 61374105, and 61533017.

ORCID

Qinglai Wei http://orcid.org/0000-0003-0501-800X

REFERENCES

1. Lewis FL, Vrabie D. Reinforcement learning and adaptive dynamic programming for feedback control. *IEEE Circuits Syst Mag*. 2009;9(3):32-50.
2. Werbos PJ. Foreword - ADP: the key direction for future research in intelligent control and understanding brain intelligence. *IEEE Trans Syst Man Cybern Part B Cybern*. 2008;38(4):898-900.
3. Wei Q, Liu D, Yang X. Infinite horizon self-learning optimal control of nonaffine discrete-time nonlinear systems. *IEEE Trans Neural Netw Learn Syst*. 2015;26(4):866-879.
4. Wei Q, Song R, Yan P. Data-driven zero-sum neuro-optimal control for a class of continuous-time unknown nonlinear systems with disturbance using ADP. *IEEE Trans Neural Netw Learn Syst*. 2016;27(2):444-458.
5. Sahoo A, Xu H, Jagannathan S. Near optimal event-triggered control of nonlinear discrete-time systems using neurodynamic programming. *IEEE Trans Neural Netw Learn Syst*. 2015;27(9):1801-1815.
6. Wei Q, Lewis FL, Shi G, Song R. Error-tolerant iterative adaptive dynamic programming for optimal renewable home energy scheduling and battery management. *IEEE Trans Ind Electron*. 2017;64(12):9527-9537.
7. Liu D, Xu Y, Wei Q, Liu X. Residential energy scheduling for variable weather solar energy based on adaptive dynamic programming. *IEEE/CAA J Autom Sin*. 2018;5(1):36-46.
8. Wei Q, Liu D, Lin Q, Song R. Discrete-time optimal control via local policy iteration adaptive dynamic programming. *IEEE Trans Cybern*. 2017;47(10):3367-3379.
9. Werbos PJ. Advanced forecasting methods for global crisis warning and models of intelligence. *Gen Syst Yearb*. 1977;22(6):25-38.
10. Wei Q, Liu D, Lewis FL, Liu Y. Mixed iterative adaptive dynamic programming for optimal battery energy control in smart residential microgrids. *IEEE Trans Ind Electron*. 2017;64(5):4110-4120.
11. Wei Q, Liu D, Lin H. Value iteration adaptive dynamic programming for optimal control of discrete-time nonlinear systems. *IEEE Trans Cybern*. 2016;46(3):840-853.

12. Wei Q, Lewis FL, Liu D, Song R, Lin H. Discrete-time local value iteration adaptive dynamic programming: convergence analysis. *IEEE Trans Syst Man Cybern: Syst*. 2016. https://doi.org/10.1109/TSMC.2016.2623766
13. Sokolov Y, Kozma R, Werbos LD, Werbos PJ. Complete stability analysis of a heuristic approximate dynamic programming control design. *Automatica*. 2015;59:9-18.
14. Wei Q, Shi G, Song R, Liu Y. Adaptive dynamic programming-based optimal control scheme for energy storage systems with solar renewable energy. *IEEE Trans Ind Electron*. 2017;64(7):5468-5478.
15. Deng T, Fu B, Wang Y. DNNs-HDP oscillation modal control based on ESD damping control. *Control Eng China*. 2017;24(6):1194-1200.
16. Li Q, Tang W. Optimization control of heat source for central heating system based on dual heuristic programming. *Control Eng China*. 2017;24(10):2016-2021.
17. Wei Q, Lewis FL, Sun Q, Yan P. Discrete-time deterministic *Q*-learning: a novel convergence analysis. *IEEE Trans Cybern*. 2017;47(5):1224-1237.
18. Prokhorov DV, Wunsch DC. Adaptive critic designs. *IEEE Trans Neural Netw*. 1997;8(5):997-1007.
19. Xue L, Sun C, Wunsch D, Zhou Y, Yu F. An adaptive strategy via reinforcement learning for the prisoner's dilemma game. *IEEE/CAA J Autom Sin*. 2018;5(1):301-310.
20. Wei Q, Liu D, Lin Q. Discrete-time local iterative adaptive dynamic programming: admissibility and termination analysis. *IEEE Trans Neural Netw Learn Syst*. 2017;28(11):2490-2502.
21. Zhang H, Liang H, Wang Z, Feng T. Optimal output regulation for heterogeneous multiagent systems via adaptive dynamic programming. *IEEE Trans Neural Netw Learn Syst*. 2015;28(1):18-29.
22. Wei Q, Liu D, Liu Y, Song R. Optimal constrained self-learning battery sequential management in microgrid via adaptive dynamic programming. *IEEE/CAA J Autom Sin*. 2017;4(2):168-176.
23. Wang X, Lemmon MD. Event-triggering in distributed networked control systems. *IEEE Trans Autom Control*. 2011;56(3):586-601.
24. Aström KJ. Event based control. In: *Analysis and Design of Nonlinear Control Systems*. Vol 3. Berlin, Heidelberg: Springer-Verlag Berlin Heidelberg; 2008:127-147.
25. Anta A, Tabuada P. To sample or not to sample: self-triggered control for nonlinear systems. *IEEE Trans Autom Control*. 2010;55(9):2030-2042.
26. Eqtami A, Dimarogonas DV, Kyriakopoulos KJ. Event-triggered control for discrete-time systems. Paper presented at: American Control Conference; 2010; Baltimore, MD.
27. Song X, Men Y, Zhou J, Zhao J, Shen H. Event-triggered H_∞ control for networked discrete-time Markov jump systems with repeated scalar nonlinearities. *Appl Math Comput*. 2017;298:123-132.
28. Shen H, Su L, Wu Z-G, Park JH. Reliable dissipative control for Markov jump systems using an event-triggered sampling information scheme. *Nonlinear Anal Hybrid Syst*. 2017;25:41-59.
29. Vamvoudakis KG. Event-triggered optimal adaptive control algorithm for continuous-time nonlinear systems. *IEEE/CAA J Autom Sin*. 2014;1(3):282-293.
30. Mazo M, Tabuada P. Decentralized event-triggered control over wireless sensor/actuator networks. *IEEE Trans Autom Control*. 2011;56(10):2456-2461.
31. Sahoo A, Xu H, Jagannathan S. Adaptive neural network-based event-triggered control of single-input single-output nonlinear discrete-time systems. *IEEE Trans Neural Netw Learn Syst*. 2016;27(1):151-164.
32. Heemels WPMH, Sandee JH, Van Den Bosch PPJ. Analysis of event-driven controllers for linear systems. *Inf Sci*. 2008;81(4):571-590.
33. Dimarogonas DV, Frazzoli E, Johansson KH. Distributed event-triggered control for multi-agent systems. *IEEE Trans Autom Control*. 2012;57(5):1291-1297.
34. Tallapragada P, Chopra N. On event triggered tracking for nonlinear systems. *IEEE Trans Autom Control*. 2013;58(9):2343-2348.
35. Liu W, Yang C, Sun Y, Qin J. Observer-based event-triggered tracking control of leader-follower systems with time delay. *J Syst Sci Complex*. 2016;29(4):865-880.
36. Zhong X, He H. An event-triggered ADP control approach for continuous-time system with unknown internal states. *IEEE Trans Cybern*. 2017;47(3):683-694.
37. Dong L, Zhong X, Sun C, He H. Adaptive event-triggered control based on heuristic dynamic programming for nonlinear discrete-time systems. *IEEE Trans Neural Netw Learn Syst*. 2017;28(7):1594-1605.
38. Al-Tamimi A, Lewis FL, Abu-Khalaf M. Discrete-time nonlinear HJB solution using approximate dynamic programming: convergence proof. *IEEE Trans Syst Man Cybern Part B: Cybern*. 2008;38(4):943-949.
39. Jiang Z-P, Wang Y. Input-to-state stability for discrete-time nonlinear systems. *Automatica*. 2001;37(6):857-869.
40. Si J, Wang Y-T. Online learning control by association and reinforcement. *Neural Netw*. 2001;12(2):264-276.
41. Liu F, Sun J, Si J, Guo W, Mei S. A boundedness result for the direct heuristic dynamic programming. *Neural Netw*. 2012;32(1):229-235.

How to cite this article: Wang Z, Wei Q, Liu D. A novel triggering condition of event-triggered control based on heuristic dynamic programming for discrete-time systems. *Optim Control Appl Meth*. 2018;39:1467–1478. https://doi.org/10.1002/oca.2421