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Distributed consensus of networked markov jump multi-agent systems with mode-dependent event-triggered communications and switching topologies

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Abstract

This paper investigates the distributed leaderless consensus problem of networked Markov jump multi-agent systems with mode-dependent switching topologies. Specifically, a novel mode-dependent sampling and event-triggered communication strategy is proposed to reduce the network burden with less conservatism. Based on model transformation and constructing the mode-dependent Lyapunov-Krasovskii functional, sufficient consensus criteria are first established. Then, the desired event triggering function parameters and the controller gains are designed in terms of linear matrix inequalities (LMIs). In the end, an illustrative example is provided to verify the effectiveness of our proposed consensus method.

Keywords Distributed consensus · Markov jump multi-agent systems · Mode-dependent event-triggered communication · Mode-dependent switching topologies

1 Introduction

The past decade has witnessed a huge development on the multi-agent systems (MASs) due to their theoretical significance and potential applications. Well-known real-world examples can be found in the unmanned vehicles, sensor networks, power systems and so on [1–6]. As a fundamental issue, the distributed consensus problem has been a focal topic, which means that the group of agents can achieve an agreement via local communications network [7–9]. Distinguish advantages can be obtained by distributed communication strategy including higher communication efficiency and lower energy consumption [10, 11]. It should be pointed out that the communication network plays a significant role for the MASs with the information exchanges, where the communication topology structure is a key factor affecting the consensus performance of the MASs. In particular, the communication topologies are always with active or passive switches due to network environment or agent dynamics, which brings more difficultites during the analysis and synthesis of MASs. Fortunately, fruitful consensus research results of MASs with switching topologies have been developed [12–15]. Furthermore, since the Markov jump systems can well describe the switching or jumping features of dynamical systems, the Markov jump MASs (MJMASs) have been receiving increasing attention [16–19]. Some remarkable results can be found in the literature and the references therein [20–22].

On the other hand, a lot of efforts have been devoted to the researches on networked control systems (NCSs) and cyber-physical systems (CPSs) [23–26]. With the rapid developments of communication network, it is practical and urgent to apply more flexible and efficient communication strategies in the real-world network. With this index, one of the most effective methods is the so-called event-triggered strategy. For the MASs with event-triggered communications, the agent states are sampled and information is exchanged only when specified conditions can be satisfied. This is more applicable in practical applications since it can considerably save the communication resources and reduce the communication frequency [27–30]. Although some

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attempts have been made towards the MASs with Markov jump topologies [31–33], it is worth mentioning that few concern has been addressed for the mode-dependent sampling and event-triggered communication strategy for the MJMASs despite its practical importance, which motivates us for this study.

Based on the aforementioned discussions, in this paper, the distributed consensus problem of networked MJMASs with mode-dependent event-triggered communications and switching topologies is studied. As compared to the relevant literature, main contributions of this paper can be summarized as two-fold. 1) A novel mode-dependent asynchronous event-triggered communication strategy for MJMASs is developed for the first time based on the mode-dependent sampling periods, such that less conservative triggering functions can be designed by detected system modes. 2) By constructing appropriate mode-dependent Lyapunov-Krasovskii functional, sufficient conditions are derived to ensure that the leaderless consensus can be reached in the mean-square sense and the agent controller gains can be designed with LMIs accordingly.

The rest of this paper is arranged as follows. In Section 2, some preliminaries are introduced and consensus problem to be investigated is formulated. Section 3 presents the main theoretical results with details. In Section 4, a numerical example is given to demonstrate the effectiveness of our obtained results. Section 5 concludes the paper and gives our future work.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n dimensional Euclidean space and the space of $m \times n$ real matrices, respectively. $A - B \succ 0$ ($A - B \prec 0$) denotes that $A - B$ is positive definite (negative definite). $A \otimes B$ stands for the Kronecker product. $(\Omega, \mathcal{F}, \mathcal{P})$ denotes a probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{\cdot\}$ represents the mathematics expectation of a stochastic process. $*$ in symmetric block matrices means an ellipsis for the symmetry terms. diag$\{\cdots\}$ denotes a block-diagonal matrix. All matrices are supposed to have compatible dimensions if not explicitly states.

2 Preliminaries and problem formulation

2.1 Networked MJMASs dynamics

Fixed $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following group of N MJMASs:

$$\dot{x}_i(t) = A(r(t))x_i(t) + B(r(t))u_i(t), i = 1, 2, \ldots, N, \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the ith agent, $u_i(t) \in \mathbb{R}^p$ is the control vector of the ith agent, $r(t)$ is a continuous-time discrete-state Markov process, $A(r(t))$ and $B(r(t))$ are constant system matrices for each fixed mode $r(t)$. $r(t)$ takes values in a finite set $\mathcal{M} = \{1, \ldots, M\}$ and the transition probability matrix $\Theta := (\pi_{kl})$ is defined by

$$\Pr(r(t+\Delta t) = l | r(t) = k) = \begin{cases} \pi_{kl}\Delta t + o(\Delta t), k \neq l, \\ 1 + \pi_{kk}\Delta t + o(\Delta t), k = l, \end{cases}$$

where $\lim(o(\Delta t)/\Delta t) = 0$ and $\pi_{kl} \geq 0, k \neq l, \forall k, l \in \mathcal{M}$ is the transition rate from mode k at time t to mode l at time $t + \Delta t$, satisfying

$$\pi_{kk} = -\sum_{l=1,\, k\neq l}^{M} \pi_{kl}, \forall k \in \mathcal{M}.$$

Without loss of generality, it is assumed that all possible modes can be detected for the MJMASs.

The consensus is said to be achieved in the mean-square sense if and only if it holds that:

$$\lim_{t\to\infty} \mathbb{E}\{\|x_i - x_j\|\} = 0, i, j = 1, 2, \ldots, N.$$

Remark 1 It is noteworthy that the system modes have played a significant role during the consensus procedure, such that they are supposed to be synchronously detectable for the MJMASs.

2.2 Graph theory

The directed graph $\mathcal{G}(r(t)) = \{\mathcal{V}(r(t)), \mathcal{E}(r(t)), \mathcal{A}(r(t))\}$ is utilized to describe the communication topology among the MASs with a fixed mode $r(t)$, where $\mathcal{V}(r(t))$ represents the sets of nodes, $\mathcal{E}(r(t))$ is the sets of edges and $\mathcal{A}(r(t)) = [a_{ij}(r(t))] \in \mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix, where

$$\begin{cases} a_{ij}(r(t)) > 0, (v_i(r(t)), v_j(r(t))) \in \mathcal{E}, \\ a_{ij}(r(t)) = 0, \text{otherwise}. \end{cases}$$

Accordingly, the Laplacian matrix $L(r(t)) = [l_{ij}(r(t))] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}(r(t))$ is defined by $l_{ii}(r(t)) = \sum_{j=1,j\neq i}^{N} a_{ij}(r(t))$ and $l_{ij}(r(t)) = -a_{ij}(r(t)), i \neq j$, which aims to describe the directed graph structure. If $\mathcal{G}(r(t))$ has a directed spanning tree, then $L(r(t))$ has a simple zero eigenvalue and all the other eigenvalues are real [1].

2.3 Mode-dependent event-triggered communication scheme

The sampler of the ith agent is assumed to be time-driven with a mode-dependent sampling period $h(r(t))$. Each agent communicates with its neighboring agents and updates its own controller according to the following asynchronous mode-dependent event triggering function:

$$\begin{aligned} t^i_{\sigma+1}h\left(r(t)\right) =\;& t^i_\sigma h\left(r(t)\right) \\ &+\min_{\kappa_i\geq 1}\{\kappa_i h\left(r(t)\right)|\chi_i^T(t^i_\sigma h \\ &+\kappa_i h\left(r(t)\right))W_1\left(r(t)\right) \\ &\times\chi_i(t^i_\sigma h\left(r(t)\right)+\kappa_i h\left(r(t)\right)) \\ \geq\;& \varepsilon\varkappa_i^T(t^i_\sigma h\left(r(t)\right) \\ &+\kappa_i h\left(r(t)\right))W_2\left(r(t)\right)\varkappa_i(t^i_\sigma h\left(r(t)\right) \\ &+\kappa_i h\left(r(t)\right))\}, \end{aligned} \tag{2}$$

where $t^i_\sigma\left(r(t)\right)$ denotes the latest σth triggering instant of the ith agent, $0<\varepsilon<1$ represents the triggering threshold, $W_1\left(r(t)\right)\succ 0$ and $W_2\left(r(t)\right)\succ 0$ are the weighting matrices, $\chi_i(t^i_\sigma h\left(r(t)\right)+\kappa_i h\left(r(t)\right)) := x_i(t^i_\sigma h\left(r(t)\right)+\kappa_i h\left(r(t)\right))-x_i(t^i_\sigma h\left(r(t)\right))$, $\varkappa_i(t^i_\sigma h\left(r(t)\right)+\kappa_i h\left(r(t)\right)) := \sum_{j\in\mathcal{N}_i}a_{ij}\left(r(t)\right)(x_i(t^i_\sigma h\left(r(t)\right)) - x_j(t^i_{\sigma^*}h\left(r(t)\right)))$ with $\sigma^*\triangleq\arg\min_\varrho\{t^i_\sigma+\kappa_i-t^j_\varrho|t^i_\sigma+\kappa_i>t^j_\varrho,\varrho\in\mathbb{N}\}$.

Moreover, define

$$e_i(\kappa h\left(r(t)\right))\triangleq x_i(\kappa h\left(r(t)\right))-x_i(t^i_\sigma h\left(r(t)\right)),\, t^i_\sigma\leq\kappa<t^i_{\sigma+1},$$

and divide $t^i_\sigma\leq\kappa<t^i_{\sigma+1}$ into $t^i_{\sigma+1}-t^i_\sigma$ sampling intervals. Then, it follows that

$$\begin{aligned} &e_i^T(\kappa h\left(r(t)\right))W_1\left(r(t)\right)e_i(\kappa h\left(r(t)\right)) \\ &\leq\varepsilon\bar{\varkappa}_i^T(\kappa h\left(r(t)\right))W_2\left(r(t)\right)\bar{\varkappa}_i(\kappa h\left(r(t)\right)),\,\kappa\in\mathbb{N}, \end{aligned}$$

where

$$\begin{aligned} \bar{\varkappa}_i(\kappa_i h\left(r(t)\right))\triangleq\;&\sum_{j\in\mathcal{N}_i}a_{ij}\left(r(t)\right)(x_i(t^i_{\sigma_i(\kappa)}h\left(r(t)\right)) \\ &-x_j(t^j_{\sigma_j(\kappa)}h\left(r(t)\right))), \end{aligned}$$

and $\sigma_i(\kappa)\triangleq\arg\min_\varrho\{\kappa-t^i_\varrho|\kappa>t^j_\varrho,\varrho\in\mathbb{N}\}$.

Remark 2 It should be pointed out that in our proposed mode-dependent event-triggered communication scheme, the sampled data of each agent is mode-dependent with mode jumpings. This can bring less conservatism than the common sampling schemes. In addition, since the event-triggered function with $W_1\left(r(t)\right)\succ 0$ and $W_2\left(r(t)\right)\succ 0$ is based on the detected system modes, further conservatism can be obtained compared with mode-independent event-triggered communication schemes.

Remark 3 Note that the triggering threshold ε and the mode-dependent sampling period $h\left(r(t)\right)$ are pre-given in the event-triggered function. When choosing smaller values of ε, more information will be exchanged among the agents. Meanwhile, smaller values of $h\left(r(t)\right)$ can also lead to more effective event-triggered communications.

In order to achieve the consensus, the following mode-dependent consensus controllers can be designed by

$$\begin{aligned} u_i(t) =\;& -K\left(r(t)\right)\sum_{j\in\mathcal{N}_i}a_{ij}\left(r(t)\right)(x_i(t^i_\sigma h\left(r(t)\right)) \\ &-x_j(t^i_{\sigma^*}h\left(r(t)\right))),\, t\in[t^i_\sigma h, t^i_{\sigma+1}h), \end{aligned} \tag{3}$$

where $K\left(r(t)\right)\in\mathbb{R}^{m\times n}$ is the mode-dependent controller gain to be determined. It can be found that the designed controllers are in the distributed form with local information exchanges.

As a result, by bringing the above control input into each agent, the closed-loop dynamics of the MASs can be obtained with the help of Kronecker product as follows

$$\begin{aligned} \dot{x}(t) =\;& (I_N\otimes A\left(r(t)\right))x(t) \\ &-(L\left(r(t)\right)\otimes B\left(r(t)\right)K\left(r(t)\right))x(\kappa h\left(r(t)\right)) \\ &+(L\left(r(t)\right)\otimes B\left(r(t)\right)K\left(r(t)\right))e(\kappa h\left(r(t)\right)), \end{aligned} \tag{4}$$

where

$$x(t) := [x_1^T(t), x_2^T(t),\ldots,x_N^T(t)]^T,$$
$$e(\kappa h\left(r(t)\right)) := [e_1^T(\kappa h\left(r(t)\right)), e_2^T(\kappa h\left(r(t)\right)),\ldots,e_N^T(\kappa h\left(r(t)\right))]^T,$$

For notational simplicity, each $r(t)$ is denoted by the index k. In addition, the following lemma is introduced for later use.

Lemma 1 [34] *For any positive symmetric constant matrix $\mathcal{R}\in\mathbb{R}^{n\times n}$, scalars h_1, h_2 satisfying $h_1<h_2$, a vector function $\phi:[h_1,h_2]\to\mathbb{R}^n$, such that the integrations concerned are well defined, then*

$$\begin{aligned} &\left(\int_{h_1}^{h_2}\phi(s)ds\right)^T\mathcal{R}\left(\int_{h_1}^{h_2}\phi(s)ds\right) \\ &\leq(h_2-h_1)\left(\int_{h_1}^{h_2}\phi^T(s)\mathcal{R}\phi(s)ds\right). \end{aligned}$$

3 Main results

In this section, sufficient consensus conditions are derived with details and the corresponding mode-dependent controller gains are designed.

Theorem 1 *With the designed event-triggered transmission scheme (2) and given mode-dependent controller gains K_k, the multi-agent system (1) can achieve the consensus, if $\mathcal{G}_k$ has a directed spanning tree and there exist mode-dependent* *matrices* $P_k \succ 0$, $W_{1k} \succ 0$, $W_{2k} \succ 0$ *and matrix* $Q \succ 0$, *such that* $\tilde{\Pi}_k \prec 0$, *where*

$$\begin{aligned}
\tilde{\Pi}_k \;:&= \begin{bmatrix} \tilde{\Pi}_{k1} & \tilde{\Pi}_{k2} \\ * & \tilde{\Pi}_{k3} \end{bmatrix}, \\
\tilde{\Pi}_{k1} \;:&= \begin{bmatrix} \tilde{\Pi}_{k11} & (\tilde{L}_k \otimes P_k B_k K_k) - (\tilde{L}_k^T \tilde{L}_k \otimes W_{2k}) \\ * & -(I_{N-1} \otimes Q) + \varepsilon(\tilde{L}_k^T \tilde{L}_k \otimes W_{2k}) \end{bmatrix}, \\
\tilde{\Pi}_{k2} \;:&= \begin{bmatrix} (\tilde{L}_k \otimes P_k B_k K_k) & \tilde{L}_k((I_{N-1} \otimes A_k) - (\tilde{L}_k^T \otimes K_k^T B_k Q)) \\ 0 & \bar{\tau}_k(\tilde{L}_k^T \otimes K_k^T B_k Q) \end{bmatrix}, \\
\tilde{\Pi}_{k3} \;:&= \begin{bmatrix} -(I_{N-1} \otimes W_{1k}) & \bar{\tau}_k(\tilde{L}_k^T \otimes K_k^T B_k Q) \\ * & -(I_{N-1} \otimes Q) \end{bmatrix}, \\
\tilde{\Pi}_{k11} &= 2(I_{N-1} \otimes P_k A_k) - 2(\tilde{L}_k \otimes P_k B_k K_k) + \varepsilon(\tilde{L}_k^T \tilde{L}_k \otimes W_{2k}) \\
&\quad + \sum_{l=1}^{M} \pi_{kl}(I_{N-1} \otimes P_l).
\end{aligned}$$

Proof By applying the input-delay approach, system (4) can be rewritten as follows:

$$\begin{aligned}
\dot{x}(t) &= (I_N \otimes A_k)x(t) - (L_k \otimes B_k K_k)x(t - \tau_k(t)) \\
&\quad + (L_k \otimes B_k K_k)e(t - \tau_k(t)), \qquad (5)
\end{aligned}$$

where $\tau_k(t) = t - \kappa h_k, t \in [\kappa h_k, (\kappa + 1)h_k)$ denotes the virtual delay during the sampling period with $0 \leq \tau_k(t) < \bar{\tau}_k, \dot{\tau}(t) = 1$.

Consequently, it can be verified that when $\mathcal{G}_k$ has a directed spanning tree, there exists a matrix W_k such that

$$W_k^{-1} L_k W_k = \begin{bmatrix} \tilde{L}_k & 0 \\ 0 & 0 \end{bmatrix},$$

where $[1, 1, \ldots, 1]^T$ is the last column of W_k.

By denoting

$$\begin{aligned}
\hat{x}(t) &= (W_k^{-1} \otimes I)x(t) \\
&= [\tilde{x}^T(t), \bar{x}^T(t)]^T, \\
\hat{e}(t - \tau_k(t)) &= (W_k^{-1} \otimes I)e(t - \tau_k(t)) \\
&= [\tilde{e}(t - \tau_k(t), \bar{e}(t - \tau_k(t)]^T,
\end{aligned}$$

it follows that

$$\begin{aligned}
\dot{\hat{x}}(t) &= (I_N \otimes A_k)\hat{x}(t) - (W_k^{-1} L_k W_k \otimes B_k K_k)\hat{x}(t - \tau_k(t)) \\
&\quad + (W_k^{-1} L_k \otimes B_k K_k)\hat{e}(t - \tau_k(t)),
\end{aligned}$$

which yields that

$$\begin{aligned}
\dot{\tilde{x}}(t) &= (I_{N-1} \otimes A_k)\tilde{x}(t) - (\tilde{L}_k \otimes B_k K_k)\tilde{x}(t - \tau_k(t)) \\
&\quad + (\tilde{L}_k \otimes B_k K_k)\tilde{e}(t - \tau_k(t)),
\end{aligned}$$

and

$$\dot{\bar{x}}(t) = A_k \bar{x}(t).$$

Then, it can be verified that the consensus can be reached if $\tilde{x}(t)$ is mean-square asymptotically stable.

The following mode-dependent Lyapunov-Krasovskii functional is selected for each mode k,

$$V(k, t) = V_1(k, t) + V_2(k, t),$$

where

$$\begin{aligned}
V_1(k, t) &:= \tilde{x}^T(t)(I_{N-1} \otimes P_k)\tilde{x}(t), \\
V_2(k, t) &:= \bar{\tau}_k \int_{-\bar{\tau}_k}^{0} \int_{t+\varphi}^{t} \dot{\tilde{x}}^T(\theta)(I_{N-1} \otimes Q)\dot{\tilde{x}}(\theta)d\theta d\varphi.
\end{aligned}$$

Moreover, the weak infinitesimal operator $\mathcal{L}$ of $V(k, t)$ is defined by

$$\mathcal{L}V(k, t) \triangleq \lim_{\Delta \to 0} \frac{1}{\Delta}\{\mathbb{E}\{V(r(t + \Delta), t + \Delta)|r(t) = k\} - V(k, t)\}.$$

As a result, it can be derived that

$$\begin{aligned}
\mathcal{L}V_1(l, t) &= \dot{\tilde{x}}^T(t)(I_{N-1} \otimes P_k)\tilde{x}(t) \\
&\quad + \tilde{x}^T(t)(I_{N-1} \otimes P_k))\dot{\tilde{x}}(t) \\
&\quad + \sum_{l=1}^{M} \pi_{kl}\tilde{x}^T(t)(I_{N-1} \otimes P_l)\tilde{x}(t) \\
&= 2\tilde{x}^T(t)(I_{N-1} \otimes P_k)((I_{N-1} \otimes A_k)\tilde{x}(t) \\
&\quad - (\tilde{L}_k \otimes B_k K_k)\tilde{x}(t - \tau_k(t)) \\
&\quad + (\tilde{L}_k \otimes B_k K_k)\tilde{e}(t - \tau_k(t))) \\
&\quad + \sum_{l=1}^{M} \pi_{kl}\tilde{x}^T(t)(I_{N-1} \otimes P_l)\tilde{x}(t) \\
&= 2\tilde{x}^T(t)(I_{N-1} \otimes P_k)((I_{N-1} \otimes A_k)\tilde{x}(t) \\
&\quad - (\tilde{L}_k \otimes B_k K_k)(\tilde{x}(t) - \int_{t-\tau_k(t)}^{t} \dot{\tilde{x}}(\theta)d\theta) \\
&\quad + (\tilde{L}_k \otimes B_k K_k)\tilde{e}(t - \tau_k(t))) \\
&\quad + \sum_{l=1}^{M} \pi_{kl}\tilde{x}^T(t)(I_{N-1} \otimes P_l)\tilde{x}(t)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{L}V_2(l, t) &= \bar{\tau}_k^2 \dot{\tilde{x}}^T(t)(I_{N-1} \otimes Q)\dot{\tilde{x}}(t) \\
&\quad - \tau_k \int_{t-\tau_k}^{t} \dot{\tilde{x}}^T(\varphi)(I_{N-1} \otimes Q)\dot{\tilde{x}}(\varphi)d\varphi.
\end{aligned}$$

By lemma 1, one has

$$\begin{aligned}
&-\tau_k \int_{t-\tau_k}^{t} \dot{\tilde{x}}^T(\varphi)(I_{N-1} \otimes Q)\dot{\tilde{x}}(\varphi)d\varphi \\
&\leq -\int_{t-\tau_k}^{t} \dot{\tilde{x}}^T(\varphi)d\varphi(I_{N-1} \otimes Q)\int_{t-\tau_k}^{t} \dot{\tilde{x}}(\varphi)d\varphi \\
&\leq -\int_{t-\tau_k(t)}^{t} \dot{\tilde{x}}^T(\varphi)d\varphi(I_{N-1} \otimes Q)\int_{t-\tau_k(t)}^{t} \dot{\tilde{x}}(\varphi)d\varphi.
\end{aligned}$$

Furthermore, by the event triggering function, it holds that

$$\begin{aligned}
&-\tilde{e}^T(t - \tau_k(t))(I_{N-1} \otimes W_{1k})\tilde{e}(t - \tau_k(t)) \\
&+\varepsilon(\tilde{x}^T(t) - \int_{t-\tau_k(t)}^{t} \dot{\tilde{x}}^T(\varphi)d\varphi)(\tilde{L}_k^T \tilde{L}_k \otimes W_{2k})(\tilde{x}^T(t) \\
&-\int_{t-\tau_k(t)}^{t} \dot{\tilde{x}}^T(\varphi)d\varphi) \geq 0.
\end{aligned}$$

Thus, it can be obtained that

$$\begin{aligned}
\mathcal{L}V(k,t) &= \mathcal{L}V_1(k,t) + \mathcal{L}V_2(k,t)\\
&\le 2\tilde{x}^T(t)(I_{N-1}\otimes P_kA_k)\tilde{x}(t)\\
&\quad -2\tilde{x}^T(t)(\tilde{L}_k\otimes P_kB_kK_k)\tilde{x}(t)\\
&\quad +2\tilde{x}^T(t)(\tilde{L}_k\otimes P_kB_kK_k)\int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}(\theta)d\theta\\
&\quad +2\tilde{x}^T(t)(\tilde{L}_k\otimes P_kB_kK_k)\tilde{e}(t-\tau_k(t))\\
&\quad +\sum_{l=1}^{M}\pi_{kl}\tilde{x}^T(t)(I_{N-1}\otimes P_l)\tilde{x}(t)\\
&\quad +\bar{\tau}_k^2\dot{\tilde{x}}^T(t)(I_{N-1}\otimes Q)\dot{\tilde{x}}(t)\\
&\quad -\int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}^T(\varphi)d\varphi(I_{N-1}\otimes Q)\int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}(\varphi)d\varphi\\
&\quad -\tilde{e}^T(t-\tau_k(t))(I_{N-1}\otimes W_{1k})\tilde{e}(t-\tau_k(t))\\
&\quad +\varepsilon(\tilde{x}^T(t)-\int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}^T(\varphi)d\varphi)(\tilde{L}_k^T\tilde{L}_k\otimes W_{2k})(\tilde{x}^T(t)\\
&\quad -\int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}^T(\varphi)d\varphi)\\
&\le \xi_k^T(t)\Pi_k\xi_k(t)+\bar{\tau}_k^2\dot{\tilde{x}}^T(t)(I_{N-1}\otimes Q)\dot{\tilde{x}}(t),
\end{aligned}$$

where $\xi_k(t) = [\tilde{x}^T(t), \int_{t-\tau_k(t)}^{t}\dot{\tilde{x}}^T(\varphi)d\varphi, \tilde{e}^T(t-\tau_k(t))]^T$ and

$$\Pi_k = \begin{bmatrix} \Pi_{k1} & (\tilde{L}_k\otimes P_kB_kK_k)-\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes W_{2k}) & (\tilde{L}_k\otimes P_kB_kK_k)\\ * & -(I_{N-1}\otimes Q)+\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes W_{2k}) & 0\\ * & * & -(I_{N-1}\otimes W_{1k})\end{bmatrix},$$

$$\Pi_{k1} = 2(I_{N-1}\otimes P_kA_k)-2(\tilde{L}_k\otimes P_kB_kK_k)+\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes W_{2k}) +\sum_{l=1}^{M}\pi_{kl}(I_{N-1}\otimes P_l).$$

It follows by Schur complement that if $\tilde{\Pi}_k \prec 0$ holds, then system (5) is asymptotically stable in the mean-square sense, which implies that the consensus can be achieved and thus completes the proof. □

Based on the derived results in Theorem 1, the controller design procedure can be given in the following theorem.

Theorem 2 *With the designed event-triggered transmission scheme (2), the multi-agent system (1) can achieve the consensus, if $\mathcal{G}_k$ has a directed spanning tree and there exist mode-dependent matrices $\tilde{P}_k \succ 0$, $\tilde{W}_{1k} \succ 0$, $\tilde{W}_{2k} \succ 0$ and matrix $\tilde{Q} \succ 0$, such that $\hat{\Pi}_k \prec 0$, where*

$$\begin{aligned}
\hat{\Pi}_k &:= \begin{bmatrix}\hat{\Pi}_{k1} & \hat{\Pi}_{k2}\\ * & \hat{\Pi}_{k3}\end{bmatrix},\\
\hat{\Pi}_{k1} &:= \begin{bmatrix}\hat{\Pi}_{k11} & (\tilde{L}_k\otimes B_kU_k)-\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes\tilde{W}_{2k}) & (\tilde{L}_k\otimes B_kU_k)\\ * & (I_{N-1}\otimes\tilde{Q})-2(I_{N-1}\otimes\tilde{P}_k)+\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes\tilde{W}_{2k}) & 0\\ * & * & -(I_{N-1}\otimes\tilde{W}_{1k})\end{bmatrix},\\
\hat{\Pi}_{k2} &:= \begin{bmatrix}\hat{\Pi}_{k21} & \hat{\Pi}_{k22}\end{bmatrix},\\
\hat{\Pi}_{k21} &:= \begin{bmatrix}\bar{\tau}_k((I_{N-1}\otimes\tilde{P}_kA_k)-(\tilde{L}^T\otimes U_k^TB_k)\\ \bar{\tau}_k(\tilde{L}^T\otimes U_k^TB_k)\\ \bar{\tau}_k(\tilde{L}^T\otimes U_k^TB_k)\end{bmatrix},\\
\hat{\Pi}_{k22} &:= \begin{bmatrix}\sqrt{\pi_{k1}}(I_{N-1}\otimes\tilde{P}_k),\ldots,\sqrt{\pi_{kM}}(I_{N-1}\otimes\tilde{P}_k)\\ 0\\ 0\end{bmatrix},\\
\hat{\Pi}_{k3} &:= \begin{bmatrix}-(I_{N-1}\otimes\tilde{Q}) & 0\\ * & -diag\{(I_{N-1}\otimes\tilde{P}_1),\ldots,(I_{N-1}\otimes\tilde{P}_M)\}\end{bmatrix},\\
\hat{\Pi}_{k11} &= 2(I_{N-1}\otimes A_k\tilde{P}_k)-2(\tilde{L}_k\otimes B_kU_k)+\varepsilon(\tilde{L}_k^T\tilde{L}_k\otimes\tilde{W}_{2k})+\pi_{kk}(I_{N-1}\otimes\tilde{P}_k)
\end{aligned}$$

Moreover, the desired mode-dependent controller gains K_k can be obtained by $K_k = U_k\tilde{P}_k^{-1}$.

Proof Let $\tilde{P}_k = P_k^{-1}$, $\tilde{Q} = Q^{-1}$, $U_k = K_k\tilde{P}_k$, $\tilde{P}_kW_{1k}\tilde{P}_k = \tilde{W}_{1k}$, $\tilde{P}_kW_{2k}\tilde{P}_k = \tilde{W}_{2k}$ and pre- and post-multiply both the sides of $\Pi \prec 0$ by diag$\{I_{N-1}\otimes\tilde{P}, I_{N-1}\otimes\tilde{P}, I_{N-1}\otimes\tilde{P}, I_{n(N-1)}, I_{n(N-1)},\ldots,I_{n(N-1)}\}$. Then, the rest of the proof can be directly obtained by Theorem 1. □

4 Illustrative example

In this section, the simulation example is provided to validate the effectiveness of our proposed method.

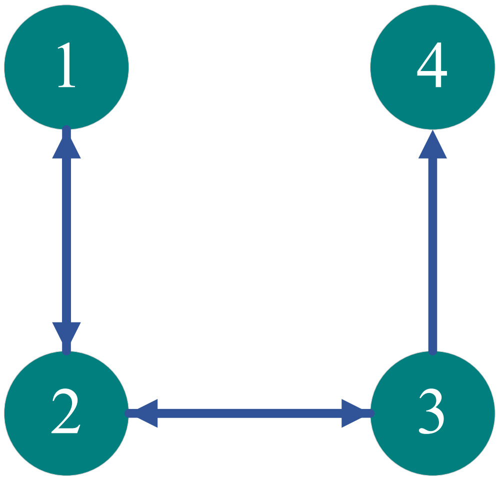

Fig. 1 The communication topology with mode 1

Consider the following MJMASs with four agents and two jumping modes, where the agent dynamics are given as

$$A_1 = \begin{bmatrix} -1.8 & 0.5 \\ 0 & -3 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},$$

and

$$A_2 = \begin{bmatrix} -1.5 & 0.4 \\ 0.3 & -2.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$

and the transition is supposed to be

$$\Theta = \begin{bmatrix} -0.6 & 0.6 \\ 0.4 & -0.4 \end{bmatrix}.$$

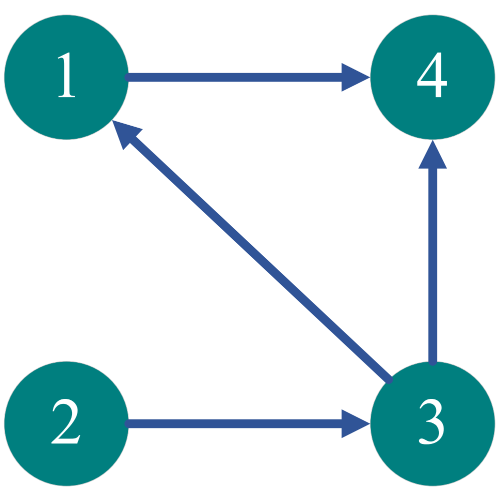

Fig. 2 The communication topology with mode 2

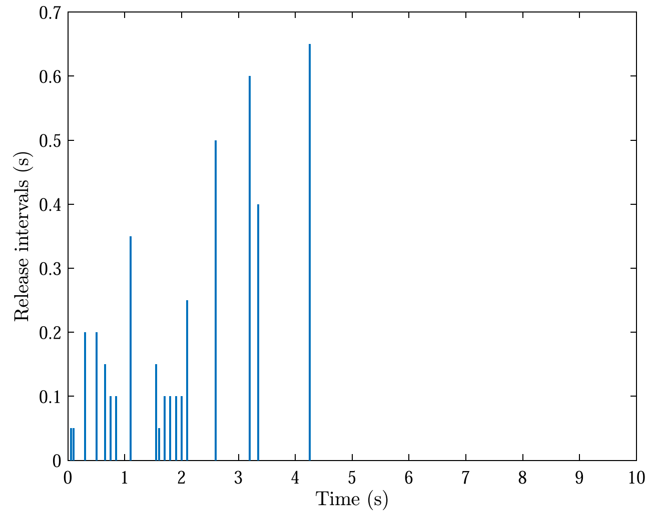

Fig. 3 Broadcasting instants and release intervals of agent 1

Moreover, the directed communication topologies are shown in Figs. 1 and 2, respectively. The corresponding Laplacian matrices are given by

$$L_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

In the simulation, the mode-dependent sampling periods are set as $h_1 = 0.05$s and $h_2 = 0.1$s. The event triggering parameter is given as $\varepsilon = 0.01$. By solving the LMIs in

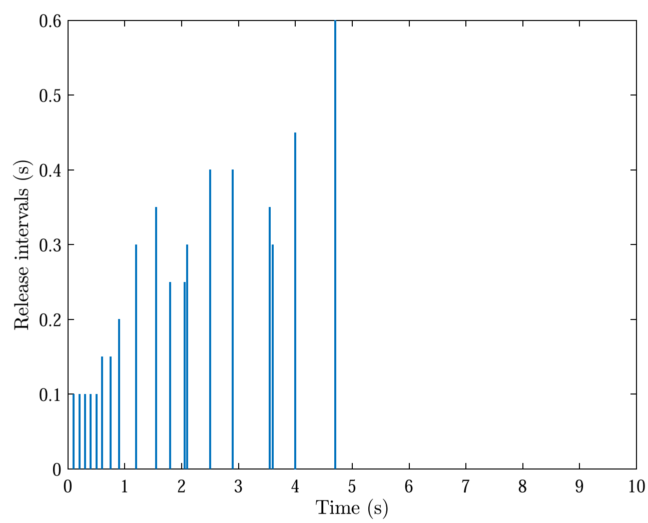

Fig. 4 Broadcasting instants and release intervals of agent 2

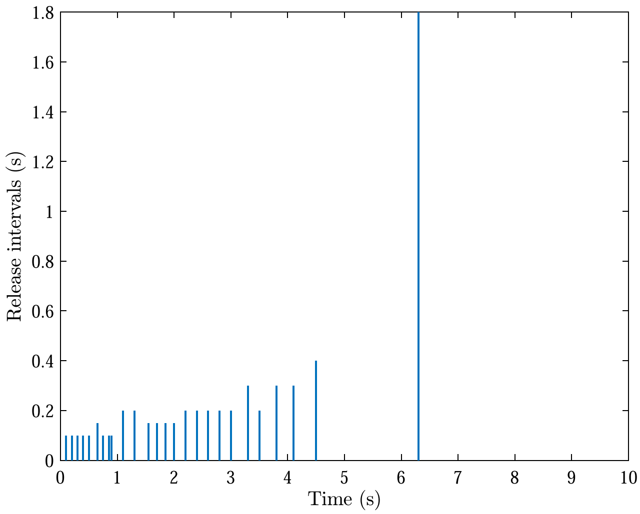

Fig. 5 Broadcasting instants and release intervals of agent 3

theorem 2, the desired mode-dependent controller gains can be obtained as follows:

$$K_1 = \begin{bmatrix} -0.1597 & -0.5800 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.4052 & -0.2635 \end{bmatrix}.$$

Moreover, the weighting matrices W_{1k} and W_{2k}, $k = 1, 2$ can be obtained by

$$W_{11} = \begin{bmatrix} 1.0843 & -0.3843 \\ -0.3843 & 2.1244 \end{bmatrix},$$
$$W_{12} = \begin{bmatrix} 0.8798 & -0.3580 \\ -0.3580 & 1.8068 \end{bmatrix},$$
$$W_{21} = \begin{bmatrix} 1.0407 & -0.3739 \\ -0.3739 & 1.9235 \end{bmatrix},$$
$$W_{22} = \begin{bmatrix} 0.8516 & -0.3515 \\ -0.3515 & 1.7441 \end{bmatrix}.$$

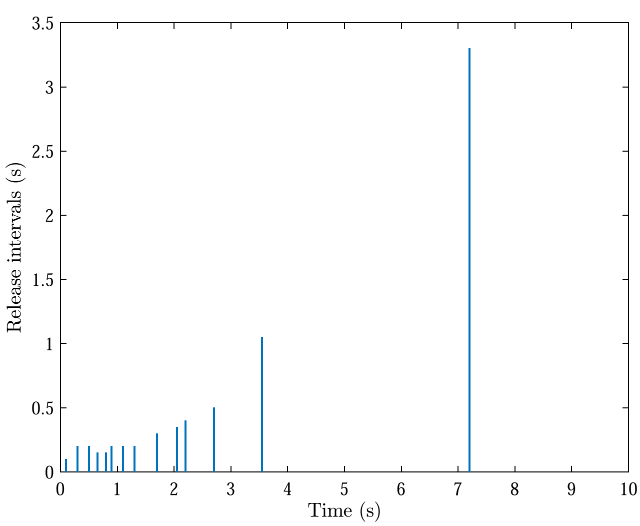

Fig. 6 Broadcasting instants and release intervals of agent 4

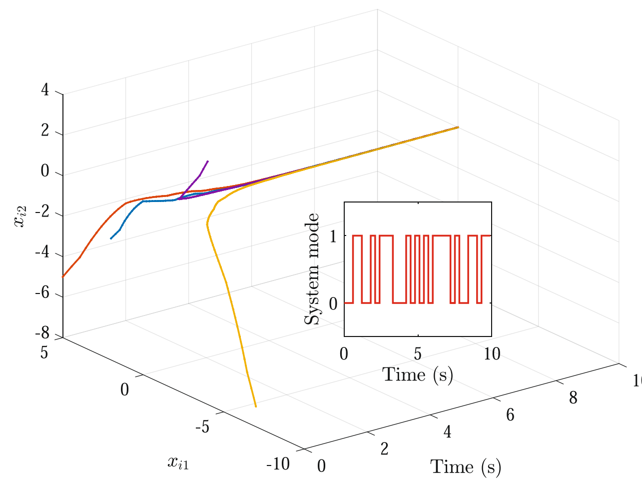

Fig. 7 State trajectories of the MJMASs

With the above parameters, Figs. 3, 4, 5, 6 and 7, depict the information transmission instants with release intervals and the state trajectories of the MJMASs, respectively.

It can be seen that different from time-triggered schemes, the agents broadcast the information only when the event-triggered function can be satisfied. Furthermore, since the energy consumption issue is related with the wireless broadcasting, the event-triggered communication scheme can also save the agent energy accordingly. Hence, the consensus can be well achieved in the mean-square sense while the signal transmissions can be effectively decreased, which supports our theoretical results.

5 Conclusion

This paper is concerned with the distributed consensus problem for networked MJMASs under leaderless framework. Especially, the mode-dependent switching topologies are considered and the mode-dependent event-triggered communication strategy is proposed with mode-dependent sampled data. By employing the mode-dependent Lyapunov-Krasovskii functional method, sufficient consensus conditions are developed, based on which the mode-dependent controller gains are further designed accordingly. Numerical simulations are given to demonstrate the availability of our theoretical results. Our future work will be extending the results to the cases with more general mode information such as time-varying or partially known transition probabilities.

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Compliance with Ethical Standards

Conflict of interests The authors declare that they have no conflict of interest.

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