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Partial correspondence based on subgraph matching

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ABSTRACT

Exploiting both appearance similarity and geometric consistency is popular in addressing the feature correspondence problem. However, when there exist outliers the performance generally deteriorates greatly. In this paper, we propose a novel partial correspondence method to tackle the problem with outliers. Specifically, a novel pairwise term together with a neighborhood system is proposed, which, together with the other two pairwise terms and a unary term, formulates the correspondence to be solved as a subgraph matching problem. The problem is then approximated by the recently proposed Graduated Non-Convexity and Graduated Concavity Procedure (GNCGCP). The proposed algorithm obtains a state-of-the-art accuracy in the existence of outliers while keeping $\mathcal{O}(N^3)$ computational complexity and $\mathcal{O}(N^2)$ storage complexity. Simulations on both the synthetic and real-world images witness the effectiveness of the proposed method.

1. Introduction

Feature correspondence, aiming to find a reasonable assignment between local feature sets of different images, is one fundamental problem in computer vision and pattern recognition and is extensively applied in many tasks including object detection and recognition, camera self-calibration, 3D reconstruction and tracking. The extracted local features can hardly be used by most current objective object recognition, 3D reconstruction algorithms, unless they are put into correspondence. Though the correspondence problem has been studied for decades, it is still a challenging problem.

Starting from using only the appearance descriptor such as SIFT descriptor [1], bag-of-words model [2] which get good results in some computer vision tasks, recently much more effort in this area has been devoted to the incorporation of structural information into the appearance cues. They thus formulate the correspondence problem as a combination of the unary term and the pairwise term which relate to the appearance similarity and geometric consistency, respectively. Inspite of some controversy over the effectiveness of the structural constraints [3,4], some most recent works [5,6] which pay more attention to the robustness and distinctive ability of the structural model witness obvious performance improvements on some benchmark datasets and reconfirm the usefulness of structural cues.

However, the correspondence is still a challenging task when there exist outliers, which are inevitable in many practical applications, e.g. when matching an object in complex background or part of the object is occluded. Such a correspondence problem with unequal feature points in two images in the presence of outliers is denoted as partial correspondence. Many current partial correspondence methods deteriorate greatly as the outlier number increases [7] and some other methods [8], especially the adjacency matrix based methods [6,9–11] cannot even deal with the partial problem.

In this paper, we propose a novel partial feature correspondence method, with two main contributions listed below:

1. A pairwise term together with a neighborhood system which describes the coherence of key points is proposed. The coherence prior means that the adjacent points more likely locate in the same image region, either in object region or background region.
2. An effective and efficient combinatorial optimization framework named Graduated-NonConvexity-and-Graduated-Concavity-Procedure (GNCGCP) [12] is introduced to solve the partial correspondence problem.

In addition, two pairwise terms based on our previously proposed directed distance and direction descriptors [6] with remarkable distinctive ability are adapted to the partial situation. Together with the appearance term, coherence term and GNCGCP, the whole scheme achieves state-of-the-art performance, and at the same time enjoys $O(N^3)$ computational complexity and $O(N^2)$ storage complexity, where N is the key point number.

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There has been other literature on the partial correspondence issue [5,7,13]. The proposed method differentiates mainly from two aspects: (1) the directed structural model based objective function; and (2) the GNCGCP based optimization algorithm.

2. Proposed method

2.1. Objective function

Given two image feature sets $G\in\mathbb{R}^{m\times s}, H\in\mathbb{R}^{n\times s}, m\le n$, where m, n are the key point numbers and s is the appearance descriptor dimension. The partial correspondence problem is formulated as finding a good assignment, or equivalently a partial permutation matrix $P\in\mathbb{R}^{m\times n}$ between the key points in G and H to get a correspondence criterion $F(P,G,H)$ minimized. In this paper, we utilize

$$F(P,G,H) = w^{app}F^{app}(P) + w^{dis}F^{dis}(P) + w^{dir}F^{dir}(P) + w^{coh}F^{coh}(P),$$
$$\text{s.t. } P\in\mathcal{P}, \mathcal{P} = \left\{ P | P_{ij} = \{0,1\}, \sum_{j=1}^{n} P_{ij} = 1, \sum_{i=1}^{m} P_{ij} \le 1, \forall i,j \right\} \tag{1}$$

where w^{app}, w^{dis}, w^{dir}, w^{coh} are weights of the four terms.

Next we will first introduce a neighborhood system, and then give the explanation for each term in (1).

When using the complete neighborhood system [6,9] where all the key points are mutually connected, the connection between two points far away from each other often brings in noise rather than makes the structural model robust, especially in the non-rigid correspondence. Instead we utilize a local neighborhood system where a key point i is connected to its K nearest neighboring points $\mathcal{N}_K(i)$, and is considered as infinitely far away from all the other points. A neighborhood matrix $N\in\mathbb{R}^{n\times n}$ is given as

$$N_{ij} = \begin{cases} 1/K & \text{if } j\in\mathcal{N}_K(i), \\ 0 & \text{if } j\notin\mathcal{N}_K(i). \end{cases} \tag{2}$$

where i, j are two key points. The local neighborhood system improves the correspondence performance as will be illustrated in Section 3. Based on the system, several pairwise terms are introduced or modified below.

$F^{app}(P)$ is a unary term which measures the appearance similarity between feature sets. We define it as

$$F^{app}(P) = \mathrm{tr}(CP^T) \tag{3}$$

where $\mathrm{tr}(\cdot)$ is the matrix trace, $C\in\mathbb{R}^{m\times n}$ is the cost matrix where c_{ij} denotes the dissimilarity between the two appearance descriptors g_i and h_j normalized by the largest value in C. SIFT descriptor, shape context descriptor, bag-of-words model and other appearance descriptors could be adopted in this term depending on their typical uses, and the dissimilarity measures vary accordingly, where the chi-square distance for histogram based appearance descriptor such as SIFT and the Euclidean distance are two common choices.

$F^{dis}(P)$ is a pairwise term which measures the geometric consistency from the distance aspect. We define it as

$$F^{dis}(P) = \|A_G^{dis} - PA_H^{dis}P^T\|_F^2 \tag{4}$$

where $\|\cdot\|$ is the Frobenius matrix norm defined as $\|A\|_F = \sqrt{\sum_i\sum_j A_{ij}^2} = \sqrt{\mathrm{tr}(A^TA)}$. $A_G^{dis}\in\mathbb{R}^{m\times m}, A_H^{dis}\in\mathbb{R}^{n\times n}$ are the adjacency matrices describing the distance attributes of feature sets defined as

$$a_{\underset{ij}{\rightarrow}}^{dis} = \begin{cases} \exp\left(-\frac{\|l_i-l_j\|^2}{\max_{j\in\mathcal{N}_K(i)}\|l_i-l_j\|^2}\right) & \text{if } j\in\mathcal{N}_K(i) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where l_i, l_j are the locations of key points i and j respectively. The distance descriptor is directed since generally $a_{\underset{ij}{\rightarrow}}^{dis} \ne a_{\underset{ji}{\rightarrow}}^{dis}$, which makes it more distinctive between i and j [6], and the descriptor normalized by the local distance maximum makes itself less affected by the outliers compared with the traditional normalization [6].

$F^{dir}(P)$ is a similar pairwise term as $F^{dis}(P)$ but from the orientation aspect. We define it as

$$F^{dir}(P) = \|A_G^{dir} - PA_H^{dir}P^T\|_F^2 \tag{6}$$

where $A_G^{dir}\in\mathbb{R}^{m\times m}, A_H^{dir}\in\mathbb{R}^{n\times n}$ are the adjacency matrices for the direction attribute defined as

$$a_{\underset{ij}{\rightarrow}}^{dir} = \begin{cases} \frac{1}{\pi}\arccos\left(\frac{(l_i-l_j)}{\|l_i-l_j\|}\frac{\overline{d}}{\|\overline{d}\|}\right) & \text{if } j\in\mathcal{N}_K(i)\wedge\|\overline{d}\|\ne 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\overline{d}$ is denoted as the object orientation which is a relatively fixed direction with respect to the object rotation. When matching two objects with clear background, an effective definition is

$$\overline{d} = \sum_{\substack{i=1\cdots n \\ l_i\ne\overline{l}}} \frac{l_i-\overline{l}}{\|l_i-\overline{l}\|} \tag{8}$$

where $\overline{l} = (1/n)\sum_{i=1\cdots n} l_i$ can be regarded as the center of the object [6]. While with complex background or given the prior that there is rare rotation between two objects, e.g. Chinese character matching, it is better to utilize horizon direction instead. Similarly to $a_{\underset{ij}{\rightarrow}}^{dis}$, $a_{\underset{ij}{\rightarrow}}^{dir}$ is also a directed descriptor which makes it more robust and distinctive [6].

$F^{coh}(P)$ is a pairwise penalty term which penalizes the correspondence status difference between neighboring key points. That means the neighboring key points should locate in the coherent region—both in the object region or both in the background [5]. We define it as

$$F^{coh}(P) = -\|PN_HP^T\|_F^2 \tag{9}$$

where N_H is the neighborhood matrix of H given by (2). Minimizing this term means that when a key point in H is selected then more of its neighboring points are preferred to be selected.

2.2. Feature correspondence algorithm

The problem (1) is a combinatorial optimization problem which could be solved by the graph matching algorithms [14]. The feature points could be viewed as the vertices of the graph model and the pairwise relations describe the edge attributes. Then the unary term and pairwise term measure the similarity of vertices and consistency of edges, respectively. However, the above problem is an NP-hard problem with factorial complexity [10]. To make the problem computationally tractable, some approximations are necessary, for which a comprehensive review is referred to, e.g., [15].

Here we will adopt the recently proposed Graduated Non-Convexity and Graduated Concavity Procedure (GNCGCP) to minimize (1). The GNCGCP was proposed as a general framework for the discrete optimization problem over the set of partial permutation matrices $\mathcal{P}$. It has been proved to realize exactly the Convex–Concave Relaxation Procedure [9,10,16] but in a much simpler manner; it does not involve explicitly the convex or concave relaxation functions, which are typically difficult to construct [7,9–11] and greatly hinder the real application of CCRP. The GNCGCP exhibited competitive or even better accuracy than traditional CCRP, and meanwhile enjoys a low computational and storage complexity as CCRP. Since GNCGCP does not need convex or concave relaxation explicitly, this makes GNGCGP applicable on subgraph matching problem.

To solve (1), the GNCGCP takes the following form:

$$F_\zeta(P)=\begin{cases}(1-\zeta)F(P,G,H)+\zeta\,\mathrm{tr}\,P^TP & \text{if } 1\ge\zeta\ge 0,\\ (1+\zeta)F(P,G,H)+\zeta\,\mathrm{tr}\,P^TP & \text{if } 0>\zeta\ge -1,\end{cases}\quad P\in\mathcal{D}, \tag{10}$$

where $\mathcal{D}\in\mathbb{R}^{m\times n}$ is the convex hull of $\mathcal{P}$ defined as

$$\mathcal{D}=\left\{D|D_{ij}\ge 0,\ \sum_{j=1}^{n}X_{ij}=1,\ \sum_{i=1}^{m}\le 1,\forall i,j\right\} \tag{11}$$

which is referred to as the set of doubly sub-stochastic matrices.

In implementation, when ζ decreases gradually from 1 to -1, the objective function $F_\zeta(P)$ becomes gradually from $\mathrm{tr}P^TP$ to $F(P,G,H)$ (Graduated NonConvexity) and then gradually to $-\mathrm{tr}P^TP$ (Graduated Concavity). Meanwhile, a partial permutation matrix solution P is finally gotten because $F_\zeta(P)$ has exactly the same local minima in $\mathcal{P}$ as (1) when $F_\zeta(P)$ becomes concave. The algorithm is rewritten by Algorithm 1, where Frank–Wolfe algorithm [17] in the inner loop is utilized to minimize $F_\zeta(P)$ on each fixed ζ.

Algorithm 1. GNCGCP based feature correspondence scheme.

Input: Two feature sets G and H
Initialization: $P=1_{N\times N}/N$, $\zeta=1$
repeat
  repeat
    $X=\arg\min\ \mathrm{tr}(\nabla F_\zeta(P)^TX), s.t. X\in\mathcal{D}$ ①
    $\alpha=\arg\min\ F_\zeta(P+\alpha(X-P)), s.t. 0\le\alpha\le 1$ ②
    $P=P+\alpha(X-P)$
  until P converges ③
  $\zeta=\zeta-d\zeta$
until $\zeta<-1\vee X\in\mathcal{P}$
Output A permutation matrix P

In this algorithm, the linear programming problem ① could be solved by the Hungarian algorithm [18], the line search problem ② could be solved by the backtracking algorithm [19], and the stop condition for ③ is given as follows: if $\mathrm{tr}(\nabla F_\eta(P)^T(P-X))<\varepsilon|F_\eta(P)+\mathrm{tr}(\nabla F_\eta(P)^T(X-P))|$, then break. $F_\zeta(P)$ is got through (10), and its gradient is given by

$$\nabla F_\zeta(P)=\begin{cases}(1-\zeta)\nabla F(P,G,H)+2\zeta P & \text{if } 1\ge\zeta\ge 0,\\ (1+\zeta)\nabla F(P,G,H)+2\zeta P & \text{if } 0>\zeta\ge -1,\end{cases}\quad P\in\mathcal{D}, \tag{12}$$

where

$$\nabla F(P,G,H)=w^{app}\nabla F^{app}(P)+w^{dis}\nabla F^{dis}(P)+w^{dir}\nabla F^{dir}(P)+w^{coh}\nabla F^{coh}(P),$$

and

$$\nabla F^{app}(P)=C,$$

$$\nabla F^{dis/dir}(P)=2P(A_H^TP^TPA_H+A_HP^TPA_H^T)-2(A_GPA_H^T+A_G^TPA_H),$$

where A_G and A_H should be replaced by A_G^{dis}, A_G^{dir} and A_H^{dis}, A_H^{dir}, respectively,

$$\nabla F^{coh}=-2P(N_H^TP^TPN_H+N_HP^TPN_H^T).$$

3. Experiments

We apply the proposed scheme to a benchmark dataset as well as the real-world images. Two experiments are carried out, with the first one to compare the proposed scheme with some state-of-the-art methods, and second to evaluate the effect of the coherence pairwise term together with the neighborhood system.

3.1. Comparison with state-of-the-art methods

Experimental setting: In this experiment, the comparison is carried out on the CMU 'House' sequence consisting of 111 frames where each frame is marked with the same 30 landmark points as [3] and some samples are shown in Fig. 1. The methods for comparison are *Spectral technique*(SPE) [13], *Factorized method* (FAC), *Path following method*(PF), *Extended path following method* (EPF), *Hungarian algorithm*(HUN) and *the proposed method*(OUR), where the structural models for SPE and FAC are the same as [7,13], for PF and EPF are undirected and directed [6], respectively, and for HUN is only the unary term with SIFT as the appearance descriptor.

We make two comparisons between these methods, with the first one to compare the correspondence accuracy with respect to $n-m$, where n is set to be 30 and m decreases from 30 to 10, and the second one with respect to the frame separation, where larger frame separation implies heavier 3D-rotation between two 'houses'. Following the law of the single variable, in the first comparison we average the accuracy with frame separations increasing from 0 to 90 by a step 10, and in the second one with $n-m$ increasing from 0 to 20 by step size 2. In both comparisons, we set $w^{app}=w^{dis}=w^{dir}=w^{coh}=0.25$ and $K=5$.

Result: The first comparison result is shown in Fig. 2(a), from which we can observe that (1) generally the accuracies for all the methods with pairwise term get worse as the $n-m$ increases; (2) the proposed method achieves the best result when $n-m>10$ implying its better robustness to outliers which is due to the coherence term. However, this term slightly affects the accuracy when $n-m<6$. Actually, when $n-m=0$, EPF, OUR and FAC get 100% accuracy in case we set $w^{coh}=0, w^{app}=w^{dis}=w^{dir}=0.33$; (3) the accuracies of PF and EPF decrease faster as $n-m$ increases, probably because it is not an equivalent transformation by transforming the original subgraph matching problem to the equal-sized adjacency matrix matching problem by adding dummy nodes which may change the global solution. Actually, it is such an in-equivalence that necessitates the direct subgraph matching algorithm; and (4) OUR and EPF outperform PF, which validates the effectiveness of directed structural model.

Fig. 2(b) shows the result of the second comparison. Generally the accuracies decrease when the frames are more separated and the proposed method gets the best result in most cases. Meanwhile, it is also observed that when the frame separation is 0 HUN and OUR get much better accuracy than the others. It is reasonable that HUN gets 100% accuracy because it only uses unary term. That OUR outperforms the other methods with pairwise terms is mainly due to the neighborhood system making the model more distinctive.

Some correspondence samples are shown in Fig. 3.

3.2. Evaluation of the coherence term

Experimental setting: The second experiment is performed on two datasets from Caltech256 [20] with 20 pairs of *revolver* images and *eiffel* images. For each pair, one image contains only the object with 20 and 35 key points in the two datasets while the other image contains both the object with 20 and 35 ground-truth

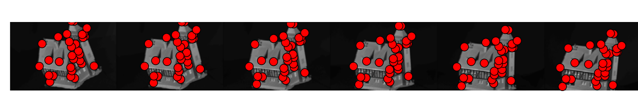

Fig. 1. Some 'House' samples with the indices as 0, 20, 40 60, 80, 100.

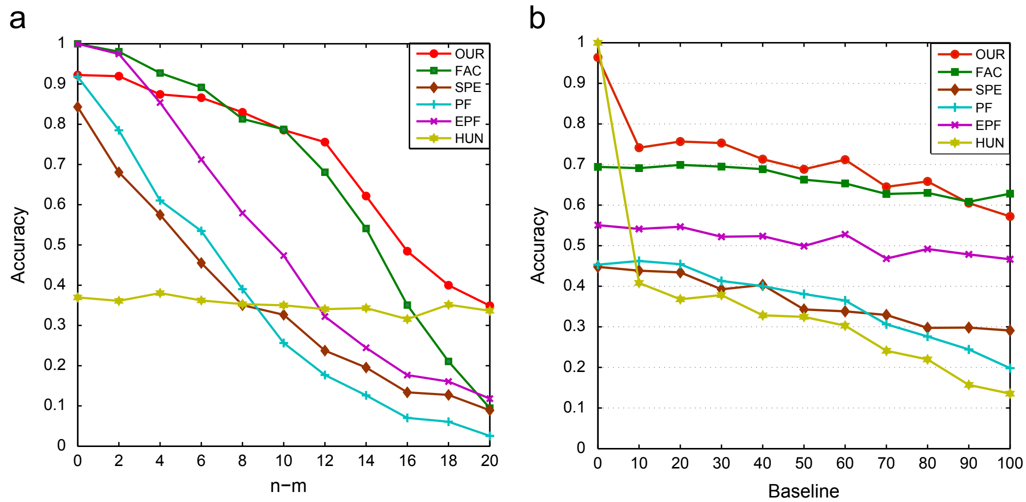

Fig. 2. Experimental results on the ‘House’ sequence. (a) Comparison with respect to $n-m$. (b) Comparison with respect to frame separation.

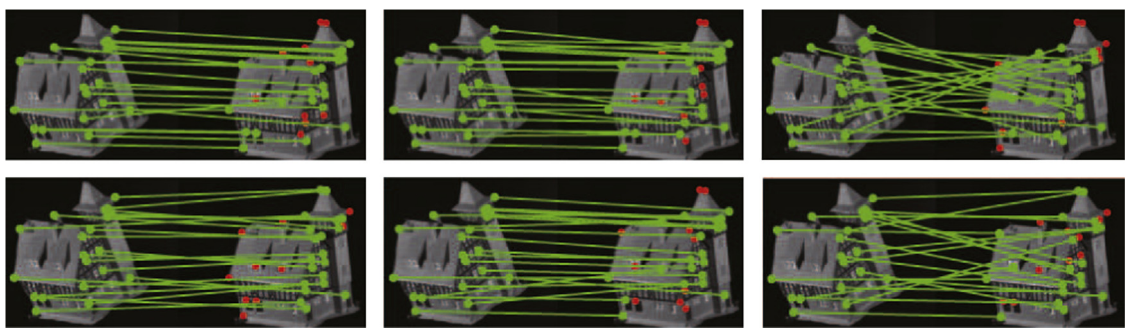

Fig. 3. Some ‘House’ correspondence samples where $m=20$, $n=30$ and frame separation is 50.

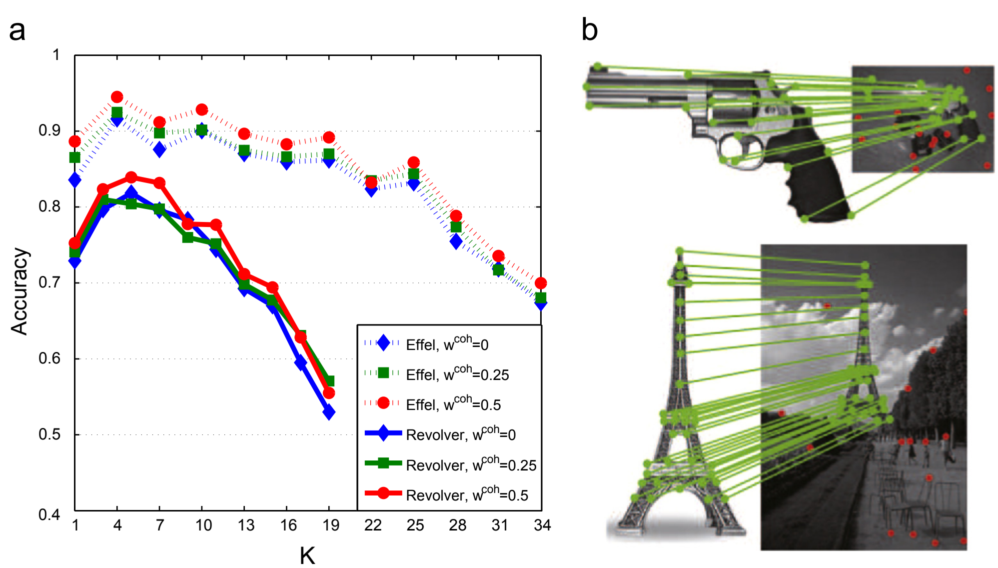

Fig. 4. Experimental results on the Caltech256 dataset. (a) Correspondence accuracy. (b) Correspondence samples.

correspondence points and the background with 20 randomly selected points. The shape context descriptor [21] is adopted as the appearance descriptor.

To evaluate the neighborhood system and coherence pairwise term, K is increased from 1 to its maximum 19(34) by a step size 2 (3) and w^{coh} is increased from 0 to 0.5 by a step size 0.25. The accuracy is averaged as the outlier number increases from 0 to 20 and we set $w^{app}=w^{dis}=w^{dir}=1-w^{coh}/3$.

Result: The result is shown in Fig. 4(a), with some typical correspondence samples given in Fig. 4(b). From Fig. 4(a) we can observe that the accuracy gets its minimum when setting $K=19$ and $K=34$ for *revolver* and *eiffel*, respectively, implying the complete neighborhood system's sensitivity to the outliers. On the other hand, it verifies the effectiveness of the neighborhood system by observing that the accuracy maximum is gotten at $K=4$~5. Meanwhile, it is also observed that the coherence term makes certain improvement to the correspondence accuracy on both datasets.

4. Conclusion

In this paper we proposed a novel partial feature correspondence method for the unequal sized feature sets, which were designed to tackle the correspondence problem with outliers and obtained a state-of-the-art accuracy while keeping low computational and storage complexities. The resulting performance owes to three aspects: (1) GNCGCP is effective and efficient in solving the combinatorial optimization problem over the partial permutation matrices; (2) the coherence pairwise term makes the algorithm less sensitive to outliers; and (3) the structural model is directed, which makes it more distinctive. In the future, we will extend the work to the correspondence of objects both in complex backgrounds, i.e., there are outliers in both images.

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