Attitude Tracking of Rigid Spacecraft With Bounded Disturbances

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***Abstract*—The problem of attitude control for a spacecraft model that is nonlinear in dynamics with inertia uncertainty and external disturbance has been investigated. Adaptive law and extended state observer are applied to estimate the disturbance, by which sliding-mode controllers are designed to combine the two approaches in order to force the state variables of the closed-loop system to converge to the reference attitude states. Also, simulation results are presented to illustrate the effectiveness of the control strategies.**

***Index Terms*—Adaptive control, attitude tracking, extended state observer (ESO), sliding-mode control (SMC).**

I. Introduction

THE ATTITUDE control problem for rigid spacecraft with highly nonlinear characteristics has attracted a great deal of interest for its important application [1]–[3]. The attitude motion of a rigid body is represented by a set of two vector equations, namely, the kinematic equation that relates to the time derivatives of the orientation angles to the angular velocity vector and the dynamic equation that describes the time evolution of the angular velocity vector [4]. In [5], passivity-based control is proposed to ensure the asymptotic convergence of attitude tracking without angular velocity measurements. In [6], the authors provide a solution to the attitude tracking problem by introducing a unit-quaternion auxiliary system which has the same structure as the actual unit-quaternion attitude model. The proposed control strategy can guarantee almost global asymptotic attitude tracking without considering uncertainty and disturbance. In [7], the attitude tracking problem without velocity measurement is also considered. A certainty-equivalence passivity-based controller is developed to guarantee the convergence with an adaptive observer to estimate the angular velocity. In [8], a robust quaternion feedback control scheme employing thrust vector control is proposed to deal with the attitude control problem, by which the closed-loop system can be guaranteed to be globally uniformly stable in the presence of uncertainties, which can be modeled to satisfy the matching condition. In [9], a general design principle of tracking problems for nonlinear systems is derived by using Fliess functional expansion with admissible constant controls. Then, the problem of attitude control is considered as a special tracking problem and solved by the control method. In [10], the inverse optimal adaptive control law combining the adaptive control approach and the optimal control method is designed to solve the attitude tracking problem of a rigid spacecraft. The designed controller can achieve the asymptotic attitude tracking with an uncertain inertia matrix and guarantee the boundedness of the tracking errors with external disturbances. In [11], attitude control is converted into a global stabilization problem of a particular type of nonlinear systems involving both disturbances and mass parameter uncertainties. An adaptive controller is designed to accomplish the stabilization problem and has achieved asymptotic rejection of a class of external disturbances by designing a compensator.

However, the precision of attitude tracking control of spacecraft orientation is not straightforward. In attitude equations, two types of uncertainties are paid attention widely, i.e., one is the external disturbance that arises from the unexpected environmental torques, and another is the model uncertainty existing in the inertia matrix of the spacecraft. The presence of external disturbances and inertia uncertainty makes the attitude tracking control problem more complicated. In most literature works, for unknown disturbances, control design schemes can only achieve disturbance attenuation [12]–[14]. Exact asymptotic disturbance rejection can only be achieved for some special disturbances [11]. Therefore, in this paper, we will further consider a more interesting attitude control problem where a spacecraft involves inertia uncertainties and unknown external disturbances.

The main contributions of this paper are as follows.

1) A new adaptive law for attitude tracking is designed. The proposed control scheme does not rely on the inertial matrix, so it can be applied in spacecraft systems with large parametric uncertainty in inertial matrix or even unknown inertial matrix.
2) This is the first paper that applies the extended state observer (ESO) [15] in the attitude control problem. Combining the sliding-mode control (SMC) method, the controller can achieve fast and accurate response via effective compensation for the external disturbance and uncertainty in inertial matrix.

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II. Nonlinear Model and Problem Formulation

Consider the rigid spacecraft system described by the following attitude kinematic and dynamic equations [16]:

$$\dot{q}_v = \frac{1}{2}\left(q_4 I_3 + q_v^\times\right)\Omega \quad \dot{q}_4 = -\frac{1}{2}q_v^T\Omega \tag{1}$$

$$J\dot{\Omega} = -\Omega^\times J\Omega + u + d. \tag{2}$$

The unit quaternion is a vector defined by $q = [q_1 \quad q_2 \quad q_3 \quad q_4]^{\mathrm{T}} = [q_v \quad q_4]^{\mathrm{T}}$ satisfying $q_v^T q_v + q_4^2 = 1$, where $q_v \in R^3$ is the vector part and $q_4 \in R$ is the scalar component. $\Omega \in R^3$ is the angular velocity of the spacecraft; $u \in R^3$ and $d \in R^3$ are the control torques and bounded external disturbances, respectively; $J \in R^{3\times3}$ is the symmetric inertia matrix of the spacecraft and expressed as

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \tag{3}$$

I_3 is the $R^{3\times3}$ identity matrix; and $^\times$ is an operator on any vector $a = [a_1 \quad a_2 \quad a_3]^{\mathrm{T}}$ such that

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{4}$$

Assumption 2.1: In spacecraft model equations (1) and (2), full states can be measured, which implies that unit quaternion q and angular velocity Ω are available in feedback control design.

We suppose that the desired attitude motion is generated by

$$\dot{q}_{dv} = \frac{1}{2}\left(q_{d4} I_3 + q_{dv}^\times\right)\Omega_d \qquad \dot{q}_{d4} = -\frac{1}{2}q_{dv}^T\Omega_d \tag{5}$$

where $q_d = [q_{d1} \quad q_{d2} \quad q_{d3} \quad q_{d4}]^{\mathrm{T}} = [q_{dv} \quad q_{d4}]^{\mathrm{T}}$ satisfying $\|q_d\| = 1$ is the unit quaternion representing the target attitude quaternion and Ω_d is the target angular velocity. As in [10], Ω_d and $\dot{\Omega}_d$ are assumed to be bounded.

In this paper, we aim at attitude tracking in the presence of inertia uncertainties and disturbances with bounded energy. The objective is to design a feedback controller such that the states of the closed-loop system (1) and (2) track the given desired attitude motion (5), which can be expressed as follows:

$$\lim_{t\to\infty}\xi_1(t) = 0, \qquad \xi_1(t) = q(t) - q_d(t) \tag{6}$$

$$\lim_{t\to\infty}\xi_2(t) = 0, \qquad \xi_2(t) = \Omega(t) - \Omega_d(t). \tag{7}$$

The objective of attitude tracking can be turned into a stabilization problem by two sets of following transformations. Considering the error quaternion $e = [e_1 \quad e_2 \quad e_3 \quad e_4]^{\mathrm{T}} = [e_v \quad e_4]^{\mathrm{T}}$ mentioned in [16]

$$e_v = q_{d4}q_v - q_{dv}^\times q_v - q_4 q_{dv} \tag{8}$$

$$e_4 = q_{dv}^T q_v + q_4 q_{d4} \tag{9}$$

$$\omega = \Omega - C\Omega_d \tag{10}$$

with $C = (1 - 2e_v^T e_v)I_3 + 2e_v e_v^T - 2e_4 e_v^\times$, we have

$$\dot{e}_v = \frac{1}{2}\left(e_4 I_3 + e_v^\times\right)\omega \quad \dot{e}_4 = -\frac{1}{2}e_v^T\omega \tag{11}$$

$$J\dot{\omega} = -(\omega + C\Omega_d)^\times J(\omega + C\Omega_d) + J(\omega^\times C\Omega_d - C\dot{\Omega}_d) + u + d. \tag{12}$$

It has been proven in [2] that the objective (6) and (7) can be achieved if there exists a control law for system (11) and (12) such that $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty}\omega(t) = 0$.

Then, taking the following coordinate transformation suggested in [11]

$$x = \omega + Ke_v \tag{13}$$

gives

$$\dot{e}_v = \frac{1}{2}\left(e_4 I_3 + e_v^\times\right)\omega \quad \dot{e}_4 = -\frac{1}{2}e_v^T\omega \tag{14}$$

$$J\dot{x} = -(\omega + C\Omega_d)^\times J(\omega + C\Omega_d) + J(\omega^\times C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}JK\left(e_4 I_3 + e_v^\times\right)\omega + u + d \tag{15}$$

where K is a positive definite matrix.

Lemma 2.1: Considering the kinematic system (14) and (15), then, for any $x(t)$ satisfying $\lim_{t\to\infty} x(t) = 0$, the solutions of the system are guaranteed $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty}\omega(t) = 0$, respectively.

Proof: It has been proven in [11] that for the kinematic subsystem (14), $\lim_{t\to\infty} e_v(t) = 0$ can be achieved if there exists a control law for (15) satisfying $\lim_{t\to\infty} x(t) = 0$ with any initial state $\|e(0)\| = 1$. This means that when $\lim_{t\to\infty} x(t) = 0$, we have $\lim_{t\to\infty} e_v(t) = 0$, which implies that $\lim_{t\to\infty}\omega(t) = 0$ as well due to (13). Therefore, the achievement of $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty}\omega(t) = 0$ can be accomplished simultaneously only by $\lim_{t\to\infty} x(t) = 0$.

III. SMC With Adaptive Method

In this section, we will propose a new control design for the attitude control problem. It is well known that SMC is a robust method to control nonlinear and uncertain systems, which has attractive features to keep the systems insensitive to uncertainties on the sliding surface [17], [18].

For solving the attitude tracking problem with inertial uncertainty and disturbance existing in the spacecraft system, the adaptive method is a natural choice and has been widely applied. Therefore, in order to decrease the impact of inertial uncertainty and disturbance more efficiently, the SMC scheme can be considered to achieve the objective with the combination of the adaptive approach and the optimal control method.

It has been claimed in Section II that the objective (6) and (7) can be achieved if there exists a control law for system (11) and (12) such that $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty}\omega(t) = 0$. Thus, a sliding surface is designed here to guarantee that the system states can be attracted from the outside to the inside of the region and finally remain inside the region in spite of uncertainty and disturbances. In order to ensure that $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty} \omega(t) = 0$, the sliding surface is selected similar as (13)

$$\tilde{S} = \omega + ke_v = 0 \tag{16}$$

where $\tilde{S} = [\tilde{S}_1, \tilde{S}_2, \tilde{S}_3]^{\mathrm{T}} \in R^3$ and $k > 0$ is a scalar.

Remark 3.1: The difference between (16) and (13) is the parameters k and K. Equation (16) is a special form of (13) in the condition of $K = \mathrm{diag}[k, k, k]$. Thus, by Lemma 2.1, it is clear that if there exists a dynamic state feedback control law such that the trajectories of the closed-loop system (11) and (12) can be driven on the sliding surface (16) and converge into the origin, then $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty} \omega(t) = 0$ can be ensured, and by which the tracking objective (6) and (7) can be achieved.

Now, consider the following reaching law:

$$\dot{\tilde{S}} = -\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S}) \tag{17}$$

where

$$\begin{aligned}
\tau &= \mathrm{diag}[\tau_1, \tau_2, \tau_3] \\
\sigma &= \mathrm{diag}[\sigma_1, \sigma_2, \sigma_3] \\
\mathrm{sgn}(\tilde{S}) &= \left[\mathrm{sgn}(\tilde{S}_1), \mathrm{sgn}(\tilde{S}_2), \mathrm{sgn}(\tilde{S}_3)\right]^{\mathrm{T}} \\
\tau_i &> 0, \quad \sigma_i > 0.
\end{aligned}$$

In [17], it has been shown that the reaching control law can guarantee the convergence of the trajectory of the closed-loop system since it is driven onto the sliding surface in finite time, and the chattering is reduced by tuning the parameters τ and σ properly.

Based on the sliding surface (16) and reaching law (17), the sliding motion can enter a neighborhood of equilibrium in finite time and remain within it by designing the reaching motion controller, which is shown in the following theorem. Before proving the theorem, the following assumptions and definition are recalled.

Assumption 3.1: The symmetric positive definite inertia matrix is assumed to satisfy the following inequality:

$$\|J\| \le \lambda_J \tag{18}$$

where $\lambda_J > 0$ is the upper bound on the norm of the inertia matrix, which is unknown due to the uncertainty existing in the inertia matrix.

Assumption 3.2: The external disturbance $d(t)$ in (2) is assumed to be bounded and satisfy the following condition:

$$\|d(t)\| \le c_{01} + k_1 \|e_v(t)\| + k_{02} \|\omega(t)\| \tag{19}$$

where c_{01}, k_1, and k_{02} are unknown bounds, which are not easily obtained due to the complicated structure of the uncertainties in practical control systems.

Assumption 3.3: There exist positive scalars c and k_2 such that the following condition is satisfied:

$$\begin{aligned}
&\left[\left\|(ke_v - C\Omega_d)^\times\right\| + \frac{1}{2}k\left\|e_4 I_3 + e_v^\times\right\| + \|C\Omega_d\|\right]\|J\|\|\omega\| \\
&\quad + \left\|(ke_v - C\Omega_d)^\times JC\Omega_d - JC\dot{\Omega}_d\right\| \\
&\quad\quad \le (k_2 - k_{02})\|\omega(t)\| + (c - c_{01}).
\end{aligned} \tag{20}$$

Note that Ω_d and $\dot{\Omega}_d$ are the desired system states that are bounded, and e_v and e_4 are also bounded due to $\|q\| = \|q_d\| = 1$, which leads to the boundness of C. Thus, the assumption is reasonable.

Definition 3.1: Consider the nonlinear system $\dot{x} = f(x, u)$, $y = h(x)$, where x is the state vector, u is the input vector, and y is the output vector. The solution is uniformly ultimately bounded (UUB) if, for all $x(t_0) = x_0$, there exist $\varepsilon > 0$ and $T(\varepsilon, x_0)$, such that $\|x(t)\| < \varepsilon$, for all $t \ge t_0 + T$ [19].

Theorem 1: With the linear sliding surface given by (16), the trajectory of the closed-loop system (11) and (12) can be driven onto the sliding surface in finite time with the adaptive controller (21) and update law (23)–(26) and finally evolves in a neighborhood around the origin

$$u_{Ada}(t) = -\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S}) - u_p(t) \tag{21}$$

where the adaptive control law u_p is defined as

$$u_p(t) = \begin{cases} \frac{\tilde{S}(t)}{\|\tilde{S}(t)\|}\hat{\rho}, & \text{if } \hat{\rho}\left\|\tilde{S}(t)\right\| > \epsilon \\ \frac{\tilde{S}(t)}{\epsilon}\hat{\rho}^2, & \text{if } \hat{\rho}\left\|\tilde{S}(t)\right\| \le \epsilon \end{cases} \tag{22}$$

and the adaptation update laws are

$$\hat{\rho} = \hat{c}(t) + \hat{k}_1(t)\|e_v(t)\| + \hat{k}_2(t)\|\omega(t)\| \tag{23}$$

$$\dot{\hat{c}}(t) = p_0\left(-\epsilon_0\hat{c}(t) + \left\|\tilde{S}(t)\right\|\right) \tag{24}$$

$$\dot{\hat{k}}_1(t) = p_1\left(-\epsilon_1\hat{k}_1(t) + \left\|\tilde{S}(t)\right\|\|e_v(t)\|\right) \tag{25}$$

$$\dot{\hat{k}}_2(t) = p_2\left(-\epsilon_2\hat{k}_2(t) + \left\|\tilde{S}(t)\right\|\|\omega(t)\|\right) \tag{26}$$

where p_0, p_1, p_2, ϵ, ϵ_0, ϵ_1, and ϵ_2 are the design parameters and $\hat{c}$, $\hat{k}_1$, $\hat{k}_2$, and $\hat{\rho}$ are used to estimate the bounds.

Proof: Consider the following Lyapunov function:

$$V_{sa} = \frac{1}{2}\left[\tilde{S}^T(t)J\tilde{S}(t) + \frac{1}{p_0}\tilde{c}^2 + \frac{1}{p_1}\tilde{k}_1^2 + \frac{1}{p_2}\tilde{k}_2^2\right] \tag{27}$$

where $\tilde{c} = c - \hat{c}(t)$ and $\tilde{k} = k - \hat{k}(t)$. Its time derivation is

$$\begin{aligned}\dot{V}_{sa} &= \tilde{S}^T(t)J\dot{\tilde{S}}(t) - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2\\
&= \tilde{S}^T(t)\Big[-(\omega + C\Omega_d)^\times J(\omega + C\Omega_d)\\
&\qquad + J(\omega^\times C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}Jk\left(e_4 I_3 + e_v^\times\right)\omega\Big]\\
&\quad + \tilde{S}^T(t)d + \tilde{S}^T(t)u_{Ada}(t) - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2\\
&= \tilde{S}^T(t)\Big[-(ke_v - \tilde{S} + C\Omega_d)^\times J(\omega + C\Omega_d)\\
&\qquad + J(\omega^\times C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}Jk\left(e_4 I_3 + e_v^\times\right)\omega\Big]\\
&\quad + \tilde{S}^T(t)d + \tilde{S}^T(t)u_{Ada}(t) - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2.\end{aligned}$$

Noting the property of operator $^\times$ that implies that $\tilde{S}^T\tilde{S}^\times = [0 \quad 0 \quad 0]$, we have

$$\begin{aligned}\dot{V}_{sa} &= \tilde{S}^T(t)\left[(ke_v - C\Omega_d)^\times J\omega + \frac{1}{2}Jk\left(e_4 I_3 + e_v^\times\right)\right]\omega\\
&\quad + \tilde{S}^T(t)J\omega^\times C\Omega_d\\
&\quad + \tilde{S}^T(t)\left[(ke_v - C\Omega_d)^\times JC\Omega_d - JC\dot{\Omega}_d\right]\\
&\quad + \tilde{S}^T(t)u_{Ada}(t) + \tilde{S}^T(t)d\\
&\quad - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2\\
&\leq \left[\left\|(ke_v - C\Omega_d)^\times\right\| + \frac{1}{2}k\left\|e_4 I_3 + e_v^\times\right\| + \|C\Omega_d\|\right]\\
&\quad \times \|J\|\,\|\tilde{S}\|\,\|\omega\| + \|(ke_v - C\Omega_d)^\times JC\Omega_d\\
&\quad - JC\dot{\Omega}_d\|\,\|\tilde{S}\| + \|\tilde{S}\|\,\|d\| + \tilde{S}^T(t)u_{Ada}(t)\\
&\quad - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2.\end{aligned}$$

Noting Assumptions 3.2 and 3.3, we obtain

$$\begin{aligned}\dot{V}_{sa} &\leq \left\|\tilde{S}(t)\right\|\left(c + k_1\|e_v(t)\| + k_2\|\omega(t)\|\right)\\
&\qquad + \tilde{S}^T(t)u_{Ada}(t) - \frac{1}{p_0}\tilde{c}\dot{\hat{c}} - \frac{1}{p_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{p_2}\tilde{k}_2\dot{\hat{k}}_2.\end{aligned}$$

If $\|\tilde{S}(t)\|\hat{\rho} > \epsilon$, with the control law defined in (21) and adaptation laws (23)–(26), we have

$$\begin{aligned}\dot{V}_{sa}(t) &= \tilde{S}^T(t)\left[-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right] - \tilde{S}^T(t)u_p\\
&\quad + \|S(t)\|\left(c + k_1\|e_v(t)\| + k_2\|\omega(t)\|\right)\\
&\quad - \tilde{c}\left(-\epsilon_0\hat{c} + \left\|\tilde{S}(t)\right\|\right) - \tilde{k}_1\left(-\epsilon_1\hat{k}_1 + \left\|\tilde{S}(t)\right\|\|e_v(t)\|\right)\\
&\quad - \tilde{k}_2\left(-\epsilon_2\hat{k}_2 + \left\|\tilde{S}(t)\right\|\|\omega(t)\|\right)\\
&\leq \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right)\\
&\quad - \left\|\tilde{S}(t)\right\|\left(\hat{c} + \hat{k}_1\|e_v(t)\| + \hat{k}_2\|\omega(t)\|\right)\\
&\quad + \left\|\tilde{S}(t)\right\|\left(c + k_1\|e_v(t)\| + k_2\|\omega(t)\|\right)\\
&\quad - \tilde{c}\left(-\epsilon_0\hat{c} + \left\|\tilde{S}(t)\right\|\right) - \tilde{k}_1\left(-\epsilon_1\hat{k}_1 + \left\|\tilde{S}(t)\right\|\|e_v(t)\|\right)\\
&\quad - \tilde{k}_2\left(-\epsilon_2\hat{k}_2 + \left\|\tilde{S}(t)\right\|\|\omega(t)\|\right)\\
&= \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right) + \epsilon_0\tilde{c}\hat{c} + \epsilon_1\tilde{k}_1\hat{k}_1 + \epsilon_2\tilde{k}_2\hat{k}_2\\
&= \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right) - \epsilon_0\left(\hat{c} - \frac{1}{2}c\right)^2\\
&\quad - \epsilon_1\left(\hat{k}_1 - \frac{1}{2}k_1\right)^2 - \epsilon_2\left(\hat{k}_2 - \frac{1}{2}k_2\right)^2\\
&\quad + \frac{1}{4}\left(\epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2\right)\\
&\leq -\Sigma_{i=1}^3\left(\tau_i\tilde{S}_i^2 + \sigma_i|\tilde{S}_i|\right)\\
&\quad + \frac{1}{4}\left(\epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2\right).\end{aligned} \tag{28}$$

Clearly, $\dot{V}_{sa}(t) < 0$ if $\|\tilde{S}\| > \sqrt{\delta_1/4\tau_{\min}}$ or $|\tilde{S}_i| > (\delta_1/4\sigma_{\min})$, where $\delta_1 = \epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2$, $\tau_{\min} = \min(\tau_i)$, and $\sigma_{\min} = \min(\sigma_i)$. The decrease of $V_{sa}(t)$ eventually drives the trajectories of the closed-loop system into $\|\tilde{S}\| \leq \sqrt{\delta_1/4\tau_{\min}}$ and $|\tilde{S}_i| \leq (\delta_1/4\sigma_{\min})$. Therefore, the trajectories of the closed-loop system are bounded ultimately as

$$\lim_{t\to\infty}\tilde{S}(t) \in \left(\|\tilde{S}\| \leq \sqrt{\frac{\delta_1}{4\tau_{\min}}}\right) \cap \left(|\tilde{S}_i| \leq \frac{\delta_1}{4\sigma_{\min}}\right) \tag{29}$$

which is a small set containing the origin of the closed-loop system.

If $\|\tilde{S}(t)\|\hat{\rho} \leq \epsilon$, with the control law (21) and adaptation laws (23)–(26), we obtain

$$\begin{aligned}\dot{V}_{sa}(t) &\leq \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right) - \frac{\left\|\tilde{S}(t)\right\|^2}{\epsilon}\hat{\rho}^2\\
&\quad + \left\|\tilde{S}(t)\right\|\left(c + k_1\|e_v(t)\| + k_2\|\omega(t)\|\right)\\
&\quad - \tilde{c}\left(-\epsilon_0\hat{c} + \left\|\tilde{S}(t)\right\|\right) - \tilde{k}_1\left(-\epsilon_1\hat{k}_1 + \left\|\tilde{S}(t)\right\|\|e_v(t)\|\right)\\
&\quad - \tilde{k}_2\left(-\epsilon_2\hat{k}_2 + \left\|\tilde{S}(t)\right\|\|\omega(t)\|\right)\\
&= \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right) - \frac{\left\|\tilde{S}(t)\right\|^2}{\epsilon}\hat{\rho}^2 + \left\|\tilde{S}(t)\right\|\hat{\rho}\\
&\quad + \epsilon_0\tilde{c}\hat{c} + \epsilon_1\tilde{k}_1\hat{k}_1 + \epsilon_2\tilde{k}_2\hat{k}_2\\
&= \tilde{S}^T(t)\left(-\tau\tilde{S} - \sigma\,\mathrm{sgn}(\tilde{S})\right) - \left(\frac{\left\|\tilde{S}(t)\right\|}{\sqrt{\epsilon}}\hat{\rho} - \frac{\sqrt{\epsilon}}{2}\right)^2\\
&\quad + \frac{\epsilon}{4} - \epsilon_0\left(\hat{c} - \frac{1}{2}c\right)^2 - \epsilon_1\left(\hat{k}_1 - \frac{1}{2}k_1\right)^2\\
&\quad - \epsilon_2\left(\hat{k}_2 - \frac{1}{2}k_2\right)^2 + \frac{1}{4}\left(\epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2\right)\\
&\leq -\Sigma_{i=1}^3\left(\tau_i\tilde{S}_i^2 + \sigma_i|\tilde{S}_i|\right) + \frac{\epsilon}{4}\\
&\quad + \frac{1}{4}\left(\epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2\right).\end{aligned} \tag{30}$$

By similar analysis for (28), the trajectory is ultimately bounded in the region

$$\lim_{t\to\infty} \tilde{S}(t) \in \left(\|\tilde{S}\| \leq \sqrt{\frac{\delta_2}{4\tau_{\min}}} \right) \cap \left(\|\tilde{S}_i\| \leq \frac{\delta_2}{4\sigma_{\min}} \right) \quad (31)$$

where $\delta_2 = \epsilon + \epsilon_0 c^2 + \epsilon_1 k_1^2 + \epsilon_2 k_2^2$.

In order to guarantee the bounded motion around the sliding surface, the positive parameters τ_i and σ_i are chosen large enough such that $\dot{V}_{sa} < 0$ when $V_{sa}(t)$ is out of a certain bounded region which contains an equilibrium point. Of course, the design parameters ϵ, ϵ_0, ϵ_1, and ϵ_2 determine the band of the bounded region, and we can choose ϵ, ϵ_0, ϵ_1, and ϵ_2 to be small enough in order to guarantee the motion along the sliding surface nearly.

It can be concluded now from (29) and (31) that all signals are UUB. In order to alleviate the undesirable chattering when system trajectories cross the switching surface, we adopt the so-called boundary layer method [20] in controller design. The continuous positive scalar-valued function $\hat{\rho}$ is estimated by a smoothed SMC control law taking account of the boundary layer effect, which is the part u_p. The benefits of this kind of smooth techniques have been stated in [20], which offers a continuous approximation to the discontinuous SMC law inside the boundary layer and guarantees the output tracking error within any neighborhood of the sliding surface. However, asymptotic stability is lost, and we cannot analyze the stability of the dynamics of the sliding mode that is restricted on the sliding surface. It can only guarantee the bounded motion around the sliding surface.

Remark 3.2: The design parameters ϵ_0, ϵ_1, and ϵ_2 determine the band of the bounded region, and we can choose ϵ_0, ϵ_1, and ϵ_2 to be small enough in order to guarantee the motion along the sliding surface nearly. However, a compromise is made between the band of the bounded region and the convergence speed of the estimated bounds $\hat{c}$, $\hat{k}_1$, and $\hat{k}_2$, which are also related to the parameters ϵ_0, ϵ_1, and ϵ_2. Too small ϵ_0, ϵ_1, and ϵ_2 will lead to a very low convergence rate of the estimated bounds $\hat{c}$, $\hat{k}_1$ and $\hat{k}_2$. Thus, the parameters ϵ_0, ϵ_1, and ϵ_2 cannot be selected too small.

Remark 3.3: The control law (21) does not rely on inertia matrix J, so the proposed controller can be applied to the spacecraft system with large uncertainty in inertia matrix or even unknown inertia matrix.

IV. SMC Design With ESO

In this section, we will propose another practical control design for the attitude control problem. Due to the great advances in nonlinear control theory, the observer-based controller has become one of the most commonly used schemes in industrial applications. The ESO mentioned in [15] and [21] has a high efficiency in accomplishing the nonlinear dynamic estimation [22]. Therefore, for solving the attitude tracking problem with inertial uncertainty and disturbance existing in the spacecraft system, a sliding-mode controller can be designed to force the state variables to converge to the reference state by compensating the total disturbances via the ESO.

Now, consider system (15) with the inertia matrix containing parameter uncertainty in the form of $J = J_0 + \triangle J$, where J_0 is the known constant matrix that is chosen nonsingular and $\triangle J$ denotes the unmatched uncertainty. Thus, the dynamic equation (15) can be rewritten as

$$\begin{aligned}(J_0+\Delta J)\dot{x} = &-(\omega+C\Omega_d)^{\times}(J_0+\Delta J)(\omega+C\Omega_d)\\ &+(J_0+\Delta J)(\omega^{\times}C\Omega_d-C\dot{\Omega}_d)\\ &+\frac{1}{2}(J_0+\Delta J)K\left(e_4I_3+e_v^{\times}\right)\omega+u+d. \end{aligned} \quad (32)$$

Note that $(J_0 + \Delta J)^{-1}$ can be expressed as

$$(J_0 + \Delta J)^{-1} = J_0^{-1} + \Delta\tilde{J} \quad (33)$$

where $\Delta\tilde{J}$ is the uncertainty as well. Thus, with some simple algebraic manipulations to (32), we have

$$\dot{x} = F + G + J_0^{-1}u + \bar{d} \quad (34)$$

where

$$\begin{aligned}F = J_0^{-1}\Big[&-(\omega + C\Omega_d)^{\times}J_0(\omega + C\Omega_d)\\ &+ J_0(\omega^{\times}C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}J_0K\left(e_4I_3 + e_v^{\times}\right)\omega\Big]\end{aligned} \quad (35)$$

$$\begin{aligned}G = J_0^{-1}\Big[&-(\omega + C\Omega_d)^{\times}\Delta J(\omega + C\Omega_d)\\ &+ \Delta J(\omega^{\times}C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}\Delta JK\left(e_4I_3 + e_v^{\times}\right)\omega\Big]\\ + \Delta\tilde{J}\Big[&-(\omega + C\Omega_d)^{\times}J(\omega + C\Omega_d)\\ &+ J(\omega^{\times}C\Omega_d - C\dot{\Omega}_d) + \frac{1}{2}JK(e_4I_3 + e_v^{\times})\omega\Big]\\ + \Delta\tilde{J}u&\end{aligned} \quad (36)$$

$$\bar{d} = J_0^{-1}d + \Delta\tilde{J}d. \quad (37)$$

System (34) contains both parameter uncertainty G and external disturbance $\bar{d}$. In this section, the uncertainty and disturbance are lumped together as the total disturbances, and thus, the further simplified model form is obtained

$$\dot{x} = F + B_0u + \tilde{d} \quad (38)$$

where $\tilde{d} = G + \bar{d}$ and $B_0 = J_0^{-1}$.

Remark 4.1: It can be seen that if there exists a dynamic state feedback control law such that the solution of the closed-loop system (38) is guaranteed $\lim_{t\to\infty} x(t) = 0$, the state of the closed-loop system (15) can be obtained $\lim_{t\to\infty} x(t) = 0$, which, by Lemma 2.1, leads to the closed-loop system (11) and (12) satisfying $\lim_{t\to\infty} e_v(t) = 0$ and $\lim_{t\to\infty} \omega(t) = 0$. Then, the tracking objective (6) and (7) can be achieved. Therefore, the attitude tracking problem is solved by the stabilization problem of the nonlinear system (38), which contains both uncertainty and external disturbance.

A. SMC

As usual, in the sliding-mode technique, the control forces the system evolution on a certain surface, which guarantees the achievement of the control requirements. A natural choice is the sliding surface

$$S = C_2 x \tag{39}$$

where $S = [S_1, S_2, S_3]^{\mathrm{T}} \in R^3$ and $C_2 \in R^{3\times 3}$. Without losing generality, we assume that matrix C_2 is of full rank and matrix $C_2 B_0$ is nonsingular.

With this choice and reaching law (17), the derivative of $S(t)$ can be also rewritten as follows:

$$\begin{aligned} \dot{S} &= C_2\dot{x} \\ &= C_2(F + B_0 u + \tilde{d}) \\ &= -\tau S - \sigma\,\mathrm{sgn}(S). \end{aligned} \tag{40}$$

Solving for $u(t)$ in (40) gives the control law

$$u(t) = (C_2 B_0)^{-1}(-\tau S - \sigma \mathrm{sgn}(S) - C_2 F - C_2\tilde{d}). \tag{41}$$

Note that the control law (41) consists of the total disturbances $\tilde{d}(t)$, which are not completely known to us, and it could not be applied to practical systems. In order to obtain the disturbance, we will introduce the ESO to estimate it.

Remark 4.2: The system (38) can be made to converge using standard SMC methods [17]. However, in order to suppress the uncertainty and disturbance, the control input may lead to violent chattering, which is normally undesirable in practice. Hence, the ESO can be adopted here to make the total disturbance estimated and compensated in the control input, which implies the decrease of the chattering and control power.

B. SMC With ESO

The ESO views the system model uncertainties and external disturbances as the extended state to be estimated. Here, the observer can be designed for estimating the total disturbances $\tilde{d}(t)$ existing in the control law (41). We add an extended state x_2 as the disturbances $\tilde{d}(t)$, and the system (38) can be written as

$$\begin{aligned} \dot{x} &= F + B_0 u + x_2 \\ \dot{x}_2 &= g(t) \end{aligned} \tag{42}$$

where the function $g(t)$ is the derivative of the disturbances $\tilde{d}(t)$, which is uncertain as well. Then the second-order ESO for systems (42) is proposed in the following:

$$\begin{aligned} E_1 &= Z_1 - x \\ \dot{Z}_1 &= Z_2 + F - \beta_{01}E_1 + B_0 u \\ \dot{Z}_2 &= -\beta_{02} fal(E_1, \alpha_1, \delta) \end{aligned} \tag{43}$$

where E_1 is the estimation error of the ESO, Z_1 and Z_2 are the observer outputs, and β_{01} and β_{02} are the observer gains. The function $fal(\cdot)$ is defined as

$$fal(E_1, \alpha_1, \delta) = \begin{bmatrix} fal_1(E_1, \alpha_1, \delta) \\ fal_2(E_1, \alpha_1, \delta) \\ fal_3(E_1, \alpha_1, \delta) \end{bmatrix} \tag{44}$$

where

$$fal_i(E_1, \alpha_1, \delta) = \begin{cases} |E_{1i}|^{\alpha_1}\mathrm{sgn}(E_{1i}), & |E_{1i}| > \delta \\ E_{1i}/\delta^{1-\alpha_1}, & \text{otherwise} \end{cases} \tag{45}$$

with E_{1i} being the ith component of vector E_1, $0 < \alpha_1 < 1$, $\delta > 0$. For appropriate values of β_{01}, β_{02}, α_1, and δ, the observer output Z_2 approaches to $\tilde{d}$, and Z_1 approaches to x.

With the disturbances $\tilde{d}(t)$ estimated by the ESO, the control law (41) is modified as

$$u_{\mathrm{ESO}}(t) = (C_2 B_0)^{-1}\left(-\tau S - \sigma \mathrm{sgn}(S) - C_2 F - C_2 Z_2\right). \tag{46}$$

Remark 4.3: Note that the third formula Z_2 in (43) is the most important. It shows that Z_2 can estimate (or track) the total action of the uncertain models and the external disturbances or the real-time action of the system disturbances. As Z_2 is the estimation for the total action of the unknown disturbances, in the feedback, Z_2 is used to compensate for the disturbances.

Remark 4.4: As soon as the values of the state variables q and Ω are measured according to Assumption 2.1, $S(t)$ is computed by (39), (13), (8), and (10), F is acquired by (35), and $Z_2(t)$ is obtained by (43), the modified control law $u_{ESO}(t)$ can be calculated ultimately.

C. Stability Analysis of Closed-Loop Dynamics

In this section, the stability of the closed-loop system (38) can be established by the following theorem.

Theorem 2: Considering plant (38), control law (46), and ESO (43), there exist observer gains β_{01}, β_{02}, α_1, and δ such that the estimated states Z_1 and Z_2 converge into a residual set of the actual states x and $\tilde{d}$, respectively, and the trajectory of the closed-loop system can be driven onto the sliding surface in finite time and converges into a neighborhood of the origin.

Proof: In order to examine the stability of the closed-loop system, one must develop an expression for the observer error dynamics. Defining the observer error $E_1 = Z_1 - x$, $E_2 = Z_2 - x_2 = Z_2 - \tilde{d}$, the observer error dynamics are expressed as

$$\begin{cases} \dot{E}_1 = E_2 - \beta_{01}E_1 \\ \dot{E}_2 = -g(t) - \beta_{02} fal(E_1, \alpha_1, \delta). \end{cases}$$

The stability of the ESO has been obtained by selecting the appropriate parameters β_{01} and β_{02} [23], [24]. When the observer is stable, the derivative of vector $\dot{E} = [\dot{E}_1 \quad \dot{E}_2]^{\mathrm{T}} = 0$, and then, the errors of estimation can be written as

$$\begin{cases} E_1 = -fal^{-1}\left(g(t)/\beta_{02}\right) \\ E_2 = -\beta_{01} fal^{-1}\left(g(t)/\beta_{02}\right). \end{cases}$$

Note that, in (45), if $|E_{1i}| > \delta$, the errors of estimation are

$$\begin{cases} |E_{1i}| = |g_i(t)/\beta_{02}|^{1/\alpha_1} \\ |E_{2i}| = \beta_{01}\, |g_i(t)/\beta_{02}|^{1/\alpha_1} \end{cases} \tag{47}$$

and if $|E_{1i}| \le \delta$, the errors of estimation can be expressed as

$$\begin{cases} |E_{1i}| = \left|g_i(t)\delta^{1-\alpha_1}\right| / \beta_{02} \\ |E_{2i}| = \beta_{01}\, \left|g_i(t)\delta^{1-\alpha_1}\right| / \beta_{02} \end{cases} \tag{48}$$

where $g_i(t)$ is the ith component of vector $g(t)$. From (47) and (48), it is clear that the estimation errors are determined by the parameters β_{01}, β_{02}, α_1, and δ. The fundamental selection of the parameters can be chosen as $\beta_{01} > 0$, $\beta_{02} > 0$, $0 < \alpha_1 < 1$, and $\delta > 0$. Furthermore, an appropriate β_{02} can be selected to be large enough such that $|g_i(t)/\beta_{02}|$ is small enough, although $g(t)$ is unknown to us. Of course, β_{01} should be small enough to make the estimation error E_2 as small as possible. In (47), the smaller the α_1 is, the smaller the steady estimation errors will be. Thus, via tuning these parameters properly, the estimation errors E_1 and E_2 can be limited to be small enough, which means that Z_1 and Z_2 converge into a neighborhood of the actual states x and $\tilde{d}$, respectively.

Having shown that the observer error converges into the residual set of zero, it remains to show that the system states converge to the origin in finite time. Considering the Lyapunov function candidate with the linear sliding surface given by (39) and the observer obtained by (43), we obtain

$$V_{so} = \frac{1}{2} S^T S. \tag{49}$$

Taking the derivative of (49) and making use of the control law (46) give

$$\begin{aligned} \dot{V}_{so} &= S^T \dot{S} \\ &= S^T C_2 (F + B_0 u_{ESO} + \tilde{d}) \\ &= S^T \left(-\tau S - \sigma \operatorname{sgn}(S) + C_2 \tilde{d} - C_2 Z_2 \right) \\ &= -\Sigma_{i=1}^3 \left(\tau_i S_i^2 + \sigma_i |S_i| \right) + S^T C_2 (\tilde{d} - Z_2) \\ &= -\Sigma_{i=1}^3 \left(\tau_i S_i^2 + \sigma_i |S_i| \right) - S^T C_2 E_2. \end{aligned} \tag{50}$$

It has been shown that $Z_2(t)$ converges into a residual set of $\tilde{d}(t)$, which means E_2 converges into a residual set of zero. By similar analysis for (28) and (30), appropriate τ_i and σ_i can be selected such that $\dot{V}_{so} < 0$ when $V_{so}(t)$ is out of a certain bounded region that contains a equilibrium point. Thus, it can be concluded that with the bounded motion around the sliding surface, the state x of the close-loop system (38) will converge into a neighborhood of the origin, which implies that state x is UUB.

Remark 4.5: Since the observer cannot track the signal completely in any practical systems, asymptotic stability is lost, and it can only guarantee the bounded motion about the sliding surface. Therefore, we cannot analyze the stability of the dynamics of the sliding mode that is restricted on the sliding surface. In (50), the boundary layer of the sliding surface is affected by the estimation error of the ESO. Thus, parameter selection of the ESO is more important because it not only determines the performance of the ESO observing the total disturbances but also impacts the behavior of the sliding surface. More information about parameter selection for the ESO can been seen in [25] and [26].

V. Simulation Results

A. SMC With Adaptive Law

In order to demonstrate the effectiveness of the proposed adaptive control schemes (21), numerical simulations have been performed and presented in this section. Consider the spacecraft model (1) and (2) with the nominal inertia matrix [11]

$$J_0 = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \tag{51}$$

and parameter uncertainties

$$\Delta J = \operatorname{diag}\left[\sin(0.1t), 2\sin(0.2t), 3\sin(0.3t)\right] \text{ kg} \cdot \text{m}^2. \tag{52}$$

The external disturbances are described as

$$d(t) = \begin{bmatrix} 0.1\sin(0.1t) \\ 0.2\sin(0.2t) \\ 0.3\sin(0.2t) \end{bmatrix} \text{ N} \cdot \text{m}. \tag{53}$$

In this numerical simulation, we suppose that the desired angular velocity is given by

$$\Omega_d(t) = 0.05 \begin{bmatrix} \sin\left(\frac{\pi t}{100}\right) \\ \sin\left(\frac{2\pi t}{100}\right) \\ \sin\left(\frac{3\pi t}{100}\right) \end{bmatrix} \text{ rad/s} \tag{54}$$

and the corresponding desired unit quaternion to be tracked is generated by (5).

The initial attitude orientation of the unit quaternion is $q(0) = [0.3, -0.2, -0.3, 0.8832]^T$, and the initial target unit quaternion is $q_d(0) = [0, 0, 0, 1]^T$. The initial value of the angular velocity is $\Omega(0) = [0, 0, 0]^{\mathrm{T}}$ rad/s.

The attitude quaternion tracking errors and angular velocity tracking errors are shown in Figs. 1 and 2, respectively, which show that the adaptive sliding-mode controller guaranteed attitude tracking in spite of the disturbance and uncertainty.

The parameters τ, σ, and k can be used to regulate the convergence rate of the state trajectory and tuned to reduce the chattering on the sliding surface. Fig. 3 shows the simulation results with $\tau = 10I_3$, $\sigma = 0.001I_3$, and $k = 2$. Obviously, the sliding mode is stable, and the trajectories of the system tend to a residual set of the origin in spite of the uncertainty and disturbance. Fig. 4 shows the input control signal. It is clear that the undesired chattering in the control input is reduced effectively due to the estimation of the uncertainty and disturbance when system state trajectories cross the sliding surface.

The design parameters ϵ, ϵ_0, ϵ_1, p_1, and p_2 are related to the convergence rate about the estimation of the bound parameters $\hat{c}$, $\hat{k}_1$, and $\hat{k}_2$, and the values are chosen as $\epsilon = 0.1$, $\epsilon_0 = 0.1$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $p_0 = 1$, $p_1 = 1$, and $p_2 = 1$. The corresponding estimated parameters are shown in Figs. 5 and 6.

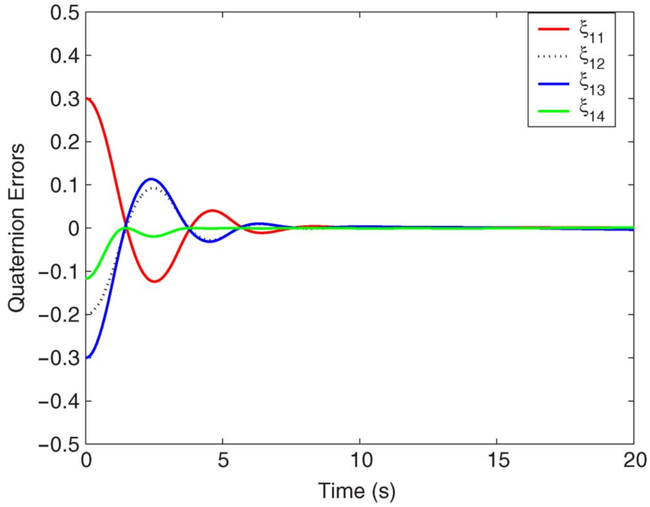

Fig. 1. Attitude quaternion tracking errors.

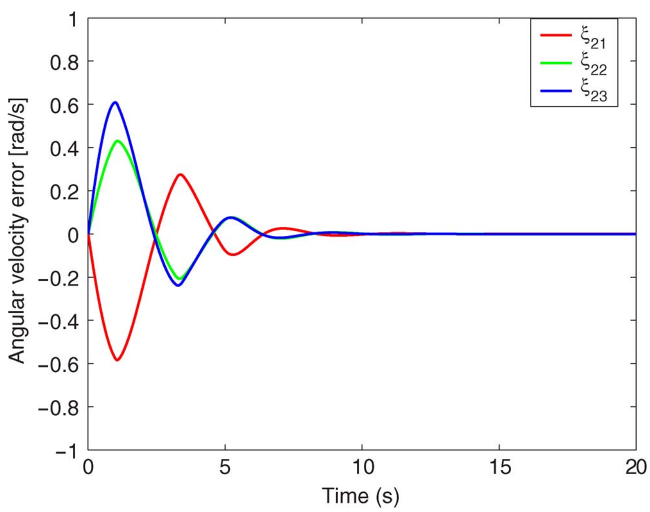

Fig. 2. Angular velocity tracking errors.

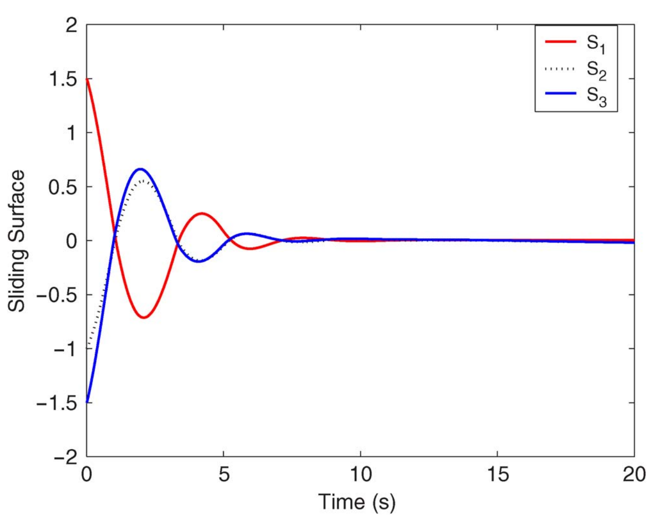

Fig. 3. Sliding surface.

It is clear that the parameters $\hat{c}$, $\hat{k}_1$, and $\hat{k}_2$ converge to constants with the initial conditions $\hat{c}(0) = 1$, $\hat{k}_1(0) = 1$, and $\hat{k}_2(0) = 1$, respectively. The estimation errors $\tilde{c}$, $\tilde{k}_1$, and $\tilde{k}_2$ do not

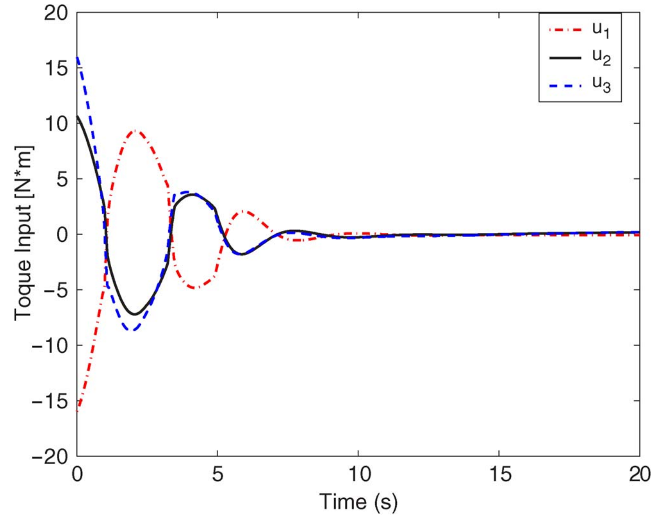

Fig. 4. Control input.

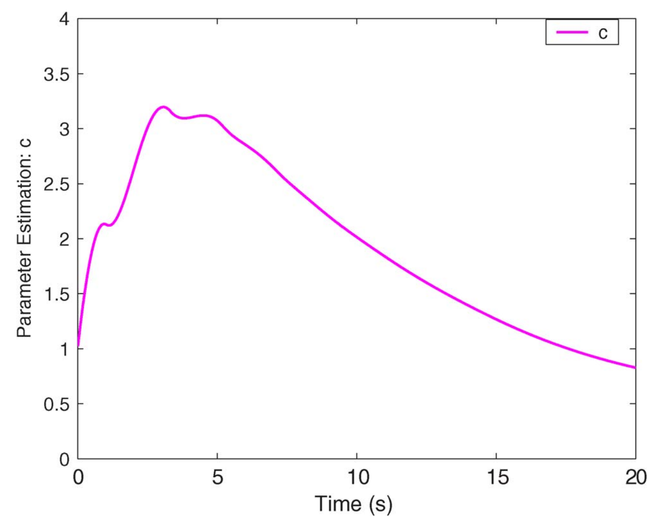

Fig. 5. Estimated parameter $\hat{c}$.

necessarily converge to zero since the derivation of (27) is not strictly negative.

In order to demonstrate that the control strategy can also work well when the desired angular velocity proceeds with abrupt changes, we suppose that the target angular velocity components are all square waves in the form of

$$\Omega_d(t) = \begin{bmatrix} 0.05 \\ 0.03 \\ 0.02 \end{bmatrix} \text{ rad/s}, \qquad 2kT < t < (2k+1)T \tag{55}$$

$$\Omega_d(t) = -\begin{bmatrix} 0.05 \\ 0.03 \\ 0.02 \end{bmatrix} \text{ rad/s}, \qquad (2k+1)T < t < (2k+2)T \tag{56}$$

where $T = 10$ s is the switch period. The parameters needed for controller, system initial states, inertia matrix, and external disturbances are all the same, as mentioned previously. However, the desired $\dot{\Omega}_d$ in this paper is assumed to bounded, and the $\dot{\Omega}_d$ does not exist in the switch point of square waves.

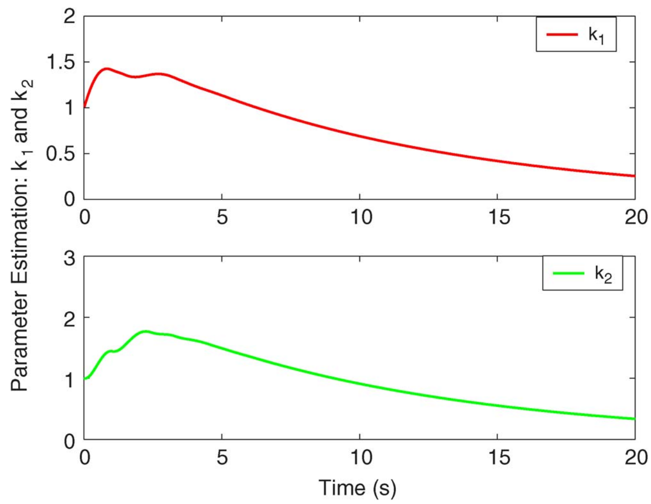

Fig. 6. Estimated parameters $\hat{k}_1$ and $\hat{k}_2$.

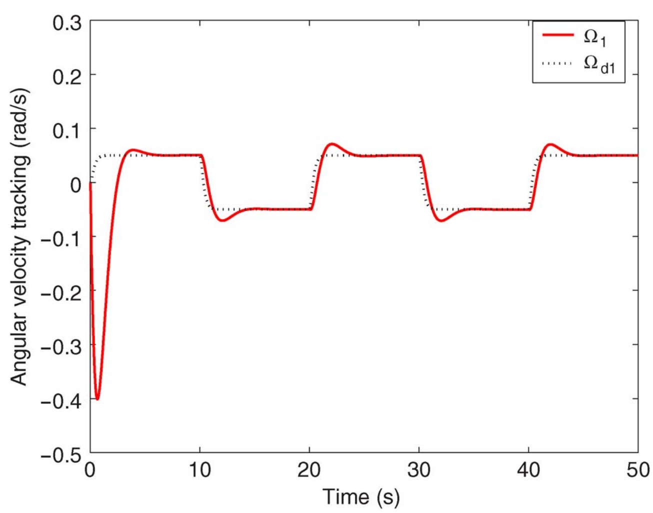

Fig. 7. Angular velocity tracking.

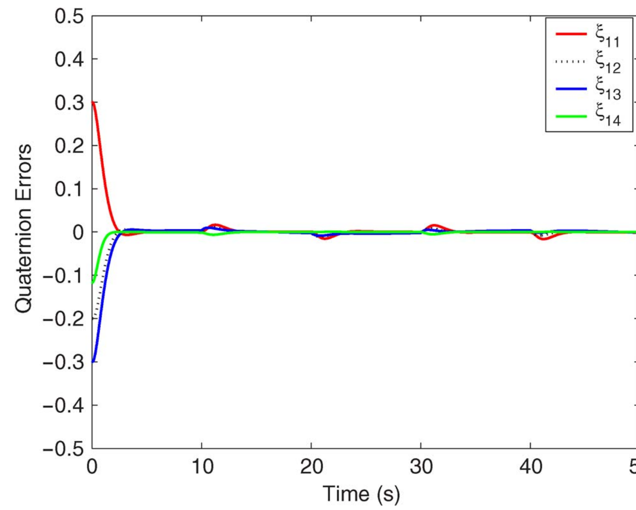

Fig. 8. Quaternion tracking errors.

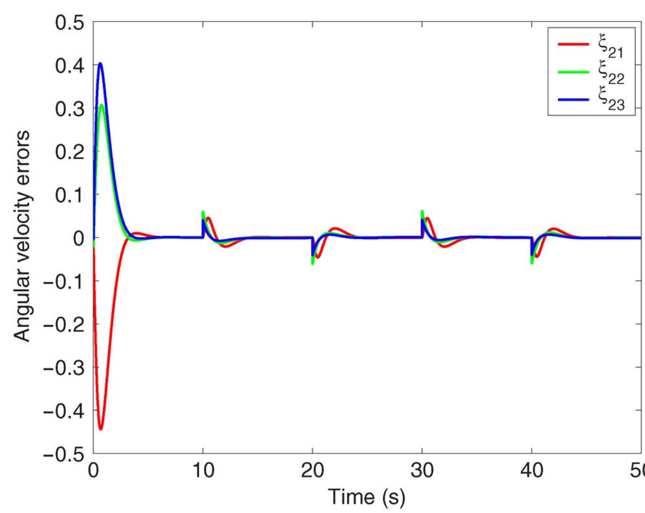

Fig. 9. Angular velocity tracking errors.

Here, a tracking differentiator (TD) [26] can be introduced as an alternative to obtain the derivative of Ω_d. The most important role of TD is its ability to obtain the derivative of a noisy signal with a good signal-to-noise ratio. It is well known that a pure differentiator is not physically implementable. The error signal is often not differentiable in practice because of the noises in the feedback and the discontinuities in the reference signal. However, a discrete-time realization of TD can improve the numerical properties and can avoid high-frequency oscillations. Further explanations of this can be found in [15]. Via the design of TD for Ω_d, the derivative $\dot{\Omega}_d$ can be obtained, and then, we suppose that the target angular velocity components are approximate to square waves, which are differentiable everywhere under the help of TD.

The actual angular velocity component tracking the reference angular velocity component that is approximate to the square wave is shown in Fig. 7. It is obvious that the angular velocity tracks the reference trajectory effectively, even though the target angular velocity changes abruptly in some points. The quaternion errors and angular velocity errors are shown in Figs. 8 and 9, and the tracking errors can converge into a neighborhood of zero except those switch points. Also, the estimation of adaptive parameters is shown in Fig. 10. Some shocks appear in the estimated parameter trajectory due to the drastic changes of the desired angular velocity.

B. SMC With ESO

The attitude tracking problem of a spacecraft is simulated in this section to demonstrate the performance of the sliding-mode controller (46) with an ESO (43). Having shown that, for proper choices of the gains given in Section IV, such as τ, σ, β_{01}, and β_{02}, output tracking of the reference trajectory will be achieved. Consider the spacecraft model (1) and (2) with the same inertia matrix (51) and (52), disturbances (53), and system initial conditions mentioned earlier. We suppose that the target angular velocity to be tracked is given by (54).

The attitude quaternion tracking errors and angular velocity tracking errors are shown in Figs. 11 and 12, which show that the sliding-mode controller achieves good performance on attitude tracking with rapid convergence.

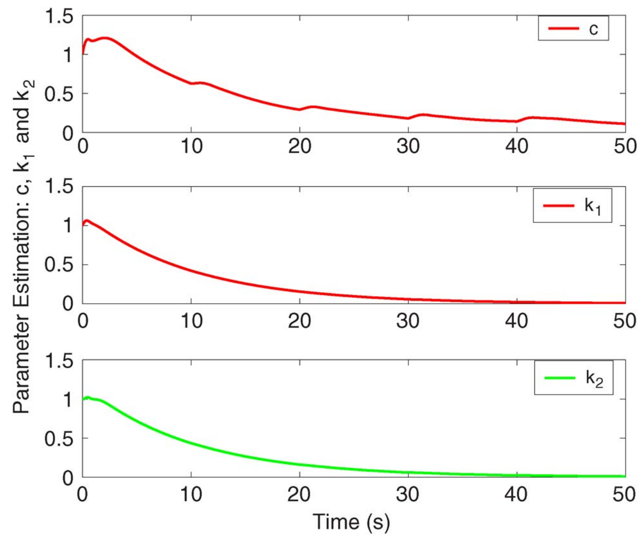

Fig. 10. Estimated parameters $\hat{c}$, $\hat{k}_1$, and $\hat{k}_2$.

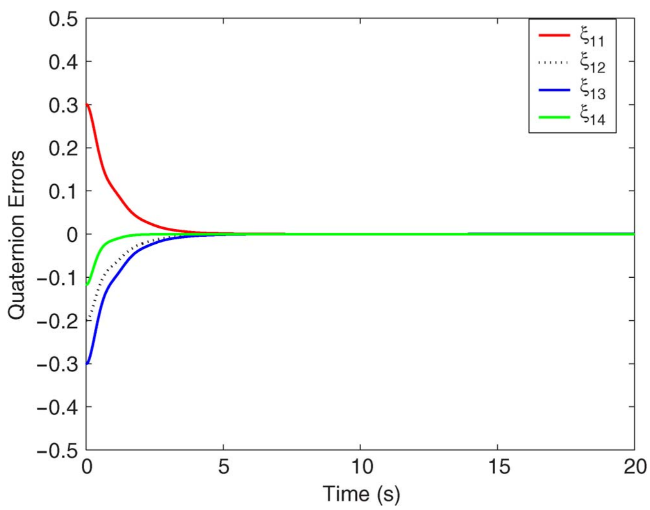

Fig. 11. Attitude quaternion tracking errors.

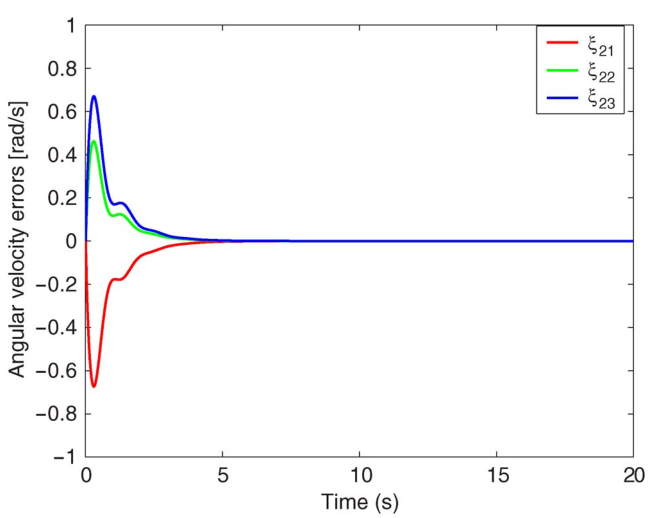

Fig. 12. Angular velocity tracking errors.

The parameters τ and σ are selected the same as aforementioned with $\tau = 10I_3$ and $\sigma = 0.001I_3$. The sliding surface is shown in Fig. 13 with $K = 2I_3$ and $C_2 = I_3$. Obviously, the sliding mode is stable, and the trajectories of the system tend to a residual set of the origin in spite of the uncertainties and disturbances. Fig. 14 shows the input control signal. It is clear that when state trajectories cross the sliding surface, the undesired chattering can also be reduced effectively with the estimation of the uncertainty and disturbance by the ESO.

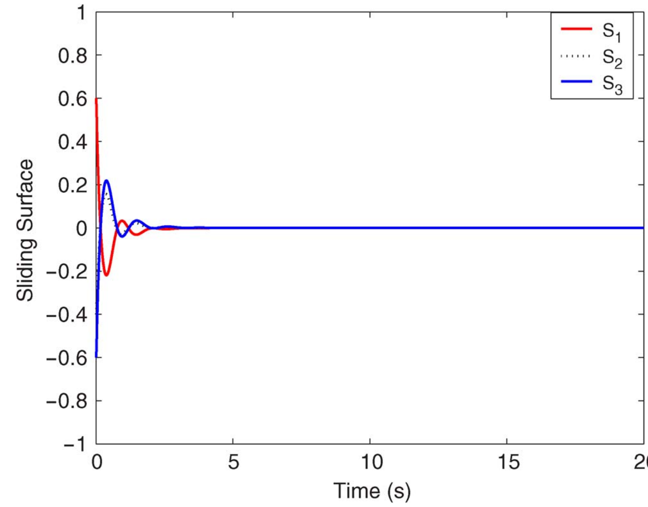

Fig. 13. Sliding surface.

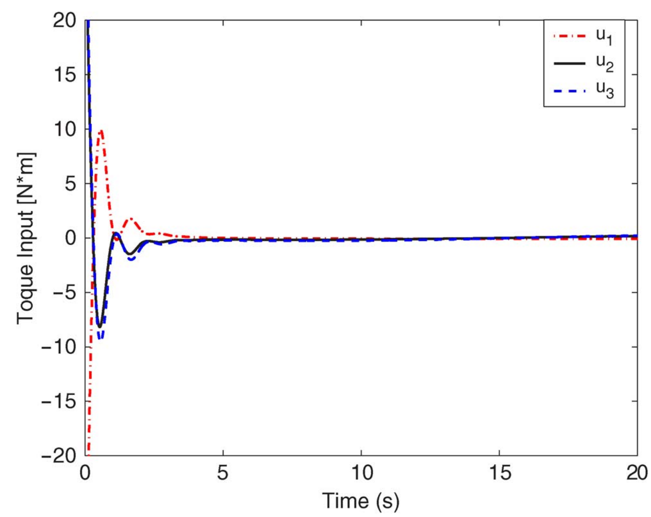

Fig. 14. Control input.

The performances of the ESO observing the disturbances $\tilde{d}(t)$ are shown in Fig. 15. By selecting appropriate values of $\beta_{01} = 5, \beta_{02} = 30, \alpha_1 = 0.25$, and $\delta = 0.2$, each component of the estimated states $Z_{2i}(t)$ converges to the actual disturbance component $\tilde{d}_i(t)$ in finite time.

The sliding-mode controller tracking approximate square waves are shown in Figs. 16–19. The desired angular velocity is the same as (55) and (56), and the design parameters are all the same as mentioned previously.

Fig. 16 shows the actual angular velocity component and the reference angular velocity component that is approximate to the square wave. It is clear that, over time, the angular velocity tracks the reference trajectory. Despite the fact that disturbance and uncertainty are applied to the plant, it is able to reject the disturbance and track the desired trajectory effectively.

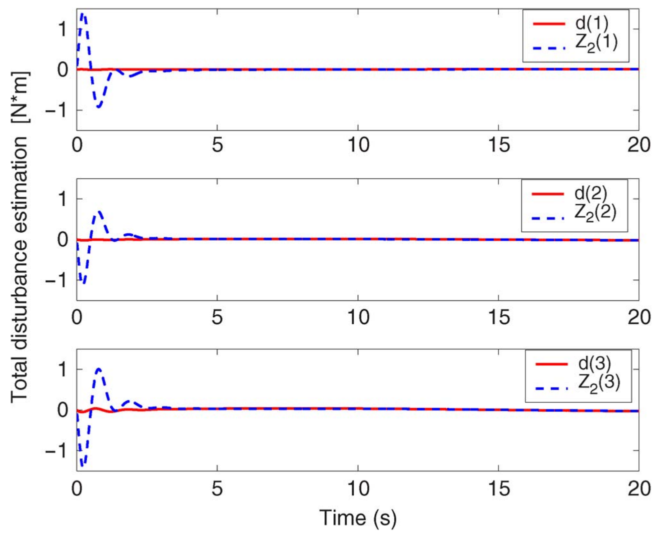

Fig. 15. Estimation of disturbances via ESO.

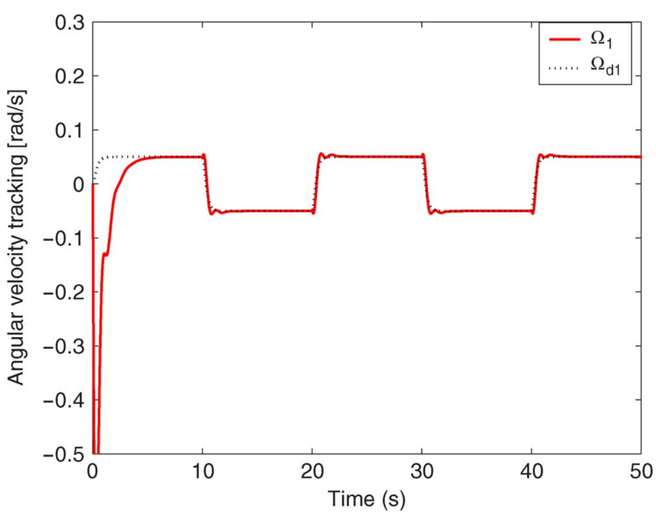

Fig. 16. Angular velocity tracking.

The quaternion errors and angular velocity errors are shown in Figs. 17 and 18, respectively, and the tracking errors can converge into a neighborhood of zero except those switch points. Also, the performance of the ESO is shown in Fig. 19. The observer can estimate the total disturbance effectively even if there exist some small shocks due to the drastic changes of the desired angular velocity.

Based on the previous simulations, we can conclude that the parameter τ in (21) and (46) is very important. It is one of the parameters determining the bounded layer when the state trajectories of (38) and (11) and (12) evolve around the sliding surface, and it also guarantees the convergence precision of the system state. This is clear that, in (50), the system state cannot converge to zero, but a larger τ will force the state to be small enough, even though there exist the estimation errors of the ESO. Similar in (28) and (30), a larger τ can be used to ensure that the system state is small enough, even though the adaptive parameters cannot be vanished in the derivative of the Lyapunov function. Thus, the attitude tracking accuracy

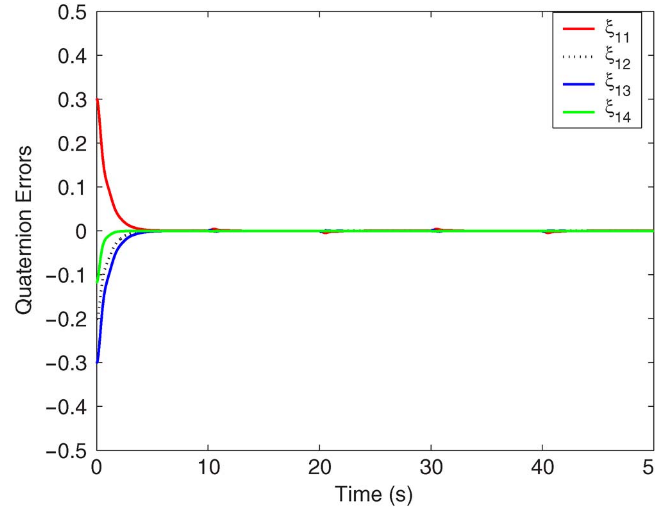

Fig. 17. Quaternion tracking errors.

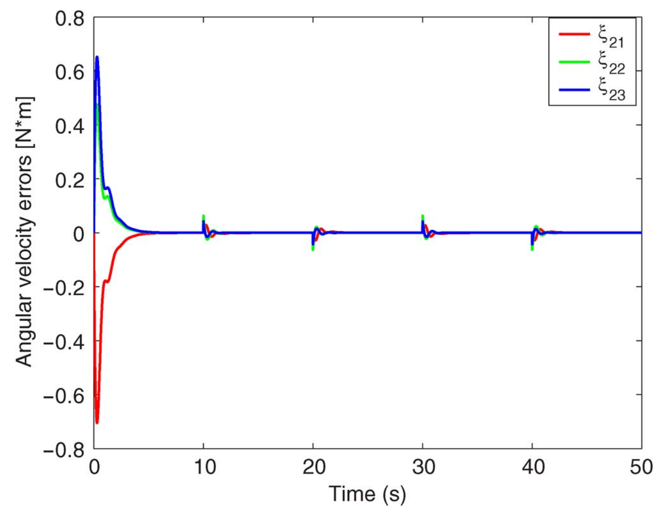

Fig. 18. Angular velocity tracking errors.

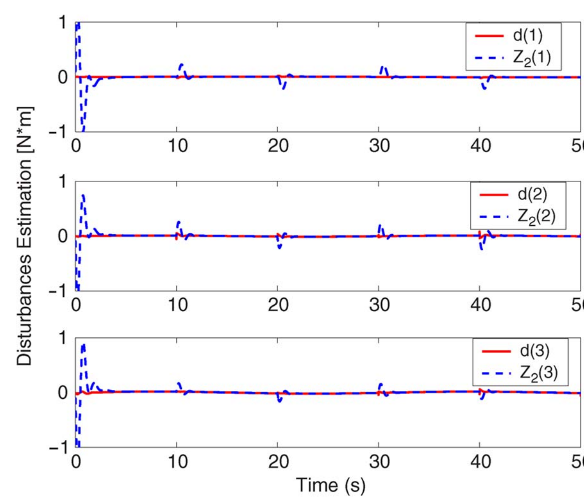

Fig. 19. Estimation of disturbances via ESO.

is determined in a great degree by the parameter τ, while the ESO and adaptive law play an auxiliary role in guaranteeing the tracking precision in the presence of external disturbance and inertial uncertainty. In this simulation, the larger parameter $\tau = 10I_3$ makes the two control strategies acquire nearly the same tracking accuracy in quaternion tracking (see Figs. 1 and 11) and angular velocity tracking (see Figs. 2 and 12) in spite of the presence of disturbance and inertial uncertainty. However, in practice, a compromise is made between the tracking accuracy and control input because a too big τ will require a very high control input, which is always bounded in reality. Thus, the parameter τ cannot be selected too large.

On the other hand, the parameter uncertainty and external disturbance are the main problems to be treated. The adaptive scheme and ESO are applied to deal with the uncertainty and disturbance, respectively. However, which method should be adopted depends on the properties of the uncertainty and disturbance. For disturbance satisfying the structure of (19), the adaptive method is prior to choose. For disturbance which is absolutely unknown, the ESO is the only selection. In addition, both control algorithms can deal with spacecraft systems with uncertainty in inertial matrix. In particular, the adaptive control law relies on no knowledge of inertia matrix, so the controller can achieve the attitude tracking with large uncertainty in inertia matrix or even unknown inertia matrix, which is superior than the controller with an ESO. The convergence speed of the sliding surface for the ESO is faster than that of the adaptive approach (see Figs. 3 and 13), which leads to the same effect on the convergence rate of quaternion errors and angular velocity errors (see Figs. 1, 2, 11, and 12). What is worth mentioning is that the convergence speed of the sliding surface is, in a great degree, dependent on the parameters K and k in (13) and (16), respectively. In this simulation, the parameters are chosen as $K = 2I_3$ and $k = 2$ to have the same impact on the sliding surface. The input power for the adaptive approach is lower than that for the ESO (see Figs. 4 and 14). This is reasonable because when the adaptive law is used, the structure of the disturbance is known, and the information that we possess about disturbance is more than that of the ESO, which is applied on the condition of absolutely unknown disturbance. Thus, we can get more precise estimation of the disturbance using the adaptive law than that using the ESO.

Remark 5.1: The system performance relies on the parameters τ and σ extremely. The system response can speed up apparently by choosing large τ and σ with the compromise such that the undesired chattering will be enhanced simultaneously in the control input by increasing the two parameters.

VI. Conclusion

In this paper, the problem of attitude tracking for a spacecraft model with inertia uncertainty and external disturbance has been investigated. Two methods, namely, adaptive law and ESO, have been introduced to estimate the disturbance. Sliding-mode controllers have been proposed to combine the two approaches in order to force the state variables of the closed-loop system to converge to the reference state. Detailed simulation results have been presented to illustrate the developed method.

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