Science in China Series F: Information Sciences

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Linguistic dynamic systems based on computing with words and their stabilities

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Linguistic dynamic systems (LDS) are the systems based on computing with words (CW) instead of computing with numbers or symbols. In this paper, LDS are divided into two types: type-I LDS being converted from conventional dynamical systems (CDS) by using extension principle and type-II LDS by using fuzzy logic rules. For type-I LDS, the method of endograph is provided to discuss the stabilities of type-I LDS and two cases of stabilities of logistic mappings: one is the states being abstracted and the other is parameters also being abstracted. For type-II LDS, the method of degree of match is used to discuss the dynamical behavior of arbitrary initial words under fuzzy rule.

computing with words (CW), extension principle, stability, fuzzy rule, degree of match

1 Introduction

One of the focuses of the current scientific society is to understand and predict the behavior of complex huge systems such as economic systems, social systems and ecological systems. For general engineer systems, some approaches are presented such as nonlinear dynamical systems modeling and chaos theory to study these systems[1−3]. A common feature to the approaches is that precise number and symbols are used to obtain precise mathematical model. However, in reality society, it is very difficult for people to analyze these complex huge systems by using differential equation or difference equation because of their complexity and incompleteness of information. That is to say, it is actually impossible to describe these systems by conventional mathematical modeling. On the other hand, even though mathematical description can be built for a complex huge system, it is extremely difficult to convert the collected datum into suitable modeling and express the eventual result in an easily understandable way.

Fortunately, people have accumulated much knowledge and experience of complex systems. Generally, this kind of knowledge is expressed in nature language, i.e. words or linguistic items. Using this knowledge, we are able to reduce the complexity of complex huge systems and then describe, predict, control and evaluate behavior of huge complex systems. And in financial area, although mathematical market forecasting has been studied for many decades, experts still assess market behavior and specify actions to be taken in linguistic terms, and take corresponding measures.

Received May 15, 2007; accepted March 20, 2008
doi: 10.1007/s11432-009-0089-6
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Supported by the National Natural Science Foundation of China (Grant Nos. 60125310, 60474498)

Citation: Mo H, Wang F Y. Linguistic dynamic systems based on computing with words and their stabilities. Sci China Ser F-Inf Sci, 2009, 52(5): 780–796, doi: 10.1007/s11432-009-0089-6

However, many efforts have been taken toward this objective in past, for example, knowledge based systems, agent systems, linguistic structures and multi-valued logic and so on[4−8], but the theories of stability analysis, control design of CDS have not been built yet.

In order to make the analysis and synthesis of complex huge systems formal, consistent, and systemize, Wang[9−11] presented the theory frame of CW and LDS by synthesize fuzzy sets, nonlinear analyse, number theory, optimal control theory and approach the fixed point of LDS preliminary[12]. In this paper, we analyze the dynamical orbits of LDS along the direction of papers[9,10] and apply CW in complex financial systems.

2 From CDS to LDS

Let X be a conventional metric space, $\Re(X)$ denoting a set of all fuzzy sets defining on X, i.e. the set of all function $w : X \to [0,1]$. For $\forall w \in \Re(X), \lambda \in (0,1]$, λ-cut set of word w is $L_\lambda w = \{x|x \in X, w(x) \geqslant \lambda\}$, when $\lambda = 0, L_0 w = \overline{\{x|x \in X, w(x) > 0\}}$, where $\overline{A}$ is the closure of A, called the support set of w.

Word w is expressed as

$$w = \{(x,\lambda)|x \in X, w(x) = \lambda\}.$$

Obviously, $w \subset X \times I$. If λ only takes 0,1, then w is called clear word, and w is written as χ_A. If $A = \{a\}$, where $a \in X$, then w is called a singleton word, written as χ_a.

A conventional control system is defined as

$$\begin{aligned} &x_{k+1} = f(x_k, u_k, k),\\ &f\colon R^n \times R^m \times T \to R^n; \end{aligned} \tag{1a}$$

$$\begin{aligned} &y_k = g(x_k, k),\\ &g\colon R^n \times T \to R^p; \end{aligned} \tag{1b}$$

$$\begin{aligned} &u_k = h(y_k, v_k, k),\\ &h\colon R^p \times R^q \times T \to R^m, \end{aligned} \tag{1c}$$

where $T = \{0,1,2,\cdots,k\}$ is a series of discrete times series, $x_k \in R^n$ is a vector representing a term for the state of the system, $y_k \in R^p$ the output term, $v_k \in R^q$ the input term, $u_k \in R^m$ the control term. f, g, h are conventional continuous mappings representing the system, control and output. Eqs. (1a), (1b), (1c) are called state equation, output equation and feedback equation respectively.

Embedding the output equation and feedback equation into the state equation, we have

$$x_{k+1} = f(x_k, h(g(x_k,k), v_k, k), k), \tag{2}$$

then (2) simplified as

$$x_{k+1} = f'(x_k, v_k), \tag{3}$$

when $v_k = 0$, (3) is

$$x_{k+1} = f'(x_k), \tag{4}$$

(4) is called automatic control systems, written as CDS(X, f'). For the precise mathematical modeling, a giving system and the giving initial state, the orbit of CDS is easily gotten.

Converting the input, output and control of (1a)–(1c) into their linguistic forms, and the same as the system, control and output mappings, we call the procedure the abstraction of CDS:

$$\begin{aligned} &X_{k+1} = \tilde{f}(X_k, U_k, k),\\ &\tilde{f} : \Re(R^n) \times \Re(R^m) \times T \to \Re(R^n); \end{aligned} \tag{5a}$$

$$\begin{aligned} &Y_k = \tilde{g}(X_k, k),\\ &\tilde{g} : \Re(R^n) \times T \to \Re(R^p); \end{aligned} \tag{5b}$$

$$\begin{aligned} &U_k = \tilde{h}(Y_k, V_k, k),\\ &\tilde{h} : \Re(R^p) \times \Re(R^q) \times T \to \Re(R^m), \end{aligned} \tag{5c}$$

where $X_k \in \Re(R^n), Y_k \in \Re(R^p), U_k \in \Re(R^m), V_k \in \Re(R^q)$ are state term, output term, control term and input term of LDS respectively. $\tilde{f}, \tilde{g}, \tilde{h}$ are fuzzy logic operators which define the system, output, and control mappings of the LDS, respectively.

Similarly, embedding eqs. (5b) and (5c) into (5a), and when $V_k = 0$, we have the following equation similar to (4)

$$X_{k+1} = \tilde{f}(X_k). \tag{6}$$

If f is continuous, $\tilde{f}$ is also continuous[13,14].

(5a)–(5c) and (6) are called type-I LDS being abstract from (1),(4).

Example 1. The automatic control system $f(x) = Ax^2 + Bx + C$, where A, B, C are 2×2 coefficient matrix, $x \in R^2$. If we convert system into its linguistic form, the corresponding type-I LDS is

$$X_{n+1} = AX_n^2 + BX_n + C,$$

where the state of $f(x) = Ax^2 + Bx + C$ is abstract as the fuzzy sets on R^2, coefficient matrixes A, B, C be kept. As in function $f(x) = Ax^2+Bx+C, A, B, C$ act on points, and in $X_{n+1} = \mathrm{A}X_n^2 + \mathrm{B}X_n + \mathrm{C}$, A, B, C act on a fuzzy sets, block letters A, B, C are used to differentiate A, B, C in CDS.

From Example 1, type-I LDS is converted from CDS, so conventional mathematical model is needed in the system. But for the systems conventional mathematical model cannot be built, people could not get the corresponding type-I LDS. If the behavior of the system can be analyzed from fuzzy rule, the system is called type-II LDS.

A type-II LDS can be represented as

$$\begin{aligned} &X_{k+1} = R_F(X_k, U_k, k), \\ &R_F\colon \Re(R^n) \times \Re(R^m) \times T \to \Re(R^n); \\ &Y_k = R_G(X_k, k) \\ &R_G\colon \Re(R^n) \times T \to \Re(R^p); \\ &U_k = R_H(Y_k, V_k, k), \\ &R_H\colon \Re(R^p) \times \Re(R^q) \times T \to \Re(R^m), \end{aligned} \tag{7}$$

where R_F, R_G, R_H are fuzzy logic operators defining the system, output and control fuzzy mappings of (7), and $X_k \in \Re(R^n), Y_k \in \Re(R^p)$, $U_k \in \Re(R^m), V_k \in \Re(R^q)$ are the state, output, control and input of type-II LDS respectively.

What the type-II LDS study are fuzzy huge systems for which mathematical modelings have not been built. When dealing with these systems, people mimic human's ability to reason with and manipulate perceptions by the use of language, build fuzzy inference rule to compute and reason with words (i.e. fuzzy sets) and so as to understand and predict the behavior of fuzzy systems. So ecological system, human body system, geographical system, society system and economical system can be analyzed by the method of type-II LDS.

Example 2. For hand-control water temperature regulating system, people can control water temperature by knobbing the knob. In form 1, three words of the first line{high, moderate, low} show the state of water temperature, three words of the first row {left, middle, right} show the direction of knob knobbed to raise, keep and lower the water temperature, Table 1 is the result of water temperature after being regulated.

Table 1 Temperature controlled table

	High	Moderate	Low
left	high	high	moderate
middle	high	moderate	low
right	moderate	low	low

For the system, it is necessary for people to know the precise degree of water temperature and the concrete position of the knob knobbed to, only know: if the water temperature is high, knob the knobbing the lower direction; if the temperature is lower, knob the knobbing to the high direction, and as so on, until the water temperature is moderate and then we can adjust the water temperature to the moderate state.

Type-I LDS, type-II LDS are all called LDS, expressed as

$$\begin{aligned} &X_{k+1} = F(X_k, U_k, k), \\ &F\colon \Re(R^n) \times \Re(R^m) \times T \to \Re(R^n); \\ &Y_k = G(X_k, k), \\ &G\colon \Re(R^n) \times T \to \Re(R^p); \\ &U_k = H(Y_k, V_k, k), \\ &H\colon \Re(R^p) \times \Re(R^q) \times T \to \Re(R^m), \end{aligned} \tag{8}$$

where F, G, H are all fuzzy logic operators defining the states mapping, output mapping and control mapping of type-II LDS, and $X_k \in \Re(R^n)$, $Y_k \in \Re(R^p)$, $U_k \in \Re(R^m), V_k \in \Re(R^q)$ the state, output, control and input respectively, k the discrete time series.

In this paper, $X_i, i \in \{0, 1, \cdots\}$ satisfy the following conditions:

1) X_i is normal, i.e. there exists a point $x \in X$, such that $\mu_{X_i}(x) = 1$;

2) X_i is a convex fuzzy set;

3) X_i is super-continuous;

4) the support set L_0X_i of X_i is compact and $0 \notin L_0X_i$.

3 Computing with words

For CDS, computing with number or symbols is used, i.e. the initial datum set and the terminal datum set are symbols or numbers. For LDS, people use CW whose computing unit is a word, i.e.

the initial condition and conclusion are linguistic items.

In the course of computing, the conclusion can be referred from the initial condition. The computing of CDS and LDS can be expressed as

$$\Big\{\text{initial data set (IDS)}\Big\} \rightarrow$$

$$\begin{cases} \big[\text{CDS: computing with number}\big] \\ \big[\text{LDS: computing with words (cw)}\big] \end{cases} \rightarrow$$

$$\Big\{\text{terminal data set (TDS)}\Big\}.$$

CW includes the following three steps[14]:

Step one is the explicitation of words. The meaning of words is made explicit by a suitable constraint definition.

Step two is the constraint propagation. The constraint propagation of CDS is a function, but for LDS, the constraint is fuzzy function or fuzzy inference rule.

Step three is the constraint translation. By using fuzzy synthetic principlewe convert the result of constraint inference to the conclusion being needed, i.e. convert to the form that people easily understand expressed by words or linguistic form.

In following, the CW of type-I and type-II LDS are presented.

3.1 Type-I LDS' CW

For the automatic type-I LDS

$$X_{k+1} = \tilde{f}(X_k), \tag{9}$$

then

$$\mu_{X_{n+1}}(y) = \max_{y=f(x)} \mu_{X_n}(x). \tag{10}$$

If f is a one-to-one mapping, we have

$$\mu_{X_{n+1}}(y) = \mu_{X_n}(x),$$

where $y = f(x)$, the computing of upper equation is easy.

In general, it is difficult to analyze (10). In this paper, a method of endogragh is given to discuss this problem.

For $\forall w \in \Re(X)$ and $\lambda \in [0,1]$,

$$\tilde{L}_\lambda(w) = \{(x,\lambda) | x \in X, w(x) \geqslant \lambda\} = L_\lambda(w) \times \lambda$$

is called the λ-endograph of w. At the time

$$w = \bigcup_{\lambda\in[0,1]} \tilde{L}_\lambda(w) = \bigcup_{\lambda\in[0,1]} L_\lambda(w) \times \lambda,$$

where for $\forall \lambda_1, \lambda_2 \in [0,1], (x,\lambda_1) \bigcup (x,\lambda_2) = (x,\bar{\lambda})$, $\bar{\lambda} = \max(\lambda_1,\lambda_2)$, $\chi_a = \bigcup_{\lambda\in[0,1]} a \times \lambda$.

$$\tilde{f}(w) = \bigcup_{\lambda\in[0,1]} \tilde{L}_\lambda(\tilde{f}(w)) = \bigcup_{\lambda\in[0,1]} f(L_\lambda(w)) \times \lambda, \tag{11}$$

and $f(L_\lambda(w)) \subseteq X$, the mapping $\tilde{f}$ defining on $\Re(X)$ can be converted to the mapping f defining on X. Obviously, if $f : X \rightarrow X$ is continuous, then $\tilde{f} : \Re(X) \rightarrow \Re(X)$ is also continuous[14], and $\tilde{f}(\tilde{L}_\lambda(w)) = \tilde{L}_\lambda(\tilde{f}(w))$. And then the property of CDS can extend to that of LDS directly.

The three steps of type-I CW are: expilicitation of words by using the method of endograph, constraint propagation by using fuzzy function, translating the inference result by using fuzzy synthetic principle.

3.2 Type-II LDS' CW

For type-II LDS, CW is computing with words or idioms from natural language. A simple proposition p is expressed as $X\ isr\ R$, where X is a constrained variable, R is a constraining relations, isr is a variable copula in which r determines the way in which R constrains X, in another way, the value of discrete variable r defines the action of R on X.

The initial data set is composed of the main factors of various constraints and compositions of complex systems. At first, the main and decisive factors for systems study should be extracted from the magnanimous information to form a fuzzy rule basis, in which every rule can be represented as the conform of simple composition, then for the initial data set, CW is used, and then terminal data set is achieved.

Three steps of type-II CW is discussed as follows.

3.2.1 Explicitation of words. In this paper, an appropriate fuzzy rule basis $\tilde{R} = \sum_{i=1}^{N} \tilde{R}_i$ is defined on the discourse of A, B, where N is a nature number, $\tilde{R}_i$ is a fuzzy rule.

$$\tilde{R}_i\text{: if } X \text{ is } A_i, \text{ then } Y \text{ is } B_i, i = 1,2,\cdots,N$$
$$\text{or } (X,Y) \rightarrow (A_1 \times B_1 + \cdots + A_N \times B_N),$$

where $A_i \in \Re(A), B_i \in \Re(B), i = 1,2,\cdots,N$, "×" is Descartes product, "+" is disjunction, $\tilde{R} = \sum_{k=1}^{N} \tilde{R}_k = \sum_{k=1}^{N} A_k \times B_k$.

The initial data X_0 is A', and A' is represented as

$$A' = \sum_{k=1}^{N} \lambda_k A_k,$$

where $\lambda_k = \sup_x(\mu_{A_k}(x) \wedge \mu_{A'}(x))$ is the degree of match of A_k and A'[15], written as $\lambda_k = A_k \nabla A'$, where

$$A_k = \{(x, \lambda)|\mu_{A_k}(x) = \lambda\}(k = 1, 2, \cdots, N),$$
$$\lambda_k A_k = \{(x, \lambda_k m)|\mu_{\lambda_k A_k}(x) = \lambda_k m\}.$$

3.2.2 Constraint propagation. The process of deriving the results from the initial data set is a constraint step. Correlative conclusions have achieved by moderate constraint and reference.

For the giving fuzzy rule basis $\tilde{R} = \sum_{i=1}^{N} \tilde{R}_i$ and $A', \tilde{R}(A')$ is called the image word of A' under the fuzzy rule basis $\tilde{R}$.

If $A' = A_k, k \in \{1, 2, \cdots, N\}$, then

$$\tilde{R}(A') = R_k(A_k) = B_k,$$

and for arbitrary initial word $A' \neq A_k, k \in \{1, 2, \cdots, N\}$, what is the imagine word?

Let $B' = \tilde{R}(A')$, then

$$\mu_{B'}(y) = \sup_x(\mu_{\tilde{R}}(x, y) \wedge \mu_{A'}(x)), \qquad (12)$$

where $\mu_{\tilde{R}}(x, y)$ and $\mu_{A'}(x)$ represent the membership function of $\tilde{R}, A'$ respectively.

In (12)

$$\mu_{\tilde{R}}(x, y) = \sum_{i=1}^{N} \mu_{A_i}(x) \wedge \mu_{B_i}(y). \qquad (13)$$

Embedding (13) into (12) and exchanging the computing order, we have

$$\mu_{B'}(y) = \sup_x \left(\sum_{i=1}^{N} (\mu_{A_i}(x) \wedge \mu_{A'}(x)) \wedge \mu_{B_i}(y) \right)$$
$$= \sum_{i=1}^{N} \sup_x(\mu_{A_i}(x) \wedge \mu_{A'}(x)) \wedge \mu_{B_i}(y), \qquad (14)$$

where

$$\lambda_i = \sup_x(\mu_{A_i}(x) \wedge \mu_{A'}(x)). \qquad (15)$$

Embedding (15) into (14), we have

$$\mu_{B'}(y) = \sum_{i=1}^{N} \lambda_i \wedge \mu_{B_i}(y). \qquad (16)$$

From (16), the imagine word of the initial word A' under the fuzzy rule $\tilde{R}$ is

$$B' = \sum_{i=1}^{N} \lambda_i \wedge B_i, \qquad (17)$$

where for every arbitrary word $B_i = \{(x, m)|\mu_{B_i}(x) = m\}(i = 1, 2, \cdots, N), \lambda_i \wedge B_i = \{(x, r)|r = \min(\lambda_i, \mu_{B_i}(x))\}$ is defined.

In (17), for every arbitrary initial words A', if the degree of match $\lambda_i \neq \phi$ of words A' and A_i, then the rule $\tilde{R}_i$ is activated. Let $\tilde{R}_i = A_i \times B_i$, then the image of A' under $\tilde{R}_i$ defines as

$$\tilde{R}_i(A') = (A_i \nabla A') \times B_i.$$

And $\tilde{R} = \sum_{i=1}^{N} \tilde{R}_i = \sum_{i=1}^{N} A_i \times B_i$, so the image of word A' under $\tilde{R}$ is

$$\tilde{R}(A') = \sum_{i=1}^{N} (A_i \nabla A') \times B_i, \qquad (18)$$

$\lambda_i = A_i \nabla A', \tilde{R}(A') = B'$, thus (18) is the same as (17).

3.2.3 Constraint translation. After constraint propagation, fuzzy conclusions are translated to natural language being easily understood by using the method of linguistic approximation. Thus, the initial data set produces the initial data set by using CW.

So, CW of type-II LDS can be concluded as follows:

First, the initial data set is represented as the normal form $X \ isr \ A$, and cover the discourse A by moderate basis words $\{A_1, \cdots, A_n\}$, and then a series fuzzy rule basis for the giving basis words should be built,

$$\tilde{R} = \sum_{i=1}^{n} \tilde{R}_i = \sum_{i=1}^{n} A_i \times B_i.$$

Second, for arbitrary initial word A_0, computing its degree of match λ_i with every basis words $A_i, i \in \{1, \cdots, n\}$, and then conclude the image words $\lambda_i \wedge B_i$ of A_0 under $\tilde{R}_i$.

Third, for all image words, we get $\sum_{i=1}^{n} \lambda_i \wedge B_i$ by the way of fuzzy synthetic principle.

Last, by the method of linguistic approximation, we have $\sum_{i=1}^{n} \lambda_i \wedge B_i \approx B_0$, where words B_0 is the image of the initial word A_0 under the fuzzy rule basis $\tilde{R}$.

Example 3. The result of Zhang San is fairly good and the result of Li Si is better than that of Zhang San, computing the linguistic value of the result of Li Si.

First, converting the proposition “the result of Li Si is better than that of Zhang San”to the normal form:

The result of Li Si is better than that of Zhang San.

In the proposition, word “fairly good” substitutes for “the result of Zhang San”, we have “the result of Li Si is better than ‘fairly good’”. According to the scores, human use five basis words {excellent, very good, good, moderate, bad} to cover the discourse [0,100]. The membership functions of basis words {excellent, very good, good, moderate, bad} and the input word “fairly good” is as follows:

$$\mu_{\text{bad}}(x) = \begin{cases} 1 & x \leqslant 65, \\ 0.2(70-x) & 65 < x \leqslant 70, \\ 0 & \text{other}; \end{cases}$$

$$\mu_{\text{excellent}}(x) = \begin{cases} 0.2(x-93) & 93 \leqslant x \leqslant 98, \\ 1 & 98 < x \leqslant 100, \\ 0 & \text{other}. \end{cases}$$

$$\mu_{\text{moderate}}(x) = \begin{cases} 0.2(x-65) & 65 \leqslant x \leqslant 70, \\ 1 & 70 < x \leqslant 75, \\ 0.2(80-x) & 75 < x \leqslant 80, \\ 0 & \text{other}; \end{cases}$$

$$\mu_{\text{good}}(x) = \begin{cases} 0.2(x-75) & 75 \leqslant x \leqslant 80, \\ 1 & 80 < x \leqslant 85, \\ 0.2(90-x) & 85 < x \leqslant 90, \\ 0 & \text{other}; \end{cases}$$

$$\mu_{\text{very good}}(x) = \begin{cases} 0.2(x-85) & 85 \leqslant x \leqslant 90, \\ 1 & 90 < x \leqslant 93, \\ 0.2(98-x) & 93 < x \leqslant 98, \\ 0 & \text{other}; \end{cases}$$

$$\mu_{\text{fairly good}}(x) = \begin{cases} 0.2(x-72) & 72 \leqslant x \leqslant 77, \\ 1 & 77 < x \leqslant 79, \\ 0.2(84-x) & 79 < x \leqslant 84, \\ 0 & \text{other}. \end{cases}$$

For the giving basis words, the corresponding fuzzy rule basis is

$\tilde{R}_1$: better than “bad”, the result is “moderate”;

$\tilde{R}_2$: better than “moderate”, the result is “good”;

$\tilde{R}_3$: better than “good”, the result is “very good”;

$\tilde{R}_4$: better than “very good”, the result is “excellent”.

Let $\tilde{R} = \sum_{k=1}^{4} \tilde{R}_k$. Basis words A_1=bad, A_2=moderate, A_3=good, A_4=very good, A_5= excellent.

Second, computing the image $\tilde{R}$(fairly good) of “fairly good” under $\tilde{R}$.

Determining the degree of match of the initial words “fairly good” and the basis words A_i, $i = 1, 2, 3, 4$, by membership function and (15), we know

$$\begin{aligned} &\lambda_1 = \lambda_4 = 0, \\ &\lambda_3 = \text{fairly good} \wedge \text{good} = 0.9, \\ &\lambda_2 = \text{fairly good} \wedge \text{moderate} = 0.8, \end{aligned}$$

where λ_2, λ_3 are shown as in Figure 1(a) and (c), respectively. The image word of “fairly good” under very rule is

$$\begin{aligned} &\tilde{R}_1(\text{fairly good}) = \tilde{R}_4(\text{fairly good}) = 0, \\ &\tilde{R}_2(\text{fairly good}) = 0.8 \wedge \text{good}, \\ &\tilde{R}_3(\text{fairly good}) = 0.9 \wedge \text{very good}, \end{aligned}$$

where 0.8 ∧ good, 0.9 ∧ very good as in Figure 1(b) and (d) respectively.

Third, computing the synthesis of word “fairly good” under every fuzzy rule

$$\begin{aligned} &\tilde{R}(\text{fairly good}) \sum_{i=1}^{4} \lambda_i \wedge B_i = 0.8 \times \text{good} \\ &+ 0.9 \times \text{very good} \end{aligned}$$

as shown in Figure 1(e).

Last, by linguistic approximation, we have

$$0.8 \wedge \text{good} + 0.9 \wedge \text{very good} = \text{good and very good}.$$

By the method of CWthe image word of the initial word “fairly good” under the fuzzy rule $\tilde{R}$ is

$$\tilde{R}(\text{fairly good}) = \text{good and very good}.$$

Thus, the computing outcome of “The result of Zhang San is fairly good and the result of Li Si is better than that of Zhang San” is “the result of Zhang San is fairly good and that of Li Si is good and very good”.

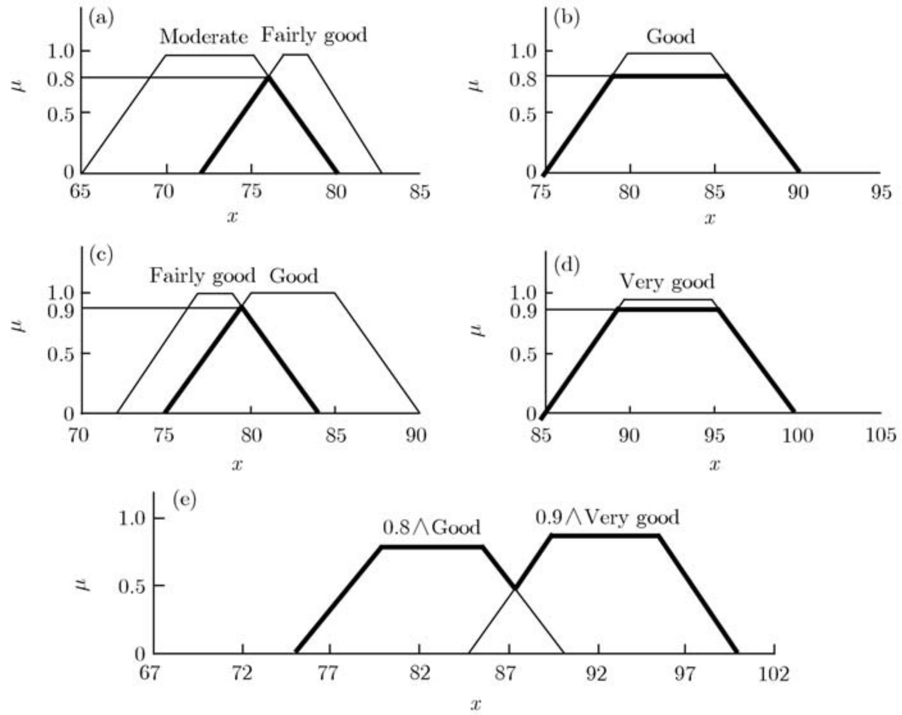


Figure 1 CW of II-LDS.

From upper computing course, we can see that the computing unit of CW of type-II LDS is linguistic item, and so is the terminal output result, thus the conclusions are easy to be accepted by the common.

4 Stability of type-I LDS and their arithmetic

Stability is the main function norm, also one of the most basic and important questions when people study various dynamical systems[16,17]. The stable questions of CDS have achieved rich results. Now we analyze the relations between stability of CDS and type-I LDS being abstract from CDS.

Let (X, d) be a complete metric space, $\Gamma(X), \Re(X)$ are sets of common subset and fuzzy set defining on X respectively. The hausdorff metric defined on $\Gamma(X)$ is

$$h(A, B) = \inf\{\varepsilon | A \subseteq N(B, \varepsilon), B \subseteq N(A, \varepsilon)\},$$

where

$$N(A, \varepsilon) = \{x \in X | H(x, A) < \varepsilon\},$$
$$H(x, A) = \inf_{a \in A} d(x, a),$$

$f : X \to X$ is a continuous mapping and so is the mapping $\tilde{f} : \Re(X) \to \Re(X)$ induced by f. The metric defined on $\Re(X)$ is

$$D(w, v) = \sup_{\lambda \in [0,1]} h(\tilde{L}_\lambda w, \tilde{L}_\lambda v) = \sup_{\lambda \in [0,1]} h(L_\lambda w, L_\lambda v).$$

Obviously, when $w = \chi_a$, $D(\chi_a, v) = H(a, L_0 v)$.

The following two lemmas are needed in this section:

Lemma 1. The continuous fuzzy mapping $\tilde{f} : \Re(X) \to \Re(X)$ is induced by the continuous mapping $f : X \to X$ defining on X and for $\forall \omega \in \Re(X)$, we have

$$\tilde{f}(\tilde{L}_\lambda(w)) = \tilde{L}_\lambda(\tilde{f}(w)).$$

Proof. By the inducing step of fuzzy mapping, for $\forall (x, \lambda) \in \Re(X)$, we have

$$\tilde{f}((x, \lambda)) = (f(x), \lambda),$$

$\forall y \times \lambda \in \tilde{L}_\lambda(\tilde{f}(\omega)) \Leftrightarrow y \in L_\lambda(\tilde{f}(w)) \Leftrightarrow \tilde{f}(\omega)(y) > \lambda \Leftrightarrow \vee_{f(x)=y} w(x) = y \Leftrightarrow \exists x \in X$ satisfies $f(x) = y$, such that $\omega(x) > \lambda \Leftrightarrow \exists x \in X$ satisfies $f(x) = y$ such that $x \in L_\lambda \omega \Leftrightarrow x \times \lambda \in \tilde{L}_\lambda \omega \Leftrightarrow \tilde{f}(x \times \lambda) \in \tilde{f}(\tilde{L}_\lambda \omega) \Leftrightarrow \tilde{f}(x) \times \lambda \in \tilde{f}(\tilde{L}_\lambda \omega) \Leftrightarrow y \times \lambda \in \tilde{f}(\tilde{L}_\lambda \omega)$.

Thus, $\tilde{f}(\tilde{L}_\lambda(w)) = \tilde{L}_\lambda(\tilde{f}(w))$.

Lemma 2. If $x = e$ is the fixed point of continuous mapping $x_{n+1} = f(x_n)$ defining on the universe of discourse X, then the singleton word χ_e is the equilibrium word of the corresponding LDS $X_{n+1} = \tilde{f}(X_n)$.

Proof. Let $e \in X$ be the fixed point of the continuous mapping $f : X \to X$, then by Lemma 1

$$\begin{aligned}\tilde{f}(\chi_e) &= \bigcup_{\lambda\in[0,1]} L_\lambda \tilde{f}(\chi_e) \times \lambda \\ &= \bigcup_{\lambda\in[0,1]} f(L_\lambda \chi_e) \times \lambda \\ &= \bigcup_{\lambda\in[0,1]} e \times \lambda = \chi_e.\end{aligned}$$

So the singleton word χ_e is the equilibrium word of $\tilde{f}$.

4.1 The stability of type-I LDS

Theorem 1. If $x = e$ is the globally asymptotically stable point of CDS $x_{n+1} = f(x_n)$, then the singleton word χ_e is the globally asymptotically stable word of the corresponding LDS $X_{n+1} = \tilde{f}(X_n)$.

Proof. Let $e \in X$ be the globally asymptotically stable point of CDS $x_{n+1} = f(x_n)$, then by Lemma 2, the singleton word χ_e is the equilibrium word of $\tilde{f}$. Following is the proof that the singleton word $\chi_e \in \Re(X)$ is the globally asymptotically stable word of mapping $\tilde{f} : \Re(X) \to \Re(X)$.

Suppose e is the globally asymptotically stable point of CDS $x_{n+1} = f(x_n)$, then

1) the point e is stable, i.e. for $\forall \varepsilon > 0, \exists \delta = \delta(\varepsilon)$, such that for all $x \in X$satisfying $d(x, e) < \delta$ and $\forall k > 0$, there is $d(f^k(x), e) < \varepsilon$, then for $\forall w \in \Re(X)$, there is $D(w, \chi_e) < \delta$, i.e. $\max_{x\in L_0 w} d(x, e) < \delta$, by the stability of point e, we have

$$\max_{x\in L_0 w} d(f^k(x), e) < \varepsilon, \text{ or } H(f^k(L_0 w), e) < \varepsilon,$$

and f is continuous, there is $L_0 \tilde{f}^k(w) = f^k(L_0 w)$, then

$$H(L_0 \tilde{f}^k(w), e) < \varepsilon,$$

and then for $\forall k \in N$, there is $D(\tilde{f}^k(w), \chi_e) < \varepsilon$, thus the singleton word χ_e is stable.

2) e is asymptotically stable, i.e. e is stable and there exists $\gamma > 0$, such that for every $x \in X$ satisfying $d(x, e) < \gamma$ and $k \to \infty, f^k(x) \to e$. For such γ, there is $D(w, \chi_e) < \varepsilon$ for $\forall w \in \Re(X)$, and when $k \to \infty$, for $\forall x \in L_\lambda w, f^k(x) \to e$, then $f^k(L_\lambda w) \to e$ and then

$$\begin{aligned}\lim_{k\to\infty} \tilde{f}^k(w) &= \lim_{k\to\infty} \bigcup_{\lambda\in[0,1]} L_\lambda \tilde{f}^k(w) \times \lambda \\ &= \lim_{k\to\infty} \bigcup_{\lambda\in[0,1]} f^k(L_\lambda w) \times \lambda \\ &= \bigcup_{\lambda\in[0,1]} e \times \lambda = (e, 1) = \chi_e,\end{aligned}$$

thus χ_e is asymptotically stable.

3) e is globally asymptotically stable, i.e. e is asymptotically stable and for $\forall x \in X$, when $k \to \infty, f^k(x) \to e$ and for $\forall w \in \Re(X), L_\lambda f^k(w) \to e$, and then

$$\begin{aligned}\lim_{k\to\infty} \tilde{f}^k(w) &= \lim_{k\to\infty} \bigcup_{\lambda\in[0,1]} L_\lambda \tilde{f}^k(w) \times \lambda \\ &= \bigcup_{\lambda\in[0,1]} e \times \lambda = (e, 1) = \chi_e.\end{aligned}$$

Thus χ_e is globally asymptotically stable.

Following is the numerical arithmetic of type-I LDS$X_{n+1} = f(X_n)$:

Step 1: initialization.

Set $i = 0, j = 1$, the initial word is $X_0 = \{(x, \lambda) | \mu_{X_0}(x) = \lambda\}$, take λ-endograph series $\{L_{\lambda_1} w_0 \times \lambda_1, L_{\lambda_2} w_0 \times \lambda_2 \cdots, L_{\lambda_n} w_0 \times \lambda_n\}$ of X_0, here $k = 1, 2, \cdots, n-1, \lambda_{k+1} > \lambda_k$.

Step 2: transformation.

λ_j-endograph of the $(i+1)$th state X_{i+1} is

$$L_{\lambda_j} X_{i+1} \times \lambda_j = f(L_{\lambda_j} X_i) \times \lambda_j.$$

Step 3: λ_j-loop.

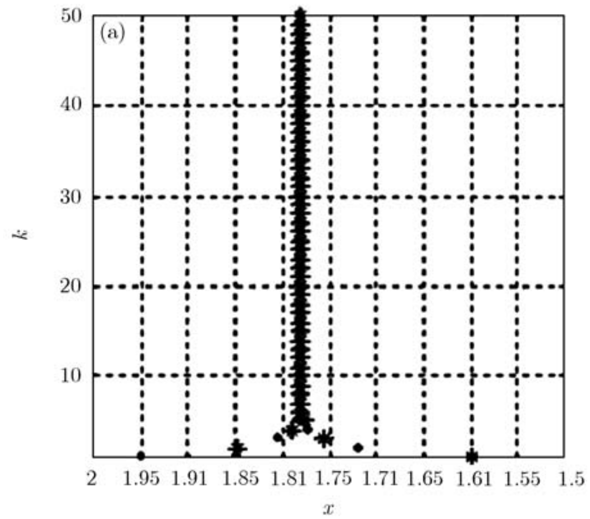
If $j < n$, then $j \leftarrow j + 1$, return to step 2. Otherwise $X_{i+1} = \bigcup_{j\in[0,1,\cdots,n]} L_{\lambda_j} X_i \times \lambda_j$.

Step 4: i-loop.

If $i < N$, then $i \leftarrow i + 1, j = 1$, return to step 2, otherwise stop.

By using the numerical arithmetic, the type-I LDS is analyzed as follows:

Example 4. Let the universe discourse be $[0, 2]$, one-dimension dynamical systems $x_{n+1} = 0.5 + \frac{3x_n}{1+x_n^2}$ is globally asymptotically stable and the globally asymptotically stable point 1.78. The initial value $x_0 = 1.6, x_1 = 1.95$, the orbit of their CDS is shown as in Figure 2(a) (expressed as "*" and "·" respectively).



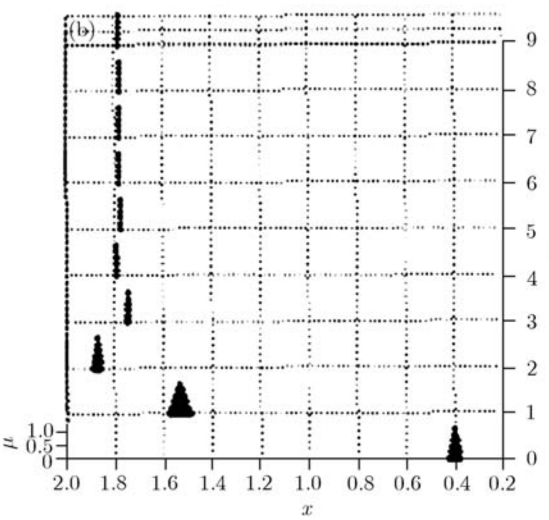


Figure 2 (a) Globally asymptotically stable CDS; (b) globally asymptotically stable LDS.

The corresponding LDS is

$$X_{n+1} = 0.5 + \frac{3X_n}{1 + X_n^2}.$$

Take the initial word $X_0 =$"near to 0.4", membership function be triangular, the radius of support set be 0.05, and their orbit is shown in Figure 2(b).

From Figure 2(a), the orbit of the initial value is near to the fixed point 1.7808, so 1.7808 is the globally asymptotically stable point of CDS$x_{n+1} = 0.5 + \frac{3x_n}{1+x_n^2}$. From Figure 2(b), after times of iterations, the orbit of X_0 is more and more close to the equilibrium word $\chi_{1.7808}$, and for any other word, we have the same conclusion, so the equilibrium word $\chi_{1.7808}$ is the globally asymptotically stable word of LDS $X_{n+1} = 0.5 + \frac{3X_n}{1+X_n^2}$. From Figure 2(a) and (b), CDS $x_{n+1} = 0.5 + \frac{3x_n}{1+x_n^2}$ and the corresponding LDS $X_{n+1} = 0.5 + \frac{3X_n}{1+X_n^2}$ have the same stability.

4.2 LDS of logistic mappings and their stability

Now we compute the linguistic dynamical orbit converted from logistic mappings $x_{n+1} = \lambda x_n - \lambda x_n^2, \lambda \in [0, 3]$ defining on the universe of discourse $X = [0, 1]$.

By the conventional dynamical property, the CDS of logistic mappings $x_{n+1} = \lambda x_n(1 - x_n), 0 < \lambda \leqslant 4$ vary with the parameter change. When $0 < \lambda \leqslant 3$, the CDS of logistic mappings is globally asymptotically stable, and when $0 < \lambda \leqslant 1$, the globally asymptotically stable point is 0, and when $1 < \lambda \leqslant 3$, the globally asymptotically stable point is $1 - \frac{1}{\lambda}$, as shown in Figure 3.

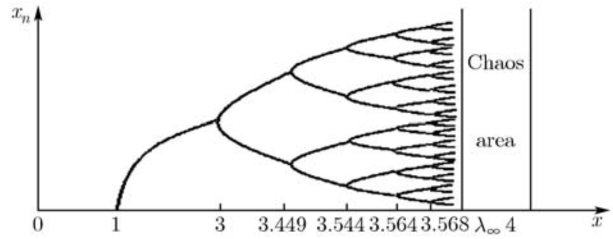


Figure 3 CDS of logistic mappings corresponding to the varying parameter.

Following two cases LDS of logistic mappings $x_{n+1} = \lambda x_n - \lambda x_n^2, \lambda \in [0, 3]$ are discussed in this paper: 1) keep the parameter λ, convert the state to the abstract form and get LDS $X_{n+1} = \lambda X_n - \lambda X_n^2, \lambda \in [0, 3]$; 2) convert the parameter and state to the abstract form and get LDS $X_{n+1} = MX_n - MX_n^2$, where M is a fuzzy set defining on $[0, 3]$, and X_n is the fuzzy set defining on the universe of discourse $X = [0, 1]$ and n is a nature number.

4.2.1 The stability of logistic mappings $X_{n+1} = \lambda X_n - \lambda X_n^2$. The parameter λ is kept, and systematic state is abstract and we have the corresponding LDS $X_{n+1} = \lambda X_n - \lambda X_n^2, \lambda \in [0, 3]$, here X_n is a fuzzy set defining on the universe discourse $[0, 1]$. The relationship between CDS and LDS of stable logistic mappings is as follows.

Example 5. When $\lambda = 0.5, x_{n+1} = 0.5x_n(1 - x_n), 0, 2$ are its equilibrium points. Take the initial value $0.6, 0.95$, and their orbits are shown in Figure 4(a).

The LDS corresponding to the logistic mapping $x_{n+1} = 0.5x_n(1 - x_n)$ is

$$X_{n+1} = 0.5X_n(1 - X_n).$$

Take the initial word $X_0 =$"point near to 0.5", membership function be triangular, the radius of support set be 0.1, and the orbit of word X_0 is as Figure 4(b). In Figure 4(b), the vertical μ-axis denotes the membership function of the state word and x-axis denoting the universe of discourse of $x(k)$ and k-axis denoting the iterative times (the same as follows).

From Figure 4(a), in CDS$x_{n+1} = 0.5x_n(1 - x_n)$, the orbits of the initial value 0.95, 0.6 are close to the fixed point 0 after limited iterations, and the same to other points. Correspondingly, from Figure 4(b), in LDS $X_{n+1} = 0.5X_n(1 - X_n)$, the radiuses of the support sets of the words of the initial word X_0 are close to 0 rapidly, by the decomposition principle of fuzzy set, and the limit word is the equilibrium word χ_0.

Example 6. For the logistic mapping $x_{n+1} = 2.5x_n - 2.5x_n^2$, the fixed point 0.6 is globally asymptotically stable. Take the initial value $x_0 = 0.4, x_1 = 0.85$, then their CDS are as Figure 5(a).

For the corresponding LDS $X_{n+1} = 2.5X_n - 2.5X_n^2$, take the initial word $X_0 =$"close to point 0.6", the LDS is as Figure 5(b).

By Theorem 1, the singleton $\chi_{0.6}$ is the globally asymptotically stable word of LDS $X_{n+1} = 2.5X_n - 2.5X_n^2$. Figure 5(b) also says that the granularity of words of orbit is smaller and smaller with the increasing of iteration times and the dynamical orbit will be close to the singleton word $\chi_{0.6}$ terminally.

Similar to CDS, the globally asymptotically stable words of the abstract logistic mapping LDS$X_{n+1} = \lambda X_n - \lambda X_n^2, \lambda \in [0, 3]$ do not vary with the change of initial state words. When $\lambda \in [0, 1]$, the globally asymptotically stable word χ_0 is kept, but when $\lambda \in [1, 3]$, the globally asymptotically stable words will vary with the change of parameter λ.

4.2.2 The stability of logistic mappings $X_{n+1} = MX_n - MX_n^2$. When $0 < \lambda \leqslant 1$, the globally asymptotically stable point of logistic mapping $X_{n+1} = \lambda X_n - \lambda X_n^2$ is the equilibrium point 0. Make the systematic parameter and state abstract, the corresponding LDS is

$$X_{n+1} = MX_n - MX_n^2,$$

where $\mathrm{M} \in \Re([0, 1])$, we analyze the LDS as follows:

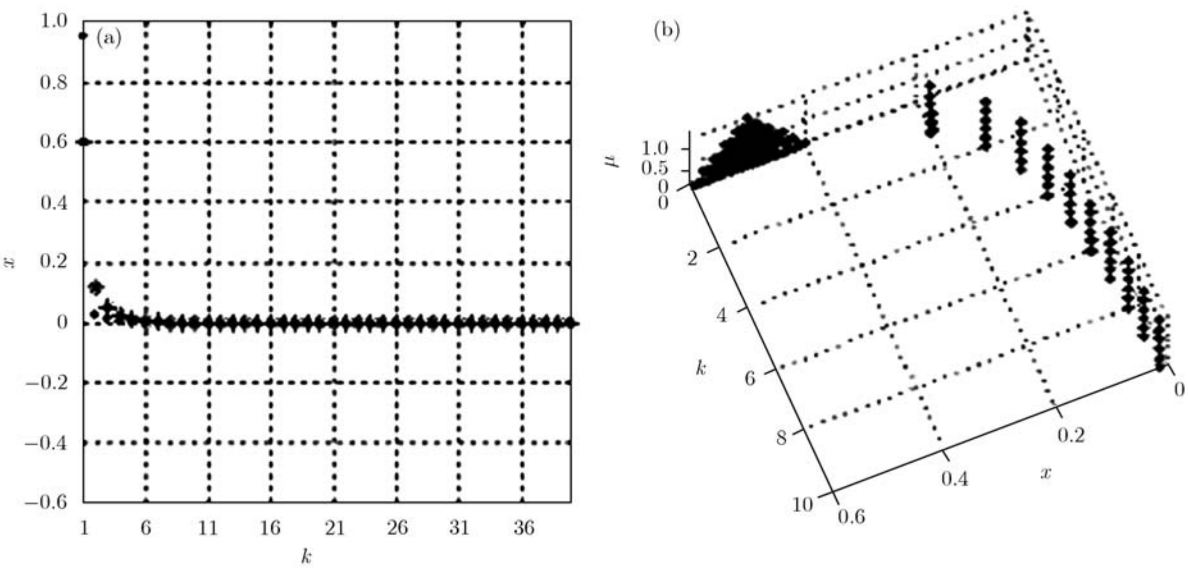


Figure 4 (a) CDS of globally asymptotically stable point 0; (b) LDS of globally asymptotically stable word χ_0.

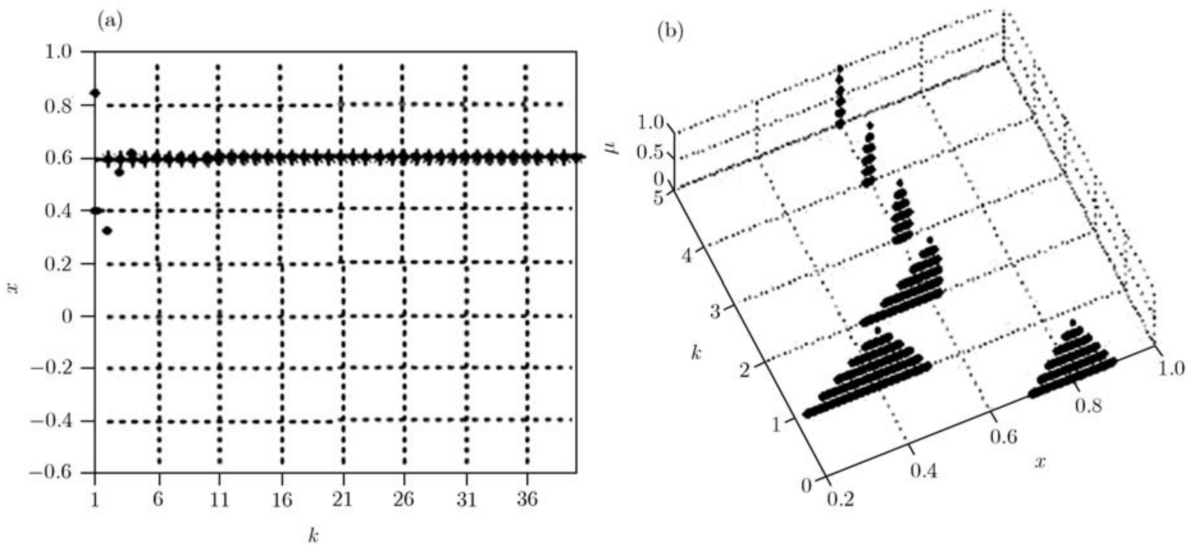


Figure 5 (a) Globally asymptotically stable CDS; (b) globally asymptotically stable LDS.

Example 7. For the logistic mapping $x_{n+1} = 0.5x_n(1 - x_n)$, take the initial value $x_0 = 0.6, x_1 = 0.95$, and the CDS is as shown in Figure 4(a). The corresponding LDS is $X_{n+1} = MX_n(1 - X_n)$, and take the initial word $X_0 =$"close to point 0.6", parameter word $M =$"about 0.5", and their membership functions are both triangular and the radius of support set are 0.4, 0.1 respectively and their LDS are shown in Figure 6.

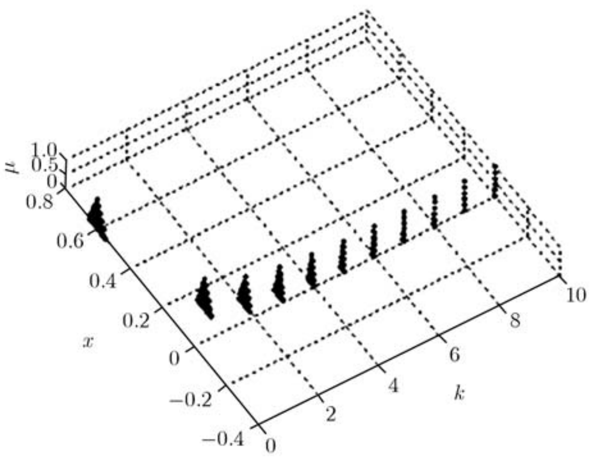


Figure 6 Globally asymptotically stable LDS.

When $M \in \Re([0,1])$ and $X_0 \in \Re([0,1])$, the globally asymptotically stable word of LDS $X_{n+1} = MX_n(1 - X_n)$ is χ_0, and this corresponds to the globally asymptotically stable of LDS $X_{n+1} = 0.5X_n(1-X_n)$. That is to say, when $M \in \Re([0,1])$, the globally asymptotically stable word of LDS does not vary with the change of parameter M and initial state X_0.

When $1 < \lambda \leqslant 3$, the globally asymptotically stable point of logistic mapping $x_{n+1} = \lambda x_n - \lambda x_n^2$ is the equilibrium point $1 - \frac{1}{\lambda}$, and another equilibrium point 0 is a repelling point. The corresponding LDS is $X_{n+1} = MX_n(1 - X_n)$, where $M \in \Re([1,3]), X_0 \in \Re([0,1])$.

Example 8. In LDS $X_{n+1} = MX_n(1 - X_n)$, when the parameter is word $M =$"about 2.5", the membership function is

$$\mu_M(\lambda) = \begin{cases} 5(\lambda - 2.3) & 2.3 \leqslant \lambda \leqslant 2.5, \\ 5(2.7 - \lambda) & 2.5 < \lambda \leqslant 2.7, \\ 0 & \text{otherwise}. \end{cases}$$

Take two initial words "point close to 0.85", "point close to 0.4" and their membership function are triangular and the radius of the support set are both 0.1 and the orbits of LDS are shown in Figure 7(a) and (b) respectively.

In Figure 7(a) and (b), keep the parameter $M =$"about 2.5", and the initial words of systematic state vary and then the globally asymptotically stable word "point near to 0.6" is kept and the membership function is

$$\mu_\Lambda(e) = \begin{cases} 5\left(\dfrac{1}{1-e} - 2.3\right) & e \in \left[\dfrac{13}{23}, 0.6\right], \\ 5\left(2.7 - \dfrac{1}{1-e}\right) & e \in \left[0.6, \dfrac{17}{27}\right], \\ 0 & \text{other}. \end{cases}$$

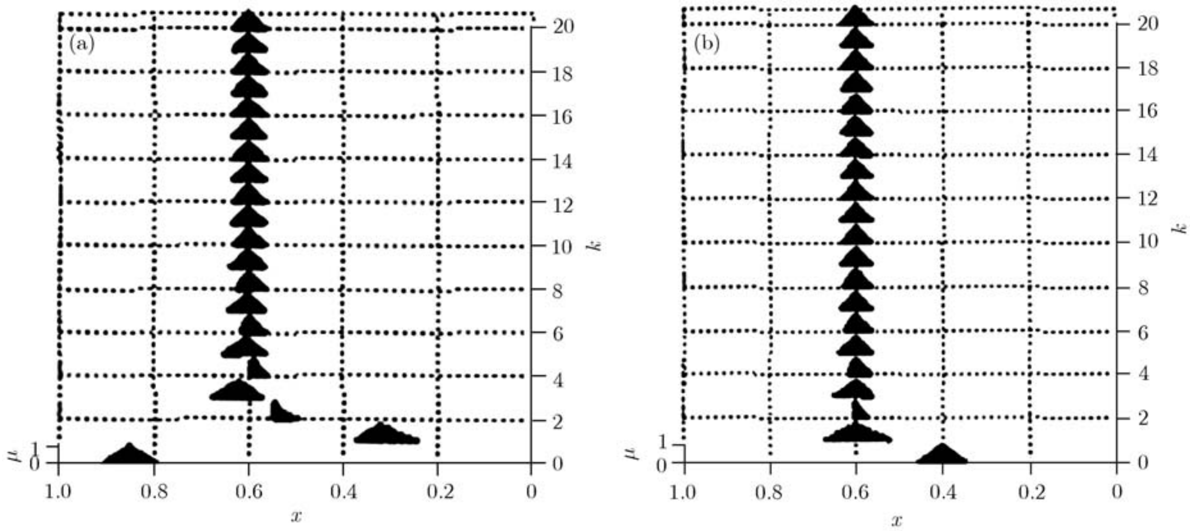


Figure 7 (a) Globally asymptotically stable LDS; (b) globally asymptotically stable LDS.

The orbits of arbitrary initial word of LDS$X_{n+1} = MX_n(1 - X_n)$ are introduced when $M \in \Re([1, 3])$ and the parameters change.

Example 9. Let the initial word X_0 ="point near to 0.4", the membership function be triangular, the radius of support set be 0.1. Let the parameter M ="about 2", membership function be triangular, the radius of support set be 0.2, and the LDS is as Figure 8.

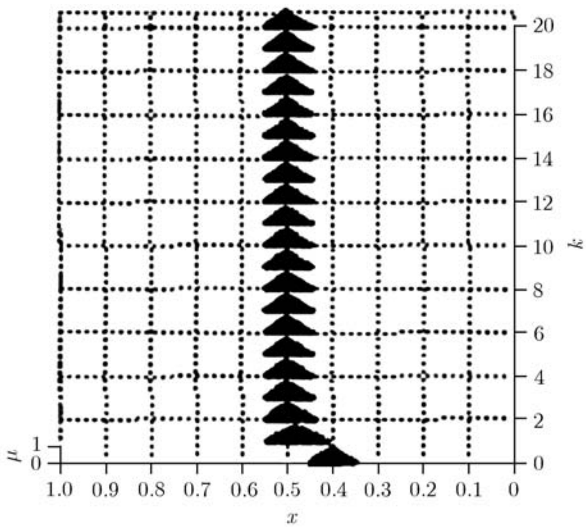


Figure 8 Globally asymptotically stable LDS.

Keep the initial word and the parameter varies. Take M ="about 1", membership function is triangular and the radius of support set is 0.1 and then the dynamical orbit of LDS $X_{n+1} = MX_n(1 - X_n)$ are as Figures 9(a) (the former ten times iterations) and 9(b) (the 110P$^{\rm th}$ to 120P$^{\rm th}$ times iterations).

From Figures 8, 9(a) and 9(b), when the parameter $M \in \Re([1, 3]), X_0 \in \Re([0, 1])$, LDS$X_{n+1} = MX_n(1 - X_n)$ is globally asymptotically stable and the globally asymptotically stable word of LDS does not vary with the change of the initial value X_0, but do with the change of the parameter M.

From Figures 6–9, when $M \in \Re([0, 1])$, the globally asymptotically stable word of LDS $X_{n+1} = MX_n(1 - X_n)$ is χ_0 not varying with the change of the parameter M and the initial word X_0, but when $M \in \Re([1, 3])$, the globally asymptotically stable words of LDS $X_{n+1} = MX_n(1 - X_n)$ are $(0, \max_{0 \leqslant \lambda \leqslant 1} \mu_M(\lambda)) \bigcup \left\{ \left(1 - \frac{1}{\lambda}, \mu_M(\lambda)\right) | 1 < \lambda \leqslant 3 \right\}$, vary with change of the parameter M but not with the change of the initial value X_0.

χ_0 is a special case of $(0, \max_{0 \leqslant \lambda \leqslant 1} \mu_M(\lambda))$, so when $M \in \Re([0, 3])$, the globally asymptotically stable words of LDS $X_{n+1} = MX_n(1 - X_n)$ are $(0, \max_{0 \leqslant \lambda \leqslant 1} \mu_M(\lambda)) \bigcup \left\{ \left(1 - \frac{1}{\lambda}, \mu_M(\lambda)\right) | 1 < \lambda \leqslant 3 \right\}$.

5 The arithmetic of type-II LDS and their dynamics

Suppose $\Re(A)$ is a set of all fuzzy sets on the universe discourse A, and $\{A_1, \cdots, A_N\}$ is a series of words covering A, called basis words series. We use a series fuzzy rule basis to represent the behavior rule:

$\tilde{R}_k$: if X_k is A_k, then X_{k+1} is $B_k; k \in \{1, 2, \cdots, N\}$

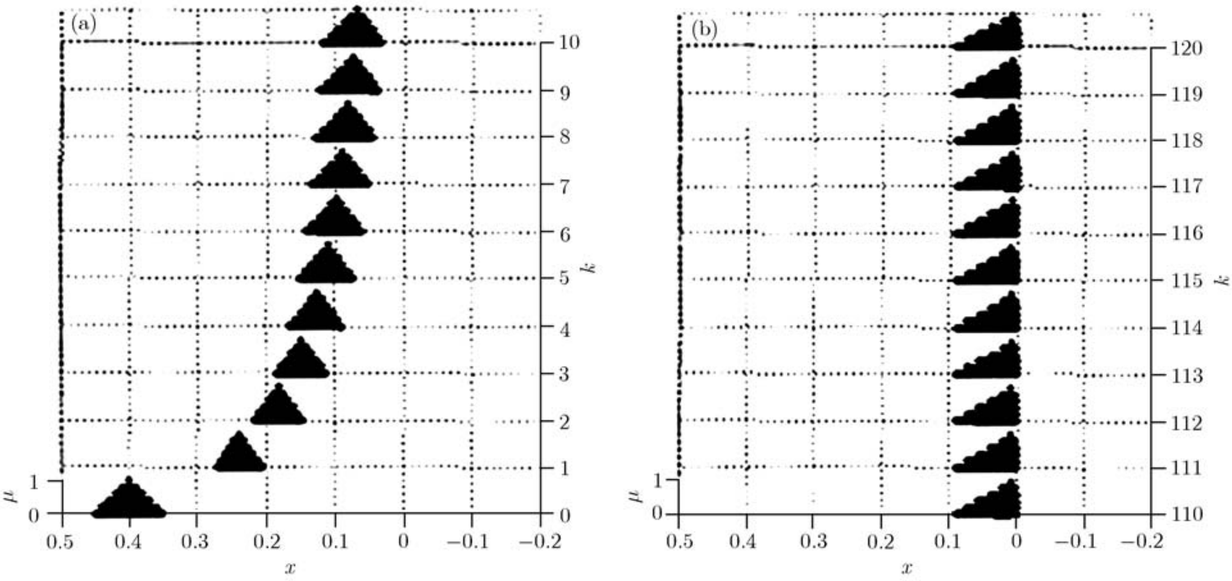


Figure 9 (a) LDS of the former 10 times iteration; (b) LDS of 110P$^{\rm thP}$-120P$^{\rm thP}$ times iterations.

or $(X_k, X_{k+1}) \to (A_1 \times B_1 + \cdots + A_N \times B_N)$,

where $A_k \in \Re(A), B_k \in \Re(A)$. Let B_k be the image word of A_k under the rule $\tilde{R}_k$, N be the number of fuzzy rule ,"×" be the Descartes product, "+" be the conjunctive symbol.

The image word of arbitrary initial word $A'_0 \in \Re(A)$ is $\tilde{R}(A'_0)$ under $\tilde{R}$.

By (18),

$$\tilde{R}(A'_0) = \sum_{i=1}^{N} \lambda_{i0} \wedge B_i,$$

where $\lambda_{i0} = \sup_x(\mu_{A_i}(x) \wedge \mu_{A'_0}(x))$.

Let $A'_1 = \tilde{R}(A'_0)$, and for the same reason, the image word of A'_1 under $\tilde{R}$ is

$$\tilde{R}(A'_1) = \sum_{i=1}^{N} \lambda_{i1} \wedge B_i, \tag{19}$$

where

$$\lambda_{i1} = \sup_x(\mu_{A_i}(x) \wedge \mu_{A'_1}(x)), \tag{20}$$

and

$$\mu_{A'_1}(x) = \sum_{j=1}^{N} \lambda_{j0} \wedge \mu_{B_j}(x), \tag{21}$$

Embedding (21) into (20), we have

$$\lambda_{i1} = \sum_{j=1}^{N} \lambda_{j0} \wedge \lambda_{ij},$$

where $\lambda_{ij} = \sup_x(\mu_{A_i}(x) \wedge \mu_{B_j}(x))$.

Written $A'_2 = \tilde{R}(A'_1)$, so as on, the image of A'_k under $\tilde{R}$ is

$$\tilde{R}(A'_k) = \sum_{i=1}^{N} \lambda_{ik} \wedge B_i,$$

where $\lambda_{ik} = \sup_x(\mu_{A_i}(x) \wedge \mu_{A'_k}(x))$, written as $A'_{k+1} = \tilde{R}(A'_k)$.

LDS of word A' under the fuzzy rule basis $\tilde{R}$ is $\{A'_k, k \in Z^+\}, Z^+$ denoting positive integer number, where $A'_k = \tilde{R}(A'_{k-1}) = \sum_{i=1}^{N} \lambda_{i(k-1)} \wedge B_i, \lambda_{i(k-1)} = \sup_x(\mu_{A_i}(x) \wedge \mu_{A'_{k-1}}(x))$.

For the type-II LDS defining on the universe of discourse A, $\{A_1, A_2, \cdots, A_N\}$ are N basis words covering A, the corresponding fuzzy rule is $\tilde{R}$.

The numerical arithmetic of CW of type-II LDS is

Step 1: initialization.

Set $i = 0$, the initial word $X'_0 = \{(x, \lambda)| x \in X, \mu_{X'_0}(x) = \lambda\}$ and let $\lambda_{j0} = \sup_x(\mu_{A_j}(x) \wedge \mu_{A'_0}(x)), j \in \{1, 2, \cdots, N\}$.

Step 2: transformation.

The $(k+1)$th state X_{k+1} is

$$X_{k+1} = \sum_{j=1}^{N} \lambda_{jk} \wedge B_j.$$

Step 3: i-loop.

If $i < M$, then $i \leftarrow i + 1$, and return to step 2, otherwise, stop.

Example 10. Under the normal atmosphere pressure, people heat the cool water, hot water and boiling water respectively, and for every t minutes, observing the change of water-temperature, have the following three fuzzy rules:

$\tilde{R}_1$: heat the cool water for about t minutes and it turns hot water;

$\tilde{R}_2$: heat the hot water for about t minutes and it turns boiling water;

$\tilde{R}_3$: heat the boiling water for about t minutes and it turns steam.

Let $\tilde{R} = \bigcup_{i=1}^{3} \tilde{R}_i$, for arbitrary initial state, we can get the LDS orbit under the fuzzy rule $\tilde{R}$. Obviously, for the initial states appearing in the fuzzy rule, their dynamics are achieved from fuzzy rule directly. For example, heating "cool water", for every t min, we observe the water temperature and the dynamical course is {cool water, hot water, boiling water, steam}.

But for the initial state not appearing in the fuzzy rule, such as "warm water", how can we depict the dynamical state of water temperature after being heated?

Supposing the membership function of cool water, hot water, boiling water and warm water are as follows:

$$\mu_{\text{cool water}}(x) = \begin{cases} 0.2(x-14) & \text{if } 14 \leqslant x \leqslant 18, \\ 1 & \text{if } 18 < x \leqslant 30, \\ 0.2(35-x) & \text{if } 30 < x \leqslant 35, \\ 0 & \text{otherwise;} \end{cases}$$

$$\mu_{\text{hot water}}(x) = \begin{cases} 0.2(x-40) & \text{if } 40 \leqslant x \leqslant 45, \\ 1 & \text{if } 45 < x \leqslant 60, \\ 0.2(60-x) & \text{if } 55 < x \leqslant 60, \\ 0 & \text{otherwise;} \end{cases}$$

$$\mu_{\text{boiling water}}(x) = \begin{cases} 0.1(x-55) & \text{if } 55 \leqslant x \leqslant 65, \\ 1 & \text{if } 65 < x \leqslant 85, \\ 0.1(95-x) & \text{if } 85 < x \leqslant 95, \\ 0 & \text{otherwise;} \end{cases}$$

$$\mu_{\text{steam}}(x) = \begin{cases} 0.1(x-85) & \text{if } 85 \leqslant x \leqslant 95, \\ 1 & \text{if } 95 < x < 105, \\ 0 & \text{otherwise;} \end{cases}$$

$$\mu_{\text{warm water}}(x) = \begin{cases} 0.2(x-15) & \text{if } 23 \leqslant x \leqslant 28, \\ 1 & \text{if } 28 < x \leqslant 38, \\ 0.2(43-x) & \text{if } 38 < x \leqslant 43, \\ 0 & \text{otherwise.} \end{cases}$$

After being heated, the dynamical course of warm water is as shown in Figure 10, where x-axis denotes the universe of discourse of water temperature and k denotes the times of iteration and μ is the membership function.

Figure 10 shows undergoing states when the warm water is heated. After one times iteration, it shows the water is hot and it has a little steam, and after two times iteration, it becomes boiling water with a fair volume of steam, and after three times iteration, heated water turn to the 4th stage, and at this time, the water has become steam and running, thus all the dynamical course accords to the reality.

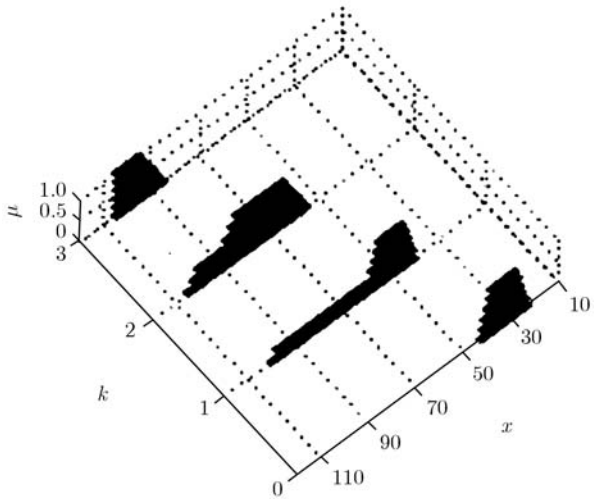


Figure 10 LDS of warm water being heated.

In type-II LDS, we introduce basis words and rule basis, and express arbitrary words as the synthetic form of basis word and then analyze, forecast the behavior of complex system. By using the degree of match of initial word and basis words, we get the image word under the fuzzy rule basis and then give the dynamical orbit of type-II LDS.

6 Application of computing with words in financial system

Financial system is one of the most complex systems created by human. In financial system, the partial behavior is not very complex when government, collective and individual take action with the external. However, their interaction can make the market floating heavily or make it crumble, and at the same time, their interaction promote the market, so financial system is a huge complex system[18,19], and the integral motion of the market display many dynamical properties, such as chaos, etc.[20,21].

In addition, the internet play an important pole on people's life, and the financial system also take a great change: the financial action is electronic and the cost of transaction drop with the amount of transaction increasing, and the over-inflated financial fictitious asset leads to financial foam and financial crisis[22,23]. All this shows that there exists a great amount nonlinear phenomena in financial system, and the problem cannot be resolved by a series of differential equations.

To analyze financial system, we can use the way of computing with words to mimic human' property to dealing with the financial problem in everyday life, this is also the method of personifying intelligent. Individual, short-time financial act is precise and be measured, however, the integral and whole act is fuzzy and incomplete. Computing with words is an effective way to dealing with precise numerical information and fuzzy language information.

Making a strategy decision, investors hope return on asset the higher the better, so if there exists difference among investing on different trades (for example, supermarket and electronic systems), or different systems of the same trade (for example, Rolls-Royce and Cherokee), then the fund being invest on low-return-on-asset trade will be transferred to the high-return-on-asset trades until the difference of return on asset among different trades has disappeared[24].

For return on asset, we have the following formula:

$$\text{return on asset} = \text{ratio of sales} \times \text{turnover of assets}. \quad (22)$$

X, Y, Z denote the universe of discourse of ratio of sales, turnover of capital and return on asset respectively, {low, high} denote the fuzzy sets of $\Re(X), \Re(Y)$, however, the fuzzy set of $\Re(X)$ is not the same as that of $\Re(Y)$ as Figure 11(a) and (b) respectively, {low, middle, high} denoting the fuzzy set of $\Re(Z)$, its degree of membership is as Figure 11(c).

From Figure 11(a)–(c) and (22), there exist the following four fuzzy rules:

$\tilde{R}_1$: if the ratio of sales is low and turnover of capital is low, then return on asset is low.

$\tilde{R}_2$: if the ratio of sales is low and turnover of capital is high, then return on asset is middle.

$\tilde{R}_3$: if the ratio of sales is high and turnover of capital is low, then return on asset is middle.

$\tilde{R}_4$: if the ratio of sales is high and turnover of capital is high, then return on asset is high.

Let $\tilde{R} = \sum_{i=1}^{4} \tilde{R}_i$, then $Z = \tilde{R}(X, Y)$, where the same symbols is used to denote the universe of discourse of return on asset, the ratio of sales and turnover of capital and the corresponding variances on the three universes of discourse.

For every $X_0 \in \Re(X), Y_0 \in \Re(Y)$, by $Z = \tilde{R}(X, Y)$, there exists $Z_0 \in \Re(Z)$ such that $Z_0 = \tilde{R}(X_0, Y_0)$.

When the invested projects are chosen, the investors' objective is to achieve the highest return on asset, so

$$\text{the highest return on assets } \wedge \tilde{R}_j = \phi, j = 1, 2, 3,$$

then

$$Z^0 = \text{the highest return on assets } \wedge \sum_{i=1}^{4} \tilde{R}_i(X, Y) = \text{the highest return on assets } \wedge \tilde{R}_4(X, Y),$$

thus

$$\mu_{Z^0}(z) = \max_{x \in X, y \in Y} (\mu_{X_0}(x), \mu_{Y_0}(y), \mu_{\tilde{R}_4}(x, y)),$$

where, $z = \tilde{R}_4(x, y)$, it is to say the investor will choose the trade that is high return on asset and high turnover of capital to invest.

In fact, because of the use of law of value, for the highly return on asset trade, ratio on sales and turnover of capital decrease and even to be a low-return-on-asset trade for heavily increasing invest, on the other hand, for the low-return-on-asset trade, ratio of sales and turnover of capital will increase because of low invest and even to be a high-return-on-asset trade. And so as on, returns on assets eventually will be equilibrium. Thus, the following two of the four rules are effective:

$\tilde{R}_2$: if the ratio of sales is low and turnover of capital is high, then return on asset is middle.

$\tilde{R}_3$: if the ratio of sales is high and turnover of capital is low, then return on asset is middle.

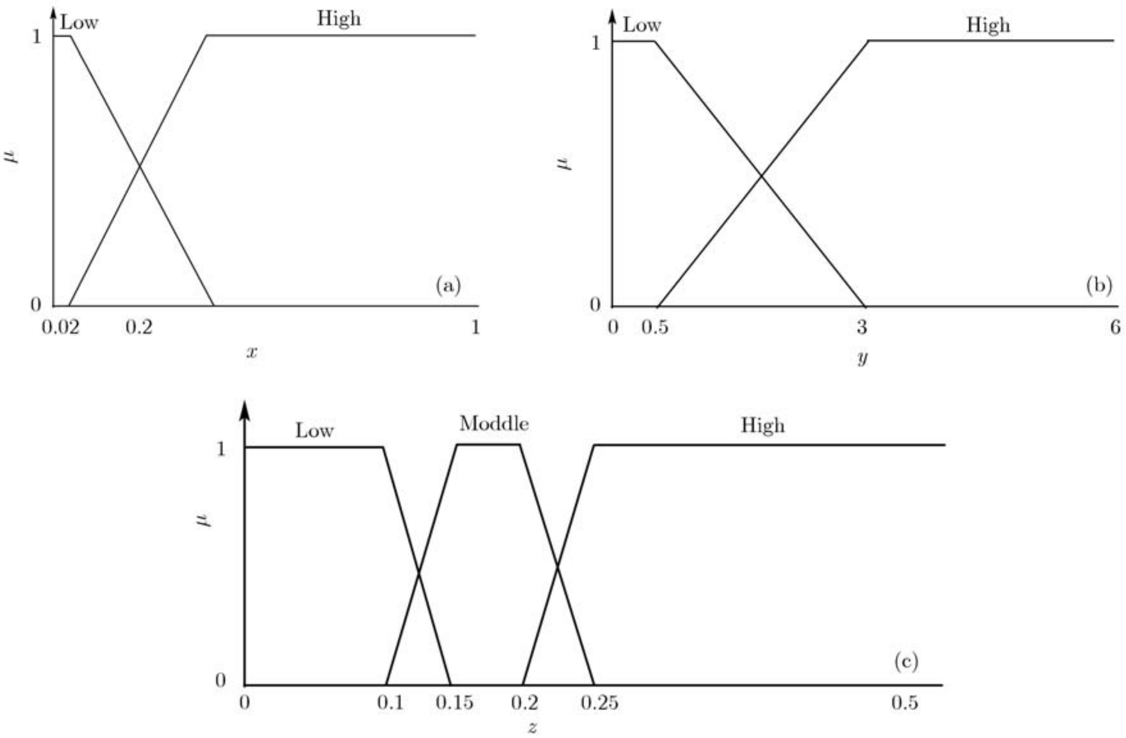


Figure 11 (a) Ratio of sales; (b) turnover of capital; (c) return on asset.

So people can understand the following phenomenon: even though the ratios on sales of the different trades have great difference, returns on assets are equal at last. This is why the supermarket is typical trade of low ratio of sales and highly turnover of capital, however, the expensive jewelry is the typical trade of high ratio of sales and low turnover of capital.

7 Conclusion

In this paper, LDS is divided into two types: type-I LDS and type-II LDS, and for different types of LDS, different computing methods are used: type-I CW and type-II CW. For type-I LDS, the way of endograph is presented for numerical value analysis, the stability of LDS is compared to that of CDS, and the stabilities of the abstractions of logistic mappings are discussed. For type-II LDS, by using the method of degree of match of the initial word and the basis words, the image words are determined, and then the orbit of LDS does. In the paper, the investor' profit and net value profit are analyzed by the way of CW.

CW is used to mimic the human's ability of manipulating language and conception to analyze and predict the behaviour of fuzzy systems. This not only simplifies the description of systemic states, dispose of fuzzy information validly, but also make the human's strategy more accord to the fact, and at the same time, it can decrease the price of making strategy. Recently, more and more people focus on type-2 fuzzy sets and their application in complex systems[25,26]. In the future, the main work is to use CW based on type-2 fuzzy sets and its application in complex systems.

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