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THE APPLICATION OF ADHDP(λ) METHOD TO COORDINATED MULTIPLE RAMPS METERING

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Abstract. *Ramp metering has been developed as a local traffic management strategy to alleviate congestion on freeways sections, however, the coordination of multiple ramps metering still need more attention of effective control strategies on a larger freeway section. The object of this paper is to use action-dependent heuristic dynamic programming based on eligibility traces (ADHDP(λ)) method to implement coordinated ramp metering. The traffic flow plant is a second order macroscopic traffic flow model. The whole coordinated ramp metering problem is considered as an approximate optimal control problem. A valid coordination performance index is proposed. ADHDP(λ) method is an effective and fast learning control method to solve such problems. With the help of eligibility traces, the learning rate is highly improved. Simulation studies on a hypothetical freeway are reported to show that the proposed control scheme is efficient.*

Keywords: Heuristic dynamic programming, Eligibility traces, Multiple ramps metering

1. **Introduction.** Ramp metering is implemented as a traffic signal that is placed at the on-ramp of a freeway as is represented in Figure 1. Ramp metering means metering the traffic allowed entering the freeway through the on-ramps, and it has been regarded as the most efficient means to alleviate the freeway traffic condition. Ramp metering can maintain uninterrupted, non-congested traffic flow on the freeway, and increase traffic volume in mainline due to avoidance or reduction of congestion duration. Ramp metering has been verified efficient both in theoretical and practical aspects [1]. Ramp metering problems are divided into local ramp metering and multiple ramps metering problems.

The existing various local ramp control algorithms can be generally classified into two categories: fixed time metering and traffic-responsive metering. The latter is proved to be more effective in dealing with freeway congestion problems than the former. Typical algorithms of this kind include occupancy algorithm [3], ALINEA (a linear local feedback control algorithm) [4], and LQR (linear quadratic regulation) [5]. These metering algorithms only provide linear control laws, so they may not be able to control the ramp system which is known as a nonlinear model. For example, ALINEA responses concussively to drastic traffic flow variations, causing unsafe effect on freeway.

Local ramp metering, only considering the local freeway traffic condition, could not handle multiple ramps metering problems, because of the interacting influence of traffic condition on the upstream and the downstream. Therefore, the coordinated ramp metering which is based on the system-wide traffic information has attracted many experts' interest. The multiple ramps metering problem is illustrated in Figure 2. The question for multiple ramps metering becomes how the ramp metering should be designed taking into account the interactions among the various ramps.

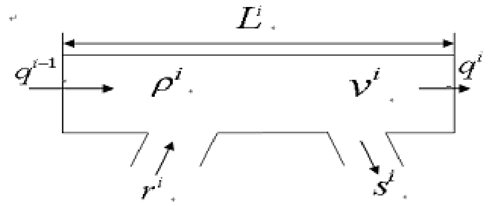

FIGURE 1. A freeway section with both on and off ramps

METALINE [30] was a coordinated feedback control method that can be used to coordinate control of several ramps. FLOW [31] was a heuristic area-wide coordinated control strategy. Model predictive control (MPC) [32] based ramp metering determined the appropriate metering rates by optimizing an objective function. Store-and-forward [27] modeling of traffic networks was first suggested by Gazis and Potts, and widely used in traffic control. There also existed some valid ideas from previous ramp metering algorithms: the simple Helper [28] and the Bottleneck [31]. Although the above algorithms achieved great success, they still possess some deficiency, e.g., most of them need prior knowledge and accurate traffic flow models.

To solve the problems presented above and achieve optimal traffic conditions by ramp metering, some researchers introduced neural network technique and further present the adaptive dynamic programming (ADP) theory [7,13-15,24-25]. We applied ADHDP(λ) theory to solve local ramp metering efficiently in previous studies [20]. In this paper, the coordinated multiple ramps metering problems are fully investigated by ADHDP(λ).

This paper is organized as follows: Section II presents the traffic flow model of multiple ramps metering. Section III applies the coordinated control method ADHDP(λ) to implement coordinated ramps metering. In Section IV, numerical simulations are conducted to demonstrate ADHDP(λ) method is effective to multiple ramps metering. Conclusions are summarized in Section V.

2. **Traffic Model of Coordinated Ramp Metering Control.** The traffic model used in this paper is originally derived by Payne [21] and modified by Cremer and May [22]. For a freeway lane which is subdivided into N sections with length $L_i (i = 1, ..., N)$, each having at most one on-ramp and one off-ramp as shown schematically in Figure 1, the evolution of the traffic flow is described by

$$\rho_{t+1}^i = \rho_t^i + \frac{T}{L^i}[q_t^{i-1} - q_t^i + u_t^i - s_t^i] \tag{1}$$

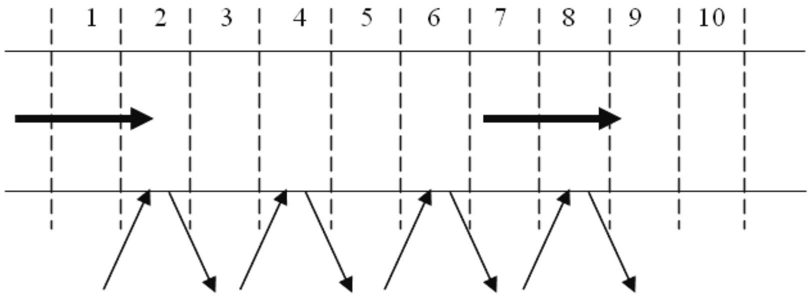

FIGURE 2. Schematic diagram of multiple ramps metering in a freeway section

$$q_t^i = \alpha\rho_t^i v_t^i + (1-\alpha)\rho_t^{i+1} v_t^{i+1} \tag{2}$$

$$v_{t+1}^i = v_t^i + \frac{T}{\Delta t}\{V_e[\rho_t^i] - v_t^{i+1}\} + \frac{T}{L^i} v_t^i [v_t^{i-1} - v_t^i] - \frac{\mu Y}{\Delta t L^i} \frac{\rho_t^{i+1} - \rho_t^i}{\rho_t^i + \kappa} \tag{3}$$

Here ρ_t^i (vehicles/km) is the average density per lane and q_t^i (vehicles/h) is the traffic flow leaving section i to enter section $i+1$. u_t^i(vehicles/h) is the entry ramp volume (i.e. the control action) and s_t^i(vehicles/h) is the off-ramp volume. T(h) is the time increment, L^i(km) is the section length. $0 < \alpha < 1$ is the weighting factor, Δt, μ, and κ are positive constants. v_t^i(km/h) is the average speed. $V_e[\rho_t^i]$ is the mean speed equilibrium modeled by

$$V^e[\rho_t^i] = v^f\{1 - [\frac{\rho_t^i}{\rho^{jam}}]^a\}^b \tag{4}$$

Here v^f, ρ^{jam}, a and b are constants to be identified for real traffic flow. The off-ramp volume s_t^i is related to the traffic volumes q_t^{i-1} through the equation $s_t^i = \varepsilon^i q_t^{i-1}$, where $0 < \varepsilon < 1$. Substituting this equation and (2) into (1) yields

$$\rho_{t+1}^i = \rho_t^i + \frac{T}{L_i}\left[\alpha(1-\varepsilon^i)\rho_t^{i-1}v_t^{i-1} + (1-2\alpha+\varepsilon^i)\rho_t^i v_t^i - (1-\alpha)\rho_t^{i+1}v_t^{i+1} + r_t^i\right] \tag{5}$$

3. **Action-Dependent Heuristic Dynamic Programming based on Eligibility Traces.** A typical structure of Action Dependent Heuristic Dynamic Programming (AD-HDP) has two components, Critic Network and Action Network. The Action Network outputs the control signal, and the Critic Network outputs an estimate of function J_t (cost to go)in the Bellman equation of dynamic programming [10,11]. ADHDP can be combined with eligibility traces to obtain a more effective learning method ADHDP(λ) scheme, as is shown in Figure 3, where λ refers to the use of eligibility traces. Eligibility traces are basic mechanisms of reinforcement learning. Action Network is the controller. When the state x_t inputs to the Action Network, the control signal u_t is output. Then both of the state and control signal are input to the Critic Network, and J_t is output. Next is to train the Critic Network with the target by $r_t + J_t$ and train the Action Network with the target of minimizing J_t. After u_t is input to the plant, x_{t+1} is obtained.

3.1. **The forward and backward view of ADHDP(λ).** There are two ways proved to be equivalent to explain ADHDP(λ). The first way is more theoretical, and the other way is more mechanistic. In the forward view, the ADHDP(λ) method uses multi-step future states. Rather than a single future state, n-step backups use a weighted average of future states as a target for learning. ADHDP(λ) is really a spectrum of algorithms, controlled by the continuous valued parameter λ. When λ=0, ADHDP(λ) uses the earliest possible state as the target for learning. While λ=1, ADHDP(λ) uses the latest possible state as the target. For other values of λ, the target for learning are distributed among all of the states along the way. In the backward view, to overcome the shortcomings of using knowledge to happen many steps later, which is involved in the forward view of ADHDP(λ), we use additional memory variables associated with each state, which are its eligibility traces. The eligibility traces for state x at time t is denoted $e_t(x)$ which is illustrated below. A detailed description can be acquired by referring to [8].

3.2. **The action network.** The weights updating equations in the Action Network are as follows. l^{action} is the learning rate of the Action Network at time t, and w_t^{action} is the weight vector in the Action Network. The training of the Action Network is the same in both ADHDP and ADHDP(λ) algorithms.

$$
\begin{aligned}
e_t^{action} &= J_t && (6)\\
E_t^{action} &= \frac{1}{2}(E_t^{action})^2 && (7)\\
w_{t+1}^{action} &= w_t^{action} + \Delta w_t^{action} && (8)\\
\Delta w_t^{action} &= l^{action}[-\frac{\partial E_t^{action}}{\partial w_t^{action}}] && (9)\\
\frac{\partial E_t^{action}}{\partial w_t^{action}} &= [\frac{\partial E_t^{action}}{\partial J_t}\frac{\partial J_t}{\partial u_t}\frac{\partial u_t}{\partial w_t^{action}}] && (10)
\end{aligned}
$$

3.3. **The critic network.** The Critic Network outputs J_t, while J_t approximates the discounted total reward-to-go. To be more quantitative, it approximates R_t at time t given by (11), where R_t is the future accumulative reward-to-go value at time t.

$$R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + ... \tag{11}$$

We use the Critic Network to generate J_t as an approximate of R_t. Our aim is minimizing the following error measure over time. l^{critic} is the learning rate of the Critic Network at time t, and w_t^{critic} is the weight vector in the Critic Network.The weight update rule for the network is given by the following equations. The difference between ADHDP and ADHDP(λ) is the training of the Critic Network.

$$
\begin{aligned}
e_t^{critic} &= \gamma J_t - [J_{t-1} - r_t] && (12)\\
E_t^{critic} &= \frac{1}{2}(e_t^{critic})^2 && (13)\\
w_{t+1}^{critic} &= w_t^{critic} + \Delta w_t^{critic} && (14)\\
\Delta w_t^{critic} &= l^{critic} e_t(x)\delta_t && (15)\\
e_t(x) &= \gamma\lambda e_{t-1}(x) + \frac{\partial J_t}{\partial w_t^{critic}} && (16)
\end{aligned}
$$

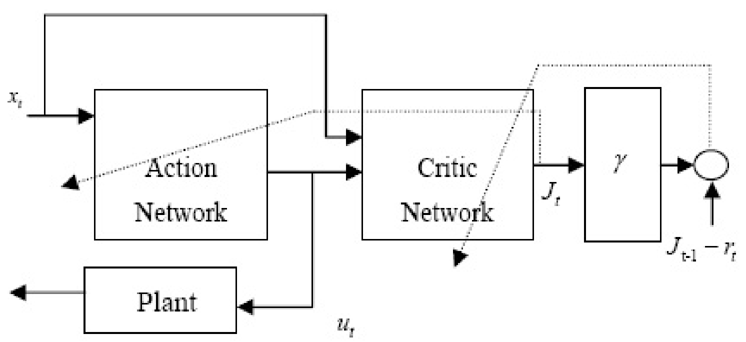

FIGURE 3. Schematic diagram for implementations of ADHDP(λ)

As we all know, the ADHDP structures learn the environment according to the Critic Network. So the learning efficiency of the Critic Network is critical. The ADHDP(λ) average n-step backups and then it integrate all the n-step backups' rewards. With the introducing of eligibility traces, the learning efficiency of the Critic Network is greatly improved, so ADHDP(λ) can efficiently handle more complex coordinated ramps metering problems.

3.4. **ADHDP(λ) for coordinated ramps metering.** In this section, we use ADHDP(λ) to solve the multiple ramps metering control problem. As shown in Figure 2, a freeway is composed of ten interactive sections. The most important control objectives for ramp metering can be summarized as follows: to maximize the traffic flow of the freeway, maximize the mean speed of the vehicles, improve the safety of traffic operation on the freeway and minimize the total time spent (TTS) by all the vehicles in the Network. Papageorgiou [33] showed that, with the condition that the network inflow is known, to keep the traffic density consistent with critical density all the time is equivalent to maximize the traffic flow of the freeway. So in this paper, we define the system performance index as (17), where ρ^{desire} is the traffic density which is little lower than the critical density. We can maintain the traffic density ρ_t^i around ρ^{desire} all the time by minimizing such a system performance through regulating the ramp metering rate u_t^i.

$$r_t = c \sum_{n=1}^{10} \zeta^n (\rho_t^n - \rho^{desire})^2 \tag{17}$$

Here, c is a positive parameter and $\zeta^n (n = 1, \cdots, 10)$ are weighing parameters. The summation of ζ is 1. These parameters are set to $\zeta^n = \frac{1}{10}$. The difference between local ramp metering and coordinated multiple ramp metering is that we have to consider the interactions among different sections in the whole freeway. As shown in Figure 2 , there are on-ramps and off-ramps in section 2, 4, 6 and 8. When applying ADHDP(λ), we consider the performances of all sections in (17). Then the coordinated ramp metering of the whole freeway is achieved.

The coordination of the metering rates of the different on-ramps ensures that the control actions taken at different locations in the network reinforce rather than cancel each other. In this way, coordination of ramp metering often leads to better results than the combination of multiple independently locally controlled ramp metering. As shown in Figure 2, there are four ramp metering controllers in this freeway. With ADHDP(λ) method, the four controllers have the same parameters. For the training of both the networks, we define K_{max} as the maximal epoch. Figure 4 shows the flowchart of an ADHDP(λ) trial, where the training details of Critic and Action Networks are given as follows:

The algorithm for the Critic training cycle

1. Initialize traffic densities, average speeds;
2. Normalize x_t^i and input it to the Action Network to obtain u_t^i;
3. Input u_t^i to the Critic Network in addition with x_t^i, obtain J_t;
4. Use eligibility traces to train the Critic Network at time $t-1$ with the target given by $r_t + J_t$;
5. Input u_t^i to the plant and obtain x_{t+1}^i;
6. If x_{t+1}^i is out of the specified range, 8;
7. If $t < K_{max}$, increment t and go to 2.;
8. Go to the Action training cycle.

The algorithm for the Action training cycle:

1. Normalize x_t^i and input it to the training Action Network to obtain new u_t^i;
2. Input u_t^i to the Critic Network in addition with x_t^i, obtain J_t;
3. Train the Action Network with the target of minimizing J_t;
4. Input x_t^i to the trained Action Network,obtain new u_t^i;
5. Input u_t^i to the plant and obtain x_{t+1}^i;
6. If x_{t+1}^i is out of the specified range, 9;
7. If $t < K_{max}$, increment k and go to 2.;
8. Check the control performance, stop if it is acceptable;
9. Go to the Critic training cycle.

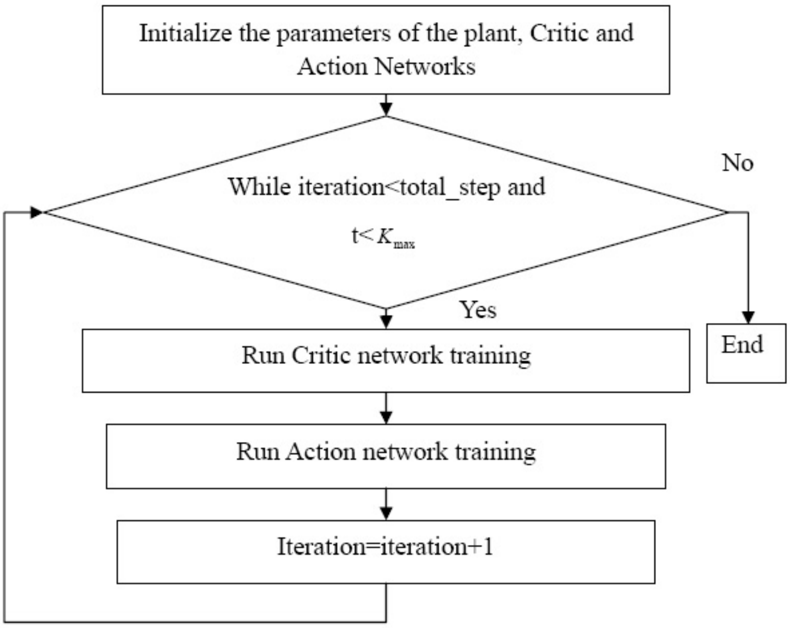

FIGURE 4. The algorithm flow chart

3.5. **ALINEA for multiple ramps metering.** ALINEA [4] is a linearized feedback control algorithm that adjusts the metering rate in order to keep the occupancy downstream of on-ramps at a desired level. ALINEA maintains the desired level of occupancy by using feedback regulation. The ALINEA closed-loop ramp metering strategy is

$$u_t^{ALINEA} = u_{t-1}^{ALINEA} + K^A \left[\rho^{desire} - \rho_t\right] \tag{18}$$

where K^A is a regular parameter and u_t^{ALINEA} is the control signal.

ALINEA is widely adopted for the control of ramp metering around the world, it is used here as a benchmark to compare the performance of ADHDP(λ).

4. **Simulation.** The performance of two different types of traffic-responsive ramp metering strategies ALINEA and ADHDP(λ) are analyzed using simulation. Then the system performance index selection for ADHDP(λ) is deeply studied.

4.1. **Tests of ADHDP(λ) for multiple ramps metering.** In this section, we will demonstrate the use of ADHDP(λ) for the coordinated control of ramp metering. The case is studied upon a hypothetical freeway consisting of ten sections with four on-ramp and four off-ramps, as shown in Figure 2. The Action Network and Critic Network are both implemented using multilayer feedforward neural Networks. The parameters of the Action Network and the Critic Network are initialized randomly.

The parameters of the traffic model are given as:

$T = 15s$, $L^i = 0.6km$, $\alpha = 0.9$, $\varepsilon^i = 0.15$, $\Delta t = 36s$, $\mu = 21.6\text{km}^2/h$, $\kappa = 20veh/km$, $v^f = 123km/h$, $\rho^{jam} = 150veh/km$. The critical density in the freeway is 47 *veh/km*. So we set $\rho^{desire} = 45veh/km$. The ramp metering rate is confined in the range of 0 to 1000. Parameters are chosen as c=0.5, γ=0.5, K^A=50.

In the first test, we study the tracking capability and transient responses of controllers in resisting stochastic variations of the traffic demand. With system states and traffic demand as shown in Figure 5, the ALINEA and ADHDP(λ) controllers are employed for control. It can be seen clearly from Figure 6 and Figure 7 that the ADHDP(λ) controller results in smaller overshoot than ALINEA do. Test results show that even if the inflow volumes are large, ALENEA and ADHDP(λ) all work well when freeway inflow volumes

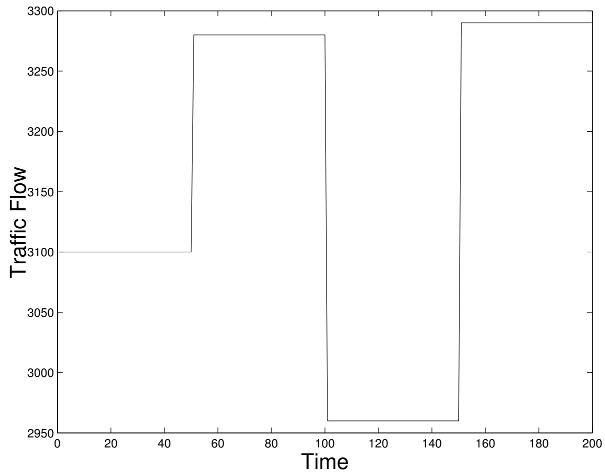

FIGURE 5. Freeway inflow volumes

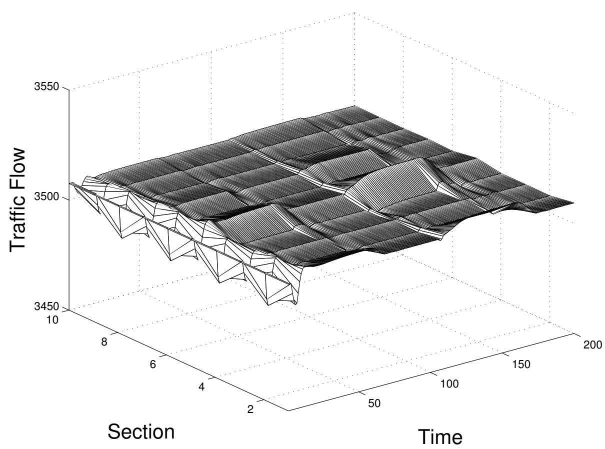

FIGURE 6. Traffic flow of ALINEA method in the first test

Table 1. Initial conditions of sectional density and speed for the second test

Section No	1	2	3	4	5	6	7	8	9	10
ρ_0^i	45	45	45	45	45	45	45	70	70	45
v_0^i	78	78	78	78	78	78	78	40	40	78

vary mildly. The performance of ALENEA relies on the tuning of K^A, but ADHDP(λ) has better self-adaptive capability.

In the second test, we check the ADHDP(λ) controller's capability of resolving congestions. Specifically, we initialize the system states as those listed in Table 1, which are calculated according to (1)-(3). With this condition, we find that the ALINEA ramp metering algorithm leads to the traffic breakdown, as shown in Figure 8. When the densities are far more than the critical density, the traffic model is not fit for this situation, so only the results in the first 30 steps are shown in Figure 8. However, as we can see from Figure 9, ADHDP(λ) has remarkable improvement in the smoothness and efficiency of keeping the maximal traffic flow; the ADHDP(λ) controller can handle traffic congestion well, and

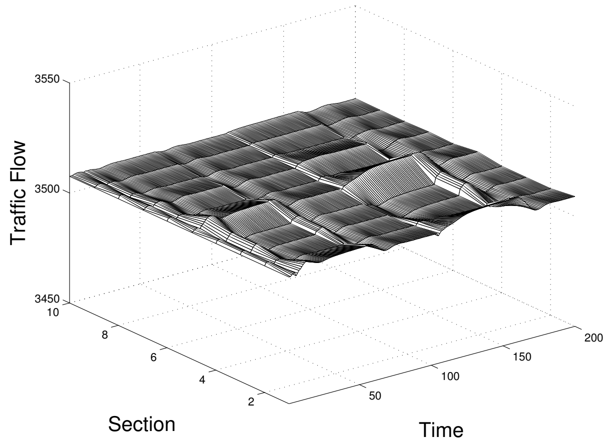

Figure 7. Traffic flow of ADHDP(λ) method in the first test

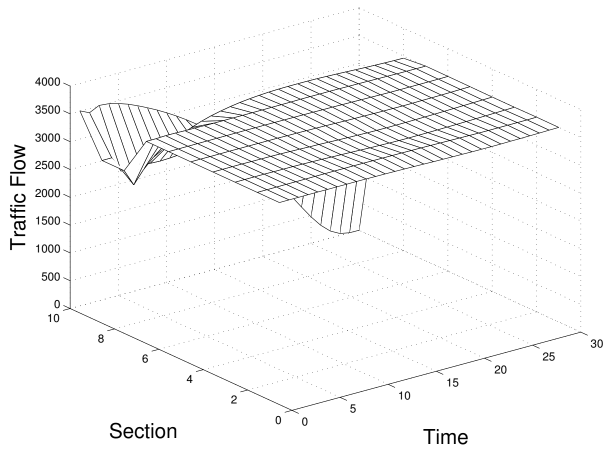

Figure 8. Traffic flow of ALINEA method in the second test

traffic density of all the sections are controlled to the desired densities by generating coordinated metering rates. ADHDP(λ) works well in the stochastic, nonlinear and varying environment, although it is introduced in a deterministic form in the previous section.

5. Conclusions.

Ramp-metering control strategy is often implemented during rush hours in heavily congested areas. In this paper, ADHDP(λ) method is used as an advanced control technique to determine the appropriate metering rates for coordinated ramp metering. The typical ALINEA feedback control methodology is adopted for comparison. ADHDP(λ) method provides many attractive features. Simulation results indicate that ADHDP(λ) method has better control performance compared to the ALINEA ramp metering algorithm. ADHDP(λ) controllers make use of all available mainstream measurements on a freeway stretch to calculate simultaneously the ramp volume values for all controllable ramps included in the same stretch. This provides potential improvements because of more comprehensive information provision and coordinated control actions. ADHDP(λ) method combines eligibility traces and ADHDP, so it can offer significantly faster learning than traditional ADHDP method. The ADHDP(λ) controller can generate more smooth control actions under drastically varying traffic inflow, which ensures more safety and also results in more efficiency in keeping the maximal traffic flow. In this article, the coordinated control of ramp metering problem is qualified as an instance, and more complex traffic control problems can be treated as well.

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REFERENCES

[1] M. Papageorgiou and Kotsialos, Freeway ramp metering: An overview, *IEEE Transactions on Intelligent Transportation Systems*, vol.3, pp.271-281, 2002.
[2] J. Robinson and M. Doctor, Ramp metering status in north America, *Final Report*, Office of Traffic Operations, Federal Highway Administration, U.S. Department of Transportation, Washington, DC.
[3] R. Wilshire, R. Black, R. Grochoske and J. Higonbotham, *ITE Traffic Control Systems Handbook*, Institute of Transportation Engineers, Washington, DC.
[4] M. Papageorgiou, H. Hadj-Salem and J. M. Blosseville, ALINEA: A local freedback control law for on-ramp metering, *Trans. Research*, pp.194-198, 1990.

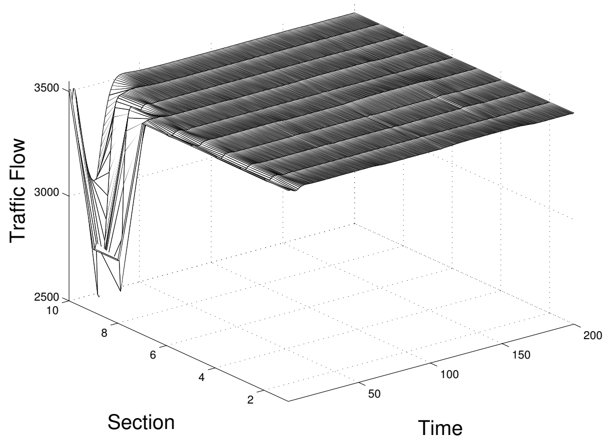

Figure 9. Traffic flow of ADHDP(λ) method in the second test

[5] N. B. Golstein and K. S. P. Kumar, A decentralized control strategy for freeway regulation, *Trans. Research Part B*, vol.16, pp.279-290, 1982.
[6] H. Zhang and S. Ritchie, Freeway ramp metering using artificial neural networks, *Trans. Res. Part C*, vol.5, pp.273-286, 1997.
[7] D. R. Liu, Action-dependent adaptive critic designs, *Proc. of the IEEE Conf. on Neural Networks*, vol.2, pp.15-19, 2001.
[8] R. Sutton and A. Barto, *Reinforcement Learning: An Introduction*, The MIT Press, 1998.
[9] L. Kaelbling, M. Littman and A. Moore, Reinforcement learning: A survey, *Journal of Artificial Intelligence Research*, vol.4, pp.237-285, 1996.
[10] J. Si and Y. T. Wang, On-line learning control by association and reinforcement, *IEEE Transactions on Neural Network*, vol.12, 2001.
[11] D. D. Prokhorov and D. C. Wunsch, Adaptive critic designs, *IEEE Transactions on Neural Networks*, vol.8, pp.997-1007, 1997.
[12] G. G. Lendaris and C. Paintz, Training strategies for critic and action neural networks in dual heuristic programming method, *Proc. of the IEEE Conf. on Neural Networks*, Houston, TX, vol.6, pp.712-717, 1997.
[13] P. J. Werbos, Consistency of HDP applied to a simple reinforcement learning problem, *Proc. of the IEEE Conf. on Neural Networks*, vol.3, pp.179-189, 1990.
[14] P. J. Werbos, Approximate dynamic programming for real-time control and neural modeling, *Handbook of Intelligent Control: Neural, Fuzzy, and Adaptive Approaches*, D. A. White and D. A. Sofge Van Nostrand Reinhold, 1992.
[15] G. G. Lendaris and C. Paintz, Training strategies for critic and action neural networks in dual heuristic programming method, *Proc. of the IEEE Conf. on Neural Networks*, Houston, TX, vol.6, pp.712-717, 1997.
[16] M. J. Lighthill and G. B. Whitham, On kinematics waves: II. A theory of traffic flow on long crowded roads, *Proc. of the Royal Society*, London, Ser. A., vol.229, pp.317-345, 1955.
[17] J. Xu, W. S. Yu, and F. Y. Wang, Ramp metering based on adaptive critic designs, *Proc. of the IEEE Conf. on Intelligent Transportation Systems*, Toronto, Canada, pp.1531-1536, 2006.
[18] S. A. Nsour, S. L. Cohen, J. E. Clark and A. J. Santiago, Investigation of the impacts of ramp metering of traffic flow with and without diversion, *Transportation Research*, vol.1365, pp.116-130, 1992.
[19] C. Jacob and B. Abdulhai, Automated adaptive traffic corridor control using reinforcement learning: approach and case studies, *Proc. of the IEEE Conf. on Transportation*, pp.1-8, 2006.
[20] X. R. Bai, D. B. Zhao and J. Q. Yi, Ramp Metering Based on Adaptive Dynamic Programming, *Proc. of the IEEE Conf. on Neural Networks*, pp.1847-1852, 2008.
[21] H. J. Payne, Models of freeway traffic and control, *Mathematical Models of Public Systems Simulation Council Proceedings*, pp.51-61, 1971.
[22] M. Cremer and A.D. May, An extended traffic model for freeway control, *Research Report UCB-ITS-RR-85-7*, 1985.
[23] P. B. Hunt, D. L. Robertson, and R. D. Bretherton, The SCOOT on-line traffic signal optimization technique, *Traffic Eng. Control*, vol.23, pp.190-192, 1982.
[24] Y. Hirashima, A Q-learning system for container transfer scheduling based on shipping order at container terminals, *Int. J. of Innovative Computing, Information and Control*, vol.4, pp.547-558, 2008.
[25] X. T. Wang, Numerical solution of optimal control for scaled systems by hybrid functions, *Int. J. of Innovative Computing, Information and Control*, vol.4, pp.849-856, 2008.
[26] A. Kotsialos and M. Papageorgiou, Nonlinear optimal control applied to coordinated ramp metering, *IEEE Trans. on Control Systems Technology*, vol.10, pp. 920-933, 2004.
[27] D. C. Gazis and R. B. Potts, The oversaturated intersection, *Proc. of the 2nd International Symposium on Trafic Theory*, London, UK, pp.221-237, 1963.
[28] L. Lipp, L. Corcoran and G. Hickman, Benefits of central computer control for the Denver ramp metering system, *Transportation Research Board Record*, January 1991.
[29] A. Alessandri, A. Di Febbaro, A. Ferrara, and E. Punta, Optimal control of freeways via speed signalling and ramp metering, *Control Eng. Practice*, vol.6, pp.771-780, 1998.
[30] M. Papageorgiou, J. M. Blosseville, and H. Hadj-Salem, Modelling and real-time control of traffic flow on the southern part of boulevard in Paris: Part II: Coordinated on-ramp metering, *Transportation Research A*, pp.361-370, 1990.

[31] L. Jacobson, K. Henry, and O. Mehyar, Real-time metering algorithm for centralized control, *Transportation Research Record*, pp.17-26, 1989.

[32] A. Hegyi, B. D. Schutter, and H. Hellendoorn, Model predictive control for optimal coordination of ramp metering and variable speed limits, *Transpn. Res. Part C*, vol.13, pp.185-209, 2005.

[33] M. Papageorgiou, Applications of automatic control concepts to traffic flow modeling and control, *Lecture Notes in Control and Information Sciences*, Berlin, Germany, 1983.