A Budget Optimization Framework for Search Advertisements Across Markets

Yanwu Yang, Jie Zhang, Rui Qin, Juanjuan Li, Fei-Yue Wang, *Fellow, IEEE*, and Wei Qi

***Abstract*—Budget optimization is one of the primary decision-making issues faced by advertisers in search auctions. A quality budget optimization strategy can significantly improve the effectiveness of search advertising campaigns, thus helping advertisers to succeed in the fierce competition of online marketing. This paper investigates budget optimization problems in search advertisements and proposes a novel hierarchical budget optimization framework (BOF), with consideration of the entire life cycle of advertising campaigns. Then, we formulated our BOF framework, made some mathematical analysis on some desirable properties, and presented an effective solution algorithm. Moreover, we established a simple but illustrative instantiation of our BOF framework which can help advertisers to allocate and adjust the budget of search advertising campaigns. Our BOF framework provides an open testbed environment for various strategies of budget allocation and adjustment across search advertising markets. With field reports and logs from real-world search advertising campaigns, we designed some experiments to evaluate the effectiveness of our BOF framework and instantiated strategies. Experimental results are quite promising, where our BOF framework and instantiated strategies perform better than two baseline budget strategies commonly used in practical advertising campaigns.**

***Index Terms*—Budget optimization, optimal strategy, search auctions, search engine marketing, sponsored search.**

I. Introduction

RECENT YEARS have witnessed a booming growth of search auctions when "economics meet search" [22]. Specifically, there is an emerging tendency to increasingly integrate Web information retrieval and online marketing techniques, leading to a targeting advertising form with some either explicit or implicit "long-tail" effects. Search auctions have now become a primary online advertising format being acknowledged as a promising business model. This is approved by the fact that it serves as the primary revenue source for major search engine companies. For example, Google [1] reported the total revenue of $8.44 billion in the fourth quarter of 2010, with the revenue via search auctions comprising about 97% of the total. According to statistics from IAB [2], more than 47% of the U.S. online advertising revenue comes from search auctions in the first half of 2010, followed by the second largest format of display advertisements (36%).

The growing prosperity of search advertising markets is vastly driven by the influx of millions of advertisers. However, most search engine companies currently provide limited number of advertising slots (e.g., 8–10) on the search engine result pages (SERPs). More and more advertisers have to advertise their products or services simultaneously across several search engines, in order to increase impressions of their advertisements and expected profits, thus to survive from the fierce competition. Consequently, many advertisements compete for space on SERPs in any given scheduling horizon [4], such that how to rationally allocate the limited advertising budget is a critical issue in search auctions, even before conducting advertising campaigns.

Search auctions fall into the category of complex systems capable of evolving with feeds from outside environments and intrainteractions [34] due to some factors, namely, the diversity of search auction mechanisms [3], the unprecedented complexity and dynamics of bidding processes [27], and the strong coupling and unpredictability of markets under conditions of imperfect information [10]. Moreover, there are plenty of uncertainties in the mapping from the budget into the advertising performance [30]. Thus, search advertisers have to face tremendous difficulties and challenges while making budget decisions. First, advertisers should take competitors' budget strategies into consideration in order to choose the best response. Second, limited advertising budgets should be wisely allocated at different levels of abstractions, in different time granularities, and ideally be adjusted in real time. Third, budget strategies heavily rely on other advertising strategies that are relevant to keyword portfolio, bids, and the prediction on advertising targets. Therefore, it is crucial to explore an integrated framework for optimal strategies of budget allocation and adjustment in search auctions. To the best of our knowledge, this is the first research effort in this direction.

This paper proposes a novel hierarchical budget optimization framework (BOF) for advertisers to allocate and adjust their advertising budgets throughout the entire life cycle of advertising campaigns in search auctions. Our BOF framework provides a basic infrastructure for various optimal budget strategies. We formulated our BOF framework and approved its

Manuscript received March 3, 2011; revised June 14, 2011; accepted August 10, 2011. Date of publication December 6, 2011; date of current version August 15, 2012. This work was supported in part by the National Natural Science Foundation of China under Contracts 71071152, 70890084, 60875049, and 60921061 and in part by the Chinese Academy of Sciences under Grant 2F08N03. The work of Y. Yang was supported by the Scientific Research Foundation for the Returned Overseas Chinese Scholars. This paper was recommended by Associate Editor M. Zhou.

Y. Yang, J. Zhang, R. Qin, J. Li, and F.-Y. Wang are with the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China (e-mail: yangyanwu.isec@gmail.com; jie.zhang@ia.ac.cn; qinrui.isec@gmail.com; lijuanjuan.isec@gmail.com; feiyue@ieee.org).

W. Qi is with the School of Management, Harbin Institute of Technology, Harbin 150006, China (e-mail: qiwei@hit.edu.cn).

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Digital Object Identifier 10.1109/TSMCA.2011.2172418

1083-4427/$26.00 © 2011 IEEE

theoretical soundness through mathematical analysis of relevant properties. Furthermore, we established a simple but illustrative instantiation of our framework and designed some experiments to evaluate our framework and instantiated strategies, with field data from real-world search advertising campaigns. Experimental results showed that our proposed BOF framework and instantiated strategies can effectively decrease the loss of effective clicks by comparing with two baseline budget strategies commonly used in real advertising campaigns.

The contributions of our work can be summarized as follows.

1) We proposed a novel hierarchical framework for budget optimization in search advertisements across several markets, with consideration of the entire life cycle of advertising campaigns.
2) We formulated our BOF framework, studied some desirable properties, and then provided an effective solution algorithm. The formulated BOF framework can integrate different optimal algorithms for budget decisions in search auctions, without adding the computational complexity.
3) We also established a simple but illustrative instantiation of our BOF framework and designed some experiments to validate its effectiveness with real-world data from search advertising campaigns.

The rest of this paper is organized as follows. The next section reviews some relevant literature. Section III analyzes the whole scenario and relevant problems related to budget decisions in search advertisements and then presents modeling details of the BOF. Section IV formulates the BOF framework, analyzes its properties, and presents a solution algorithm. Section V provides an instantiation of the BOF framework. Section VI reports some experimental settings and results to validate the BOF framework and instantiated strategies. Section VIII concludes this work.

II. Literature Review

Budget decisions such as allocation and adjustment have long been important subjects in marketing and advertisements, attracting plenty of well-established and continuing research efforts. The pioneering work of Vidale and Wolfe [33] took the initiative to define the concept of advertising effectiveness and equations of sales response dynamics and provided a solution for optimal allocation of limited budgets. Another important concept is the advertising goodwill introduced by Nerlove and Arrow [31], which can be considered as the current aggregated advertising effectiveness that can influence budget decisions later on. Based on this concept, a dynamic adjustment framework was proposed for optimal advertising strategies and price policies in their work, which was further generalized by Sethi [32] into the case with limited budgets. Krishnan and Jain [25] investigated the optimal advertising policy for new products under the influence of the diffusion phenomenon and concluded that optimal advertising strategies are determined by the advertising effectiveness, discount rate, and the ratio of advertisement to profits.

Due to the dynamic nature of search auction markets, various optimal programming algorithms have been used to improve advertising budget strategies. Integer programming and nonlinear programming are effective to find optimal solutions for budget allocation over keywords [24], [36]. Results from Zluk and Cholette [36] showed that price elasticities of the click-through rate (CTR) and response functions are key factors for budget decisions, and investing on more keywords under a certain threshold can help improve advertisers' profits. The search process for optimal budget allocation strategies can be modeled as an optimal control problem, and the optimal control theory was used to study the optimal budget allocation problem among Web portals [15]. They used dynamic programming to derive the analytical solution to the optimal budget allocation problem, and their conclusions indicated that budget allocation strategies rely nonlinearly on the targeted audiences, average CTRs, and adverting effectiveness of websites. Thus, advertisers are advised to switch more budgets into specialized Web portals in order to maximize click volumes in the long term.

In search auctions, how to rationally allocate the limited budget is a significant issue, because results of budget decisions are important inputs to other advertising strategies such as keyword portfolio and bidding determination. Building effective allocation strategies amid so many parameters is quite a challenge. Thus, turning to pursue best response becomes a feasible issue, i.e., to find the best strategy while fixing other competitors'. The best response can be abstracted as a stochastic budget optimization problem: how to spread a given budget over keywords to maximize the expected profits [16], which can be depicted as a Markov decision process (MDP) [6], [12]. Three stochastic versions of budget optimization model (including proportional, independent, and scenario) were presented in [13] and [30], where some special cases identified were solvable in polynomial time or with improved approximative ratios. Their approximation and complexity results showed that simple prefix strategies that bid on all cheap keywords up to some level were either optimal or good approximation for many cases. Archak *et al.* [6] formulated the budget allocation as a constrained optimal control problem for an MDP. Their main results showed that, under a reasonable assumption that the online advertising has positive carryover effects on the propensity and the form of user interactions with the same advertiser in the future, there exists a simple greedy algorithm for the budget allocation with the worst case running time cubic in the number of model states (e.g., keywords). The budget optimization problem can also be cast as an online (multiple-choice) knapsack problem to achieve a provably optimal competitive ratio for advertisers [8], [11].

Cooptimization of keyword bid prices and budget allocation strategies is an emerging interesting topic. Zhou *et al.* [35] modeled the noncooperative dynamic keyword auction game with limited budgets as an online multiple-choice knapsack problem and designed some deterministic and randomized algorithms that can achieve a provably optimal competitive ratio. Feldman *et al.* [16], [17] proposed a two-bid uniform bidding strategy with limited budgets, by which advertisers can obtain at least 1-1/e fraction of the maximum clicks possible.

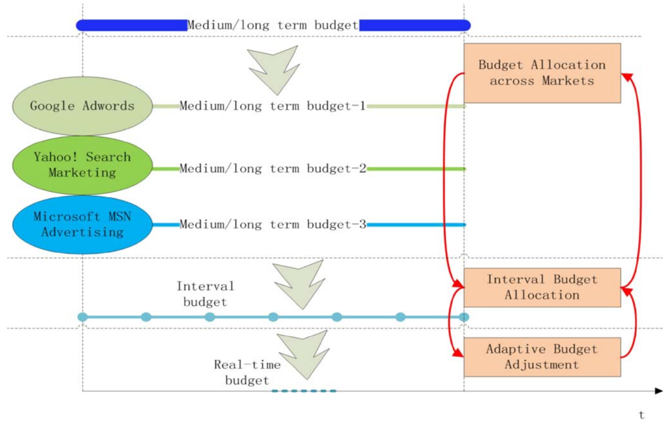

Fig. 1. Multilevel framework of budget decisions in search advertisements.

The current research efforts on search advertising budget decisions mainly focus on the keyword level, except for Fruchter and Dou [15] with attention to the website/system level. Many researches [13], [16], [30] took keywords as first-class objects to distribute advertising budgets. However, these works cannot directly fit to practical scenarios in search auctions, as currently provided by major search engine companies. On the contrary, these works fall into the category of bidding strategies. Therefore, we argue that it is of necessity to propose a novel framework for budget optimization in search auctions, taking into account the entire life cycle of advertising campaigns. This framework should, in the meanwhile, provide an open environment for various research proposals about budget allocation and adjustment, with flexible designs of complementary models, strategies, and algorithms. This intuition directly motivates our research on budget optimization.

III. Budget Allocation for Search Advertisement

A. Problem Statement

A bidding is triggered once an information request is submitted. High volume of search demands makes the bidding a continuous infinite process. The ranking results and prices will be different when any advertiser changes his/her keywords and/or bids at any time, such that an efficient advertising strategy, e.g., for budget allocation, should be capable of dynamically allocating and adjusting advertising budgets on the fly according to states of the marketing environment. In the entire life cycle of advertising campaigns in search auctions, there mainly exist three different budget decision scenarios, as shown in Fig. 1.

First, an advertiser has to allocate his/her search advertising budgets across several markets, supposing that the overall budget for search auctions is determined.

Second, an advertiser has to set budget constraints for a series of temporal slots (e.g., daily budgets) during a certain promotion period of search advertising campaigns, supposing that the budget allocated in a search market is determined. If necessary, the advertiser should coarsely tune budget constraints for the coming slots according to the advertising performance in historical slots.

Third, in an ongoing slot of advertising campaigns, an advertiser has to dynamically adjust the remaining budget, in order to avoid either too quickly wasting his/her budgets or missing golden opportunities in the future, according to real-time advertising effects. For example, when search demands are relatively higher than usual but lead to a lot of invalid clicks as detected, carefully keeping lower the remaining budget is a reasonable strategy.

Correspondingly, it is of necessity to explore an integrated framework for budget optimization with consideration of the entire life cycle of search advertising campaigns. Such framework should be capable of dealing with such structured budget problems in search auctions.

B. Definition

The following gives a definition to formally specify budget optimization problems in search auctions.

Definition 1 (Budget Optimization): **Input**: Given an overall search advertising budget B to an advertiser, a set of search markets SE, $n_1 = |SE|$, a series of temporal slots in a promotion period TS, $n_2 = |TS|$, and a series of real-time adjustments of the remaining budget in a temporal slot RA, $n_3 = |RA|$.

Output: A distribution structure for search advertising budgets $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$ with $\mathcal{A}_1$ over market-budget vectors $\boldsymbol{x} \in \mathcal{R}^{n_1}$, $\mathcal{A}_2$ over interval-budget vectors $\boldsymbol{y} \in \mathcal{R}^{n_2}$, and $\mathcal{A}_3$ over real-time-budget vectors $\boldsymbol{z} \in \mathcal{R}^{n_3}$, such that expenditure$(\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3) \le B$ and loss$(\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$ is minimized.

C. BOF

This paper provides a hierarchical BOF, with consideration of the entire life cycle of advertising campaigns in search auctions (Fig. 1). The BOF framework consists of three levels, corresponding to these three budget decision scenarios discussed in Section III-A, with attentions to advertising system, campaign, and keyword, respectively. Specifically, the **system/account level** concerns budget allocation across several search advertising markets in medium/long term (e.g., over half year). The **campaign level** focuses on budget distribution over a series of temporal slots (e.g., day or month) during a promotion period. The **keyword level** aims to adaptively adjust the remaining budget during a temporal slot of advertising campaigns in order to keep valuable expenditure for potential clicks in the future. Budget strategies at these three levels complement each other, thus forming an integrated chain of budget optimization in search auctions. That is, results of higher levels constrain activities at lower levels, and inversely, operational results at lower levels create feedbacks to activities at higher levels.

The notations used in this paper are listed in Table I.

1) System Level: Generally, budget decisions across several advertising markets are the first issue faced by an advertiser in search auctions. Letting B^l denote the overall budget for search advertisements for an advertiser $l, l = 1, 2, \ldots, r$, across

TABLE I
LIST OF NOTATIONS

Notation	Definition
B	the overall budget for search advertisements
n_1	the number of search markets
n_2	the number of temporal slots during a promotion period
n_3	the number of realtime adjustments in each temporal slot in every search market
x_i	the budget allocated in the ith search market, $i = 1, 2, \ldots, n_1$
$y_{i,j}$	the budget allocated in the jth temporal slot in the ith search market, $j = 1, 2, \ldots, n_2$
$z_{i,j,k}$	the budget allocated for the kth real-time adjustment in the jth temporal slot in the ith search market, $k = 1, 2, \cdots, n_3$
$d_{i,j,k}$	the optimal budget for the kth real-time adjustment in the jth temporal slot in the ith search market
$c_{i,j,k}$	clicks per unit cost of the kth real-time adjustment in the jth temporal slot in the ith search market
$p_{i,j,k}$	the effective CTR of the kth real-time adjustment in the jth temporal slot in the ith search market (below the optimal budget)
$p'_{i,j,k}$	the effective CTR of the kth real-time adjustment in the jth temporal slot in the ith search market (above the optimal budget)
$I^+_{i,j,k}$	the exceeded section (the allocated budget minus the optimal budget) of the kth real-time adjustment in the jth temporal slot in the ith search market
$I^-_{i,j,k}$	the insufficient section (the optimal budget minus the allocated budget) of the kth real-time adjustment in the jth temporal slot in the ith search market

n_1 markets, then budget allocation at the system level can be given as

$$\xi : B^l \to x_1^l, \ldots, x_i^l, \ldots, x_s^l, \qquad i \in \{1, 2, \ldots, n_1\}$$

where x_i^l denotes the budget allocated to a given search advertising market i. Advertising costs are positively proportional to the sum of budgets allocated to a search advertising market by all advertisers. That is, the more budget is allocated to a market, the more is the competitive bidding and then the higher are the advertising costs. For a given type of competitive advertisers (e.g., a group of advertisers with similar advertising targets), search demands from relevant keywords are finite. Therefore, from the point of view of the marketing efficiency, the search advertising efficiency of a market is not a rigid monotonically increasing function of the allocated budget. Although all advertisers have common knowledge that the budget allocated to a market should not exceed a certain amount in order to keep the marketing efficiency at a certain degree, no advertiser knows exactly his/her competitors' budget strategies, e.g., the amount of these competitors' budget allocated, while manipulating his/her budget at the system level. In this sense, budget allocation across search advertising markets can be viewed as a game with incomplete information.

2) Campaign Level: Budget decisions at the campaign level aim to distribute x_i^l over a series of temporal slots (e.g., day) during an advertising period, which can be given as

$$\tau : x_i^l \to y_{i,1}^l, \ldots, y_{i,j}^l, \ldots, y_{i,n_2}^l, \qquad j \in \{1, 2, \ldots, n_2\}$$

where $y_{i,j}^l$ represents the budget allocated to the jth slot during a period in search advertising market i by an advertiser l, which acts as constraints for relevant bidding strategies. Budget decisions at the campaign level should consider various performance indicators, including the distribution of search demands, total clicks, ineffective clicks, cost per click, bids, the conversion rate, and revenue per click. At this level, advertisers are also supposed to make some coarse adjustment for budgets in future slots according to historical advertising effects (particularly in the immediately previous temporal slot). An advertiser makes budget decisions at the campaign level with the outcome from the system level as constraints. Apparently, the former will also provides valuable feedbacks to the latter.

3) Keyword Level: Budget decisions at keyword level aim to dynamically adjust the remaining budget (with the initial value as $y_{i,j}^l$) for advertising campaigns, during a given temporal slot, which can be given as

$$\gamma : y_{i,j}^l(t) \to y_{i,j}^l(t+1)$$

where $y_{i,j}^l(t+1)$ represents the remaining budget (through some possible adjustments on $y_{i,j}^l(t)$) at time $t+1$ in the jth slot in search advertising market i by an advertiser l. Budget adjustment at the keyword level is made mainly according to some performance indicators of keywords, and bidding strategies as well. Most works discussed in Section II were done at this level, focusing on the bidding determination but ignoring the adjustment of the remaining budget of a temporal slot. Again, the adjustment at this level takes the budget allocated to a temporal slot as constraints. Meanwhile, the former provides valuable feedbacks to the latter.

IV. MATHEMATICS OF THE BOF

A. Formulation

We formulate the BOF framework as a hierarchical programming model. In related literatures, various hierarchical optimization techniques were developed to model decentralized planning problems with multiple decision makers in a hierarchical organization [5], [26]. In this section, we establish a three-level programming model for budget optimization throughout the entire life cycle of advertising campaigns in search auctions.

Model 1 (System Level Model): Letting $h^{(1)} : \mathbb{R}^{n_1+p} \to \mathbb{R}$ be the loss function at the system level, $\boldsymbol{f}^{(1)} : \mathbb{R}^{n_1} \to \mathbb{R}^{m_1}$ be the budget constraints at the system level, and $\boldsymbol{g}^{(1)} : S_1 \to \mathbb{R}^p$ be the optimal loss function at the campaign level, $S_1 \subset \mathbb{R}^{n_1}$, then the budget optimization at the system level can be modeled as

$$\begin{aligned} g^{(0)} := \min_{\boldsymbol{x}} \quad & h^{(1)}\left(\boldsymbol{x}, \boldsymbol{g}^{(1)}(\boldsymbol{x})\right) \\ \text{s.t.} \quad & \boldsymbol{f}^{(1)}(\boldsymbol{x}) \leq \boldsymbol{0}, \\ & \boldsymbol{x} \in X \subset \mathbb{R}^{n_1}. \end{aligned} \tag{1}$$

Model 2 (Campaign Level Model): Letting $h_i^{(2)} : \mathbb{R}^{n_2+q} \to \mathbb{R}$ be the loss function at the campaign level, $\boldsymbol{f}_i^{(2)} : \mathbb{R}^{n_1+n_2} \to \mathbb{R}^{m_2}$ be the budget constraints at the campaign level, and $\boldsymbol{g}_i^{(2)} : S_2 \to \mathbb{R}^q$ be the optimal loss function at the keyword level, where $i \in \{1, \ldots, p\}$ and $\boldsymbol{x} \in S_1 = \{\boldsymbol{x} \in X, \boldsymbol{f}^{(1)}(\boldsymbol{x}) \leq \boldsymbol{0}\}$, $S_2 \subset \mathbb{R}^{n_2}$, then the budget optimization at the campaign level can be modeled as

$$\begin{aligned} g_i^{(1)}(\boldsymbol{x}) := \bar{g}_i^{(1)}\left(\boldsymbol{x}, \boldsymbol{g}_i^{(2)}\right) = \min_{\boldsymbol{y}_i} \quad & h_i^{(2)}\left(\boldsymbol{y}_i, \boldsymbol{g}_i^{(2)}(\boldsymbol{y}_i)\right) \\ \text{s.t.} \quad & \boldsymbol{f}_i^{(2)}(\boldsymbol{x}, \boldsymbol{y}_i) \leq \boldsymbol{0}, \\ & \boldsymbol{y}_i \in Y \subset \mathbb{R}^{n_2}. \end{aligned} \tag{2}$$

Model 3 (Keyword Level Model): Letting $h_{ij}^{(3)} : \mathbb{R}^{n_3} \to \mathbb{R}$ be the loss function at the keyword level and $\boldsymbol{f}_{ij}^{(3)} : \mathbb{R}^{n_2+n_3} \to \mathbb{R}^{m_3}$ be the budget constraints at the keyword level, where $i \in \{1, \ldots, p\}$, $j \in \{1, \ldots, q\}$, and $\boldsymbol{y} \in S_2 = \{\boldsymbol{y} \in Y, \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0}$ for $\boldsymbol{x} \in S_1\}$, then the budget optimization at the keyword level can be modeled as

$$\begin{aligned} g_{ij}^{(2)}(\boldsymbol{y}) := \min_{\boldsymbol{z}} \quad & h_{ij}^{(3)}(\boldsymbol{z}_{ij}) \\ \text{s.t.} \quad & \boldsymbol{f}_{ij}^{(3)}(\boldsymbol{y}, \boldsymbol{z}_{ij}) \leq \boldsymbol{0}, \\ & \boldsymbol{z}_{ij} \in Z \subset \mathbb{R}^{n_3}. \end{aligned} \tag{3}$$

These three models consider budget decision problems at three interactive levels together in a hierarchical way, taking into account the entire life cycle of search advertising campaigns. **System Level Model**, **Campaign Level Model**, and **Keyword Level Model** form the overall formulated structure of our BOF framework, i.e., an integrated closed-loop chain model of budget optimization in search auctions. Our BOF framework considers not only budget decision problems at each of the three levels but also interactive relationships between these levels. Feasible regions of upper models constrain lower models. Conversely, solutions of lower models also affect optimal solutions of upper models. Furthermore, notice that our framework will not degenerate even if considering more complicated situations in search auctions.

B. Properties

In the following, we discuss some convex properties of our BOF framework.

Theorem 1: If $h(\boldsymbol{z})$ and $\boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z})$ are convex functions, $g(\boldsymbol{y}) = \min_{\boldsymbol{z}}\{h(\boldsymbol{z}) : \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z}) \leq \boldsymbol{0}\}$, then $g(\boldsymbol{y})$ is also a convex function.

Proof: Let $S = \{(\boldsymbol{y}, \boldsymbol{z}) | \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z}) \leq \boldsymbol{0}\}$. Since $\boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z})$ is a convex function, it can be deduced that S is a convex set.

Define $I_S(t)$ as

$$I_S(t) = \begin{cases} 0, & \text{if } t \in S \\ \infty, & \text{else.} \end{cases}$$

We can prove that $I_S(t)$ is a convex function.

Define $\hat{h}(\boldsymbol{y}, \boldsymbol{z}) = h(\boldsymbol{z}) + I_S(\boldsymbol{y}, \boldsymbol{z})$. Because $h(\boldsymbol{z})$ and $I_S(t)$ are convex functions, $\hat{h}(\boldsymbol{y}, \boldsymbol{z})$ is also a convex function. Hence, we have $\min_{\boldsymbol{z}} \hat{h}(\boldsymbol{y}, \boldsymbol{z})$ convex and

$$\hat{h}(\boldsymbol{y}, \boldsymbol{z}) = \begin{cases} h(\boldsymbol{z}), & \text{if } (\boldsymbol{y}, \boldsymbol{z}) \in S \\ \infty, & \text{otherwise.} \end{cases}$$

Thus, $\min_{\boldsymbol{z}} h(\boldsymbol{z}) = \min_{\boldsymbol{z}} \hat{h}(\boldsymbol{y}, \boldsymbol{z})$ for $(\boldsymbol{y}, \boldsymbol{z}) \in S$. Therefore, $g(\boldsymbol{y})$ is convex. ■

Theorem 2: If $h(\boldsymbol{y}, \hat{\boldsymbol{y}})$, $\boldsymbol{g}(\boldsymbol{y})$, and $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$ are convex functions, $h(\boldsymbol{y}, \hat{\boldsymbol{y}})$ is nondecreasing on $\hat{\boldsymbol{y}}$, and $\bar{g}(\boldsymbol{x}) = \min_y\{h(\boldsymbol{y}, \boldsymbol{g}(\boldsymbol{y})) : \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0}\}$, then $\bar{g}(\boldsymbol{x})$ is also a convex function.

Proof: Let $S = \{(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0}\}$. Since $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$ is a convex function, it can be deduced that S is a convex set. Hence, I_S is a convex function.

Define $\hat{h}(\boldsymbol{x}, \boldsymbol{y}) = h(\boldsymbol{y}, \boldsymbol{g}(\boldsymbol{y})) + I_S(\boldsymbol{x}, \boldsymbol{y})$. Because $h(\boldsymbol{y}, \hat{\boldsymbol{y}})$ is convex and nondecreasing on $\hat{\boldsymbol{y}}$, $\hat{h}(\boldsymbol{y}, \boldsymbol{z})$ is also a convex function. Hence, we have $\min_{\boldsymbol{y}} \hat{h}(\boldsymbol{x}, \boldsymbol{y})$ convex and

$$\hat{h}(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} h\left(\boldsymbol{y}, \boldsymbol{g}(\boldsymbol{y})\right), & \text{if } (\boldsymbol{x}, \boldsymbol{y}) \in S \\ \infty, & \text{otherwise.} \end{cases}$$

Thus, $\min_{\boldsymbol{y}} h(\boldsymbol{y}, \boldsymbol{g}(\boldsymbol{y})) = \min_{\boldsymbol{y}} \hat{h}(\boldsymbol{x}, \boldsymbol{y})$ for $(\boldsymbol{x}, \boldsymbol{y}) \in S$. Therefore, $\bar{g}(\boldsymbol{x})$ is convex. ■

From Theorems 1 and 2, we can obtain the following corollaries.

Corollary 1: If $h^{(3)}(\boldsymbol{z})$ and $\boldsymbol{f}^{(3)}(\boldsymbol{y}, \boldsymbol{z})$ are convex functions, then the optimization problem (3) is a convex programming problem.

Corollary 2: If $h^{(3)}(\boldsymbol{z})$, $\boldsymbol{f}^{(3)}(\boldsymbol{y}, \boldsymbol{z})$, $h^{(2)}(\boldsymbol{y}, \hat{\boldsymbol{y}})$, and $\boldsymbol{f}^{(2)}(\boldsymbol{x}, \boldsymbol{y})$ are convex functions and $h^{(2)}(\boldsymbol{y}, \hat{\boldsymbol{y}})$ is nondecreasing on $\hat{\boldsymbol{y}}$, then the optimization problem (2) is a convex programming problem.

Corollary 3: If $h^{(3)}(\boldsymbol{z})$, $\boldsymbol{f}^{(3)}(\boldsymbol{y}, \boldsymbol{z})$, $h^{(2)}(\boldsymbol{y}, \hat{\boldsymbol{y}})$, $\boldsymbol{f}^{(2)}(\boldsymbol{x}, \boldsymbol{y})$, $h^{(1)}(\boldsymbol{x}, \hat{\boldsymbol{x}})$, and $\boldsymbol{f}^{(1)}(\boldsymbol{x})$ are convex functions, $h^{(2)}$ is nondecreasing on $\hat{\boldsymbol{y}}$, and $h^{(1)}$ is nondecreasing on $\hat{\boldsymbol{x}}$, then model (1) is a convex programming problem.

C. Solution

Here, we provide a general solution process for our BOF framework, described as follows.

- Step 1) Solve each **Keyword Level Model**, and interpolate $g^{(2)}(\boldsymbol{y})$ (e.g., 1-D interpolation for a piecewise linear objective function $h^{(3)}$), denoted by $\eta(\boldsymbol{y})$.
- Step 2) Substitute $g^{(2)}(\boldsymbol{y})$ with $\eta(\boldsymbol{y})$, solve each **Campaign Level Model**, and interpolate $g^{(1)}(\boldsymbol{x})$, denoted by $\phi(\boldsymbol{x})$.
- Step 3) Substitute $g^{(1)}(\boldsymbol{x})$ with $\phi(\boldsymbol{x})$, and minimize $h^{(1)}(\boldsymbol{x}, \phi(\boldsymbol{x}))$ under constraints from model (1).

Finally, it comes to a solution for our framework, namely, optimal budget decisions in search auctions.

V. Framework Instantiation

A. Basis

In this section, we propose a simple but illustrative instantiation for our BOF framework. In search auctions, a click is an action initiating a visit to a website via a sponsored link. If a click is an intentional click that has a realistic probability of generating values once the visitor arrives at the website, then it is a *valid click*; otherwise, it is *invalid* [21]. In this paper, we consider the generated value obtained through some kinds of user behaviors, including purchase, registration, staying on the landing page for more than 5 s, surf more than two links, bookmarking, and downloading relevant pages. Then, we give a concept of effective CTR as follows.

Definition 2 (Effective CTR): Effective CTR is the ratio of valid clicks and total clicks, i.e.,

$$\text{Effective } CTR = \frac{\text{valid clicks}}{\text{total clicks}}.$$

In the objective function, we consider to minimize the loss in terms of effective clicks. In this paper, we make the following assumptions.

1) If the budget allocated is less than the optimal budget, the effective CTR is denoted by a constant $p_{i,j,k}$.
2) If the budget allocated is larger than the optimal budget, the exceeded section (the allocated budget minus the optimal budget) will be used up. The effective CTR of the exceeded section is denoted by a constant $p'_{i,j,k}$, which is smaller than $p_{i,j,k}$. This can be justified by the law of diminishing marginal utility [28]. Specifically, when advertisers invest more and more budgets to a search market, total effective clicks increase in high rates until the total budget arrives at a certain amount (e.g., the optimal budget); when it exceeds a certain amount, total effective clicks increase in comparatively lower rates.

For each i and j, where $i=1,2,\ldots,n_1$ and $j=1,2,\ldots,n_2$, the loss of the kth real-time adjustment contains the following three parts.

1) If $I^+_{i,j,k} = I^-_{i,j,k} = 0$, then the loss concerns ineffective clicks generated from $z_{i,j,k}$, i.e.,

$$c_{i,j,k} z_{i,j,k}(1 - p_{i,j,k}).$$

2) If $I^+_{i,j,k} > 0$, then the loss includes ineffective clicks generated from $z_{i,j,k} - I^+_{i,j,k}$ and from $I^+_{i,j,k}$, i.e.,

$$\begin{aligned} &c_{i,j,k}\left(z_{i,j,k} - I^+_{i,j,k}\right)(1 - p_{i,j,k}) + c_{i,j,k} I^+_{i,j,k}\left(1 - p'_{i,j,k}\right) \\ &\quad - c_{i,j,k} I^+_{i,j,k} p'_{i,j,k} \\ &= c_{i,j,k} z_{i,j,k}(1 - p_{i,j,k}) + c_{i,j,k} I^+_{i,j,k}\left(p_{i,j,k} - 2p'_{i,j,k}\right). \end{aligned}$$

3) If $I^-_{i,j,k} > 0$, then the loss is the ineffective clicks generated from $z_{i,j,k}$ and the lost effective clicks by $I^-_{i,j,k}$, i.e.,

$$c_{i,j,k} z_{i,j,k}(1 - p_{i,j,k}) + c_{i,j,k} I^-_{i,j,k} p_{i,j,k}.$$

B. Model

We establish a model with the notations in Table I as follows.

The system level concerns minimizing the total loss in terms of effective clicks across n_1 search markets

$$\begin{aligned} \min \quad & \sum_{i=1}^{n_1} g_i^{(1)}(\boldsymbol{x}) \\ \text{s.t.} \quad & \sum_{i=1}^{n_1} x_i - B \le 0 \\ & \boldsymbol{x} \ge \boldsymbol{0} \end{aligned} \tag{4}$$

where $g_i^{(1)}$ is the minimum of the loss in terms of effective clicks in the ith search market at the campaign level, given as

$$\begin{aligned} g_i^{(1)}(\boldsymbol{x}) := \min \quad & \sum_{j=1}^{n_2} g_{i,j}^{(2)}(\boldsymbol{y}) \\ \text{s.t.} \quad & \sum_{j=1}^{n_2} y_{i,j} - x_i \le 0 \\ & \boldsymbol{y} \ge \boldsymbol{0} \end{aligned} \tag{5}$$

where $g_{i,j}^{(2)}$ is the minimum of the loss in terms of effective clicks in the jth temporal slot in the ith search market at the keyword level, given as

$$\begin{aligned} g_{i,j}^{(2)}(\boldsymbol{y}) := \min \quad & \sum_{k=1}^{n_3} c_{i,j,k}\Big[z_{i,j,k}(1 - p_{i,j,k}) + I^-_{i,j,k} p_{i,j,k} \\ & \qquad + I^+_{i,j,k}\left(p_{i,j,k} - 2p'_{i,j,k}\right)\Big] \\ \text{s.t.} \quad & \sum_{k=1}^{n_3} z_{i,j,k} - y_{i,j} \le 0 \\ & I^+_{i,j,k} = [z_{i,j,k} - d_{i,j,k}] \vee 0 \\ & I^-_{i,j,k} = [d_{i,j,k} - z_{i,j,k}] \vee 0 \\ & z_{i,j,k} \ge 0, \qquad k = 1, 2, \ldots, n_3. \end{aligned} \tag{6}$$

VI. Experiments and Validation

A. Data Description

We collected field reports and logs from practical search advertising campaigns by several enterprises and organizations in two search markets during the period from Sep. 2008 to Aug. 2010 and designed experiments to validate the proposed BOF and instantiated strategies. In addition, we also did some approximate treatments on the statistical data in order to support intelligible experimental settings. We made independent budget optimization experiments in different temporal granularities (year/month/week/day). This paper reports experimental settings and some relevant results following the framework instantiation and solutions given in the previous section. We also compare optimal values with the performance of two baseline budget strategies commonly used in practical advertising campaigns.

TABLE II
VALUES OF $c_{i,j}$ FROM PROMOTION REPORTS OF AN ADVERTISER

$c_{i,j}$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	0.7	0.65	0.75	0.68	0.72
$i=2$	0.68	0.62	0.78	0.72	0.80

TABLE III
VALUES OF $\boldsymbol{p}_{i,j}$ AND $\boldsymbol{p}'_{i,j}$ FROM PROMOTION REPORTS OF AN ADVERTISER

	$\boldsymbol{p}_{1,j}$	$\boldsymbol{p}'_{1,j}$
$j=1$	(0.80,0.75,0.78,0.70)	(0.20,0.25,0.21,0.24)
$j=2$	(0.82,0.84,0.80,0.78)	(0.26,0.18,0.24,0.26)
$j=3$	(0.81,0.76,0.65,0.60)	(0.15,0.19,0.21,0.16)
$j=4$	(0.79,0.82,0.80,0.78)	(0.25,0.23,0.18,0.12)
$j=5$	(0.86,0.75,0.80,0.76)	(0.24,0.26,0.18,0.27)
	$\boldsymbol{p}_{2,j}$	$\boldsymbol{p}'_{2,j}$
$j=1$	(0.88,0.79,0.75,0.80)	(0.16,0.14,0.25,0.27)
$j=2$	(0.76,0.84,0.68,0.61)	(0.24,0.13,0.24,0.18)
$j=3$	(0.79,0.82,0.76,0.78)	(0.12,0.17,0.23,0.27)
$j=4$	(0.85,0.79,0.82,0.75)	(0.19,0.22,0.26,0.17)
$j=5$	(0.87,0.82,0.76,0.68)	(0.19,0.24,0.16,0.26)

B. Experimental Design

We have conducted preliminary computational evaluations of our approach. For comparison purposes, we implemented two baseline search strategies that are commonly applied in practical search advertising campaigns. The first benchmark, called BASE1-Fixed, represents the budget strategy from a type of advertisers who set a fixed daily budget according to experiential or survey knowledge, however without any adjustments on the remaining budget. The second benchmark is called BASE2-Heuristics with some necessary adjustments based on the fixed strategy. In other words, the middle-term (e.g., monthly) budget is equally distributed over a series of short-term temporal slots (e.g., daily). Then, the advertiser adjusts the daily budget through some heuristic rules: If the loss of effective clicks for the current day is less than the average loss computed from the historical data, then the daily budget for the next day is increased proportionally; if the loss for the current day is more than the average loss, then the daily budget for the next day is decreased proportionally; otherwise, the daily budget is kept unchanged.

The evaluation focuses on two-fold purposes. The first purpose is to prove some properties of the BOF framework as analyzed in Section IV-B. The second is to evaluate the performance of the framework instantiation in the crisp case given in Section V-B and instantiated strategies given in Section V. In the following, we provide details about our experimental evaluation results.

The experimental scene is described as follows. An advertiser takes an advertising schedule of 8 h (e.g., 9:00–17:00) to participate in search auctions each day; then, he/she plans to adjust the remaining budget four times, e.g., once every 2 h. Clicks per unit cost and the effective CTR given in Tables II and III are collected from field logs of practical advertising campaigns during five days, where $c_{i,j,1} = \cdots = c_{i,j,4}$ ($c_{i,j}$ for short) reflects that these four segments of real-time adjustment have the same clicks per unit cost in the jth day in the ith search market, and $\boldsymbol{p}_{i,j} = (p_{i,j,1}, p_{i,j,2}, p_{i,j,3}, p_{i,j,4})$ and $\boldsymbol{p}'_{i,j} = (p'_{i,j,1}, p'_{i,j,2}, p'_{i,j,3}, p'_{i,j,4})$. Suppose that the overall budget during five days is $B = \$500.000$. The optimal budget $d_{i,j,k}$ of every 2 h can be obtained through statistical analysis from historical logs of advertising campaigns, $\boldsymbol{d}_{i,j} = (d_{i,j,1}, d_{i,j,2}, d_{i,j,3}, d_{i,j,4})$, as shown in Table IV. Notice that these optimal budgets somewhat reflect budget constraints in historical campaigns, which can be viewed as the reference for the optimal procedure for budget manipulation for advertising campaigns in the future, since they are independent to budget constraints of the coming days.

TABLE IV
OPTIMAL BUDGET REFERENCE $\boldsymbol{d}_{i,j}$ (UNIT: \$)

	$\boldsymbol{d}_{1,j}$	$\boldsymbol{d}_{2,j}$
$j=1$	(15,19,11.5,16.5)	(11.5,16.5,18.5,13.5)
$j=2$	(18.5,11.5,17,16)	(10,12.5,17,22)
$j=3$	(20.5,12,14,14.5)	(12,8.5,17.5,16)
$j=4$	(16.5,13,11,9)	(12,9.5,8,14)
$j=5$	(13,18.5,15,11.5)	(16,18.5,11.5,9)

TABLE V
OPTIMAL SOLUTIONS AT THE CAMPAIGN LEVEL (UNIT: \$)

day (j)	Search market-1	Search market-2
$j=1$	27.065	59.124
$j=2$	56.096	41.571
$j=3$	60.124	54.108
$j=4$	49.098	43.085
$j=5$	54.617	55.110

C. Experimental Results

The optimal solutions are obtained through the BOF framework instantiation and solution proposed in Section IV-C. We employed the sequential least square quadratic programming method to solve the budget optimization model (as described in Section V-B) embedded in our BOF framework. The main experimental results are described as follows.

1) At the system level, the optimal budget allocated to search market-1 is \$247.00, and that allocated to search market-2 is \$253.00.
2) At the campaign level, optimal budgets for each day in these two markets are shown in Table V.
3) At the keyword level, optimal budgets for every 2 h in each day in these two markets are shown in Table VI.
4) The overall optimal value (e.g., the cumulative loss of effective clicks) for this case by our BOF framework and instantiated strategies is 106.032.
5) As shown in Figs. 2 and 3, budget decisions abstracted at all these three levels in our BOF framework are convex programming problems. Therefore, the overall budget optimization problem in the hierarchical BOF framework is convex.

TABLE VI
OPTIMAL SOLUTIONS AT THE KEYWORD LEVEL (UNIT: $)

search market i, day j	kth two-hour			
	$k=1$	$k=2$	$k=3$	$k=4$
$i=1, j=1$	15.000	0.565	11.500	0
$i=1, j=2$	18.500	11.500	17.000	9.096
$i=1, j=3$	20.500	12.000	14.000	13.624
$i=1, j=4$	16.500	13.000	11.000	8.598
$i=1, j=5$	13.000	15.117	15.000	11.500
$i=2, j=1$	11.500	16.500	17.624	13.500
$i=2, j=2$	10.000	12.500	17.000	2.071
$i=2, j=3$	12.000	8.500	17.500	16.000
$i=2, j=4$	12.000	9.500	8.000	13.585
$i=2, j=5$	16.000	18.500	11.500	9.000

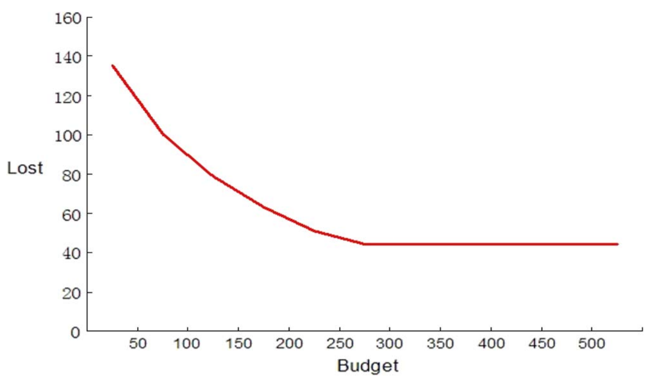

Fig. 2. Loss–budget curve at the campaign level by the BOF strategy.

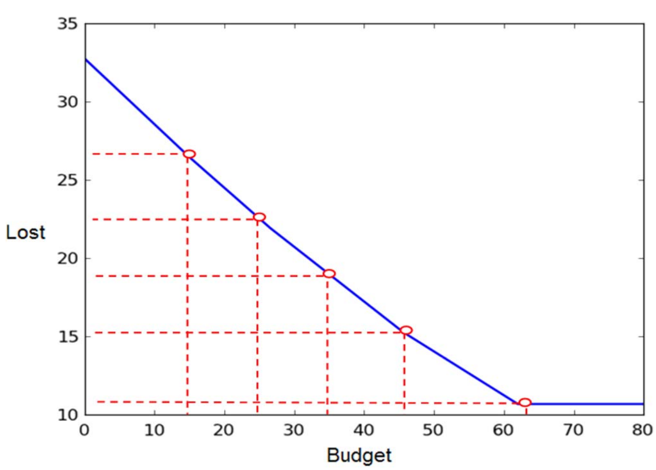

Fig. 3. Loss–budget curve at the keyword level by the BOF strategy.

In typical scenarios of budget manipulation with the fixed strategy, most advertisers evenly divide the overall budget in the two search markets (i.e., $250.00 in each), keep budgets distributed in a series of temporal slots (e.g., five days) unchanged (i.e., $50.00 as daily budget), and ignore the necessity for real-time adjustments of (remaining) daily budgets but just allocate the same amount of budget for every 2 h (i.e., $12.50). Some cautious advertisers would like to take chances to adjust the daily budget (e.g., either increase or decrease), but without consideration of real-time adjustments probably due to the fact that the latter is time consuming and sophisticatedly complex. The

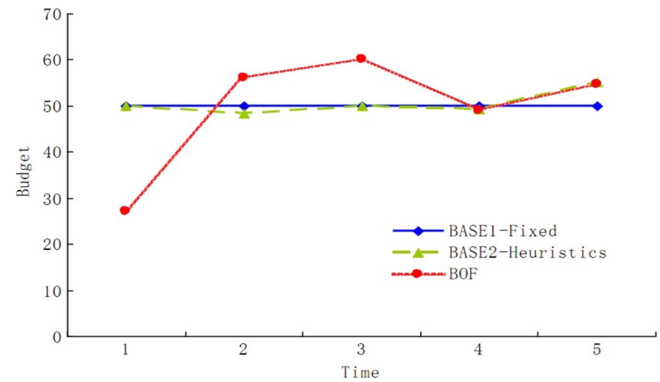

Fig. 4. Comparison of budget distribution over time in market-1.

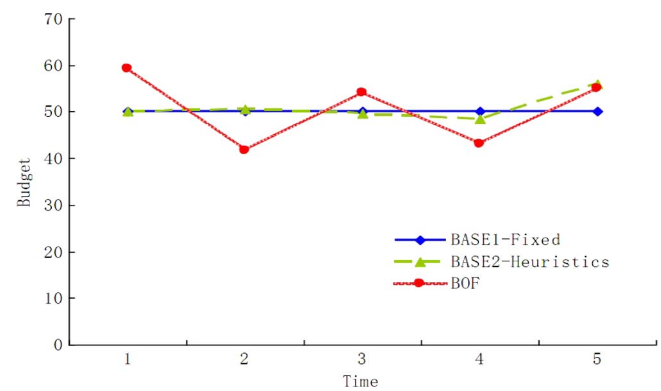

Fig. 5. Comparison of budget distribution over time in market-2.

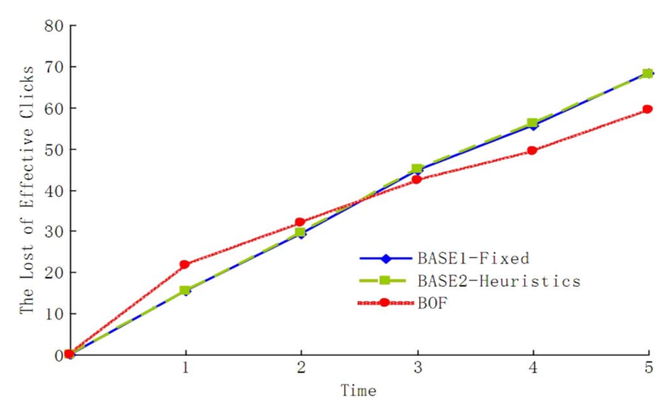

Fig. 6. Comparison of the accumulated loss of effective clicks in market-1.

cumulative losses of effective clicks are 134.567 and 133.729 for the fixed strategy and the heuristics strategy, respectively. Figs. 4 and 5 show the budget distribution over time in these two search advertising markets, respectively. Figs. 6 and 7 show the loss of effective clicks over time in these two search advertising markets, respectively. Several interesting findings are given as follows.

1) From Figs. 4 and 5, we note that the amplitude variation of budget adjustment by our BOF framework is larger than those of the other two BASE strategies. This indicates that our BOF framework is more sensitive to dynamics of advertising markets.

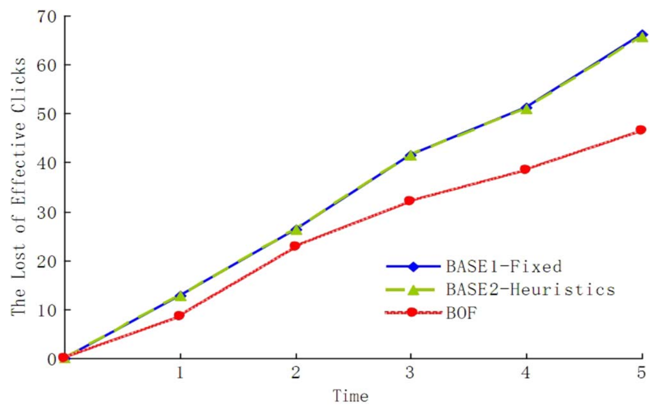

Fig. 7. Comparison of the accumulated loss of effective clicks in market-2.

2) We also notice that the budget allocated initially to the search market-1 by our BOF framework is much less than those by the other two BASE strategies; then, the loss of effective clicks is higher. The cumulative loss of effective clicks by our BOF framework gradually decreases and then becomes less than those by the other two BASE strategies.
3) In the search market-2, we observe that our BOF framework always performs better than the other two BASE strategies in terms of the loss of effective clicks.
4) From Figs. 6 and 7, we note that our BOF framework performs better than the other two BASE strategies in these two search markets in terms of the cumulative loss of effective clicks. In detail, our BOF framework and instantiated strategies can effectively decrease the loss of effective clicks (about 21.21% and 20.71%, respectively) over the performance of the fixed strategy and the heuristics strategy in practical search advertising campaigns.
5) We also notice that the heuristic strategy outperforms the fixed one. The possible reason might be that it adapts to dynamics of the advertising environment through considering the historical knowledge of advertising performance.

VII. Discussion

From the budget point of view, limited resource capacity significantly influences perceptions of the environment and, thus, the marketing performance of small businesses because marketing exercises and expenses tend to have lesser priority over other elements [14]. For small businesses, budget constraints and lack of time and expertise may lead to limited and often *ad hoc* or irrational promotional decisions. On the one hand, budget constraints limit the feasible space of various optimal strategies (e.g., bidding and keyword strategies) and thus complicate operational situations in different contexts (e.g., search auctions). On the other hand, budget constraints naturally lead to an important problem: how to allocate and adjust the limited budget in a rational way to maximize the expected profit.

The state-of-the-art budget-related work, in the context of search auctions, usually takes the budget as constraints to determine bids over keywords (see, e.g., [13], [16], and [30]). We categorize such work under the term of budget-constrained bidding strategies. To the best of our knowledge, there are few, if any, research efforts on budget optimization in an integrated way in search auctions. Moreover, we argue that the budget in search auctions is structured as we analyzed in Section III-A. Our work is the first initiative to consider budget decision problems in various decision-making situations of search auctions as a whole and to propose a novel integrated framework that could be viewed as an environment/testbed for various budget optimization strategies.

Our research provides key managerial insights for advertisers in search auctions. Advertisers usually take the budget as simple constraints and put a lot of efforts to find more effective ways for possible operations as defined by various kinds of markets (e.g., search auctions). This work indicates that a simple strategy for budget allocation and adjustment can significantly minimize the loss in terms of effective clicks in search auctions. Note that we do not expect that our BOF framework is the only way to deal with budget-related problems in search auctions. We hope that our work could raise peers' interests in budget decision problems at different levels, in different situations (e.g., goals and schedules), with different settings of various parameters (e.g., auction mechanisms and processes).

VIII. Conclusion

In this paper, we have proposed a novel hierarchical BOF with consideration of the entire life cycle of advertising campaigns in search auctions. We formulated our BOF framework, made mathematical analysis on some desirable properties, and presented an effective solution algorithm. Furthermore, we provided a simple but illustrative instantiation of our BOF framework and made some experiments to validate our work with real-world data from advertising campaigns. Experimental results are quite promising, where our BOF framework and instantiated strategies perform better than two other typical budget strategies commonly used in practical search advertising campaigns.

This paper has reported some preliminary research on our BOF framework. It not only provides an open context for possible efforts on budget strategies but also is valuable to help advertisers in real practices in search auctions. In an ongoing work, we extend the BOF framework to more complicated situations with uncertainties in search auctions. Another interesting but challenging perspective is to explore game-theoretical budget decisions at the system level and to study optimal social efficiency. Third, we also intended to extend our BOF framework in the direction of cooptimization with various advertising strategies (e.g., bidding and keyword strategies), thus to facilitate advertising performance in an omnibearing way.

Acknowledgment

The authors would like to thank the anonymous reviewers who provided valuable suggestions that led to a considerable improvement in the organization and presentation of this paper.

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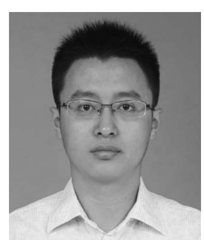
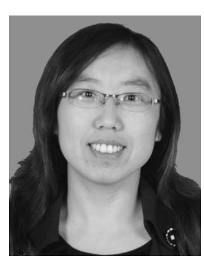
Yanwu Yang received the Ph.D. degree in computer science from the Graduate School of the Ecole Nationale Superieure de Arts et Metiers, Paris, France, in 2006.

He is currently an Associate Professor and the Head of the ISEC Research Group (Internet Sciences and Economic Computing). He is currently with the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China. His current research interests include user modeling, web personalization, and online auctions.



Jie Zhang received the M.Sc. degree in operation research from Renmin University of China, Beijing, China, in 2009.

He is currently a Researcher with the ISEC Research Group (Internet Sciences and Economic Computing). He is currently with the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing. His current research interests include online auctions and optimal control.

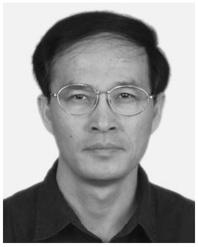


Rui Qin received the M.Sc. degree in operation research from Hebei University, Baoding, China, in 2010.

She is currently a Researcher with the ISEC Research Group (Internet Sciences and Economic Computing). She is currently with the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China. Her current research interests include online auctions.

Juanjuan Li received the M.Sc. degree in economics from Renmin University of China, Beijing, China, in 2010.

She is currently a Researcher with the ISEC Research Group (Internet Sciences and Economic Computing). She is currently with the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing. Her current research interests include online auctions.



Fei-Yue Wang (S'87–M'89–SM'94–F'03) received the Ph.D. degree in computer and systems engineering from Rensselaer Polytechnic Institute, Troy, NY, in 1990.

He is currently a Professor and the Director of the State Key Laboratory of Intelligent Control and Management of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China.

Dr. Wang was the Founding Editor-in-Chief of the *International Journal of Intelligent Control and Systems* from 1995 to 2000, the *Series on Intelligent Control and Intelligent Automation* from 1996 to 2004, and IEEE INTELLIGENT TRANSPORTATION SYSTEMS. He was the Editor-in-Chief of IEEE INTELLIGENT SYSTEMS and IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS from 2009 to 2011. He was the President of the IEEE Intelligent Transportation Systems Society from 2005 to 2007, the Chinese Association for Science and Technology (CAST, USA) in 2005, and the American Zhu Kezhen Education Foundation from 2007 to 2008. Currently, he is the Vice President and Secretary General of the Chinese Association of Automation. He is a member of Sigma Xi and an elected fellow of INCOSE, IFAC, ASME, and AAAS.

Wei Qi received the M.Sc. degree in management science from Harbin Institute of Technology, Harbin, China, in 1999.

She is currently an Associate Professor with the School of Management, Harbin Institute of Technology. Her research interests include customer behavior analysis, CRM, data mining, and business process and information system modeling.