Robust level set image segmentation via a local correntropy-based K-means clustering

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It is still a challenging task to segment real-world images, since they are often distorted by unknown noise and intensity inhomogeneity. To address these problems, we propose a novel segmentation algorithm via a local correntropy-based K-means (LCK) clustering. Due to the correntropy criterion, the clustering algorithm can decrease the weights of the samples that are away from their clusters. As a result, LCK based clustering algorithm can be robust to the outliers. The proposed LCK clustering algorithm is incorporated into the region-based level set segmentation framework. The iteratively re-weighted algorithm is used to solve the LCK based level set segmentation method. Extensive experiments on synthetic and real images are provided to evaluate our method, showing significant improvements on both noise sensitivity and segmentation accuracy, as compared with the state-of-the-art approaches.

1. Introduction

Image segmentation plays an important role in both image and video based pattern recognition systems. However, it is still a challenging problem since images are often distorted by complex noise and intensity inhomogeneity. In the past decades, a number of researchers have been attracted to solve these difficulties and have proposed many effective methods. Among them, level set based methods have proved to be a successful branch.

The early level set based segmentation methods [1–9] utilize edge information to attract active contours toward the object boundaries. However, these methods are sensitive to noise. To solve this problem, region-based methods are proposed in [10–28]. Different from edge-based methods, region-based methods exploit the statistical information inside and outside the contour to guide the contour evolution. Hence, these methods are not sensitive to image noise and weak edges.

One of the most famous region-based method is the CV model proposed by Chan et al. [10]. This method formulates the image segmentation problem as a K-means clustering. As pointed out in [19], as a global method, the CV model cannot solve intensity inhomogeneity well. To tackle this difficulty, many local methods have been proposed recently. A popular one is the local binary fitting (LBF), which is proposed by Li et al. [19]. The core idea behind the LBF is that it considers the K-means clustering problem locally. Motivated by [19], the local image fitting (LIF) and the local Gaussian fitting (LGF) have been presented in [21,29], respectively.

Wang et al. [30] point out that the LBF method is sensitive to initialization. To prevent this, they propose a local order method in [30] (LBF + Order). Besides improving global methods into local versions, many researchers focus on the convexity of segmentation models. They present many convex region-based methods [12,31,32]. For example, in [12], Nikolova et al. propose the Rudin–Osher–Fatemi total variation based CV model.

In the above level set segmentation models, image noise is always assumed to be Gaussian. In practice, for many types of images, such as synthetic aperture radar (SAR) images, it is insufficient to assume that the noise is Gaussian. To make a segmentation algorithm still work under unknown complex noise, we propose a new segmentation method via the local correntropy-based K-means (LCK) clustering. The main advantages of our segmentation method can be highlighted as follows.

1. Due to the correntropy criterion, the clustering algorithm adaptively increases the weights of samples that are close to their clusters, and decreases the weights of samples that are away from their clusters. As a result, our proposed LCK based segmentation algorithm is robust to outliers.

2. In the experiments on synthetic images, our method is more robust than the state-of-the-art approaches. Especially, in our results, foreground and background cluster images preserve edge features well even when the noise is large. In the experiments on real images, our method can provide accurate segmentation results even when the object and the background are very similar.

The rest of this paper is organized as follows. Section 2 introduces the correntropy-based K-means clustering algorithm. Section 3 describes the proposed segmentation method. The optimization is

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described in Section 4. Experimental results are provided in Section 5. The discussions and conclusions are given in Section 6.

2. Correntropy-based K-means

Given N samples \( X = \{x_i\}_{i=1}^N \), where each sample is a d-dimensional vector, the K-means clustering algorithm aims to find K clusters \( \{\mu_i\}_{i=1}^K \) by minimizing the objective function

\[
\min_{\{\mu_i\}_{i=1}^K} \sum_{i=1}^N \theta(i,c) \|x_i - \mu_c\|^2, 
\tag{1}
\]

where \( \theta(i,c) \) is the membership, which satisfies two conditions, i.e., \( \theta(i,c) \in [0,1] \) and \( \sum_{c=1}^K \theta(i,c) = 1 \). The objective function of Eq. (1) is sensitive to outliers, because the mean square error (MSE) criterion is used to measure the distance between samples and cluster centers. To tackle this difficulty, the correntropy criterion [33] is adopted instead of the MSE criterion. Thereby, the objective function is reformulated as

\[
\min_{\{\mu_i\}_{i=1}^K} - \sum_{i=1}^N \sum_{c=1}^K \theta(i,c) x_i^T \sigma^2 g(|x_i - \mu_c|), 
\tag{2}
\]

where \( g(x) = \exp(-x^2/2\sigma^2) \) is a Gaussian kernel function parameterized with the kernel width \( \sigma \). Due to the Gaussian kernel, the correntropy-based K-means clustering algorithm can adaptively emphasize the samples that are close to their corresponding cluster centers. As a result, the effect of outliers is reduced.

3. The proposed segmentation method

The CV model treats segmentation as a global K-means problem. Combining with the contour length term, the segmentation model is

\[
\mathcal{C}^* = \min_{\mathcal{C}, \{\mu_i\}_{i=1}^K} \sum_{i=1}^N \sum_{c=1}^K \theta(i,c) \|x_i - \mu_c\|^2 + \nu |\mathcal{C}|, 
\tag{3}
\]

where \( \mathcal{C} \) is the segmentation contour, and \( \nu \) is a weighting constant of the contour length term \( |\mathcal{C}| \). When the input image is distorted by intensity inhomogeneity, pixel values from different clusters overlap, which makes the global method fail. Fig. 1 presents an example to compare the global and local methods. When the image is inhomogeneous, pixel values are hard to separate globally, since they have large overlap. Fortunately, if we consider the pixels in a local region, for example, the pixels contained in the circle, their pixel values can be separated. The local region-based segmentation methods in [19,21] are proposed based on this observation. Li et al. [19] construct the local K-means clustering model:

\[
\min_{\{\mu_i\}_{i=1}^K} \sum_{i=1}^N \sum_{c=1}^K \theta(i,c) k(j,i) \|x_i - \mu_c\|^2, 
\tag{4}
\]

where \( i \in \mathcal{N}_j \) represents a local pixel around the j-th pixel, and

\[
k(j, i) = \frac{1}{2\pi\sigma^2} \exp \left\{ -\frac{d(j, i)^2}{2\sigma^2} \right\}
\]

is the Gaussian kernel introduced to capture local information, in which \( d(j, i) \) is the spatial distance between the j-th pixel and the j-th pixel. \( \{\mu_c\}_{c=1}^K \) are the local clusters of the j-th pixel. The segmentation model is constructed by summing all local models together, given by

\[
\mathcal{C}^* = \min_{\mathcal{C}, \{\mu_i\}_{i=1}^K} \sum_{i=1}^N \sum_{c=1}^K \theta(i,c) k(j,i) \|x_i - \mu_c\|^2 + \nu |\mathcal{C}|. 
\tag{5}
\]

where \( N \) is the number of pixels in the whole image.

3.1. The proposed LCK based segmentation model

Our LCK based segmentation model is

\[
\mathcal{C}^* = \min_{\mathcal{C}, \{\mu_i\}_{i=1}^K} \sum_{i=1}^N \sum_{c=1}^K \theta(i,c) \|x_i - \mu_c\|^2 + \nu |\mathcal{C}|. 
\tag{6}
\]

Apparently, the main difference between the traditional LBF model (refer to Eq. (5)) and our model (refer to Eq. (6)) is the utilization of pixel-to-cluster distance. This difference is essentially derived from the noise type definition. In LBF model, the noise in each pixel should be Gaussian, while in our model, the noise can be extended to non-Gaussian case. Hence, our LCK based segmentation model is more general.

3.2. Level set formulation

In the level set based segmentation method, the segmentation contour \( C \) is represented as the zeroth level of a level set function \( \phi \). We focus on the two-phase segmentation problem.\footnote{By introducing more than one level set function, our model can be extended to the multi-phase case.} In such a case, the cluster number K is set to 2. The membership is defined as

\[
\theta(i, 1) = H(\phi), \quad \theta(i, 2) = 1 - H(\phi), 
\tag{7}
\]

where \( H(\cdot) \) is the Heaviside function. In practice, the Heaviside function \( H(\cdot) \) is approximated by the \( \varepsilon \)-Heaviside function

\[
H_\varepsilon(x) = \frac{1}{2} \left[ 1 + \frac{2}{\varepsilon} \arctan \left( \frac{x}{\varepsilon} \right) \right],
\tag{8}
\]

and the corresponding derivation is defined by

\[
\delta_\varepsilon(x) = H_\varepsilon'(x) = \frac{\varepsilon}{\pi(\varepsilon^2 + x^2)^2}. 
\tag{9}
\]

In our implementation, the parameter \( \varepsilon \) is set to 1.0.
The objective function of our LCK based segmentation model is
\[
\phi^* = \min_{\phi, \{\mu_1, \mu_2\}^N_{j=1}} E(\phi, \{\mu_1, \mu_2\}^N_{j=1})
\]
\[
= \min_{\phi, \{\mu_1, \mu_2\}^N_{j=1}} - \sum_{j=1}^{N} \sum_{i \in X_j} k(i, j) \sigma^2(\phi_i) g(||x_j - \mu_{j,1}||_2)
+ (1 - H_r(\phi_i)) g(||x_j - \mu_{j,2}||_2) + \sum_{j=1}^{N} \frac{\sigma^2}{\nu} ||\nabla H_r(\phi_i)|| + \kappa \sum_{j=1}^{N} \frac{1}{\nu} ||\nabla \phi_i|| - 1)^2.
\]
(10)

In the above equation, the second term is the contour length term, while the third term is the regularization term proposed in [34], which restricts the level set function to a signed distance function.

4. Model optimization

The objective function of Eq. (10) is nonlinear, which derives from the nonlinear property of the Gaussian kernel. To solve this problem, we adopt the iteratively re-weighted (IR) algorithm.

4.1. Weight computing in the IR algorithm

We first consider the local region around the j-th pixel. In the IR algorithm, the correntropy-based distance is reformulated by the weighted MSE at the t-th iteration, given by
\[
\begin{align*}
\sigma^2 g(||x_j - \mu_{j,1}||_2) &= -w_{j,1,1} g(||x_j - \mu_{j,1}||_2), \\
\sigma^2 g(||x_j - \mu_{j,2}||_2) &= -w_{j,2,2} g(||x_j - \mu_{j,2}||_2).
\end{align*}
\]
(11)
The two weights, $w_{j,1,1}$ and $w_{j,2,2}$ are determined by
\[
\begin{align*}
w_{j,1,1} &= g(||x_j - \mu_{j,1}^{t-1}||_2), \\
w_{j,2,2} &= g(||x_j - \mu_{j,2}^{t-1}||_2),
\end{align*}
\]
(12)
where $\mu_{j,1}^{t-1}$ and $\mu_{j,2}^{t-1}$ are the clusters calculated in the previous time $t-1$. Eq. (12) shows that each pixel has two weights, namely, $w_{j,1,1}$ and $w_{j,2,2}$, which correspond to the two clusters. To obtain a unitary weight for each pixel, we use the following combination:
\[
w_{j} = H_r(\phi_i^{t-1}) w_{j,1,1} + (1 - H_r(\phi_i^{t-1})) w_{j,2,2}.
\]
(13)
The final weight of i-th pixel is calculated by summing all local weights together, given by $w_i = \sum_i k(i, j) w_j$. Combining with Eqs. (12) and (13), we get
\[
w_i = \sum_{i \in X_j} (H_r(\phi_i^{t-1}) g(||x_i - \mu_{i,1}^{t-1}||_2) + (1 - H_r(\phi_i^{t-1})) g(||x_i - \mu_{i,2}^{t-1}||_2)).
\]
(14)
The weight $w_i$ is given by
\[
w_i = \begin{cases} w_i, & w_i \geq \tau \\
0, & w_i < \tau
\end{cases}
\]
(15)
In our implementation, the threshold $\tau$ is set to 0.1.

4.2. Local clusters calculation and level set function updating

In Eqs. (14) and (15), each pixel is assigned with a weight. Therefore, the objective function at the t-th iteration is
\[
\phi^t = \min_{\phi, \{\mu_1, \mu_2\}^N_{j=1}} E(\phi, \{\mu_1, \mu_2\}^N_{j=1})
\]
\[
= \min_{\phi, \{\mu_1, \mu_2\}^N_{j=1}} \sum_{j=1}^{N} \sum_{i \in X_j} k(i, j) (H_r(\phi_i) w_i ||x_j - \mu_{j,1}||^2_2
+(1 - H_r(\phi_i)) w_i ||x_j - \mu_{j,2}||^2_2) + \sum_{j=1}^{N} \frac{\sigma^2}{\nu} ||\nabla H_r(\phi_i)|| + \kappa \sum_{j=1}^{N} \frac{1}{\nu} ||\nabla \phi_i|| - 1)^2.
\]
(16)
The level set function and the local clusters are determined from the new objective function of Eq. (16) by the standard gradient descent method. The procedure has the following two sub-steps.

Step1: For the fixed level set function $\phi_i^{t-1}$, the clusters $\mu_{1,1}^{t-1}$, $\mu_{2,1}^{t-1}$ are calculated by setting the derivatives of Eq. (16) with respect to $\mu_{1,1}$, $\mu_{2,1}$ equal to zero.
\[
\mu_{1,1}^t = \sum_{i \in X_j} k(i, j) H_r(\phi_i^{t-1}) w_i x_i, \quad \mu_{2,1}^t = \sum_{i \in X_j} k(i, j) (1 - H_r(\phi_i^{t-1})) w_i x_i.
\]
(17)
From Eq. (17), we can see that the local clusters are obtained by calculating the weighted mean. By introducing the weights, the effect of outliers is suppressed.

Step2: Keeping the clusters $\mu_{1,1}^t$, $\mu_{2,1}^t$ fixed, the level set function $\phi_i^t$ is updated by solving following gradient flow equation:
\[
\phi_i^t = \phi_i^{t-1} - \eta \frac{dE(\phi, \{\mu_{1,1}^t, \mu_{2,1}^t\}^N_{j=1})}{d\phi_i}
\]
(18)
where $\eta = 0.1$ is the time-step. The gradient of $dE(\phi, \{\mu_{1,1}^t, \mu_{2,1}^t\}^N_{j=1})/d\phi_i$ is
\[
\begin{align*}
dE(\phi, \{\mu_{1,1}^t, \mu_{2,1}^t\}^N_{j=1}) &= H_r(\phi_i) \delta \phi_i w_i \sum_{j=1}^{N} \sum_{i \in X_j} k(j, i) ||x_i - \mu_{i,1}^{t-1}||^2_2 - ||x_i - \mu_{i,2}^{t-1}||^2_2
- \nu \delta \phi_i \text{div} (\frac{\nabla \phi_i}{\phi_i}) - \kappa (\Delta \phi_i - \text{div} (\frac{\nabla \phi_i}{\phi_i})).
\end{align*}
\]
(19)

Algorithm 1. LCK based segmentation.

- **Data:** The input image $\{x_i\}_{i=1}^N$, and parameter $\nu$ and $\kappa$.
- **Result:** The level set function $\phi$.
- **1:** Initializing the level set function $\phi = \phi^0$ by Eq. (20);
- **2:** Initializing the weights $\{w_j\}_{j=1}^N$;
- **3:** **For** $t \leftarrow 1$ **to** $\text{maturity}$ **do**
  - **4:** Calculating the weights $\{w_j\}_{j=1}^N$ by Eq. (14);
  - **5:** Shrinking the weights $\{w_j\}_{j=1}^N$ by Eq. (15);
  - **6:** Computing local clusters $\{\mu_{1,1}^{t-1}, \mu_{2,1}^{t-1}\} = \{\mu_{1,1}^{t-1}, \mu_{2,1}^{t-1}\}$ by Eq. (17);
  - **8:** $\mu_{1,1}^{t} = \min \{\mu_{1,1}^{t-1}, \mu_{2,1}^{t-1}\}$, $\mu_{2,1}^{t} = \max \{\mu_{1,1}^{t-1}, \mu_{2,1}^{t-1}\}$;
  - **9:** Updating the level set function $\{\phi_i^{t}\}_{t=1}^N$ by Eq. (18);
  - **11:** if $\phi^t = \phi^{t-1}$ then
    - **break**;
    - **end**
- **end**

4.3. Segmentation procedures

The procedures of the LCK based segmentation method are summarized in Algorithm 1. The contour is initialized manually.
The initial level set function $\phi^0$ is obtained by

$$\phi^0 = \phi(i, t = 0) = \begin{cases} -\rho & i \in \text{rectangle inner} \\ 0 & i \in \text{rectangle boundary} \\ \rho & i \in \text{rectangle outer} \end{cases}$$

(20)

As shown in Line 7 of Algorithm 1, the local order regularization proposed in [30] is utilized to improve the stability of our segmentation algorithm.

5. Experimental results

Extensive experiments are performed to evaluate our LCK based segmentation method. We compare it with the following approaches, i.e., CV [10], LBF [19], LIF [21], and LBF+Order [30], quantitatively and qualitatively.

5.1. Our results on real images

We select five medical images with different acquisition techniques to evaluate our method. The segmentation results are shown in Fig. 2. Although the acquisition techniques are different, our method can still segment the objects successfully. For example, the detailed information is correctly segmented in the MRI brain image and the retinal image, and the weak boundaries are detected in the ultrasound images. Moreover, with the help of the robust correntropy-based measurement, similar background in each image can be suppressed well. Fig. 3 presents our results with different types of images, including medical image, remote sensing image and astronomical image. The results further demonstrate the effectiveness of our algorithm.

5.2. Noise sensitivity comparisons

In this experiment, we compare our method with LBF and LBF+Order on the segmentation of a blood vessel image. The input image is corrupted by the salt and pepper noise with

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5 Available at www.mathworks.com/matlabcentral/fileexchange/23445.
6 Available at www.engr.uconn.edu/~cml/.
7 Available at www4.comp.polyu.edu.hk/~cskhzhang/.
0.5 noise level. That is, half of pixels in this image are corrupted. The input image is preprocessed by performing the median filtering operation. The comparative results are illustrated in Fig. 4.

As shown in the first row, despite the input image is greatly corrupted by noise, our LCK based segmentation method can still segment the blood vessel out. However, LBF and LBF+Order both fail. The second and third rows are two clusters images, i.e., background cluster image and foreground cluster image. The cluster images provided by our method can preserve edge features better so that our segmented contour can fit the object more

Fig. 4. The comparison of our method with LBF and LBF + Order. The images in the first column from top to bottom are the original image, the noise distorted image (the noise level is 0.5) and the manually segmentation result. The remaining three columns are the results of LBF, LBF+Order and our method. For each column, the first row is the segmentation result, while the second and third rows are the two cluster images.

Fig. 5. More comparisons of our method with LBF. The images in first row are the results of LBF, while the image in the second row are our results.

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8 All the local clusters \( \mu_{j,1}^{\text{N}} \) are reshaped as the background cluster image (refer to the images in the second row of Fig. 4). Similarly, all the local clusters \( \mu_{j,2}^{\text{N}} \) are reshaped as the foreground cluster image (refer to the images in the third row of Fig. 4).
accurately. Moreover, without the local order regularization, the clusters in the two cluster images are fused together. That is, we can observe large clusters (white dots) and small clusters (black dots) in one cluster image. Due to this, the segmented contour may locate on the smooth regions (refer to the discussions in [30]).

Fig. 5 presents more comparisons of our method with LBF. The input images are all distorted by the salt and pepper noise with 0.4 noise level. From this figure, our method provides more accurate segmentation results.

In this experiment, we compare our method with LBF+Order on another blood vessel image under seven noise levels, i.e., (0.1, 0.2, ..., 0.7). Comparative results are shown in Fig. 6. From this figure, we conclude that our segmentation results are more accurate than those of LBF+Order, especially when the noise level is large. When the noise level reaches 0.7, our segmented contour cannot fit object edges. Fortunately, as shown in Fig. 7, the two cluster images of our method can preserve the edge features well. On the contrary, it is still impossible for human beings to detect the object from the two cluster images provided by LBF+Order.

5.3. Segmentation comparisons on real images

In this experiment, we compare our method with CV, LIF, LBF and LBF+Order on a SAR image that is distorted by a strong noise. The comparative results are illustrated in Fig. 8. From this figure, we see that our method segments out the road successfully, yet introduces smaller other clutter background. The success is derived from the utilization of the robust correntropy-based distance measure. Due to this measure, some sporadic pixels, which are not similar with the pixels on the road, are regarded as outliers in the contour evolution. Unfortunately, in LIF, LBF and LBF+Order, these pixels are considered as foreground at the beginning of the contour evolution, and hence they diffuse to other neighboring pixels during the contour evolution. As a result, the segmentation results of these methods all present undesired background.

More comparisons are illustrated in Figs. 9 and 10. From the results on the SAR images in Fig. 9, we see that if the object and the background are similar, both the LBF+Order and our method can provide good segmentation results. However, if the object and the background are similar, the LBF+Order fails to segment out
Fig. 8. The comparison with CV, LIF, LBF and LBF+Order on a SAR image.

Fig. 9. Comparisons on two SAR images.

Fig. 10. The comparison on a ultrasound image.
the object. The similar observation can be found in a ultrasound image (see Fig. 10).

5.4. Quantitatively comparisons on a verification image database

We also evaluate our method on a verification image database, which contains 200 verification images. In practice, the verification images are generated by adding some simulated noise and illumination to character images. Fig. 11 gives an example from this database. To compare the segmentation accuracy quantitatively, we utilize the F-score value as a evaluation criterion, given by

\[
F-score = \frac{2 \cdot TP}{2 \cdot TP + FN + FP}
\]

where TP, FP, and FN are the true positive, false positive, and false negative, respectively. A good segmentation result should give a high F-score value.

We compare our method with CV, LIF, LBF and LBF+Order. As shown in Fig. 12, when the testing images are corrupted by Gaussian noise, our F-score values are the same with those of LBF+Order. When the noise type is salt and pepper, our method is better than others, especially when the noise level is large. This comparative result indicates that our LCK based segmentation method can perform well for images with multiple noise types, especially it performs well with salt and pepper noise.

6. Discussions and conclusions

In this paper, we propose a robust region-based level set segmentation algorithm based on LCK. Due to LCK, the proposed method is robust to unknown complex noise. Furthermore, as a local method, our method has desirable performance for images with intensity inhomogeneity.

Although our LCK based segmentation method is robust to complex noise and intensity inhomogeneity, it also has some limitations.

1. Similar to many traditional region-based level set segmentation methods, the proposed LCK model in our segmentation method is not convex. Hence, our method is a bit sensitive to the initialization, which prevents our method from achieving good results for the automatic segmentation task.

2. In LCK based segmentation, the weights of edge pixels are often decreased faster than those of region pixels during the iterative re-weighting process. Therefore, edge pixels are easier to be classified as outliers than region pixels, which may introduce inaccurate results along image edges. Hence, our method cannot obtain good results for images that contain wispy and clutter targets, such as hair.

In the future, we plan to add shape constraints so that our segmentation method can perform well for special objects as well as handle more complex images. Moreover, as a general method, LCK based clustering can be used for other related applications, such as matting and classification.

Conflict of Interest statement

None declared.
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References


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