



# Adaptive Critic Designs of Optimal Control for Ice Storage Air Conditioning Systems

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**Abstract.** In this paper, the optimal control scheme for ice storage air conditioning system is solved via an adaptive critic design method. Adaptive critic design is also called adaptive dynamic programming (ADP). First, the operation of the air conditioning system is analyzed. Next, adaptive critic method is designed to realize the optimal control for the air conditioning system. Numerical results show that using the data-based ADP optimal control method can reduce the operation costs.

**Keywords:** Ice storage air conditioning · Adaptive critic design  
Adaptive dynamic programming · Neural network · Optimal control

## 1 Introduction

Recently, ice storage air conditioning has been widely used in the world due to its outstanding characteristics [1–3]. It has become an important issue on how to realize the optimal control for the air conditioning system and give full play to its advantages of shifting peak and filling valley, so as to help the users achieve the greatest benefit in the economy.

Adaptive critic design (ACD) is one of the effective methods to solve the problems of nonlinear systems optimal control, and it can also efficiently conquer the “curse of dimensionality” in general dynamic programming. Adaptive dynamic programming (ADP) is another name of adaptive critic design. ADP was first proposed by Werbos to solve the forward-in-time problems of optimal control [4]. The main principle of ADP is to approximate the control law and the performance index function in dynamic programming equation by using function approximation structure, and obtain the system’s optimal performance [5]. Werbos used two neural networks in function approximation structure, to implement the ADP method. So ADP is often called neuro-dynamic programming [6]. Now ADP has been introduced in the field of energy management [7–11].

Developing a new self-learning scheme of optimal control for ice storage air conditioning via a data-based ADP method is the main focus of this paper. A self-learning optimal control method is designed to manage ice storage of

the air conditioning system, in order to save money and meet the cooling load demand, simultaneously. Compared with the current control strategies [12, 13], it is emphasized that the air conditioning system can realize self-learning by the load demand and real-time electricity rate, without requiring a mathematical model of the system. Numerical results by the method will show the effectiveness.

## 2 Self-learning Optimal Control Scheme for Ice Storage Air Conditioning System via Data-Based ADP

### 2.1 Ice Storage Air Conditioning System

The system is made up of air conditioning refrigerator, cooling load demand, cold storage equipment system (including cooling converter) and cooling management system. The management system is connected to the cold storage equipment through the converter. In this ice storage air conditioning system, the cold storage equipment adopts different control strategies to meet the cooling load demand. For the air conditioning system, three operational modes are considered.

- (1) Store mode: when the electricity rate is high and the cooling load demand is low, the air conditioning refrigeration system will meet the load demand directly and store cooling into the cold storage equipment.
- (2) Idle mode: the air conditioning refrigeration system will meet the load demand directly at certain hours, while the quantity of cold storage keeps constant.
- (3) Release mode: considering load demand and electricity rate, when the cost rate is high, the cold storage equipment releases cooling to supply the load at hours.

### 2.2 Air Conditioning Dynamics

The cooling capacity stored in the cold storage equipment can be expressed as

$$P_I(t+1) = P_I(t) - p_I(t)\eta(p_I(t)), \quad p_I(t) < 0. \quad (1)$$

Let  $t$  denote the time index. Let  $P_I(t)$  (kWh) denote the residual cooling capacity in the cold storage equipment. Let  $p_I(t)$  (kW) denote the cooling capacity output of the cold storage equipment, and let  $\eta(p_I(t))$  denote the conversion efficiency. The cooling capacity released from the cold storage equipment can be expressed as

$$P_I(t+1) = P_I(t) - p_I(t)\eta(p_I(t)), \quad p_I(t) > 0. \quad (2)$$

The load demand is shared between the air conditioning refrigerator and the cold storage equipment, which is expressed as

$$p_L(t) = p_I(t) + p_C(t). \quad (3)$$

Let  $p_L(t)$  denote the cooling load demand, and let  $p_C(t)$  denote the cooling capacity output of the refrigerator. The optimization problem can be described as minimizing the performance index function, shown as

$$\begin{cases} \min J(t) = \alpha \sum_{t=1}^{\infty} C(t) \times p_C(t) \\ \text{s.t. Physical Constrains} \end{cases} \quad (4)$$

where  $\alpha$  is the coefficient of power consumption. Let  $J(t)$  denote the performance index function, and let  $C(t)$  denote the electricity rate.

### 2.3 Data-Based Adaptive Dynamic Programming

The delays in  $p_L(t)$  and  $p_C(t)$  are introduced for convenience of analysis, so the load balance as  $p_L(t-1) = p_I(t-1) + p_C(t)$  can be defined. Then, we let  $x_1(t) = p_C(t)$  and  $x_2(t) = P_I(t)$ . Let  $x(t) = [x_1(t), x_2(t)]^T$  and  $u(t) = p_I(t)$ . According to the air conditioning model, the discrete nonlinear system is given by

$$x(t+1) = F[x(t), u(t), t] = \begin{pmatrix} p_L(t) - u(t) \\ x_2(t) - u(t)\eta(u(t)) \end{pmatrix}. \quad (5)$$

We let

$$J[x(t), t] = \sum_{k=t}^{\infty} \gamma^{k-t} U[x(k), u(k), k] \quad (6)$$

denote the performance index function, where the utility function is defined as  $U[x(k), u(k), k] = \alpha \times C(t) \times x_1(t)$ , and  $\gamma$  is the discount factor between pre and post stages of the system with  $0 < \gamma \leq 1$ . Finding the sequence of control actions  $u(k)$  is the research object of dynamic programming. And it helps to minimize  $J[x(t), t]$  in (6). The optimal performance index function according to Bellman's principle is equals to

$$J^*[x(t), t] = \min_{u(t)} (\gamma J^*[x(t+1), t+1] + U[x(t), u(t), t]). \quad (7)$$

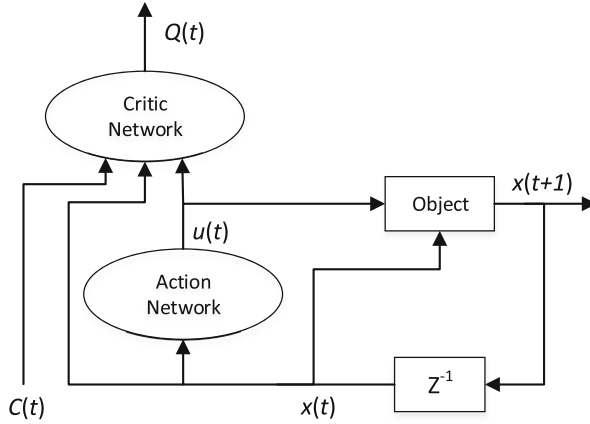
We let  $u^*(t)$  denote the optimal control actions. It can be expressed as

$$u^*(t) = \arg \min_{u(t)} (\gamma J^*[x(t+1), t+1] + U[x(t), u(t), t]). \quad (8)$$

A data-based ADP is employed to obtain the optimal control without constructing the dynamic of the cold storage system in this work. This kind of ADP is called action-dependent heuristic dynamic programming (ADHDP). Figure 1 shows the scheme of ADHDP, which minimizes the following error to train the critic network.

$$\|E_p\| = \sum_t E_p(t) = \sum_t \frac{1}{2} [Q(t-1) - U(t) - \gamma Q(t)]^2. \quad (9)$$

Let  $Q(t)$  denote the critic network output.



**Fig. 1.** The ADHDP scheme

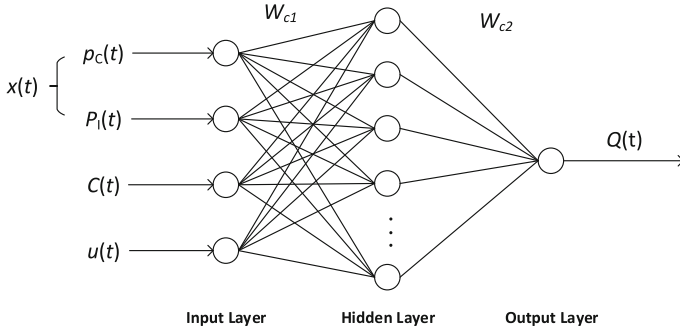
If for all time  $t$ , there is  $E_p(t) = 0$ , then it implies from (9) that

$$\begin{aligned}
 Q(t-1) &= \gamma Q(t) + U(t) \\
 &= \gamma[\gamma Q(t+1) + U(t+1)] + U(t) \\
 &= \dots \\
 &= \sum_{k=t}^{\infty} \gamma^{k-t} U[k].
 \end{aligned} \tag{10}$$

## 2.4 Self-Learning Scheme for Air Conditioning System

The optimal control scheme for air conditioning system is based on ADHDP. It includes action module and critic module. In the critic module, the performance index function is approximated by a BP neural network. The BP neural network has 3 layers with 4 nodes of input layer, 1 node of output layer and 9 nodes of hidden layer. As Fig. 2 shows,  $x(t)$ ,  $C(t)$  and  $u(t)$  are the inputs of the network, while  $x(t)$  denotes the system state that includes the residual cooling capacity  $P_I(t)$  and the cooling capacity output of refrigerator  $p_C(t)$ ,  $u(t)$  denotes the control action and  $C(t)$  denotes the electricity rate. Besides,  $Q(t)$  is the critic network output,  $W_{c1}$  and  $W_{c2}$  are the corresponding weight matrices. The object of the critic module is to minimize its error function in (9).

In the action module, three actions are defined as follow: release with  $u(t) = -1$ , idle with  $u(t) = 0$ , or store with  $u(t) = 1$ . The value of  $p_I(t)$  is related to  $u(t)$  and the parameters of the cold storage equipment, which needs to be discussed. Figure 3 shows the self-learning process of the optimal control scheme. When a load demand is received, it will find which action can minimize the critic network output, and then select this action as the current control action. The above process is based on the successful training of the critic network.



**Fig. 2.** The schematic diagram of critic network

Based on the above preparation, *Algorithm 1* describes the optimal control scheme for the ice storage air conditioning system.

*Algorithm 1. ADP Implementation*

① Data collecting. In this stage, when a cooling load demand happens, the random action  $u(t)$  with same probability will be adopted. At the same time, corresponding to each action, the electricity rate  $C(t)$  and the state  $x(t)$  should be collected, and the utility function will be calculated. In this work, the normalized utility function can be given by

$$U(t) = \frac{\text{the cost of power consumption}}{\text{the possible maximum cost}}. \quad (11)$$

② Critic network training. In this stage, the data collected in the last stage is used to train the critic network.

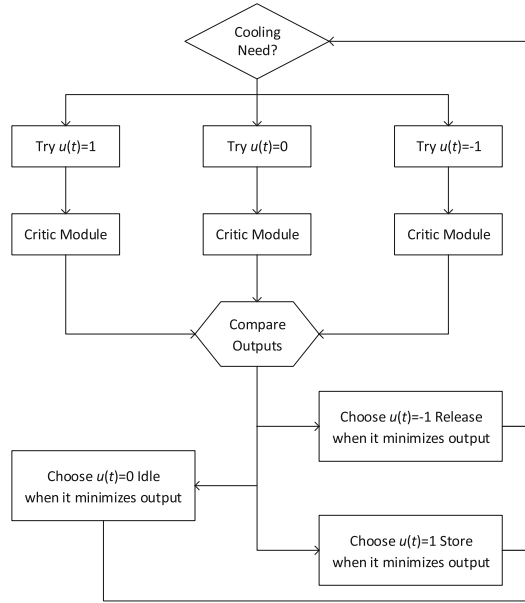
③ Applying the critic network that is trained successfully into the process as in Fig. 3. And the actions which minimizes the output will be selected in the system. The progress will continue until the number of total iterations is reached.

### 3 Numerical Experiment

#### 3.1 Experiment Preparation

The target of experiment is to minimize the cost of the air conditioning system by using the self-learning optimal control scheme, meeting the load demand and system constraints. Before the implementation, some settings are required.

- (1) The air conditioning system should meet the demand of cooling load at any time.
- (2) The capacity of the cold storage equipment used in the simulation is 50000 kWh and a minimum of 20% of the storage is required to maintain cooling load. The maximum rate of store/release is 8000 kW. The initial cold storage capacity is 20000 kWh.



**Fig. 3.** Block diagram of the self-learning scheme

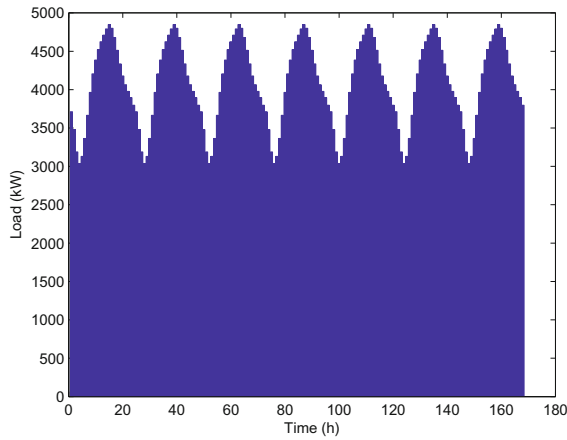
- (3) It is assumed that the cold storage equipment and the air conditioning refrigerator will not supply cooling at the same time. The demand of cooling load is supplied by either cold storage equipment or refrigerator at any time.
- (4) Using the load prediction model to predict the hourly cooling load in a certain day.

The training set for the cooling load demand data is chosen in [14,15], which are the load data of Yonyou Software Park No. 2 R & D Center in 2004. The cooling load demand data is given in Fig. 4. The daily real-time electricity rate is chosen by Beijing commercial electricity [16], which is shown in Fig. 5.

### 3.2 Results and Analysis

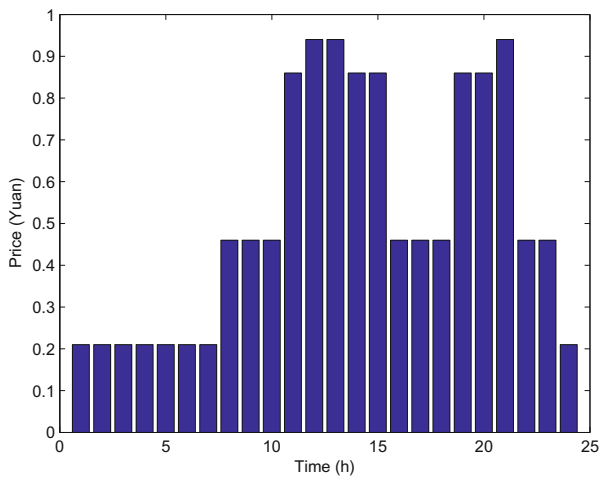
Based on the daily real-time electricity rate and the cooling load demand, the optimal control scheme can be implemented by Algorithm 1, where the structure of the critic network is set as 4–9–1. The optimal cooling storage/release control law for the air conditioning system in a week is shown in Fig. 6.

From the daily real-time price, we can see that, in the period of 11:00–15:00 and 19:00–21:00, there are two price peaks occurring. According to the cooling load curve, it can be found that the peak of the load occurs in the afternoon while the real-time price is high. So it is obvious that the peak of the cost happens when both peaks of load demand and real-time price occur. As can be seen from Fig. 6, when the real-time price is cheap and the load demand is



**Fig. 4.** The cooling load demand data in a week

low, the cooling capacity are fully stored. After that, cold storage equipment releases cooling during the peak cost hours, and stores cooling again during the valley cost hours. As a result, the total cost in a week without optimal control is 187881.04\$. Implementing the optimal control via data-based ADP method, the corresponding total cost reduces to 116487.71\$. So the optimal control scheme help the user to save 71393.33\$ in a week, and the saving rate is 38.0%.



**Fig. 5.** The daily real-time electricity rate

According to the numerical results, the saving rate has reached a very high standard. In a real-time application, it is not easy to implement the optimal control scheme because it has heavy neural network computations, which require fast microcontrollers.

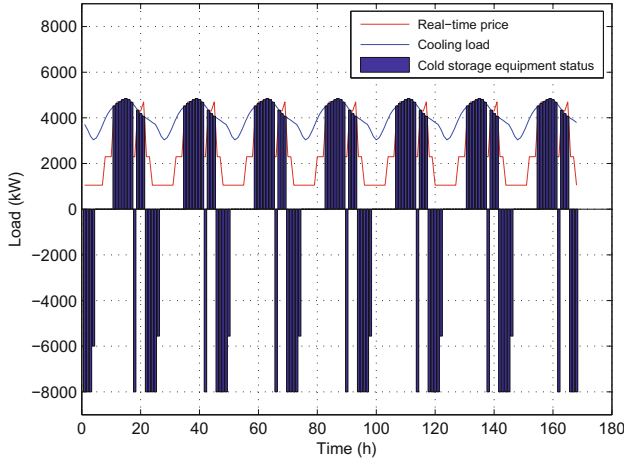


Fig. 6. Optimal scheduling of cold storage equipment in one week

## 4 Conclusion

In this paper, a self-learning optimal control scheme is developed based on ADP for ice storage air conditioning system. The main contents of this work include data-based ADP design and numerical experiment. The numerical results indicate that the developed ADP method is effective in minimizing the system cost. Compared with the current control strategies, it is of higher economy efficiency and does not need a mathematical model of the system.

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