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Optimal UAVs Formation Transformation Strategy based on Task Assignment and Particle Swarm Optimization

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***Abstract* – A nested optimization strategy based on task assignment and Particle Swarm Optimization (PSO) is proposed to solve the optimization problem of formation transformation. Both the corresponding assigned target positions of interchangeable UAVs from initial formation to target formation and the relative position relationship between two formations are investigated in this paper. The Hungarian algorithm is used to rapidly solve the assignment problem, while the PSO is adopted to compute the optimal relative position relationship iteratively. To demonstrate the effectiveness and the universality of the proposed strategy, simulation results considered different situations are presented.**

Index Terms* – *formation transformation; unmanned aerial vehicle; multi-UAV system; task assignment; particle swarm optimization (PSO).

I. INTRODUCTION

As a kind of multi-robot system with excellent flexibility and expansibility, the formation of unmanned aerial vehicles (UAVs) has developed rapidly in recent years. Compared with a single UAV, the multi-UAV system has several significant advantages, such as the expansion of reconnaissance range, the improvement of robustness and efficiency. The essential factors that bring these ascensions are the various formation configurations and the simplification of the communication topology as well as the progress from centralized to distributed computing structure, which have attracted the attention of many researchers.

Many related works on formation flight have been done and achieved fruitful achievements. Desai and Kumar used an underlying graph theoretical framework to control a team of robots in [1][2]. Using virtual structure and motion synchronization, a formation controller was proposed in [3]. Kuriki [4] presented a leader-follower formation controller and considered collision avoidance by applying the artificial potential approach. However, most of the methods mentioned above focus on the formation generating process under given initial conditions, and they treated this as a cooperative control problem.

With the formation flight showing more and more potential in various fields, such as formation reconnaissance, formation combat, and formation lighting performance, many problems are explicitly defined and studied independently, such as formation generating, formation maintaining, formation transformation and the obstacle avoidance problem. The design of formation plays a crucial role in formation flight, which directly affecting the success rate and efficiency of task completion. For example, in military applications, the horizontal formation is conducive to expanding the scope of detection search, while the longitudinal formation is conducive to reducing the radar reflection area and makes the system more secure. In the field of flight performance, large-scale UAV group needs to complete the generation, maintenance, and transformation of a variety of complex formations. Various design of formations and transformation are the basis for the performance. The problem of formation transformation is a key point since rapid and efficient completion of this process guarantees the high performance of the multi-UAV system and a considerable number of formation transformation needs to perform during the execution of tasks. Therefore, efficient formation transformation occupies a pivotal position in multi-UAV formation flight.

In the process of formation transformation, the first step is to determine the initial formation and the target formation. Then, the configuration of the multi-UAV system requires solving a task assignment problem in which each target position is assigned to a UAV. In [5], discrete and continuous particle swarm optimization algorithms were presented to solve the transformation problem of formation optimally. However, the discrete PSO needs too much computational time and only focus on polygon formation. Turpin first proposed the concept of the concurrent assignment and planning of trajectories (CAPT) in [6]. They used a centralized algorithm and an improved distributed algorithm to solve this problem and proved the trajectories were collision-free, but they did not consider the global optimization of the formation transformation problem. The reconfiguration problem of formation is treated as an optimization problem in [7], and the time to achieve the desired formation is set to be optimized. However, it is only feasible in 2-dimensional environments, which limits its application range.

In this paper, an optimal formation transformation strategy is proposed. The total distance cost in the transformation process is treated as the optimization target. Meanwhile, in order to meet the system requirements of high

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efficiency and rapidity, the well-known Hungarian Algorithm is adopted to solve the assignment problem of multi-UAV. Besides, particle swarm optimization (PSO) is used to iteratively optimize the position parameters since its low computational cost and rapid solution convergence.

The rest of this paper is organized as follows. In Section II, the description of the formation transformation problem and the mathematical expressions are given. The introduction and implement of the proposed strategy with some elementary proof as well as analysis are presented in Section III. To demonstrate the effectiveness and universality of the proposed strategy, plenty of representative simulation results are presented in Section IV. Finally, the conclusion and the future work expectations are put forward in Section V.

II. PRELIMINARIES

A. Problem description

In this paper, we primarily focus on rapid formation transformation between any given initial formation shape to target shape. In this scenario, the condition when and where to form the ultimate target formation is not explicitly assigned. This design flexibility motivates us to control the transformation process with some optimal specifications. To accelerate this process, all UAVs are homogeneous and interchangeable with no preference of any target location.

The formation transformation for multi-UAV can be decomposed into three sub-problems:

1) Determine the shape parameters of the initial formation and target formation, which is given in Section III.
2) Find the optimal explicit position of the ultimate target formation, which is solved by PSO.
3) Calculate the mapping relationship for each UAV in these two formations, which is solved by the Hungarian algorithm.

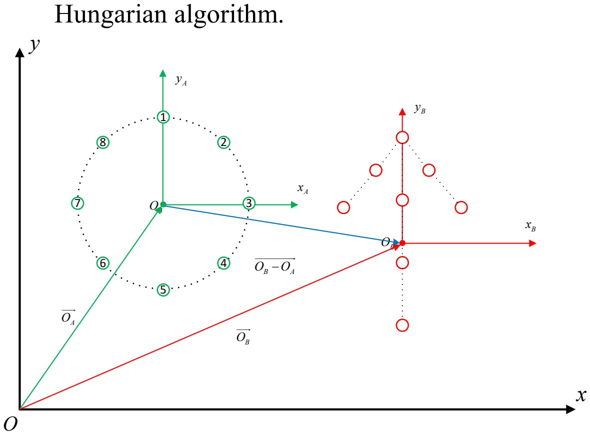


Fig. 1 A description of the formation transformation problem.

For a better understanding of this problem, a specific example is given in Figure 1. The green circle-shaped formation represents the initial formation. The red arrow-shaped formation represents the unfixed-position target formation. Our purpose is to transform the UAV formation shape from circle to arrow. Specifically, the first step is to get the coordinate positions of these two formations. The second step is to optimize the relative position relationship between the two formations, which minimizes the total distance required for the formation transformation process. Step 3 is to obtain the corresponding position in the red arrow-shaped formation for each UAV in the green round formation when a relative position relationship is determined. Actually, Step 3 is nested into step 2 in the process of optimization.

B. Definition

For simplicity, each UAV is regarded as a point in this paper and the generated paths are defined as straight lines from initial to target positions. This simplification focuses our research on the optimization of the formation transformation, and by adding relevant restrictions [6] the designed algorithm also works in a more practical situation where each UAV is taken as a circle with radius R.

Consider N UAVs moving from initial formation to target formation in an n-dimensional Euclidean space. The location of the ith UAV is specified by $\mathbf{x}_i \in \mathbb{R}^n$, $i \in \mathcal{I}_N$, $\mathcal{I}_N = \{1,2,3\ldots N\}$, and similarly, the jth target location is specified by $\mathbf{t}_j \in \mathbb{R}^n$, $j \in \mathcal{I}_N$. We then define the Nn dimensional formation system state vector, $\mathbf{X} \in \mathbb{R}^{Nn}$:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1^T & \mathbf{x}_2^T & \cdots & \mathbf{x}_N^T \end{bmatrix}^T,$$

and similarly we define the target formation system state vector $\mathbf{T} \in \mathbb{R}^{Nn}$:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T & \mathbf{t}_2^T & \cdots & \mathbf{t}_N^T \end{bmatrix}^T.$$

Define the assignment vector $\boldsymbol{\phi} \in \mathbb{R}^N$, which assigns UAVs to the corresponding positions,

$$\boldsymbol{\phi} = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_N],$$

where $\phi_i = j$ means the ith UAV is assigned to the jth target position.

For a particular initial formation $\mathbf{X}$ and target formation $\mathbf{T}$, $\boldsymbol{\phi}$ has N! possible solutions. Define $\boldsymbol{\phi}^*$ as the optimal solution which minimizes the total distance of the transformation process.

$$\boldsymbol{\phi}^* = \underset{\boldsymbol{\phi}}{\mathbf{argmin}} \sum_{i=1}^{N} \left\| \mathbf{x}_i - \mathbf{t}_{\phi_i} \right\|, \tag{1}$$

since the position of the target formation is changeable while maintaining the shape of the formation. We define the optimized target formation system state vector $\tilde{\mathbf{T}} \in \mathbb{R}^{Nn}$:

$$\tilde{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{t}}_1^T & \tilde{\mathbf{t}}_2^T & \cdots & \tilde{\mathbf{t}}_N^T \end{bmatrix}^T.$$

Furtherly, we choose the average center as a formation's reference center, and then define $\mathbf{d}$ as the relative position vector between the initial and target formations:

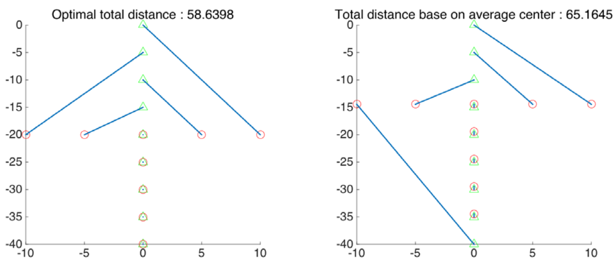
$$\mathbf{d} = \frac{1}{n}\left(\sum_{i=1}^{n} \mathbf{x}_i - \sum_{j=1}^{n} \mathbf{t}_j \right) + \boldsymbol{\alpha},$$

where $\boldsymbol{\alpha}$ is a variable parameter to be optimized, and $\tilde{\mathbf{T}} = \mathbf{T} + \mathbf{d}$.

Then the total cost of the transformation process for $\mathbf{X}$ and $\tilde{\mathbf{T}}$ is defined by C:

$$
\begin{aligned}
C = f\left(\mathbf{X}, \mathbf{T}, \mathbf{d}, \boldsymbol{\phi}\right) &= \sum_{i=1}^{n} \left\| \mathbf{x}_i - \left(\mathbf{t}_{\phi_i} + \mathbf{d}\right) \right\| \\
&= f\left(\mathbf{X}, \tilde{\mathbf{T}}, \boldsymbol{\phi}\right) = \sum_{i=1}^{n} \left\| \mathbf{x}_i - \tilde{\mathbf{t}}_{\phi_i} \right\|
\end{aligned} \quad . \tag{2}
$$

Once the initial formation $\mathbf{X}$ and the target formation $\mathbf{T}$ are determined, $\tilde{\mathbf{T}}$ as well as $\boldsymbol{\phi}$ can be obtained from $\mathbf{d}$. Therefore, we abbreviate the function as $f\left(\mathbf{d}\right)$.



(a) (b)

Fig. 2 An example to illustrate the optimal total distance.

Apparently, the average center of the two formations coincide when $\boldsymbol{\alpha} = \mathbf{0}$. However, it cannot guarantee that the total cost C is minimum in such situation. As shown in Figure 2, the total transformation distance is 65.1645 in (b) when the average centers coincide while the optimal total distance is 58.6398 in (a).

We define $\boldsymbol{\alpha}^*$ the optimal parameter which makes the target formation minimize the global distance.

$$
\boldsymbol{\alpha}^* = \underset{\boldsymbol{\alpha}}{\mathbf{argmin}} \sum_{i=1}^{N} \left\| \mathbf{x}_i - \tilde{\mathbf{t}}_{\phi^*_i} \right\| \ , \tag{3}
$$

where $\boldsymbol{\phi}^*$ is calculated by (1).

Our purpose is to calculate the optimal relative position vector for two given formation shapes and reach the global optimal of the formation transformation problem. Moreover, the method has to be efficient for the reason that nested relationship exists between the sub-problem 2) and sub-problem 3).

III. GLOBAL OPTIMAL STRATEGY

A. Formation settings

Seven basic formations are presented in this paper, including horizontal formation, longitudinal formation, T-shaped formation, X-shaped formation, circle formation, triangle formation and arrow-shaped formation. We mainly deal with the transformation problem between any two of these formations in this paper.

Since the design of formation is not our primary research object, all the formations in this paper are generated by given parameters, and it's unnecessary to go into details.

B. Hungarian algorithm

As mentioned above, our purpose is optimally to complete multi-UAV's task assignment at the same time, which means determining each UAV's corresponding position and making sure the total distance is minimum among all choices. More specifically, a process of N initial positions to N target positions includes N! possible solutions, and our task is to find the optimal one. The exhaustive method is possible for a multi-UAV system with a few robots. However, with the increase of UAV number, the amount of calculation increases exponentially, which makes it infeasible for multi-UAV system. Coincidentally, the task assignment problem occurs in some other disciplines such as operations research, distributed system, and mathematics.

While there are multiple classes of task assignment problem, the process of calculating the corresponding relationship for the initial formation and the target formation is a linear assignment problem since the numbers of UAVs and target positions are equal and the total cost of the assignment for all tasks is equal to the sum of the costs for each UAV. This problem can also be regarded as a perfect matching problem of bipartite graph, and the well-known Hungarian Algorithm [8][9] solves the linear assignment problem within time bounded by a polynomial expression of the number of agents, and is the most efficient and fastest known method to solve the linear task assignment problem optimally.

The sum of the each UAV's cost is treated as the total cost of the multi-UAV system, and this leads to two advantages:

1) The linear calculation has remarkably little computation. The computation of cost function is hardly affected by the increase in the number of individuals.

2) Collision free paths are generated using the linear sum of costs for each UAV, and the proof is below.

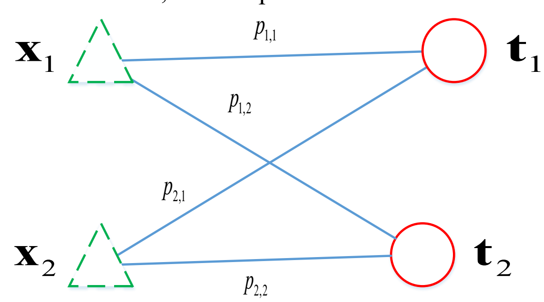


Fig. 3 A illustration of the proof.

Proof. Assume that the optimal assignment based on minimum total distance leads to collision paths. There are at least two generated paths intersecting. Might as well let them be $p_{1,2}$ and $p_{2,1}$, as shown in Figure 3. Using the triangle inequality, it's obvious that $p_{1,1} + p_{2,2} < p_{1,2} + p_{2,1}$. While the other assigned relationships remain the same, we can obtain a lower cost by replacing $p_{1,2}$ and $p_{2,1}$ with $p_{1,1}$ and $p_{2,2}$, which means the above assignment is not optimal. This is inconsistent with the hypothesis, which proves the hypothesis is wrong.

Similarly, the same result can be obtained in 3-dimensional. Therefore, the assignment based on minimum total distance never results in intersecting paths.

The pseudo-code of Hungarian algorithm is presented below, and the complexity of the algorithm is shown in Figure 3.

***Algorithm 1** Hungarian algorithm*

1. **for** Each initial UAV **do**
   for Each target position **do**
   $M(i,j) = \|x_i - t_j\|$
   end for
   end for
2. Subtract the smallest entry in each row from all the entries of its row and get $\tilde{M}$
3. Subtract the smallest entry in each column from all the entries of its column and get $\tilde{\tilde{M}}$
4. **for** Each row and column **do**
   Draw lines through appropriate rows and columns
   until All the zero entries of the cost matrix are covered
   The minimum number of such lines is m
   end for
5. **if** $m = n$ **then**
   finish
   end if
   if $m \le n$ **then**
   Determine the smallest entry not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Return to Step4
   end if
   finally An optimal assignment is met

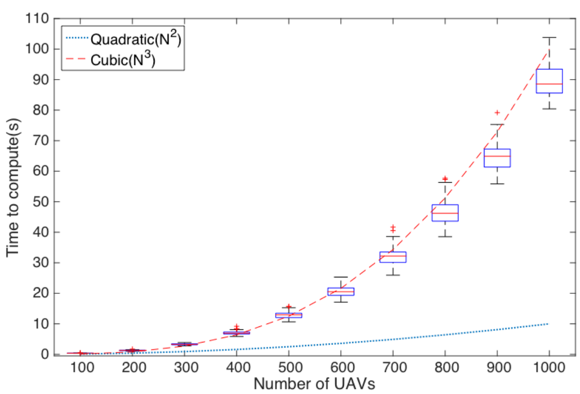


Fig. 4 The complexity of the Hungarian algorithm.

In order to verify the complexity of the algorithm, ten experiments are presented. The numbers of UAVs are set equidistantly in the range of 100 to 1000. Each experiment is conducted 100 times, and the expression of box graph is given in Figure 4. The red dashed line in the figure represents the complexity of cubic level, while the green dashed line represents the quadratic level. According to the box graph, we can find that the complexity of the Hungarian algorithm is equivalent to the cubic level, which shows that the algorithm is of low complexity.

C. Basic PSO algorithm

Particle Swarm Optimization (PSO) is proposed by Kennedy and Eberhart in 1995 [10] [11], mainly to solve the optimization problem of combination of parameters. Particle swarm algorithm is essentially a stochastic optimization algorithm based on the foraging behavior of birds and fish, etc. It works by maintaining a swarm of particles that are searching potential solutions in the feasible solution space. The direction is affected by both the improvements discovered by the other particles (social behavior) and the improvements found by the particle so far (cognition behavior). Due to its high efficiency and ease of implementation, PSO algorithm has been widely used in many fields, such as artificial intelligence, robotics, intelligent computing, and control.

Compared with other evolutionary algorithms, PSO presents two characteristics, which have a significant advantage in our particular problem: low computational cost and rapid solution convergence [12]. These features make the iterative speed fast enough and ensure an optimal or approximate optimal result.

D. Hybrid optimization algorithm

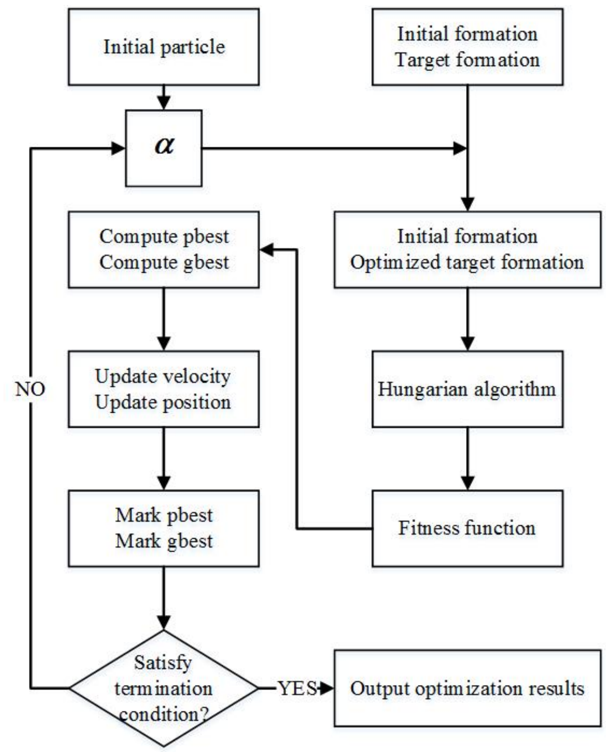


Fig. 5 The hybrid optimization algorithm.

A hybrid optimization algorithm is proposed in this paper to solve the for optimal formation transformation problem. As shown in Figure 5, the parameter α is defined as before to be optimized iteratively. More specifically, α represents a n-dimensional position vector. Each α corresponds to a new relationship between the initial formation $\mathbf{X}$ and the optimized target formation $\tilde{\mathbf{T}}$. The task assignment algorithm needs to rerun due to the change in the position of the target formation.

For each particle in each iteration of PSO, a task assignment problem is computed, and ϕ^* of this particle is obtained as well as the optimal total distance of the current relative position between the initial and the target formations. Then we take the current particle's optimal result as its fitness, and mark the optimal fitness of current iteration and the global optimal fitness. As the algorithm iterates, the parameters are continually optimized. In order to achieve the global optimization, we set a relatively larger number of iterations and observe the experimental results. The pseudo-code of the hybrid optimal algorithm is given below.

Algorithm 2 *Hybrid optimal algorithm*

for Each particle **do**

  Initialize each particle's state vector $\boldsymbol{x_i}$ randomly between the upper bound and lower bound

  Initialize particle best state vector $p_i \leftarrow x_i$, using Hungarian algorithm to obtain the optimal assignment $\boldsymbol{\phi}^*$, and compute the cost by function:

$$f\left(\mathbf{X}, \tilde{\mathbf{T}}, \mathbf{d}, \boldsymbol{\phi}^*\right) = \sum_{i=1}^{n} \left\| \mathbf{x}_i - \tilde{\mathbf{t}}_{\phi_i} + \mathbf{d} \right\|$$

  Since $\mathbf{X}$ and $\tilde{\mathbf{T}}$ are determined, and $\boldsymbol{\phi}^*$ can be obtained from $\mathbf{d}$, we abbreviated the function as $f(\mathbf{d})$

  If $f(p_i) < f(g)$ update the swarm best state vector $g \leftarrow x_i$

  Initialize each particle's velocity vector v_i randomly

end for

repeat

  for each particle **do**

    Pick random numbers r_g r_p with $rand(0,1)$

    Update the particle's velocity:

    $v_i \leftarrow \omega v_i + \phi_p r_p (p_i - x_i) + \phi_g r_g (g - x_i)$

    Update the particle's state vector:

    $x_i \leftarrow x_i + v_i$

    if $f(x_i) < f(p_i)$ **then**

      Update the particle's best-known state vector

      if $f(x_i) < f(g)$ **then**

      Update the swarm's best-known state vector $g \leftarrow x_i$

      end if

    end if

  end for

until Global optimal parameters is met

IV. Simulation

In this section, simulation of the proposed global optimal formation transformation strategy is presented with different setup conditions. The simulation has been conducted in MATLAB R2015a, and specific details are as follows.

In order to demonstrate the feasibility and the universality of various situations, different numbers of UAVs and various shapes of formations in 2-dimensional are given in the first set of simulations. As shown in Figure 6, (a) and (b) represent the process of transformations from random formation to a circle or a square formation. (c) - (h) show the optimal strategies of the formation transformation between several different formation shapes.

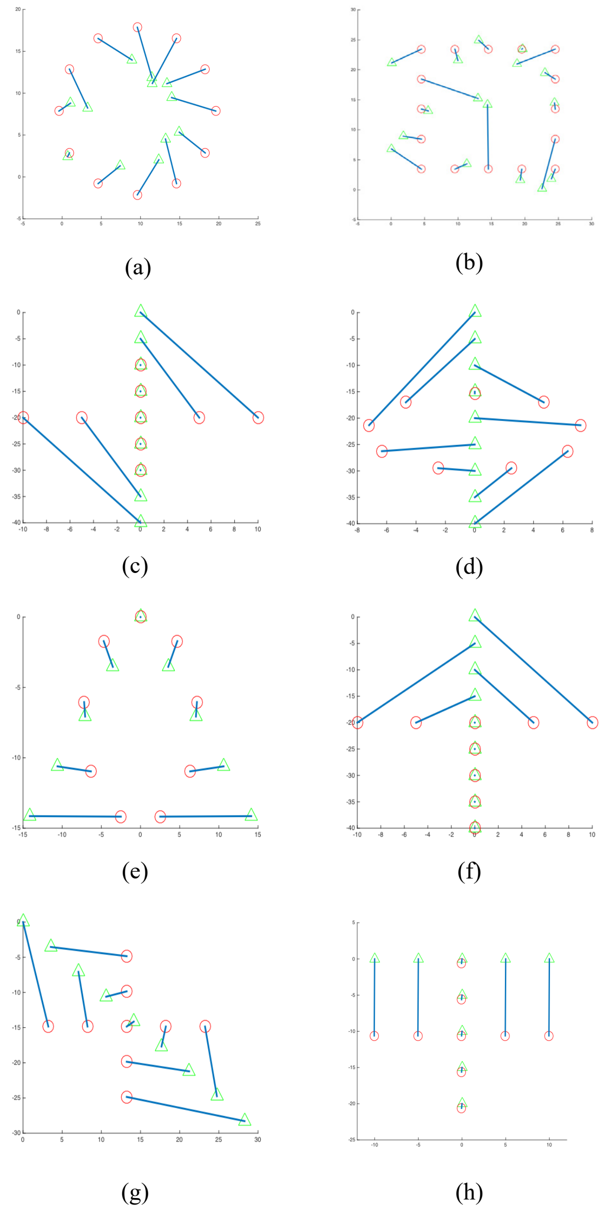
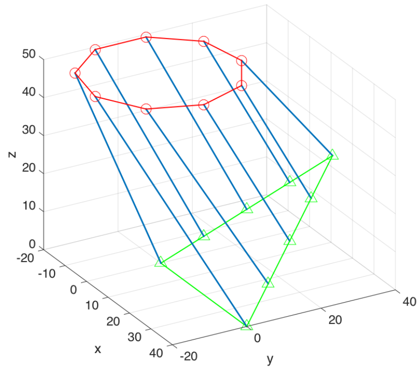
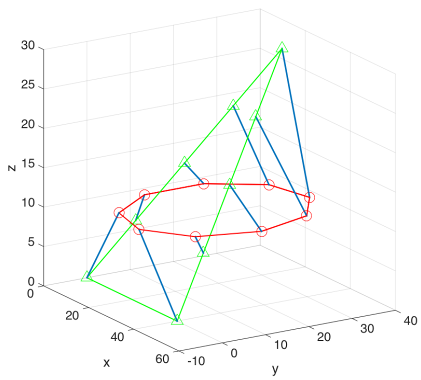


Fig. 6 Simulation results of different numbers of UAV and different formation shapes.

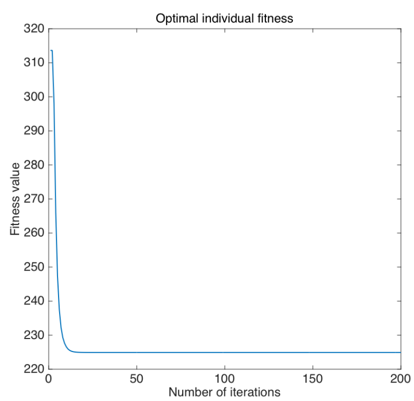
According to the simulation results shown in Figure 6, the proposed strategy implements the assignment of each UAV, and it is effective for different kinds of multi-UAV formations. The process of the formation transformation is optimized and the generated paths is collision-free.



(a)



(b)



(c)

Fig. 7 Simulation results of triangle formation transform to circle formation.

Next, the simulation of the 3-dimensional case is presented. The initial formation is set to be a triangle formation. The target formation is set to be a circle formation. To highlight the necessity of the strategy, both the unoptimized result, optimized result and the corresponding fitness function of the PSO algorithm are given.

In Figure 7, (a) represents an unoptimized formation transformation trajectory while (b) represents the optimized trajectory. Compared with (a), the total distance of the transformation process in (b) has been significantly reduced, which greatly improves efficiency of multi-UAV system.

Figure 7 (c) shows the fitness value with the number of iterations. We find that the optimization is usually completed within 50 iterations, which shows the fast convergence of PSO algorithm. Therefore, it is demonstrated that the optimization can be solved rapidly.

V. Conclusion And Expectation

In order to solve the problem of optimal formation transformation of UAVs, a strategy based on the Hungarian algorithm and PSO is proposed in this paper. The simulation results show that the strategy has a good performance in solving the nested optimization problem. The high efficiency and universality have been verified in the simulation as well. We are currently conducting practical flight experiment based on a quadrotor platform. Uncertainties and disturbances will be considered in the future, yielding an online optimization algorithm framework.

Acknowledgment

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