A Hybrid Multiagent Collision Avoidance Method for Formation Control

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Abstract. Collision avoidance in formation control is an essential and challenging problem in multiagent filed. Specifically, the agents have to consider both formation maintenance and collision avoidance. However, this problem is not fully considered in existing works. This paper presents a hybrid collision avoidance method for formation control. The formation control is designed based on consensus theory while the collision avoidance is achieved by utilizing optimal reciprocal collision avoidance (ORCA). Furthermore, the stability of the multiagent systems is proved. Finally, a simulation demonstrates the effectiveness of the proposed method.

Keywords: Collision avoidance · Formation control · Multiagent

1 Introduction

In recent years, swarm of agents is applied in various areas, including surveillance, disaster rescue and so on. More and more researchers have been attracted in this field. As a typical scenario of swarm intelligence, formation control has also received increasing attention from researchers. In order to improve the performance of the swarm, it is essential to investigate the formation control. In fact, three typical frameworks have been widely investigated for formation control, including leader-follower based approach [1], behavior based method [2], and virtual structure based strategy [3]. Although significant progress has been made in the formation control, collision avoidance in formation control is not fully investigated. In a leader follower formation architecture, one agent is chosen as the leader which decides the whole movement of the formation, while the others are followers which need to follow the leader. Actually, the formation shape will change due to different mission requirement, which may lead to collisions in the formation. Thus, collision avoidance in formation is a representative crucial issue during the mission process.

Traditional collision avoidance algorithms mainly include off-line methods [4], force-field methods [5], and sense-and-avoid methods [6]. Off-line methods is aimed at computing the collision free trajectory in advance with many constraints, but this method is computationally expensive and time consuming. The force-field methods solve the problem of collision by introducing virtual fields around obstacles and agents. However, they may encounter the problems of local minima and unreachable targets.

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The sense-and-avoid methods prevent collisions by sensing the around information and changing the immediate action accordingly, which are widely used.

Existing works on formation control with collision avoidance usually take the collision factor as an input in controller design procedure. By combining artificial potential field (APF) [7] method, an adaptive leader-following formation control with collision avoidance strategy is developed in [8]. APF is also utilized in [9] to solve obstacle avoidance problem. In [10], the collision avoidance constraint is imposed by the 2-norm of a relative position vector at each discrete time step. However, with the number of the agents increasing, those methods with collision avoidance constraints may increase the complexity and reduce the robustness of the multiagent system.

This paper presents a hybrid collision avoidance method in formation control to achieve an arbitrary transformation of formation shape based on optimal reciprocal collision avoidance (ORCA) [11] and formation control. In our proposed method, each agent only obtains information from its neighbors, and the formation controller generates the desire velocity for each agent. Afterwards, the collision avoidance module takes the preferred velocities as inputs and outputs the collision-free velocities at next step. To confirm the effectiveness of the proposed method, a numerical simulation is conducted.

The rest of the paper is organized as follows. In the next section, the preliminaries are presented. In Sect. 3, the basic ORCA method and formation control design are provided. And the hybrid method is presented. In Sect. 4, numerical simulation is provided.

2 Preliminaries

2.1 Modeling an Agent

Generally, the dynamics of an agent can be treated as a second-order system as follows,

$$\begin{cases} \dot{\mathbf{P}}(t) = \mathbf{V}(t) \\ \dot{\mathbf{V}}(t) = \mathbf{u}(t) \end{cases},$$

where $\mathbf{P}(t)$ represents the position, $\mathbf{V}(t)$ represents the velocity, and $\mathbf{u}(t)$ is the input of the system.

2.2 Graph Theory

In this paper, we define a multiagent system consisting of N agents is a system where each agent shares information with other agents via certain communication architecture. Generally, a graph denoted by $G = (V, E, A)$ is used to describe the information topology among agents. We define a single agent as node v_i, then $V = \{v_1, v_2, \ldots, v_n\}$ is the set of agents, and $E \subseteq V \times V$ represents the set of edges, where E is defined such that if $(v_j, v_i) \in E, j \neq i$, there is an edge from agent j to agent i, which means that agent j can send information to agent i. In addition, $A = [a_{ij}] \in R^{N \times N}$ is the associated adjacency matrix with $a_{ij} \geq 0$. We set $a_{ij} > 0, j \neq i$ if and only if $(v_j, v_i) \in E$; otherwise

$a_{ij} = 0$. Agent j is said to be the neighbor of agent i if and only if $a_{ij} > 0$, and $N_i = \{v_j \in V : (v_j, v_i) \in E\}$ represents the neighbor set of agent i. An undirected graph is called connected if there is a path between any two agents of graph G. Define $D = \text{diag}\{d_1, d_2, \ldots, d_N\} \in R^{N \times N}$ as the in-degree matrix, where $d_i = \sum_{v_j \in N_i} a_{ij}$. Then, the Laplacian matrix of graph G is defined as $L = D - A$.

3 Hybrid Collison Avoidance Method

3.1 Formation Control

Considering the following second-order dynamics of the ith follower

$$\begin{cases} \dot{\boldsymbol{P}}_i^F(t) = \boldsymbol{V}_i^F(t) \\ \dot{\boldsymbol{V}}_i^F(t) = \boldsymbol{u}_i^F(t), \end{cases} \tag{1}$$

where $\boldsymbol{P}_i^F = [P_{i,x}^F, P_{i,y}^F]^T$, $\boldsymbol{V}_i^F = [V_{i,x}^F, V_{i,y}^F]^T$ and $\boldsymbol{u}_i^F = [u_{i,x}^F, u_{i,y}^F]^T$ represent position, velocity and control input vector of the ith follower respectively.

Let $\boldsymbol{P}^L = [P_x^L, P_y^L]^T$ and $\boldsymbol{V}^L = [V_x^L, V_y^L]^T$ represent position and velocity vector of the leader. In addition, the follower is supposed to track the trajectory of the leader while keeping a certain distance, and $\boldsymbol{H}_{i,P} = [H_{i,x}, H_{i,y}]^T$ stands for the expected relative offset vector of $\boldsymbol{P}_i^F$ with respect to $\boldsymbol{P}^L$.

Let $\boldsymbol{e}_{i,P} = \boldsymbol{P}_i^F - \boldsymbol{P}^L - \boldsymbol{H}_{i,P}$, $\boldsymbol{e}_{i,V} = \boldsymbol{V}_i^F - \boldsymbol{V}^L$. The followers and leader are said to achieve formation tracking if for any given bounded initial states

$$\begin{aligned} \lim_{t \to \infty} \boldsymbol{e}_{i,P} = \boldsymbol{0} \\ \lim_{t \to \infty} \boldsymbol{e}_{i,V} = \boldsymbol{0} \end{aligned} \tag{2}$$

Design the following control protocol for the ith follower:

$$\begin{aligned} \boldsymbol{u}_i^F(t) = & -\sum_{i=1}^{N} a_{ij}\Big[k_1\Big(\boldsymbol{P}_i^F(t) - \boldsymbol{H}_{i,P} - \Big(\boldsymbol{P}_j^F(t) - \boldsymbol{H}_{j,P}\Big)\Big) + k_2\Big(\boldsymbol{P}_i^F(t) - \boldsymbol{H}_{i,P} - \Big(\boldsymbol{P}_j^F(t) - \boldsymbol{H}_{j,P}\Big)\Big)\Big] \\ & - b_i\big[k_1\big((\boldsymbol{P}_i^F(t) - \boldsymbol{P}^L(t) - \boldsymbol{H}_{i,P}\big) + k_2\big(\boldsymbol{V}_i^F(t) - \boldsymbol{V}^L(t)\big)\big] + \dot{\boldsymbol{V}}^L(t), \end{aligned} \tag{3}$$

where $k_1, k_2 > 0$ are control gains. a_{ij} is the elements of the adjacent matrix $\boldsymbol{A}$ among the followers, $b_i > 0$ if and only if the ith follower can receive the states information from the leader; otherwise $b_i = 0$.

Lemma 1 [12]**:** Matrix $\boldsymbol{L} + \boldsymbol{B}$ is a positive stable matrix if and only if the communication topology G has a spanning tree with the leader being the root node, where matrix $\boldsymbol{L}$ is the corresponding Laplacian matrix among the followers, $B = \text{diag}(b_1, b_2, \ldots, b_N)$.

Lemma 2 [13]**:** Matrix M is Hurwitz, if matrix $\boldsymbol{L}+\boldsymbol{B}$ is positive stable, where

$$\boldsymbol{M} = \begin{bmatrix} 0 & \boldsymbol{I}_N \\ -k_1(\boldsymbol{L}+\boldsymbol{B}) & -k_2(\boldsymbol{L}+\boldsymbol{B}) \end{bmatrix} \otimes \boldsymbol{I}_2,$$

and $\otimes$ represents Kronecker product and $\boldsymbol{I}_2$ is a two-dimensional identity matrix.

Theorem: If the communication topology G has spanning tree with the leader being the root node, then under the control protocol (3), formation tracking for the followers and leader can be achieved.

Proof: Let $\boldsymbol{e}_{i,P} = \boldsymbol{P}_i^F - \boldsymbol{P}^L - \boldsymbol{H}_{i,P}$ and $\boldsymbol{e}_{i,V} = \boldsymbol{V}_i^F - \boldsymbol{V}^L$ be the formation tracking position and velocity error, respectively. Then system (1) can be rewritten as

$$\begin{cases} \dot{\boldsymbol{e}}_{i,P} = \boldsymbol{e}_{i,V} \\ \dot{\boldsymbol{e}}_{i,V} = \boldsymbol{u}_i^F - \dot{\boldsymbol{V}}_i^F. \end{cases} \tag{4}$$

Let $\boldsymbol{e}_p = \left[\boldsymbol{e}_{1,P}^T, \boldsymbol{e}_{2,P}^T, \ldots, \boldsymbol{e}_{N,P}^T\right]^T$, $\boldsymbol{e}_V = \left[\boldsymbol{e}_{1,V}^T, \boldsymbol{e}_{2,V}^T, \ldots, \boldsymbol{e}_{N,V}^T\right]^T$ and $\boldsymbol{e} = \left[\boldsymbol{e}_P^T, \boldsymbol{e}_V^T\right]^T$. Submitting (3) into (1), then system (1) can be further rewritten as

$$\dot{\boldsymbol{e}} = \left(\begin{bmatrix} 0 & \boldsymbol{I}_N \\ -k_1(\boldsymbol{L}+\boldsymbol{B}) & -k_2(\boldsymbol{L}+\boldsymbol{B}) \end{bmatrix} \otimes I_2\right)\boldsymbol{e} = \boldsymbol{M}\boldsymbol{e}. \tag{5}$$

According to Lemmas 1 and 2, the matrix $\boldsymbol{M}$ is Hurwitz. Therefore, there exists a positive definite matrix $\boldsymbol{P}$ such that

$$\boldsymbol{M}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q}, \tag{6}$$

where matrix $\boldsymbol{Q}$ is an arbitrary positive definite matrix.

Construct the following Lyapunov function:

$$V = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{P}\boldsymbol{e}. \tag{7}$$

Taking the derivative of (7) with respect to t, we obtain

$$\begin{aligned} V &= \boldsymbol{e}^T\mathbf{P}\dot{\boldsymbol{e}} \\ &= \boldsymbol{e}^T\boldsymbol{P}\boldsymbol{M}\boldsymbol{e} \\ &= -\tfrac{1}{2}\boldsymbol{e}^T\boldsymbol{Q}\boldsymbol{e} \le 0 \end{aligned}. \tag{8}$$

Then system (5) is asymptotic stable, that is, $\boldsymbol{e} \to \boldsymbol{0}$, as $t \to 0$. Thus, the followers and the leader can achieve formation tracking under the control protocol (3).

3.2 Collision Avoidance Method

In this part, the collision avoidance module is presented. In the process of performing missions, the agents in the formation need to change their positional relationship due to the mission requirements. Thus, collision avoidance between agents need to be taken in consideration. In order to comprehensively consider the factors of the velocity information and the interactions among agents, the ORCA method is adopted here.

For two agents A and B, we define an open disc of radius r centered at $\mathbf{p}$ as follows,

$$\mathrm{D}(\mathbf{p}, r) = \{\mathbf{q} | \|\mathbf{q} - \mathbf{p}\| < r\}.$$

Then the velocity obstacle is $VO^{\tau}_{A|B}$,

$$VO^{\tau}_{A|B} = \{\mathbf{v} | \exists t \in [0, \tau] :: t\mathbf{v} \in D(\mathbf{p}_B - \mathbf{p}_A, r_A + r_B)\},$$

where $\mathbf{p}_A$ and $\mathbf{p}_B$ are the position of agents A and B; r_A and r_B represent the radius of their safe zone. τ is the time windows in which agent A and B will not collide. $VO^{\tau}_{A|B}$ means the set of all relative velocities of A with respect to B that may collide at some moment before τ.

Let $\mathbf{v}^{now}_A$ and $\mathbf{v}^{now}_B$ be the current velocities of agent A and B. and let $\mathbf{u}$ be the vector from $\mathbf{v}^{now}_A - \mathbf{v}^{now}_B$ to the closet point on the boundary of $VO^{\tau}_{A|B}$.

$$\mathbf{u} = \left(\underset{\mathbf{v} \in \partial VO^{\tau}_{A|B}}{\arg\min} \left\| \mathbf{v} - \left(\mathbf{v}^{now}_A - \mathbf{v}^{now}_B\right) \right\| \right) - \left(\mathbf{v}^{now}_A - \mathbf{v}^{now}_B\right).$$

$ORCA^{\tau}_{A|B}$ is defines as follows,

$$ORCA^{\tau}_{A|B} = \left\{ \mathbf{v} | \left(\mathbf{v} - \left(\mathbf{v}^{now}_A + \frac{1}{2}\mathbf{u} \right) \right) \cdot \mathbf{n} \geq 0 \right\},$$

where $\mathbf{n}$ is the normal vector of $\mathbf{u}$. By choosing the velocities for agent A and B using $ORCA^{\tau}_{A|B}$ and $ORCA^{\tau}_{B|A}$ respectively, both agents will take half of the responsibility to avoid collision with each other, which ensures the generation of a collision-free trajectory.

When more than one other agent exists around agent A, collision avoidance among multiagent need to be considered. The n-body collision avoidance is introduced here. By calculating $ORCA^{\tau}_{A|B}$ for each agent around agent A, $ORCA^{\tau}_{A}$ is defined as follows,

$$ORCA^{\tau}_{A} = D\left(0, \mathbf{v}^{\max}_A\right) \cap \bigcap_{B \neq A} ORCA^{\tau}_{A|B}.$$

It is clear that the set $ORCA_A^\tau$ contains the interaction effect among agents. Next, the agent selects a new velocity $\mathbf{v}_A^{new}$ which is closest to its preferred velocity $\mathbf{v}_A^{pref}$ as follows

$$\mathbf{v}_A^{new} = \underset{\mathbf{v} \in ORCA_A^\tau}{\arg\min} \left\| \mathbf{v} - \mathbf{v}_A^{pref} \right\|,$$

where $\mathbf{v}_A^{pref}$ is the preferred velocity given by global planning strategy, which mainly guide the global movement of the agent. In addition, the selection procedure can be efficiently solved by linear programming.

3.3 Hybrid Strategy

In order to deal with the formation control and collision avoidance comprehensively, a hybrid method based on formation control law and collision avoidance method ORCA is presented in this paper. We take the output of the formation controller as the preferred velocity in the ORCA module. Then the ORCA algorithm generates the collision avoidance velocity for each agent in formation. In this way, the formation will eventually be stable when there is no possibility of collision in a certain range. The formation controller is designed based on consensus theory and the stability analysis is given as above. The diagram of this method is shown as Fig. 1.

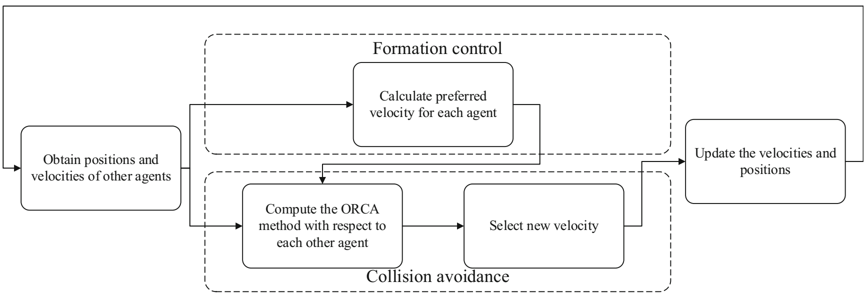

Fig. 1. The diagram of our hybrid method.

4 Simulation

In this part, we chose two typical scenarios to demonstrate the effectiveness of our method. In the first scenario, four agents are supposed to keep a square formation in the horizontal X-Y plane. In addition, the virtual leader is set as the formation center. At the beginning, the positions of agents 1–4 are $(-10, -10)$, $(10, -10)$, $(-10, 10)$, $(10, 10)$ respectively. The side length of the square is 20, which is supposed to be 10 in our formation control design. Besides, the interaction graph is shown in Fig. 2, and the control and collision avoidance parameters are set as Table 1. For a better demonstration of the proposed method, two potential collision are set in this simulation.

Firstly, during the forming process, agent 3 and 4 may get collide. Secondly, the desired formation shape will change at 100 s. Specifically, agent 1 and 4 will exchange their position, as well as agent 2 and 3. In this scene, the middle area will form a particularly crowded state. In the second scenario, four agents are supposed to keep a square formation as well, but the leader' movement is a sinusoid. Besides, four agents' initial positions and target positions are set to generate collisions among them.

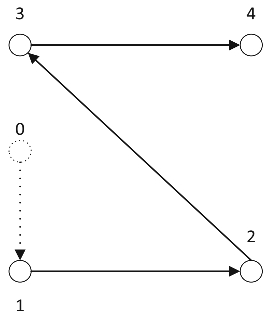

Fig. 2. The interaction graph.

Table 1. Parameters

Parameters of control		Parameters of ORCA	
k_1	1.5	Neighbor disc	5
k_2	3.5	Max neighbors	5
		Time horizon	5
		Time horizon obstacle	5
		Radius	0.3
		Max speed	10

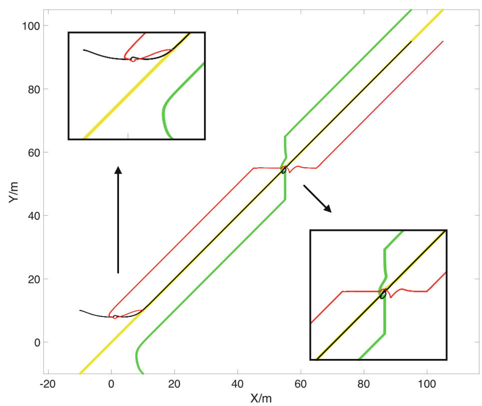

Fig. 3. The trajectories of the four agents in scenario1.

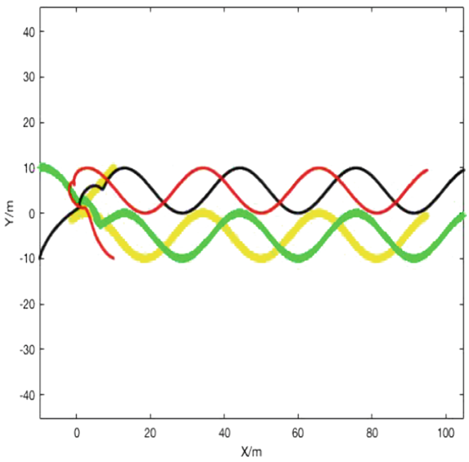

Fig. 4. The trajectories of the four agents in scenario2.

As shown in Figs. 3 and 4, the following phenomena can be observed: (i) the four agents successfully formed a square formation at the beginning; (ii) the virtual leader agent 0 moves along a straight line, the other followers also moves along the leader; (iii) during the two potential collision zones, the agents are able to adjust their velocities to avoid the potential collisions; (iv) After the collision avoidance movement, the formation is able to reform the formation shape.

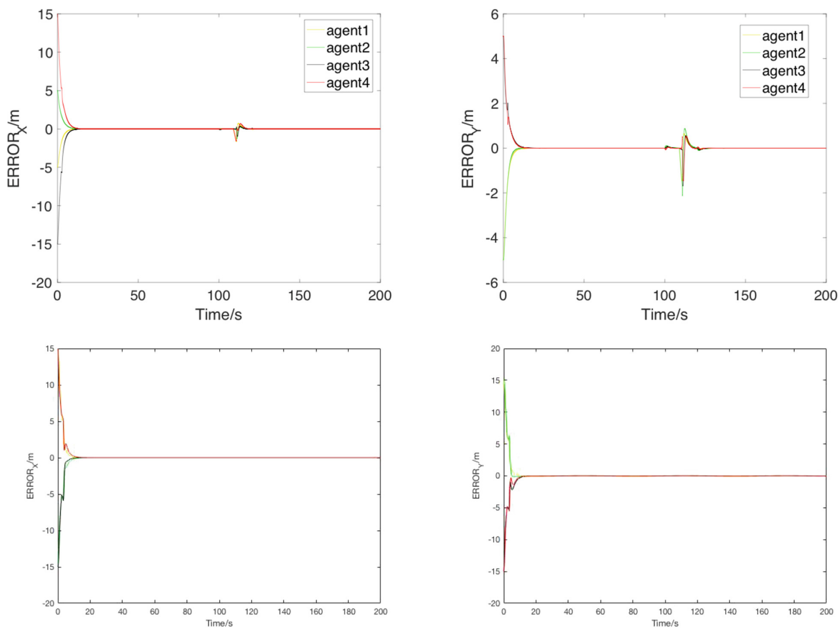

Fig. 5. The formation position tracking error on X-axis and Y-axis.

As shown in Fig. 5, the position error in both X-axis and Y-axis are small enough such that the formation will not be broken. And it implies that all agents can follow the reference position after finishing the collision avoidance. Therefore, the simulation results demonstrate that the proposed method performs well in the process of collision avoidance as well as formation control.

5 Conclusion

This paper presents a hybrid collision avoidance method in formation control to achieve arbitrary transformation of positions in formation based on optimal reciprocal collision avoidance and consensus theory. Specifically, the output of the formation control is treated as the input of the collision avoidance module, which guarantees the generation of collision free velocities. The simulation demonstrates the effectiveness of the presented method. In the further, we are looking forward to applying our method in multi ground-robot system.

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