Short-Term Solar Power Generation Forecasting Via Continuous Conditional Random Fields

Cheng Pan*
Institute of Automation, Chinese Academy of Sciences
University of Chinese Academy of Sciences
Beijing 100190, P. R. China
e-mail: pancheng2014@ia.ac.cn

Dandan Feng
State Key Laboratory of Advanced Power Transmission Technology
Global Electricity Interconnection Research Institute Co. Ltd.
Beijing 102209, P. R. China
e-mail: nobody.fd@163.com

Jie Tan
Institute of Automation, Chinese Academy of Sciences
Chinese Academy of Sciences
Beijing 100190, P. R. China
e-mail: tan.jie@tom.com

***Abstract*—Solar power generation with highly variable mode brings adverse effects on the grid. In order to reduce the negative impact on the grid, we use continuous conditional random fields (CCRF) to forecast solar generation. The CCRF is a powerful tool for relationship learning, which can capture the interaction between predicted solar generation. The potential function of the CCRF is designed as quadratic forms, which can transform the learning problems of the CCRF to convex optimization problems. In addition, it can perform probabilistic forecasting. To avoid over-fitting, the regularization of the weight is added to the loss function. We conduct the experiments on the freely available dataset to evaluate the forecasting performance. Experimental results show that the CCRF forecasting model can further improve the forecasting accuracy, compared with benchmarking forecasting method.**

Keywords-continuous conditional random fields; solar generation forecasting; regularization.

I. INTRODUCTION

Due to global warming, solar generation, one of promising renewable and clean energy, has attracted more and more attention in the last decades. Solar generation is the process of converting solar irradiance into electricity. Unlike traditional power sources, solar generation has highly-variable patterns because solar irradiance is affected by weather conditions. This makes large-scale photovoltaic grid-connected system bring adverse effects on the grid. Therefore, an accurate solar generation prediction can effectively reduce the negative impact on the grid.

Solar generation can be classified into three types, i.e., ultra-short-term forecasting, short-term forecasting, medium-term forecasting, and long-term forecasting, in terms of time horizon. Short-term forecasting refers to predict solar generation for one hour, several hours, one day or several days. At present, many researchers have done a lot of research on short-term solar generation forecasting. The main prediction methods include persistence method (PM), artificial neural network (ANN), support vector machine (SVM) and so on. In [1], a least square support vector machine (LSSVM) was proposed for short-term solar generation prediction. The forecasting performance was verified by using the data obtained from the National Solar Radiation Database (NSRDB). The results show that the LSSVM outperforms the autoregressive model and the radial basis function neural network. The LSSVM with RBF kernel function was also used in [2] to predict the next day's solar insolation. The simulation results show it is an effective and feasible method for estimating solar insolation. ANN is an information processing system that is build to mimic the structure and function of the brain neural network. It has powerful learning capabilities and is adaptable to many complex problems. Many modified ANN, i.e., Radial Basis Function Neural Network (RBFNN) [3], Extreme Learning Machine Neural Network (ELMNN) [4], Wavelet Neural Network (WNN) [5], etc., were used for short-term solar generation forecasting. Besides the machine learning method mentioned above, deep learning is also used to forecast solar generation [6]-[11]. These methods have achieved a promising prediction performance. However, most of these methods do not consider the interaction between predicted solar generation.

To make good use of these interactions, conditional random fields (CRF) were proposed in [12]. Originally, CRF was used to segment and label sequence data by establishing a probability model, which can only be used for discrete problems. In order to use CRF to solve regression problems, Qin *et al.* [13] proposed continuous conditional random fields (CCRF). Since then, CCRF has been widely used to solve various problems and achieved promising results. In [14], a CCRF model was used to estimate Aerosol Optical Depth (AOD). Guo [15] adopted a CCRF model to load forecasting. However, to our knowledge, prediction of solar

generation using a CCRF model has not been investigated. Following this motivation, we use the CCRF to forecasting solar generation. In addition, the regularization of the weight is added to the loss function to avoid the over-fitting.

The remainder of this paper is organized as follows. In Section II, the basic theory of the CCRF is introduced. The designed model for solar generation forecasting is presented in Section III. Experimental results are presented and discussed in Section IV. Finally, the conclusion of this paper is given in Section V.

II. CONTINUOUS CONDITIONAL RANDOM FIELDS

Conditional random fields are a probabilistic undirected graphical model of random variable Y under a given random variable X, which directly models the conditional distribution P(Y|X). The Linear chain CRF is a widely used CRF. According to the Hammersley-Clifford theorem, it can be defined in the following form:

$$\begin{aligned} P(\mathbf{Y}\mid\mathbf{X}) &= \frac{1}{Z(\mathbf{X})}\cdot\exp(\sum_{i=1}^{K} w_k f_k(\mathbf{Y},\mathbf{X})), \\ Z(\mathbf{X}) &= \sum_{\mathbf{Y}}\exp(\sum_{i=1}^{K} w_k f_k(\mathbf{Y},\mathbf{X})), \end{aligned} \tag{1}$$

where K is the number of potential function, w_k represents a weight vector, $f_k(\mathbf{Y},\mathbf{X})$ is a potential function, $Z(\mathbf{X})$ is the normalization factor.

CRF is first presented to build probabilistic models to segment and label sequence data. It has achieved promising results in relational learning. Conventional CRF is a discrete model since its output variable is discrete. In order to deal with continuous regression problems, continuous conditional random fields (CCRF) is proposed. In CCRF, conditional distribution $P(\mathbf{Y}\mid\mathbf{X})$ can be described as:

$$P(\mathbf{Y}\mid\mathbf{X}) = \frac{1}{Z(\mathbf{X})}\cdot\exp(\sum_{i} H(\alpha,Y_i,X_i) + \sum_{i\sim j} G(\beta,Y_i,Y_j,\mathbf{X})), \tag{2}$$

where $H(\alpha,Y_i,X_i)$ is a node potential function which is used to capture the relationships between output and input, $G(\beta,Y_i,Y_j,\mathbf{X})$ is a edge potential function which is used to model the interactions between outputs y_i and y_j, α is the weights of the node potential function, β is the weights of the edge potential function. For CCRF, $Z(\mathbf{X})$ (the normalization factor) is represented as follows:

$$Z(\mathbf{X}) = \int_y \exp(\sum_{i} H(\alpha,Y_i,X_i) + \sum_{i\sim j} G(\beta,Y_i,Y_j,\mathbf{X}))dy. \tag{3}$$

Suppose training data are defined as $D=\{(\mathbf{X}_m,\mathbf{Y}_m)\}_{m=1}^{M}$ (M is the number of training sample). The learning task of the CCRF is to determine proper weights of the potential function to maximize the conditional log-likelihood of the training dataset. It can be written as:

$$(\hat{\alpha},\hat{\beta}) = \arg\max_{\alpha,\beta}(L(\alpha,\beta)), L(\alpha,\beta)=\sum_{m=1}^{M}\log P(\mathbf{Y}^{(m)}\mid\mathbf{X}^{(m)}). \tag{4}$$

where $L(\alpha,\beta)$ is the conditional log-likelihood of the training dataset.

After obtaining the weights of the potential function, we can infer the output $\mathbf{Y}$ with the largest conditional probability $P(\mathbf{Y}\mid\mathbf{X})$ given an input $\mathbf{X}$. Mathematically, it can be defined as:

$$\mathbf{Y} = \arg\max_{\mathbf{Y}}(P(\mathbf{Y}\mid\mathbf{X})). \tag{5}$$

If the potential functions, including the node potential function and the edge potential function, are defined as the quadratic form of output $\mathbf{Y}$, then the CCRF can be transformed to the form of a multivariate Gaussian distribution. Thus, we design the potential functions as the quadratic form of the output $\mathbf{Y}$. The design process is described in detail in the following section.

III. CCRF FOR SOLAR GENERATION FORECASTING

A. Potential function design

We first introduce the node potential function. The node potential function aims to capture the interactions between target output (solar generation) and information source (weather forecast variables). It can be written as:

$$H(\alpha,Y_i,X_i) = \sum_{k=1}^{K}\alpha_{k,i}H_k(Y_i,X_i) = -\sum_{k=1}^{K}\alpha_{k,i}(Y_i - H_k(X_i))^2, \tag{6}$$

where Y_i is the i th target output, $H_k(X_i)$ represents the prediction of the i th target output through the k th predictor, K is the number of predictors.

Next, the design of the edge potential function is introduced. The edge potential function is designed for capturing the relationships between target output Y_i and Y_j. In solar forecasting, the interaction is considered when Y_i and Y_j are related. The mathematical formula of the designed edge potential function can be described as follows:

$$G(\beta,Y_i,Y_j,X) = \sum_{l=1}^{L}\frac{1}{2}\beta_l S_{i,j}^{l}(Y_i - Y_j)^2, \tag{7}$$

where $S_{i,j}^{l}$ is a Boolean variable, which indicates whether the target output is considered in the edge potential function. Mathematically, it can be represented in the following form:

$$S_{i,j}^{l} = \begin{cases} 1, & \text{if i, j} \in \{\text{l, l+1}\}, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Finally, the CCRF model can be represented as

$$\begin{aligned} P(\mathbf{Y} \mid \mathbf{X}) = \frac{1}{Z(\mathbf{X})} \cdot \exp(-\sum_{i}^{N}\sum_{k=1}^{K}\alpha_{k,i}(Y_i - H_k(X_i))^2 \\ -\sum_{i\sim j}\sum_{l=1}^{L}\frac{1}{2}\beta_l S_{i,j}^{l}(Y_i - Y_j)^2), \end{aligned} \tag{9}$$

where N is the number of target outputs. In our case, we set $N = 24$ since the CCRF model is utilized for day-ahead hourly solar generation forecasting.

Due to the quadratic forms of the potential function, we can further map (9) to a multivariate Gaussian distribution which can be represented as (10).

$$P(\mathbf{Y} \mid \mathbf{X}) = \frac{1}{(2\pi)^{N/2} \mid \Sigma \mid^{\frac{1}{2}}} \cdot \exp\{-\frac{1}{2}(\mathbf{X} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{X} - \boldsymbol{\mu})\}, \tag{10}$$

where μ is a n-dimensional mean vector, Σ is a $n \times n$ covariance matrix, $|\Sigma|$ represents the determinant of Σ.

According to (9) and (10), we can further derive the following formula

$$\begin{aligned} &\sum_{i}\sum_{k=1}^{K}\alpha_{k,i}(Y_i - H_k(X_i))^2 - \sum_{i\sim j}\sum_{l=1}^{L}\frac{1}{2}\beta_l S_{i,j}^{l}(Y_i - Y_j)^2 \\ &= \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} & 0 & \cdots & 0 \\ Q_{21} & Q_{22} & Q_{23} & \cdots & 0 \\ 0 & Q_{32} & Q_{33} & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & Q_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} - \\ &\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}^T \begin{bmatrix} \sum_{k=1}^{K}\alpha_{k,1}H_k(X_1) \\ \vdots \\ \sum_{k=1}^{K}\alpha_{k,n}H_k(X_n) \end{bmatrix} + const \\ &= \frac{1}{2}\mathbf{Y}^T\boldsymbol{\Sigma}^{-1}\mathbf{Y} - \mathbf{Y}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + const. \end{aligned} \tag{11}$$

where Q is a symmetric matrix with element

$$\begin{aligned} Q_{ii} &= \sum_{k=1}^{K}\alpha_{k,i} + \sum_{l=1}^{L}\beta_l\sum_{j=1}^{N}S_{i,j}^{l}, \\ Q_{ij} &= \begin{cases} -\sum_{l=1}^{L}\beta_l S_{i,j}^{l}, & \text{if } |\text{i-j}|=1, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{12}$$

According to (11), we can obtain the following equation

$$\begin{aligned} \boldsymbol{\Sigma}^{-1} &= 2\mathbf{Q}, \\ \boldsymbol{\mu} &= \boldsymbol{\Sigma}\mathbf{d}. \end{aligned} \tag{13}$$

In order to make the model feasible, we set the constraint that all weights ($\boldsymbol{\alpha}, \boldsymbol{\beta}$) have to be greater than zero. This can ensure the matrix $\mathbf{Q}$ is positive semi-definite. Thus the learning problem of the CCRF model becomes a convex optimization problem. In the next subsection, we will describe the learning of the CCRF model in detail.

B. Learning of the CCRF

In order to improve the computational efficiency, we use stochastic gradient descent (SGD) to calculate the parameters of the CCRF model. Also, the conditional log-likelihood is optimized with respect to $\log\alpha_{k,i}$ and $\log\beta_l$, which can guarantee that the learning problem of the CCRF is a convex optimization problem. In the SGD, the gradient of $\log\alpha_{k,i}$ and $\log\beta_l$ is calculated, which has the following forms

$$\begin{aligned} \nabla_{\log\alpha_{k,i}} &= \frac{\partial \log P(\mathbf{Y} \mid \mathbf{X})}{\partial \log\alpha_{k,i}} = \alpha_{k,i}\frac{\partial \log P(\mathbf{Y} \mid \mathbf{X})}{\partial \alpha_{k,i}}, \\ \nabla_{\log\beta_l} &= \frac{\partial \log P(\mathbf{Y} \mid \mathbf{X})}{\partial \log\beta_l} = \beta_l\frac{\partial \log P(\mathbf{Y} \mid \mathbf{X})}{\partial \beta_l}. \end{aligned} \tag{14}$$

To avoid over-fitting, the L2-norm of the weights is added to the conditional log-likelihood. It can be written as follows:

$$\begin{aligned} \log\alpha_{k,i} &= \log\alpha_{k,i} + \eta\cdot(\nabla_{\log\alpha_{k,i}} - \lambda_1\alpha_{k,i}^2), \\ \log\beta_l &= \log\beta_l + \eta\cdot(\nabla_{\log\beta_l} - \lambda_2\beta_l^2), \end{aligned} \tag{15}$$

where λ_1 and λ_2 are the weight of regularization terms.

After determining the weights of the CCRF model, the solar generation forecasting of new prediction day can be easily inferred by the mean vector, which has the following forms

$$\mathbf{Y} = \arg\max_{\mathbf{Y}} P(\mathbf{Y} \mid \mathbf{X}) = \boldsymbol{\mu} = \boldsymbol{\Sigma}\mathbf{d}. \tag{16}$$

IV. EXPERIMENTAL STUDIES

A. Data Description and Problem statement

The data in our case are obtained from the Global Energy Forecasting Competitions of 2014 (GEFCom14). The GEFCom14 provides solar generation and weather forecasts with hourly resolution. The solar generation is normalized by the nominal capacity of the corresponded solar power plant. Fig. 1 shows the hourly solar generation from 1 April 2012 to 30 April 2012. It depicts that solar generation has highly-variable patterns. The weather forecasts provided include 12 weather variables summarized in Table I. In order to verify the forecasting performance of the CCRF model, the data from 1 April 2012 to 31 July 2013 are selected. More specifically, we divide the data into training set and test set. Among them, the data from 1 April 2012 to 30 March 2013 are used as training set to train the CCRF model. The data from 1 April 2013 to 31 July 2013 are used as the test set to evaluate the forecasting performance of the CCRF model.

In this work, we use the CCRF model to predict the solar generation. More specifically, suppose we are currently at day d, we use the hourly weather forecast of the day $d+1$ as the inputs of the CCRF model to predict the hourly solar generation of the day $d+1$.

B. Evaluation Metrics

Three evaluation metrics, namely, nMBE, nRMSE, and forecast skill [16], are considered. They are given as follows:

$$nMBE = \frac{1}{M}\sum_{m=1}^{M}\frac{1}{N}\sum_{i=1}^{N}(Y_{mi} - \hat{Y}_{mi}) \times 100\%,$$
$$nRMSE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\frac{1}{N}\sum_{i=1}^{N}(Y_{mi} - \hat{Y}_{mi})^2} \times 100\%, \quad (17)$$
$$Skill = 1 - \frac{nRMSE}{nRMSE_{Persistence}},$$

where Y_{mi} is the i th normalized solar generation (target output) of m th sample, $\hat{Y}_{mi}$ represents the i th forecast solar generation of m th sample, N denotes the number of the solar generation of a day (in our case, N equals 24), M is the number of samples.

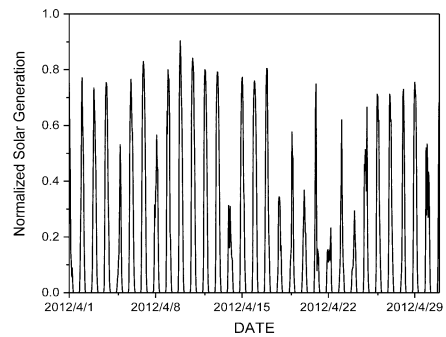

Figure 1. Hourly solar generation from 1 April 2012 to 30 April 2012.

TABLE I. WEATHER VARIABLES AND DESCRIPTION.

Variable Name	Unit	Description
Total column liquid water (tclw)	$kg{\cdot}m^{-2}$	Vertical integral of cloud liquid water content
Total column ice water (tciw)	$kg{\cdot}m^{-2}$	ertical integral of cloud ice water content
Surface pressure (SP)	Pa	
Relative humidity at 1000 mbar (r)	%	Relative humidity is defined with respect to saturation of the mixed phase, i.e. with respect to saturation over ice below -23°C and with respect to saturation over water above 0°C. In the regime in between a quadratic interpolation is applied.
Total cloud cover (TCC)	(0-1)	Total cloud cover derived from model levels using the model's overlap assumption
10 metre U wind component (10u)	$m{\cdot}s^{-1}$	
10 metre V wind component (10V)	$m{\cdot}s^{-1}$	
2 metre temperature (2T)	K	
Surface solar rad down (SSRD)	$J{\cdot}m^2$	Accumulated field
Surface thermal rad down (STRD)	$J{\cdot}m^2$	Accumulated field
Top net solar rad (TSR)	$J{\cdot}m^2$	Net solar radiation at the top of the atmosphere. Accumulated field.
Total precipitation (TP)	m	Convective precipitation + stratiform precipitation (CP +LSP). Accumulated field

C. Experimental Results and Analysis

In order to verify the forecasting performance of the CCRF, k-Nearest Neighbor (KNN) and Random Forests (RF) are used as the node potential functions, respectively.

1) Node potential function: KNN: In this subsection, KNN is used as the node potential function to verify the forecasting performance of the CCRF. The number of neighbors of the KNN is set to 25. To avoid over-fitting, we set the weight of regularization terms λ_1 and λ_2 to 1 and 10, respectively. In addition, the learning rate for the gradient ascent in the CCRF is set to 0.0001, and the number of iterations is set to 300. The iteration process of α and β is depicted in Fig. 2. It is clear that the values of α and β converge at iteration 200. Table II presents the forecasting performance of the CCRF (KNN is used as the node potential function). According to Table II, the CCRF is better than the other two benchmarking method. Thus, we can infer that the CCRF can further improve forecasting performance.

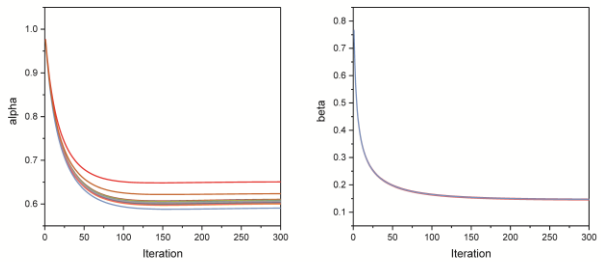

Figure 2. Iteration process of α and β (node potential function: KNN).

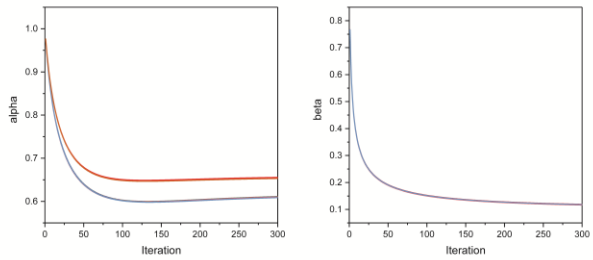

Figure 3. Iteration process of α and β (node potential function: RF).

2) Node potential function: RF: Besides KNN, we also use RF as the node potential function of the CCRF to forecast solar generation. In our experiments, the number of trees in the forest is set to 100. The learning rate for the gradient ascent in the CCRF, the number of iterations, the weight of regularization terms λ_1 and λ_2 are set to 0.0001, 300, 1 and 10 respectively, which is the same as when KNN is used as the node potential function. According to Fig. 3, we can also conclude that the weight of regularization terms λ_1 and λ_2 are almost unchanged after 200 iterations. The forecasting performance of the CCRF (RF is used as the node potential function) is shown in Table III. The nRMSE of the CCRF is 9.82% which is the lowest. Thus, we can also infer that the CCRF can further improve forecasting performance when RF is used as node potential function.

TABLE II. FORECASTING PERFORMANCE OF THE CCRF WHEN KNN IS USED AS THE NODE POTENTIAL FUNCTION

Forecasting method	***nMBE (%)***	***nRMSE(%)***	***Forecast Skill***
KNN	-2.81	12.45	0.05
Persistence Model	0.02	13.04	-
CCRF	-2.81	12.43	0.05

TABLE III. FORECASTING PERFORMANCE OF THE CCRF WHEN RF IS USED AS THE NODE POTENTIAL FUNCTION.

Forecasting method	***nMBE (%)***	***nRMSE(%)***	***Forecast Skill***
RF	-1.91	9.97	0.24
Persistence Model	0.02	13.04	–
CCRF	-1.92	9.82	0.25

The above two experiments verify that the CCRF can effectively improve forecasting accuracy. In addition, we can use the trained CCRF to perform probabilistic forecasting. According to (10) and (11), the 95% confidence intervals can represent as $Y \pm 1.96 \times diag(\Sigma)$. This shows that the CCRF can provide more information and can help photovoltaic power plants operate efficiently.

V. CONCLUSIONS

In this paper, we use the CCRF to forecast solar generation, which can capture the interaction between the predicted solar generation. In order to avoid over-fitting, the regularization terms of the weight are added to the loss function. And KNN and RF are used as the node potential function of the CCRF, respectively, to forecast solar generation. The experimental results show that the CCRF model can further improve forecasting accuracy. What's more it can provide probabilistic predictions. Feature extraction is critical for machine learning. In our future work, we will perform feature extraction of weather variables and solar generation to further improve prediction accuracy.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant U1701262 and U1801263.

REFERENCES

[1] J. Zeng and W. Qiao, "Short-term solar power prediction using a support vector machine," *Renewable Energy*, vol. 52, no. 2, pp. 118–127, 2013.

[2] B. B. Ekici, "A least squares support vector machine model for prediction of the next day solar insolation for effective use of pv systems," *Measurement*, vol. 50, no. 4, pp. 255–262, 2014.

[3] Z. Yue, M. Beaudin, R. Taheri, H. Zareipour, and D. Wood, "Day-ahead power output forecasting for small-scale solar photovoltaic electricity generators," *IEEE Transactions on Smart Grid*, vol. 6, no. 5, pp. 2253–2262, 2015.

[4] M. Hossain, S. Mekhilef, M. Danesh, L. Olatomiwa, and S. Shamshirband, "Application of extreme learning machine for short term output power forecasting of three grid-connected pv systems," *Journal of Cleaner Production*, vol. 167, pp. 395 – 405, 2017.

[5] V. Sharma, D. Yang, W. Walsh, and T. Reindl, "Short term solar irradiance forecasting using a mixed wavelet neural network," *Renewable Energy*, vol. 90, pp. 481–492, 2016.

[6] J. F. Torres, A. Troncoso, I. Koprinska, Z. Wang, and F. Martinezalvarez, "Big data solar power forecasting based on deep learning and multiple data sources," *Expert Systems*, 2019.

[7] A. Gensler, J. Henze, B. Sick, and N. Raabe, "Deep learning for solar power forecasting-an approach using autoencoder and lstm neural networks," in *2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, Oct 2016, pp. 002 858–002 865.

[8] A. P. Yadav, A. Kumar, and L. Behera, "Rnn based solar radiation forecasting using adaptive learning rate," in *Swarm, Evolutionary, and Memetic Computing*, B. K. Panigrahi, P. N. Suganthan, S. Das, and S. S. Dash, Eds. Cham: Springer International Publishing, 2013, pp. 442–452.

[9] K. Wang, X. Qi, and H. Liu, "A comparison of day-ahead photovoltaic power forecasting models based on deep learning neural network," *Applied Energy*, vol. 251, p. 113315, 2019.

[10] M. Abdel-Nasser and K. Mahmoud, "Accurate photovoltaic power forecasting models using deep lstm-rnn," *Neural Computing and Applications*, Oct 2017.

[11] H. Wang, H. Yi, J. Peng, G. Wang, Y. Liu, H. Jiang, and W. Liu, "Deterministic and probabilistic forecasting of photovoltaic power based on deep convolutional neural network," *Energy Conversion and Management*, vol. 153, pp. 409 – 422, 2017.

[12] J. Lafferty, A. Mccallum, and F. C. N. Pereira, "Conditional random fields: Probabilistic models for segmenting and labeling sequence data," *Proceedings of Icml*, vol. 3, no. 2, pp. 282–289, 2001.

[13] T. Qin, T. Y. Liu, X. D. Zhang, D. S. Wang, and H. Li, "Global ranking of documents using continuous conditional random fields," 2008.

[14] V. Radosavljevic, S. Vucetic, and Z. Obradovic, "Continuous conditional random fields for regression in remote sensing." in *ECAI*, 2010, pp. 809–814.

[15] H. Guo, "Accelerated continuous conditional random fields for load forecasting," *IEEE Transactions on Knowledge and Data Engineering*, vol. 27, no. 8, pp. 2023–2033, 2015.

[16] R. Marquez and C. F. Coimbra, "Proposed metric for evaluation of solar forecasting models," *Journal of solar energy engineering*, vol. 135, no. 1, p. 011016, 2013.