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CONTROL ALLOCATION BASED RECONFIGURABLE FLIGHT CONTROL SYSTEM DESIGN

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Abstract. *A new reconfigurable flight control system design scheme is proposed for multi-effector aircraft. The entire reconfigurable flight control system is composed of two parts: flight control law and control allocation. The flight control law is designed by nonlinear dynamic inversion method with two-time-scale separation, which produces the desired moment for given attitude command. A dynamic control allocation algorithm is adopted in the control allocation part, where the moment command is distributed into individual actuators under position and rate constraints. In the face of actuator faults, flight reconfiguration is fulfilled based on control allocation method, and the flight control law does not need to be changed. The reconfigurable flight control scheme is evaluated by a nonlinear six degree-of-freedom aircraft model accounting for actuator floating fault. Simulation results show the validity of the reconfigurable control scheme.*
Keywords: Reconfigurable flight control, Dynamic control allocation, Nonlinear dynamic inversion

1. **Introduction.** There is an increasing demand for the reliability and survivability of modern high performance aircraft. A reconfigurable flight control system needs to be designed to provide higher reliability and survivability through automatic reconfiguration of control system even if faults occur during flight. In certain situations, such as control surface damage or control surface jam, and so on, the aircraft dynamics is changed and unexpected nonlinearities are generated, which may deteriorate the system performance drastically. Reconfigurable flight control, which can redistribute and coordinate the control effort among remaining control surfaces, is desired to retain stability or satisfactory flight performance of the aircraft when it is physically possible.

Control allocation problem in flight control system arises from modern high-performance aircraft with redundant actuator configuration [1]. Control allocation is used to gain reasonable distribution of aerodynamic moments into individual actuators under position and rate constraints. Control allocation technique has inherent potential to perform reconfigurable flight control for modern aircraft with multi-effector configuration [2]. The idea is to use the redundancy of control surfaces to eliminate the effect of actuator failures and still provides the same (or almost the same) desired control effort. The greater the control redundancy is, the easier the reconfiguration via control allocation becomes.

In this paper, a new reconfigurable flight control system scheme is proposed, where flight control law and control allocation are combined. Different from other reconfigurable flight control systems based on adaptive control [3, 4], reconfigurable control is fulfilled by control allocation, and the original control law does not need not to be changed in the case of actuator faults. The overall system design, including the control law and control allocation, is given out in detail. The validity of the scheme is demonstrated by a six degree-of-freedom (DOF) nonlinear aircraft model.

2. **Reconfigurable Flight Control System Scheme.** The reconfigurable flight control system structure based on control allocation is shown in Figure 1.

The reference command of the flight control system are attitude angles of aircraft, i.e., roll angle, pitch angle and yaw angle, respectively. The flight control law is designed by nonlinear dynamic inversion (NDI) method [5]. For given attitude angle command, an NDI controller with two-time-scale separation is first designed to generate required moments, namely virtual control of control allocation module. Tracking error between the reference trajectory and the corresponding system states is used to update the NDI control law. Then, optimal control calculated by the control allocation module is applied to the aircraft.

An effective onboard fault detection and identification (FDI) system is assumed to provide accurate and timely fault information of actuators. When faults occur, parameters of the control allocation module are tuned according to the fault information to perform reconfigurable control, and the NDI controller need not be changed.

3. **Flight Control Law Design by Nonlinear Dynamic Inversion.** Nonlinear dynamic inversion is a straightforward and simple flight control method where nonlinear dynamics of the aircraft are canceled by feedback linearization. NDI approach has been widely used in flight control [6, 7].

3.1. **Nonlinear dynamic inversion in flight control.** The dynamics of aircraft can be formulated in the form of an affine nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where x is state vector, u is control vector, and $f(x)$, $g(x)$ are nonlinear functions of x. If $g(x)$ is invertible, the dynamic inversion of the system can be obtained by:

$$u = g^{-1}(x)[\dot{x} - f(x)] \tag{2}$$

In general, the desired response of the system is given in the form of a first-order system:

$$\dot{x} = \omega(x_c - x) \tag{3}$$

where ω is frequency bandwidth, and x_c is the commanded value of x.

3.2. **Control law design of nonlinear dynamic inversion.** It is well known that direct dynamic inversion needs to compute total inverse of a system, where the number of control variables is equal to the number of state variables. This fact prevents direct application of dynamic inversion to flight control system. To avoid the problem, a two-time-scale separation method based on singular perturbation theory [8] is introduced, where attitude angles and angular rates of aircraft are separated into slow state variables and fast state variables, respectively.

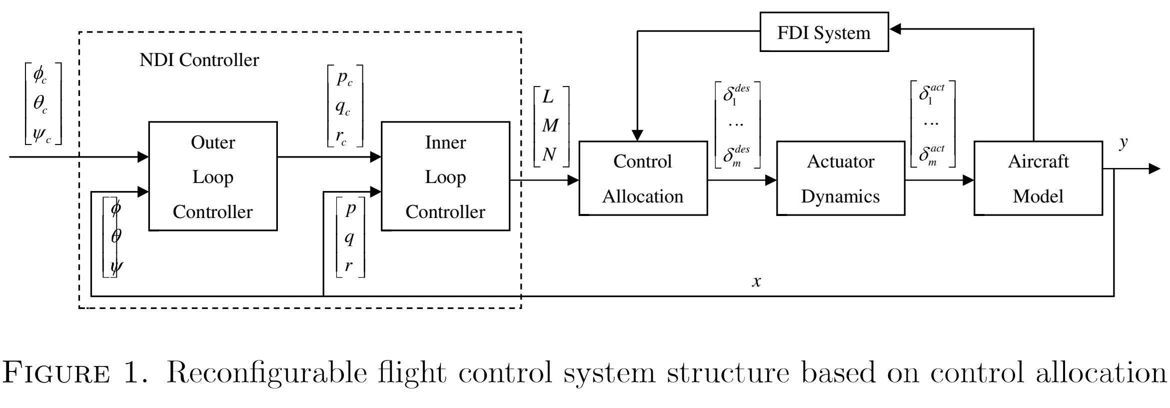


FIGURE 1. Reconfigurable flight control system structure based on control allocation

Based on the time-scale separation theory, the NDI attitude controller is composed of two control loops: the inner loop (the fast states loop) and the outer loop (the slow states loop). The outer loop controller is designed according to kinematic equations of aircraft:

$$\begin{aligned}\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ \dot{\theta} &= q\cos\phi - r\sin\phi \\ \dot{\psi} &= q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta}\end{aligned} \tag{4}$$

For the commanded attitude angles $[\phi_c, \theta_c, \psi_c]^T$, the corresponding angular rates command $[p_c, q_c, r_c]^T$ can be obtained through the outer loop controller.

Taking the input vector from the outer loop controller, the inner loop controller is used to generate the aerodynamic moment vector $[L, M, N]^T$, which serves as the input of the control allocation module. The inner loop controller is designed according to kinetic equations of aircraft:

$$\begin{aligned}\dot{p} &= c_1qr + c_2pq + c_3L + c_4N \\ \dot{q} &= c_5pr - c_6(p^2 - r^2) + c_7M \\ \dot{r} &= c_8pq - c_2qr + c_4L + c_9N\end{aligned} \tag{5}$$

where $c_1 = [(I_y - I_z)I_z - I_{xz}^2]/\Sigma$, $c_2 = [(I_x - I_y + I_z)I_{xz}]/\Sigma$, $c_3 = I_z/\Sigma$, $c_4 = I_{xz}/\Sigma$, $c_5 = (I_z - I_x)/I_y$, $c_6 = I_{xz}/I_y$, $c_7 = 1/\Sigma$, $c_8 = [(I_x - I_y)I_x + I_{xz}^2]/\Sigma$, $c_9 = I_x/\Sigma$, $\Sigma = I_xI_z - I_{xz}^2$

The dynamics of attitude angles and angular rates take the form of (3).

4. Control Allocation Based Reconfigurable Flight Control Design.

Most existing methods [9] assume a static mapping from virtual control $v(t)$ to true control $\delta(t)$, which means the resulting control distribution only depends on the present control demand. A dynamic control allocation algorithm is designed based on [10], where the control allocation result depends on not only present control distribution but also previous distribution.

4.1. Dynamic control allocation method.

The dynamic control allocation approach is an extended quadratic programming method by also penalizing the actuator rates.

$$\begin{aligned}J &= \min_{\delta(t)\in\Omega} \|W_1[\delta(t) - \delta_p(t)]\|_2 + \|W_2[\delta(t) - \delta(t-T)]\|_2 \\ \Omega &= \arg\min_{\underline{\delta}(t)<\delta<\overline{\delta}(t)} \|W_v[B\delta(t) - v(t)]\|_2\end{aligned} \tag{6}$$

where

$$\begin{aligned}\underline{\delta}(t) &= \max[\delta_{min}, \delta(t-T) + \dot{\delta}_{min}T] \\ \overline{\delta}(t) &= \min[\delta_{max}, \delta(t-T) + \dot{\delta}_{max}T]\end{aligned} \tag{7}$$

$B \in R^{n\times m}$ is the control effectiveness matrix with rank n $(m > n)$, $\delta \in R^m$ is actuator deflection vector, $v \in R^n$ is the desired virtual control vector, $v = [L, M, N]^T$ in flight control allocation case, δ_p represents some preferred positions of the actuators, W_1, W_2, W_v are weighting matrices, usually chosen as diagonal matrices of proper dimensions, T is sampling time.

Different weights in W_2 are chosen according to frequency property of different actuators. Fast/slow actuators are used to produce the high/low frequency components of the moment command, thus the chances of rate saturation are naturally reduced.

The tradeoff between the two objectives in (6) is governed by W_1, W_2. A larger diagonal matrix W_1 will make a quick convergence to the desired actuator positions, whereas a larger W_2 will prevent the actuators moving too fast.

W_1, W_2 allow for actuator prioritization to decide which actuators should be used primarily. Similarly, W_v allows for prioritization among the moments produced in pitch, roll, and yaw.

4.2. **Reconfigurable flight control design.** In the event of actuator faults, the aircraft dynamics is changed and unexpected nonlinearities are generated, which may deteriorate the system performance drastically. Generally speaking, the ith actuator faults mainly consist of being jammed at a position c_i, $\lambda_i(0 \le \lambda_i \le 100\%)$ percentage damage and actuator saturation. For the extreme case of damage fault, $\lambda_i = 100\%$ stands for the ith actuator being fully damaged or floating. Obviously, $\lambda_i = 0$ represents the nominal case. The mathematical statement of actuator faults can be formulated by:

$$\delta_i = \begin{cases} \delta_{i0} & \text{nonminal} \\ (1-\lambda_i)\delta_{i0} & \text{damaged} \\ c_i & \text{jammed} \\ \delta_{i\min} \text{ or } \delta_{i\max} & \text{saturation} \end{cases} \tag{8}$$

$$i = 1,\ldots,\mathrm{m}$$

In the face of actuator failures, the parameters of control allocation are accordingly tuned to perform reconfigurable control. When the ith actuator is damaged, its aerodynamic effectiveness is degraded, and the ith column vector in B is timed $(1-\lambda_i)$. For the extreme case $\lambda_i = 1$, the position and rate limits of the ith actuator should be set zeros additionally, i.e., $\delta_i = \dot{\delta}_i = 0$. When the ith actuator is jammed at a position c_i, the constraints of the ith actuator are changed as: $\delta_i = c_i$, $\dot{\delta}_i = 0$.

5. **Numerical Experiment.** The reconfigurable flight control system design scheme is evaluated on a nonlinear six-DOF aircraft model. The aircraft is configured with six actuators, which are left and right ailerons (δ_{al}, δ_{ar}), left and right rudders (δ_{rl}, δ_{rr}), and left and right elevators (δ_{el}, δ_{er}), respectively. The position and rate limits of actuator models are summarized in Table 1.The coefficients of the aircraft model are given as follows:

$$I_x = 39170 \text{ kg.m}^2,\ I_y = 244660 \text{ kg.m}^2,\ I_z = 107800 \text{ kg.m}^2,\ I_{xy} = I_{zy} = I_{xz} = 0.$$

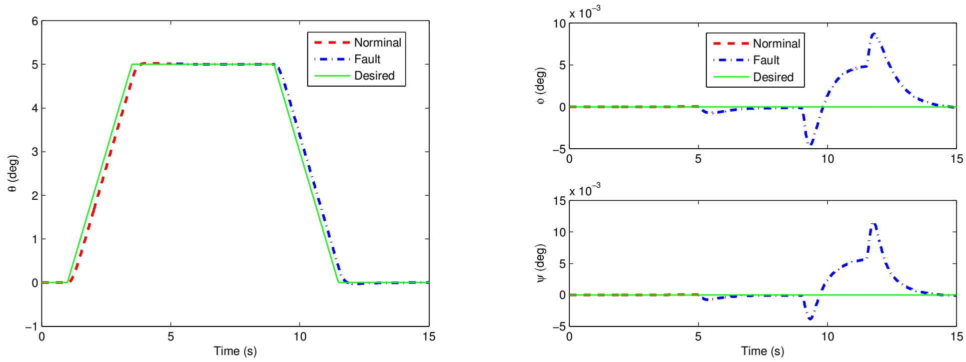
Table 1. Position and rate limits of actuators

	Position limit, deg	Rate limit, deg/s
Ailerons	(-30, 30)	(-80, 80)
Rudders	(0, 30)	(-120, 120)
Elevators	(-30, 30)	(-60, 60)

Flight condition for simulation is given as 200 m/s airspeed and 5000 m attitude. The control effectiveness matrix B under the flight condition is computed according to the wind tunnel data.

$$B = 10^6 \cdot \begin{bmatrix} 1.3965 & -1.3965 & -0.0828 & 0.0828 & 0.6831 & -0.6831 \\ -0.3593 & -0.3593 & 0.0059 & 0.0059 & -0.3549 & -0.3549 \\ 0.0257 & -0.0257 & -0.3773 & 0.3773 & 0.0448 & -0.0448 \end{bmatrix}$$

In accord with B, the actuator vector $\delta = [\delta_{al}, \delta_{ar}, \delta_{rl}, \delta_{rr}, \delta_{el}, \delta_{er}]^T$.



(a) Tracking result of pitch angle command

(b) Outputs of roll angle and yaw angle

FIGURE 2. Command-tracking results of attitude angles

To illustrate the effectiveness of the reconfigurable flight control system, take the longitudinal attitude angle command-tracking for an example here, the other two latitudinal attitude angles are set zeros during simulation. Suppose the first actuator, i.e., δ_{al} floats at t=5s during the pitch angle command-tracking process.

The weighting matrices of nominal case and fault case used in the dynamic control allocation module are: $W_1 = W_2 = I_{6\times6}$, $W_{1f} = \text{diag}([10, 1, 1, 1, 1, 1])$, $W_{2f} = \text{diag}([30, 1, 1, 1, 1, 1])$, $W_{vf} = W_v = I_{3\times3}$.

Weights of the fault actuator are enlarged in the fault weighting matrices W_{1f}, W_{2f} so as to reduce its use in the control allocation algorithm.

The command-tracking results of attitude angles in nominal and fault cases are shown in Figure 2. It can be clearly seen from Figure 2(a) that the pitch angle command θ_c is well tracked both in healthy and fault cases. The longitudinal motion produces little impact on the latitudinal motion, the outputs of the other two attitude angles are nearly zeros, as is shown in Figure 2(b).

The reconfigurable control allocation results are shown in Figure 3. In Figure 3(a), the desired moments produced by the NDI controller are depicted with solid lines, the actual moments computed by the DCA algorithm in nominal and fault cases are in dashed lines and dashdotted lines, respectively. The reconfigurable control allocation algorithm performs good moment-tracking in both cases. The good tracking performance of moments demonstrates the good attitude-tracking results in Figure 2 on the other hand. The deflection positions and deflection rates of the control surfaces are shown in Figure 3(b), Figure 3(c), respectively. When δ_{al} floats at t=5s, its output becomes zero and the desired moments are redistributed among the remained actuators. As is shown, all the actuators work within the position and rate constraints, which are listed in Table 1.

6. **Conclusions.** A new reconfigurable flight control system design scheme is proposed in this paper, based on nonlinear dynamic inversion and dynamic control allocation. The entire design procedure, mainly including two modules, is given out in detail. In the case of actuator faults, the parameters of the control allocation algorithm are correspondingly changed to perform flight reconfiguration, while the flight control law remains unchanged. The validity of the scheme is demonstrated by a nonlinear six-DOF aircraft model, numerical simulation results are analyzed in terms of effectiveness and rationality.

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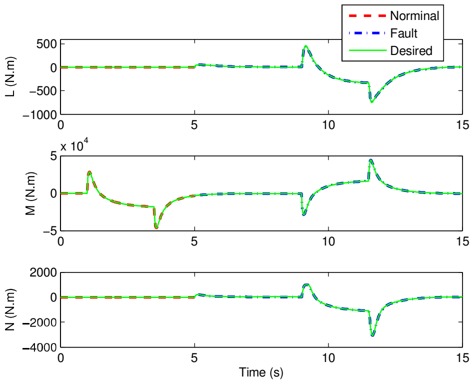
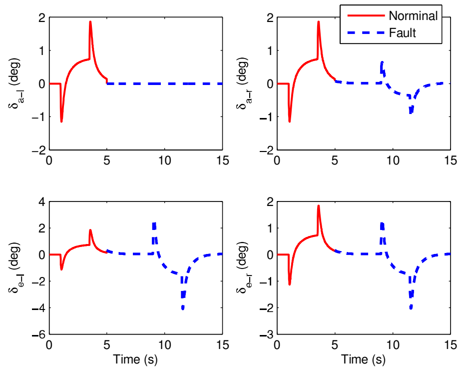
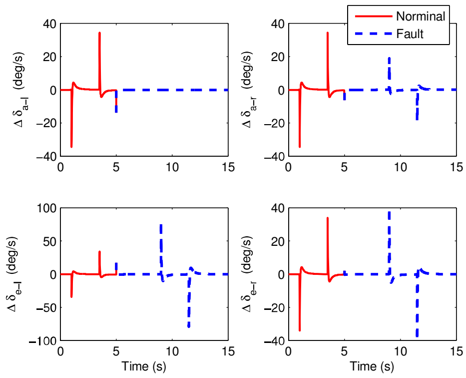


(a) Desired moments and actual moments



(b) Control surface deflection positions



(c) Control surface deflection rates

FIGURE 3. Reconfigurable control allocation results when δ_{a-l} floats at t=5s

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