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FUZZY IDENTIFICATION BASED ON MONOTONICITY AND INTERVAL TYPE-2 FUZZY LOGIC

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Abstract. *The paper presents the method of combining the prior knowledge of monotonicity with a Multi-Input-Single-Output (MISO) unnormalized Interval Type-2 (IT2) Takagi-Sugeno-Kang (TSK) Fuzzy Logic System (FLS) and applies the method to an NARX model such that the model can approach a given monotonic nonlinear system. We first provide sufficient conditions to guarantee the monotonicity of the NARX model with reference to its each input. And then based on the conditions, we show how to design the unnormalized interval type-2 fuzzy monotonic NARX models by means of a constrained least squares algorithm. Finally, an application to identify a coupled-tanks liquid-level system is given to illustrate the usefulness and advantages of the method under noisy circumstances.*

Keywords: System identification, Prior knowledge, Type-2 fuzzy logic, NARX model

1. **Introduction.** In many system identification problems, prior knowledge, such as continuity, monotonicity and convexity, etc., can play an important role, especially when mathematical models of target systems are unknown or the input-output data are not informative enough or corrupted by noise. A lot of excellent work has been done on how to use the prior knowledge [2-5]. Lindskog [2] have proposed a fuzzy model structure to ensure monotonic gain characteristics in identified models. In recent years, interval type-2 fuzzy logic theory has gained considerable concern from different research areas [1,3,4,7]. In [3], Li have studied the issue of the SISO monotonic normalized IT2 FLSs. But, at present, to the authors' knowledge, there are no literatures that incorporate prior knowledge into MISO Unnormalized Interval Type-2 Takagi-Sugeno-Kang Fuzzy Logic Systems (UIT2FLSs). In this paper, we combine the prior knowledge of monotonicity with MISO UIT2FLSs and apply the method to an NARX model. And then, we solve optimization problem to obtain the model parameters via a constrained least squares algorithm such that the model can approach a given monotonic target system.

2. **Multi-input Zeroth-order UIT2FLS.** An UIT2FLS is depicted by fuzzy IF-THEN rules. It has a complete rule base, each having M antecedents, where the ith rule R^i is denoted as

$$R^i: \text{ IF } x_1 \text{ is } \widetilde{A}_1^{j_1} \text{ and } x_2 \text{ is } \widetilde{A}_2^{j_2} \text{ and } \dots \text{ and } x_N \text{ is } \widetilde{A}_N^{j_N}, \text{ THEN } Y^i = [\underline{w}^i, \overline{w}^i],$$

where $i = j_1, j_2, \dots, j_N$ is a grid-oriented multi-index, $j_l = 1, 2, \dots, M_l$, $i = 1, 2, \dots, M$, $M = \prod_{k=1}^{N} M_k$, which is the number of fuzzy rules in this base; $\widetilde{A}_l^{j_l}$ $(l = 0, 1, \dots, N)$ are the interval type-2 fuzzy sets shown in Figure 1. For an input vector $\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$, the firing set $F^i(\boldsymbol{x})$ of rule R^i, is an interval type-1 set under product operator, i.e.,

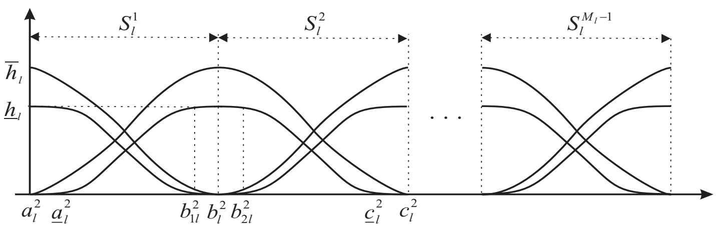


Figure 1. Interval type-2 fuzzy partition

$F^i(\boldsymbol{x}) = \left[\underline{f}^i(\boldsymbol{x}), \overline{f}^i(\boldsymbol{x})\right]$, where $\underline{f}^i(\boldsymbol{x}) = \underline{\mu}_{\widetilde{A}_1^{j_1}}(x_1)\underline{\mu}_{\widetilde{A}_2^{j_2}}(x_2)\ldots\underline{\mu}_{\widetilde{A}_N^{j_N}}(x_N)$ and $\overline{f}^i(\boldsymbol{x}) = \overline{\mu}_{\widetilde{A}_1^{j_1}}(x_1)$ $\overline{\mu}_{\widetilde{A}_2^{j_2}}(x_2)\ldots\overline{\mu}_{\widetilde{A}_N^{j_N}}(x_N)$. Here, $\overline{\mu}_{\widetilde{A}_l^{j_l}}(x_l)$s and $\underline{\mu}_{\widetilde{A}_l^{j_l}}(x_l)$s, which are described by the following equations, are the upper and lower membership functions of the interval type-2 fuzzy set $\widetilde{A}_l^{j_l}$, respectively.

$$\overline{\mu}_{\widetilde{A}_l^i}(x_l) = \begin{cases} 0 & x_l \le a_l^i \\ 2\overline{h}_l\left(\frac{x_l - a_l^i}{b_l^i - a_l^i}\right)^2 & a_l^i \le x_l \le \frac{a_l^i + b_l^i}{2} \\ \overline{h}_l - 2\overline{h}_l\left(\frac{b_l^i - x_l}{b_l^i - a_l^i}\right)^2 & \frac{a_l^i + b_l^i}{2} \le x_l \le b_l^i \\ \overline{h}_l - 2\overline{h}_l\left(\frac{x_l - b_l^i}{c_l^i - b_l^i}\right)^2 & b_l^i \le x_l \le \frac{b_l^i + c_l^i}{2} \\ 2\overline{h}_l\left(\frac{c_l^i - x_l}{c_l^i - b_l^i}\right)^2 & \frac{b_l^i + c_l^i}{2} \le x_l \le c_l^i \\ 0 & x_l \ge c_l^i \end{cases},$$

$$\underline{\mu}_{\widetilde{A}_l^i}(x_l) = \begin{cases} 0 & x_l \le \underline{a}_l^i \\ 2\underline{h}_l\left(\frac{x_l - \underline{a}_l^i}{b_{1l}^i - \underline{a}_l^i}\right)^2 & \underline{a}_l^i \le x_l \le \frac{\underline{a}_l^i + b_{1l}^i}{2} \\ \underline{h}_l - 2\underline{h}_l\left(\frac{b_{1l}^i - x}{b_{1l}^i - \underline{a}_l^i}\right)^2 & \frac{\underline{a}_l^i + b_{1l}^i}{2} \le x_l \le b_{1l}^i \\ \underline{h}_l & b_{1l}^i \le x_l \le b_{2l}^i \\ \underline{h}_l - 2\underline{h}_l\left(\frac{x_l - b_{2l}^i}{\underline{c}_l^i - b_{2l}^i}\right)^2 & b_{2l}^i \le x_l \le \frac{b_{2l}^i + \underline{c}_l^i}{2} \\ 2\underline{h}_l\left(\frac{\underline{c}_l^i - x_l}{\underline{c}_l^i - b_{2l}^i}\right)^2 & \frac{b_{2l}^i + \underline{c}_l^i}{2} \le x_l \le \underline{c}_l^i \\ 0 & x_l \ge \underline{c}_l^i \end{cases}$$

where $\tilde{a}_l \le a_l$, $\tilde{c}_l \le c_l$, $b_{1l}^i \le b_l^i$, $b_{2l}^i \ge b_l^i$. According to Theorem 13-2 in [1], the overall output of the unnormalized type-2 TSK model is inferred as

$$\widehat{y}(\boldsymbol{x}, \boldsymbol{w}) = \frac{1}{2}\sum_{j_1=1}^{M_1}\cdots\sum_{j_N=1}^{M_N}\left(\prod_{l=1}^{N}\underline{\mu}_{\widetilde{A}_l^{j_l}}(x_l)\underline{w}^{j_1\ldots j_N} + \prod_{l=1}^{N}\overline{\mu}_{\widetilde{A}_l^{j_l}}(x_l)\overline{w}^{j_1\ldots j_N}\right) \tag{1}$$

where $\boldsymbol{w}$ is a column vector which comprises $2\prod_{i=1}^{N} M_i$ elements and

$$\begin{aligned}\boldsymbol{w} = &\Big[\underline{w}^{11\ldots 11}, \underline{w}^{21\ldots 11}, \ldots, \underline{w}^{M_1 1\ldots 11}, \underline{w}^{12\ldots 11}, \ldots, \underline{w}^{M_1 2\ldots 11}, \ldots, \underline{w}^{M_1 M_2\ldots M_{N-1} M_N}, \\ &\overline{w}^{11\ldots 11}, \overline{w}^{21\ldots 11}, \ldots, \overline{w}^{M_1 1\ldots 11}, \overline{w}^{12\ldots 11}, \ldots, \overline{w}^{M_1 2\ldots 11}, \ldots, \overline{w}^{M_1 M_2\ldots M_{N-1} M_N}\Big]^{\mathrm{T}}.\end{aligned}$$

Next, we give the sufficient conditions which ensure multiple-input single-output UIT2F LSs are monotonic without proof.

Theorem 2.1. *Assume that an UIT2FLS is N-input single-output, x_l $(l = 1, 2, \ldots, N)$ denotes an input variable of the UIT2FLS, and that the corresponding input domain $U_l = [\underline{u}_l, \overline{u}_l]$ is partitioned by M_l triangular IT2FSs $\widetilde{A}_l^1, \widetilde{A}_l^2, \ldots, \widetilde{A}_l^{M_l}$. The UIT2FLS represented by (1) monotonically increases in the input vector $\boldsymbol{x} \in \mathbb{R}^N$, if the following conditions are satisfied:*

1. *Two fuzzy rules are fired, i.e.,* $a_l^1 = \underline{a}_l^1 = b_{1l}^1 = b_l^1 = \underline{u}_l$, $b_l^M = b_{2l}^M = \underline{c}_l^M = c_l^M = \overline{u}_l$, $b_l^i = a_l^{i+1}$, $c_l^i = b_l^{i+1}$ *and* $\underline{a}_l^i \geq a_l^i$, $b_l^i > b_{1l}^i$, $b_{2l}^i > b_l^i$ *for* $i = 1, 2, \ldots, M-1$*;*
2. $\overline{\mu}_{\widetilde{A}_l^i}(x_l) + \overline{\mu}_{\widetilde{A}_l^{i+1}}(x_l) = \overline{h}_l$ *and* $\underline{\mu}_{\widetilde{A}_l^i}(x_l) + \underline{\mu}_{\widetilde{A}_l^{i+1}}(x_l) = \underline{h}_l$, *for* $x_l \in S_l^i$, $i = 1, \ldots, M_l - 1$*;*
3. *for all combinations of* $j_1, \ldots, j_{l-1}, j_{l+1}, \ldots, j_N$, *it holds that* $\underline{w}^{j_1 \ldots j_l \ldots j_N} \leq \underline{w}^{j_1 \ldots j_l+1 \ldots j_N}$ *and* $\overline{w}^{j_1 \ldots j_l \ldots j_N} \leq \overline{w}^{j_1 \ldots j_l+1 \ldots j_N}$.

2.1. **Interval type-2 fuzzy NARX model.** NARX models are widely applied in many nonlinear identification solutions. Based on the interval type-2 fuzzy logic, the NARX model is given as follows:

$$\widehat{z}(t) = \widehat{y}(\boldsymbol{\varphi}(t), \boldsymbol{w}), \tag{2}$$

where $\widehat{z}(t)$ is the predicted value of the model output $z(t)$ at time t; $\widehat{y}(.)$ is a predictor which is a multi-input single-output UIT2FLS parameterized by $\boldsymbol{w}$ as shown in (1), and $\boldsymbol{\varphi}(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_N(t)]^{\mathrm{T}} = [y(t-1), \ldots, y(t-n_y), u(t-1), \ldots, u(t-n_u)]^{\mathrm{T}}$ represents the regressors, where $N = n_y + n_u$.

Remark 2.1. *Just changing the input vector* $\boldsymbol{x}$ *in (1) into* $\boldsymbol{\varphi}(t)$, *we can obtain the interval type-2 fuzzy NARX model, meanwhile we should note that the monotonicity that UIT2FLSs satisfy is identically fit for the NARX model.*

3. **Designing UIT2FLSs Via Constrained Least Squares Algorithm.** In this part, we will design monotonic NARX fuzzy model to identify target systems by means of a constrained least squares algorithm.

It is quite obvious that Equation (1) can be written as

$$\widehat{y}(\boldsymbol{\varphi}(t), \boldsymbol{w}) = \boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{\varphi}(t))\boldsymbol{w} \tag{3}$$

where the column vector $\boldsymbol{\phi}(\boldsymbol{\varphi}(t))$ consists of $2\prod_{i=1}^N M_i$ elements, and the orders of its elements $\underline{\phi}^{j_1 j_2 \ldots j_N}(\boldsymbol{\varphi}(t))$ and $\overline{\phi}^{j_1 j_2 \ldots j_N}(\boldsymbol{\varphi}(t))$ are the same as the orders of the elements $\underline{w}^{j_1 j_2 \ldots j_N}$ and $\overline{w}^{j_1 j_2 \ldots j_N}$ in $\boldsymbol{w}$, and $\underline{\phi}^{j_1 j_2 \ldots j_N}(\boldsymbol{\varphi}(t)) = \frac{1}{2}\prod_{i=1}^N \underline{\mu}_{\widetilde{A}_i^{j_i}}(\varphi_i(t))$, $\overline{\phi}^{j_1 j_2 \ldots j_N}(\boldsymbol{\varphi}(t)) = \frac{1}{2}\prod_{i=1}^N \overline{\mu}_{\widetilde{A}_i^{j_i}}(\varphi_i(t))$. From (3), we conclude that the output of the UIT2FLS is linear with its consequent parameters.

Suppose that there exist the input-output training data $\big(\boldsymbol{\varphi}(1), z(1)\big), \ldots, \big(\boldsymbol{\varphi}(r), z(r)\big)$ which are used to train the consequent parameters of the interval type-2 fuzzy NARX model. In the event that the fuzzy partitions of the model are determined, the interval weights of the consequent parts can be optimized by a least squares algorithm under the following training criteria, $E = \min_{\boldsymbol{w}} \sum_{t=1}^r |\widehat{z}(t) - z(t)|^2$, where $\widehat{z}(t)$ is the output of the NARX predictor. Noting that $\widehat{z}(t)$ is linear with its consequent parameters, the training criteria is rewritten as:

$$E = \min_{\boldsymbol{w}} \left(\Phi\boldsymbol{w} - \boldsymbol{z}\right)^{\mathrm{T}} \left(\Phi\boldsymbol{w} - \boldsymbol{z}\right), \tag{4}$$

where $\boldsymbol{z} = [z(1), z(2), \ldots, z(r)]^{\mathrm{T}}$, $\Phi = \left[\boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{\varphi}(1)), \boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{\varphi}(2)), \ldots, \boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{\varphi}(r))\right]^{\mathrm{T}}$.

The prior knowledge of monotonicity can be abstractly expressed as $P\boldsymbol{w} \leq 0$, where P is a constrained matrix. It is clear that the optimization problem in (4) can be transformed into the following constrained least squares optimization problem:

$$\begin{cases} \min\limits_{\boldsymbol{w}} (\Phi\boldsymbol{w} - \boldsymbol{z})^{\mathrm{T}} (\Phi\boldsymbol{w} - \boldsymbol{z}) \\ \text{subject to} \quad P\boldsymbol{w} \leq 0 \end{cases} \tag{5}$$

For two-input case, the constraint inequalities $P\boldsymbol{w} \leq 0$ can be rewritten as

$$\begin{bmatrix} H_1^2 & \mathbf{0}_{(M_1-1)M_2\times M_1M_2} \\ H_2^2 & \mathbf{0}_{M_1(M_2-1)\times M_1M_2} \\ \mathbf{0}_{(M_1-1)M_2\times M_1M_2} & H_1^2 \\ \mathbf{0}_{M_1(M_2-1)\times M_1M_2} & H_2^2 \\ I_{M_1M_2} & -I_{M_1M_2} \end{bmatrix} \boldsymbol{w} \leq \mathbf{0}_{(5M_1M_2-2M_1-2M_2)\times 1}, \tag{6}$$

where

$$H_1^2 = \Theta_{M_1} \in \mathbb{R}^{(M_1-1)M_2\times M_1M_2}, \qquad H_2^2 = \Psi_{M_1} \in \mathbb{R}^{M_1(M_2-1)\times M_1M_2},$$

$$\Theta_n = \begin{bmatrix} \Omega_{(n-1)\times n} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \Omega_{(n-1)\times n} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \Omega_{(n-1)\times n} \end{bmatrix}, \ \Psi_n = \begin{bmatrix} I_n & -I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & -I_n & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & I_n & -I_n \end{bmatrix}$$

in which $\Omega_{(M-1)\times M} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix}$ and I_M is the $M \times M$ identify matrix.

In conclusion, we can obtain an MISO interval type-2 fuzzy NARX via the following steps:

1. Determine the membership functions of IT2FSs $\widetilde{A}_l^1, \widetilde{A}_l^2, \dots, \widetilde{A}_l^M$ according to the first two conditions in Theorem 2.1;
2. Solve the following constrained least squares optimization problem (5), and obtain the consequent parameter vector $\boldsymbol{w}$ by means of numerical optimization methods.

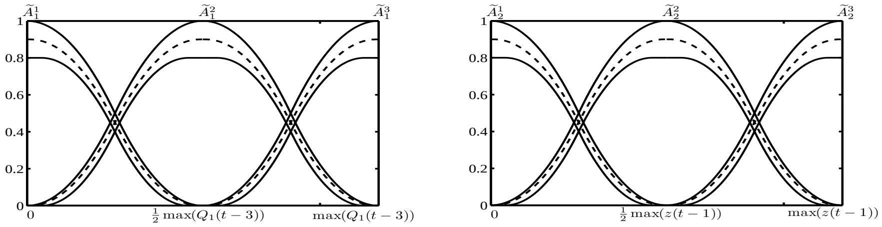
4. **Simulation.** In this part, we will consider a simulation example to identify a coupled-tanks liquid-level system, which is modeled by the state-space model equations [6,7]:

$$A_1 \frac{dH_1(t)}{dt} = Q_1(t-\tau) - \alpha_1\sqrt{H_1(t)} - \alpha_3\sqrt{H_1(t) - H_2(t)},$$
$$A_2 \frac{dH_2(t)}{dt} = \alpha_3\sqrt{H_1(t) - H_2(t)} - \alpha_1\sqrt{H_1(t)}, \qquad z(t) = H_2(t), \tag{7}$$

where $A_1 = A_2 = 36.52$, $\alpha_1 = \alpha_2 = 5.6186$, $\alpha_3 = 15$, $\tau = 3$ second. $0 \leq H_1,\ H_2 \leq 60$cm. Q_1 is the input flow rate and $Q_1 \leq 90\text{cm}^3/\text{s}$. $z(t)$ is the output of the system. Also, the regressors of the NARX model are chosen as $\boldsymbol{\varphi}(t) = [Q_1(t-3), z(t-1)]$.

It can be proved that if Q_1 is a certain constant value then the output $z(t)$ eventually keeps a constant liquid level. Simulations show this system output monotonically increases when the constant input is greater than 49 and the initial status vector $[H_1; H_2]$ is $[25; 16]$cm. Starting from such a state condition we also can justify that an increase in the inflow results in the liquid level to increase in a non-oscillatory manner and settle at a higher level. We can consider the property of the coupled-tank liquid-level system as the prior knowledge of monotonicity. Therefore, we can use the IT2 fuzzy monotonic NARX model above to identify the system.

The identification problem here is to predict the liquid levels $z(t)$ with respect to the input flow rate Q_1. From (7), we can obtain $r = 500$ input-output data pairs $(\boldsymbol{\varphi}(t), z(t))$ $(t = 1, 2, \dots, r)$ to identify this nonlinear system. In this section, we do two numerical simulation and the input signal of the first simulation is uniformly distributed random numbers on interval [0, 90], and the second one is monotonically increasing signal. In these simulations, the output data are corrupted by uniformly distributed additive noise in $U = \big[-n_b \max\big(H_2\big), n_b \max\big(H_2\big)\big]$ to obtain the training data pairs $(\widetilde{\boldsymbol{\varphi}}(t), \widetilde{z}(t))s$, i.e., $\widetilde{\boldsymbol{\varphi}}(t) = [Q_1(t-3), \widetilde{z}(t-1)]$, $\widetilde{z}(t) = z(t) + noise$, where $noise \in U$, $\max\big(H_2\big)$ denotes



(a) the membership functions of $Q_1(t-3)$

(b) the membership functions of $z(t-1)$

Figure 2. Interval type-2 membership functions of $Q_1(t-3)$ and $z(t-1)$

Table 1. Fuzzy rules of the four NARX models

Antecedent part		Consequent part $z(t)$			
$Q_1(t-3)$	$z(t-1)$	MUIT2FLS	NMUIT2FLS	MT1FLS	NMT1FLS
$\widetilde{A}_1^1$	$\widetilde{A}_2^1$	$[18.20, 21.36]$	$[-7.12, 34.89]$	20.74	20.73
$\widetilde{A}_1^2$	$\widetilde{A}_2^1$	$[18.20, 22.75]$	$[-24.08, 49.08]$	21.13	21.26
$\widetilde{A}_1^3$	$\widetilde{A}_2^1$	$[18.20, 22.75]$	$[-28.26, 53.89]$	21.13	21.02
$\widetilde{A}_1^1$	$\widetilde{A}_2^2$	$[18.20, 22.03]$	$[-39.05, 65.22]$	20.74	21.26
$\widetilde{A}_1^2$	$\widetilde{A}_2^2$	$[18.78, 22.75]$	$[-20.55, 49.99]$	21.44	21.68
$\widetilde{A}_1^3$	$\widetilde{A}_2^2$	$[18.78, 22.75]$	$[-8.81, 37.16]$	21.44	20.92
$\widetilde{A}_1^1$	$\widetilde{A}_2^3$	$[18.20, 22.03]$	$[13.17, 13.17]$	20.74	18.85
$\widetilde{A}_1^2$	$\widetilde{A}_2^3$	$[20.21, 22.75]$	$[3.30, 33.35]$	21.98	21.92
$\widetilde{A}_1^3$	$\widetilde{A}_2^3$	$[22.75, 22.75]$	$[11.49, 30.23]$	23.04	23.22

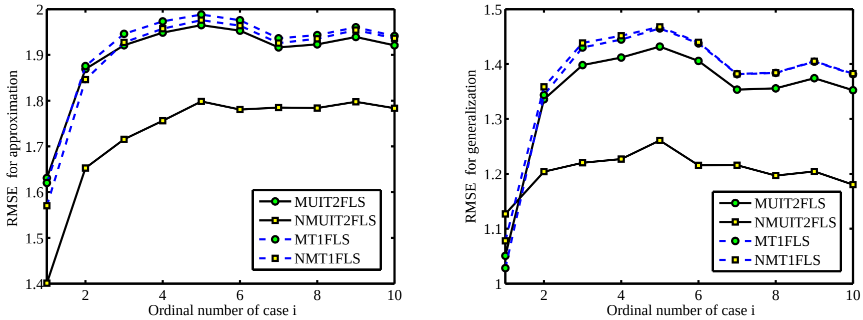
the maximum value of the r output sample data; n_b is the level of noisy disturbance; $n_b = 10\%$ in the first simulation and $n_b = 15\%$ in the second simulation. In each noisy circumstance, ten cases are considered. In case i, the training data set is chosen as $\mathfrak{D}_i = \{(\widetilde{\boldsymbol{\varphi}}(1), \widetilde{z}(1)), \ldots\ (\widetilde{\boldsymbol{\varphi}}(r_i), \widetilde{z}(r_i)\}$ where $r_i = 50i$, $(i = 1, \ldots, 10)$. Moreover, for each case, the evaluation data set is chosen as $\mathfrak{E}_i = \{(\boldsymbol{\varphi}(1), z(1), \ldots\ (\boldsymbol{\varphi}(r_i)), z(r_i))\}$ which can be used to check whether the trained model could follow the characteristics of the original signal.

Thought the data, we use the following two RMSE performance indexes in case i

$$ap_i = \left(\frac{1}{r_i}\sum_{t=1}^{r_i}\left(\widehat{y}(\widetilde{\boldsymbol{\varphi}}(t), \boldsymbol{w}) - \widetilde{z}(t)\right)^2\right)^{\frac{1}{2}}, \quad gp_i = \left(\frac{1}{r_i}\sum_{t=1}^{r_i}\left(\widehat{y}(\widetilde{\boldsymbol{\varphi}}(t), \boldsymbol{w}) - z(t)\right)^2\right)^{\frac{1}{2}}, \quad (8)$$

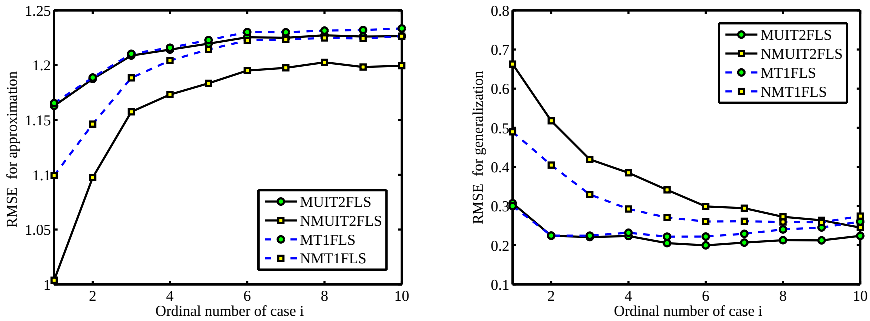
where r_i is the size of the training data set $\mathfrak{D}_i$ and the evaluation data set $\mathfrak{E}_i$ in case i; ap_i can reflect the approximation ability of fuzzy NARX models for the training data, and gp_i can reflect the generalization ability of the fuzzy models for original noise-free signal. Also, the statistical indexes, ap_{si} and gp_{si}, which are the arithmetic mean of the corresponding indexes obtained in 50 run times, are shown in Figures 3 and 4.

In the simulations, in order to demonstrate the superiority of the prior knowledge of monotonicity, we use two fuzzy models to identify the coupled-tanks liquid-level system: Monotonic UIT2FLS (MUIT2FLS), UIT2FLS without Monotonic constraints (NMUIT2F LS), Monotonic Type-1 FLS (MT1FLS) and Type-1 FLS without the constraints (NMT1F LS). For each input variables, we use 3 membership functions. The membership functions



(a) approximation performance (b) generalization performance

FIGURE 3. The performance curve using rand data when $n_b = 10\%$



(a) approximation performance (b) generalization performance

FIGURE 4. The performance curve using monotonic signal when $n_b = 15\%$

of their antecedent parts are showed in Figure 2, and note that the antecedent parameters satisfy the first two conditions of Theorem 2.1. In this study, we can obtain the consequent parameters according to the developed solution. These consequent parameters of the two fuzzy systems are shown in Table 1.

In Figure 3, it can be seen that the approximation performances ap_{si} $(i = 1, 2, \ldots, 10)$ and the generalization performances gp_{si} $(i = 2, 3, \ldots, 10)$ of NMUIT2FLS perform best than other FLSs' counterpart. The reason for this is that type-2 FLSs have more parameters than type-1 FLSs and the FLSs without constraints have more freedom than the FLSs with constraints.

In Figure 3, gp_{s1} of MUIT2FLS outperforms that of NMUIT2FLS because the beginning of the output curve of the coupled-tanks system monotonically increases, which is triggered by the initial values $H_1 = 40$cm and $H_2 = 20$cm in the simulation. When the input signal monotonically increases, the system output monotonically increases. From Figure 4, the approximation performances ap_{si} $(i = 1, 2, \ldots, 10)$ of NMUIT2FLS are still superior to the other FLSs', whereas, the generalization performances gp_{si} $(i = 1, 2, \ldots, 10)$ of the MUIT2FLS is best. Also, in Figures 3 and 4, in general, the UIT2FLS with monotonic constraints performs best among the four FLSs. The reason for this is that the prior knowledge provides the information that curve-fitting needs, and when input-output data is not enough, the prior information become more important.

In conclusion, NMUIT2FLS has better approximation performance than MUIT2FLS, whereas the generalization performance of MUIT2FLS is superior to other FLSs', when target systems take on monotonicity or training data are not enough. Therefore, combining the prior knowledge of monotonicity and unnormalized interval type-2 fuzzy logic NARX model gives more satisfactory overall performance.

5. **Conclusions.** In this paper, we encode the prior knowledge of monotonicity into an MISO unnormalized interval type-2 TSK FLS, and then, apply the method to an NARX model such that the model can approach a given monotonic system. A numerical simulation example is provided to verify the usefulness of the derived theorems and methods. The simulation results have demonstrated that when the target system is monotonic or training data are not enough, the unnormalized interval type-2 fuzzy logic NARX model with monotonic constraints outperforms other FLSs on the whole. In practice, the prior knowledge may be obvious and input-output data may always not be sufficient. Under this circumstances, the information from prior knowledge may be an excellent supplementary to the information from input-output data. Hence, it is quite important to incorporate the prior knowledge into the system identification.

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