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Robust adaptive neural network control for a class of uncertain nonlinear systems with actuator amplitude and rate saturations

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ABSTRACT

An adaptive controller which is designed with a priori consideration of actuator saturation effects and guarantees H^{∞} tracking performance for a class of multiple-input–multiple-output (MIMO) uncertain nonlinear systems with extern disturbances and actuator saturations is presented in this paper. Adaptive radial basis function (RBF) neural networks are used in this controller to approximate the unknown nonlinearities. An auxiliary system is constructed to compensate the effects of actuator saturations. Furthermore, in order to deal with approximation errors for unknown nonlinearities and extern disturbances, a supervisory control is designed, which guarantees that the closed loop system achieves a prescribed disturbance attenuation level so that H^{∞} tracking performance is achieved. Steady and transient tracking performance are analyzed and the tracking error is adjustable by explicit choice of design parameters. Computer simulations are presented to illustrate the efficiency of the proposed controller.

1. Introduction

All physical actuators in control systems have amplitude and rate limitations. For example, the elevator of an aircraft can only provide a limited force or torques in a limited rate. Actuator amplitude limitation or rate limitation constitutes a fundamental limitation on many linear or nonlinear control design techniques and has attracted the attention of numerous researchers. The controllers that ignore actuator limitations may cause the closed loop system performance to degenerate or even make the closed system unstable, and decrease the lifetime of the actuators, or damage the actuators. Higher performance may be expected if a controller is designed with a priori considering of the actuator saturation effects.

The design of stabilizing controllers with a priori consideration of the actuator saturation effects for nonlinear systems with unknown nonlinearities and external disturbances is a challenging problem. Zhou [1] proposed an adaptive backstepping scheme to design an adaptive controller for a class of uncertain nonlinear single-input–single-output (SISO) systems in the presence of input saturations. To deal with saturations, an auxiliary system with the same order as that of the plant was constructed to compensate the effect of saturation. Farrell et al. [2–5] presented an adaptive backstepping approach and an online approximation based adaptive backstepping approach for unknown nonlinear systems with known magnitude, rate, bandwidth constraints on intermediate states or actuators without disturbance. Those approaches also used auxiliary systems for generating a modified tracking error to guarantee stability during saturation. Command filtered adaptive backstepping approaches [6–9] were also proposed to deal with the constraints on the control surfaces and the control states. For single input uncertain nonlinear systems in the presence of input saturation and unknown external disturbance, robust adaptive backstepping control algorithms were also developed by introducing a well defined smooth function and using a Nussbaum function which was used to compensate for the nonlinear term arising from the input saturation [10].

Dynamic inversion [11,12] approach is a widely used nonlinear control technique. However, the effects of actuator saturations have not been addressed with nominal dynamic inversion algorithm, so certain modifications are required. Tandale [13] proposed an adaptive dynamic inversion controller for a class of nonlinear systems with control saturation constraints. Enomoto [14] investigated the dynamic inversion control for nonlinear systems with control saturation constraints by Lyapunov synthesis. For a class of uncertain nonlinear dynamical systems in Brunovsky form, Lavretsky [15] proposed a dynamic inversion based adaptive control framework to provide stable adaptation in the presence of input constraints. The proposed design methodology can protect the control law from actuator position saturation. For a class of nonlinear systems which, in the presence of saturation, were controlled by nonlinear dynamic inversion

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controllers, an anti-windup compensation scheme was also proposed [16–18]. Neural network technique [19,20] was also used to handle actuator saturations problem. Calise [21] introduced a neural network based method termed as Pseudo-Control Hedging (PCH) for addressing a wide class of plant input characteristics such as actuator position limits, actuator rate limits, time delay, and input quantization. Chen et al. [22] also introduced a radial basis function neural network based controller for uncertain MIMO nonlinear systems with input saturations. The control design for nonlinear systems with actuator saturations was also be investigated by optimal control [23], nearly optimal control [24], nonlinear model predictive control [25] and fault tolerant scheme [26], etc. However, there are still few results for the control of uncertain nonlinear systems by taking actuator saturations into account in the controller design and analysis.

In [27], the authors proposed an adaptive controller for MIMO nonlinear systems with control input limitations by using an auxiliary system and extended tracking errors which were used in neural network parameter update laws to compensate the effects of control input limitations. In this paper, for the control of a class of MIMO uncertain nonlinear systems in the presence of disturbances and actuator saturations, dynamic inversion [11] based controller which can generate constrained control signal is designed. Adaptive RBF neural networks are used to approximate unknown nonlinearities. An auxiliary system is constructed to compensate the effects of actuator amplitude and rate saturations. This auxiliary system and compensation scheme are different from [27], so that the extended tracking error in [27] is no longer needed. A supervisory control is designed to attenuate the effects of approximation errors and external disturbance so as to guarantee a H^{∞} tracking performance. The performance of the closed loop system is obtained through Lyapunov analysis. The bounds of tracking errors can be adjusted by tuning the design parameters. The proposed controller can generate control signals satisfying actuator amplitude and rate limitations, and guarantee a H^{∞} tracking performance of the closed loop system.

The rest of this paper is organized as follows. In Section 2, the problem statement is presented. In Section 3, the adaptive control scheme is discussed, and the closed loop system performance is analyzed. A numerical example is shown in Section 4. Section 5 concludes the paper. Throughout this paper, $|\cdot|$ indicates the absolute value, $\|\cdot\|$ indicates the Euclidean vector norm, and $\|\cdot\|_2$ indicates the L_2 norm.

2. Problem formulation

Consider the class of MIMO systems described by the following differential equations:

$$\begin{aligned}
&\dot{x}_{i1} = x_{i2}\\
&\vdots\\
&\dot{x}_{i,r_i-1} = x_{ir_i}\\
&\dot{x}_{ir_i} = f_i(\mathbf{x}) + \sum_{j=1}^{m} g_{ij}(\mathbf{x})u_j + d_i\\
&y_i = x_{i1}, \quad i = 1,\dots,m
\end{aligned} \tag{1}$$

which also can be rewritten in the following compact form:

$$\mathbf{y}^{(n)} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} + \mathbf{d} \tag{2}$$

where $\mathbf{y} = [y_1,\dots,y_m]^T \in \mathbf{R}^m$ is the output vector; $\mathbf{y}^{(n)} \overset{\text{def}}{=} [y_1^{(r_1)},\dots,y_m^{(r_m)}]^T \in \mathbf{R}^m$, $\sum_{i=1}^{m} r_i = n$; $y_i^{(r_i)} = d^{r_i}y_i/dt^{r_i}$; $\mathbf{x} = [x_{11},\dots,x_{1r_1},\dots,x_{m1},\dots,x_{mr_m}]^T \in \mathbf{R}^n$ is the state vector available for measurement; $\mathbf{u} = [u_1,\dots,u_m] \in \mathbf{R}^m$ is the control vector with

$$|u_i| \le u_{i\max}, \quad \dot{u}_i \le v_{i\max} \tag{3}$$

where $u_{i\max}$ and $v_{i\max}$ denote the actuator amplitude and rate limits respectively. $\mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}),\dots,f_m(\mathbf{x})] \in \mathbf{R}^m$, $\mathbf{G}(\mathbf{x}) = [g_{ij}(\mathbf{x})]_{m\times m} \in \mathbf{R}^{m\times m}$ ($[\cdot]_{m\times m}$ represents a $m\times m$ matrix) are continuous unknown functions of the state $\mathbf{x}$. d_i denotes the external disturbances which is unknown but bounded and satisfies $\int_0^T d_i^2\,dt < \infty$. $\mathbf{d} = [d_1,\dots,d_m] \in \mathbf{R}^m$.

The control objective is to force y_i to follow a given bounded reference signal y_{id} in the presence of actuator saturations and extern disturbances. For (2) to be controllable, we assume that $\sigma(\mathbf{G}(\mathbf{x})) \neq 0$ for $\mathbf{x}$ in certain controllability region $\mathbf{U}_c \in \mathbf{R}^n$, where $\sigma(\mathbf{G}(\mathbf{x}))$ denotes the minimum singular value of the matrix $\mathbf{G}(\mathbf{x})$.

3. Design of adaptive controllers

To begin, define $\tau_1,\dots,\tau_m$ as follows:

$$\tau_i = y_{id}^{(r_i)} + \sum_{j=1}^{r_i} \lambda_{ij} e_i^{(j-1)}, \quad i = 1,\dots,m$$

where y_{id}, $i = 1,\dots,m$ are the reference signals, $e_i = y_{id} - y_i$ $(i = 1,\dots,m)$ are the tracking errors, $\lambda_{i1},\dots,\lambda_{ir_i}$ are parameters which make sure that the roots of the equation $s^{r_i} + \lambda_{ir_i}s^{r_i-1} + \cdots + \lambda_{i2}s + \lambda_{i1} = 0$ are all in the open left-half complex plane.

If $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are known and the constrains on control inputs are ignored, then based on dynamic inversion algorithm, the control law:

$$\mathbf{u}_{c'} = \mathbf{G}^{-1}(\mathbf{x})(-\mathbf{F}(\mathbf{x}) + \boldsymbol{\tau}) \tag{4}$$

can be applied to (2) to achieve the following asymptotically stable tracking:

$$\begin{bmatrix} e_1^{(r_1)} + \sum_{j=1}^{r_1} \lambda_{1j} e_1^{(j-1)} \\ \vdots \\ e_m^{(r_m)} + \sum_{j=1}^{r_m} \lambda_{mj} e_m^{(j-1)} \end{bmatrix} = \mathbf{0} \tag{5}$$

in the case of no external disturbances.

Because $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are unknown vector and matrix respectively, the above control law (4) cannot be implemented in practice. Besides, there is no guarantee that $\mathbf{u}_{c'}$ satisfies the actuator constraints (3). It is well known that neural networks [19,20] can be used as universal approximators to approximate any continuous functions at any arbitrary accuracy as long as the network is big enough. In this work, in order to treat this tracking control design problem, radial basis function (RBF) neural networks are used to approximate the unknown functions, that is, $f_i(\mathbf{x})$, $i = 1,\dots,m$, and $g_{ij}(\mathbf{x})$, $i,j = 1,\dots,m$ are approximated as follows:

$$f_i(\mathbf{x}) \approx \hat{f}_i(\mathbf{x}|\boldsymbol{\Theta}_{f_i}) = \boldsymbol{\Theta}_{f_i}^T \boldsymbol{\Phi}_{f_i}(\mathbf{x}), \quad i = 1,\dots,m \tag{6}$$

$$g_{ij}(\mathbf{x}) \approx \hat{g}_{ij}(\mathbf{x}|\boldsymbol{\Theta}_{g_{ij}}) = \boldsymbol{\Theta}_{g_{ij}}^T \boldsymbol{\Phi}_{g_{ij}}(\mathbf{x}), \quad i,j = 1,\dots,m \tag{7}$$

where $\boldsymbol{\Theta}_{f_i} \in \mathbf{R}^{M_{f_i}}$, $\boldsymbol{\Theta}_{g_{ij}} \in \mathbf{R}^{M_{g_{ij}}}$ are weight vectors, and $\boldsymbol{\Phi}_{f_i}(\mathbf{x}) \in \mathbf{R}^{M_{f_i}}(\mathbf{x})$, $\boldsymbol{\Phi}_{g_{ij}}(\mathbf{x}) \in \mathbf{R}^{M_{g_{ij}}}(\mathbf{x})$ are radial basis vectors, M_{f_i}, $M_{g_{ij}}$ are the corresponding dimensions of the basis vectors. Denote

$$\hat{\mathbf{F}}(\mathbf{x}|\boldsymbol{\Theta}_{\mathbf{F}}) = \begin{bmatrix} \hat{f}_1(\mathbf{x}) \\ \vdots \\ \hat{f}_m(\mathbf{x}) \end{bmatrix}, \quad \hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_{\mathbf{G}}) = \begin{bmatrix} \hat{g}_{11}(\mathbf{x}) & \cdots & \hat{g}_{1m}(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ \hat{g}_{m1}(\mathbf{x}) & \cdots & \hat{g}_{mm}(\mathbf{x}) \end{bmatrix} \tag{8}$$

Then $\hat{\mathbf{F}}(\mathbf{x}|\boldsymbol{\Theta}_{\mathbf{F}})$ is an estimation of $\mathbf{F}(\mathbf{x})$, and $\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_{\mathbf{G}})$ is an estimation of $\mathbf{G}(\mathbf{x})$.

Using the approximation (8), and considering the control inputs constraints and extern disturbances, we modify the control law (4) as follows:

$$\mathbf{u}_c = \hat{\mathbf{G}}^{\#}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})(-\hat{F}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{F})+\boldsymbol{\tau}+\boldsymbol{\eta}+\mathbf{u_d}) \tag{9}$$

$$\mathbf{u} = \mathbf{sat}(\mathbf{u}_c) \tag{10}$$

$$\boldsymbol{\eta} = -\mathbf{C}\xi - \dot{\xi} \tag{11}$$

where $\mathbf{sat}(\mathbf{u}_c)$ represents the amplitude and rate limitations on $\mathbf{u}_c$. $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_m)^T$ is the robust control term. $\mathbf{u_d} \in \mathbf{R}^m$, which will be determined later, is a supervisor control used to attenuate the extern disturbance $\mathbf{d}$. $\mathbf{G}^{\#}(\mathbf{x})$ represents the generalized inversion [28] of $\mathbf{G}(\mathbf{x})$. $\xi = (\xi_1, \ldots, \xi_m)^T \in \mathbf{R}^m$ is the state of the following constructed auxiliary system (12) which uses the difference of $\mathbf{u}_c$ before and after amplitude and rate limitations as input:

$$\dot{\xi} = -\mathbf{C}\xi + \hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})\Delta\mathbf{u} \tag{12}$$

$\mathbf{C} = \mathrm{diag}(c_1, \ldots, c_m) \in \mathbf{R}^{m\times m}$, c_i $(i = 1, \ldots, m)$ are positive parameters, $\Delta\mathbf{u} = \mathbf{u} - \mathbf{u}_c$. ξ is used to compensate the effect of actuator saturations.

$\mathbf{u}_c$ is obtained according to *certainty equivalence principle* [29] which is widely used in adaptive control schemes. However, there is also no guarantee that $\mathbf{u}_c$ satisfies the constraints (3), hence amplitude and rate limitations $\mathbf{sat}(\mathbf{u}_c)$ are imposed on $\mathbf{u}_c$ to generate $\mathbf{u}$ which satisfies the constraints (3).

The amplitude and rate limitations on $\mathbf{u}_c$, i.e., $\mathbf{sat}(\mathbf{u}_c)$, can always be implemented by assuming a first-order model or second-order model for the dynamics of each component of $\mathbf{u}_c$, for example, $\dot{u}_i = \mathrm{sat}_R(\omega_i(\mathrm{sat}_A(u_{ci}) - u_i))$ or $\ddot{u}_i = \mathrm{sat}_R(\omega_i^2(\mathrm{sat}_A(u_{ci}) - u_i) - 2\zeta_i\omega_i\dot{u}_i)$, where u_i, u_{ci} are the i-th elements of $\mathbf{u}$, $\mathbf{u}_c$ respectively, ω_i, ζ_i are positive constants, $\mathrm{sat}_R(\cdot)$, $\mathrm{sat}_A(\cdot)$ represent the rate and amplitude saturation functions respectively. The function $\mathrm{sat}_R(x)$ is defined as follows:

$$\mathrm{sat}_R(x) = \begin{cases} R & \text{if } x \geq R \\ x & \text{if } |x| < R \\ -R & \text{if } x \leq -R \end{cases}$$

and $\mathrm{sat}_A(x)$ is defined similarly. Fig. 1 gives visual descriptions for the first-order and second-order models respectively. In the linear range of the function $\mathrm{sat}_A(x)$ and $\mathrm{sat}_R(x)$, the transfer function for the first-order model is

$$\frac{U_i}{U_{ci}} = \frac{\omega_i}{s+\omega_i} \tag{13}$$

and the transfer function for the second model is

$$\frac{U_i}{U_{ci}} = \frac{\omega_i^2}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \tag{14}$$

This means that by choosing appropriate parameters ω_i, ζ_i, not only the actuators amplitude and rate saturations but also the dynamics of the actuators, like dampings and frequencies, can be integrated into the controller.

Remark 1. We known that for the first-order system:

$$\dot{x} = f(x) + u \tag{15}$$

if $f(x)$ is known, then we can construct a new system

$$u = \dot{\nu} - f(\nu) \tag{16}$$

with ν as input and u as output. Eqs. (15) and (16) yield a closed-loop system:

$$\dot{x} - f(x) = \dot{\nu} - f(\nu) \tag{17}$$

For system (17), the output x is totaly the same as the input ν if and only if $x(0) = \nu(0)$. The transform function from ν to x is 1. Actually, the zeros of (16) is the poles of (15), and (16) is the zero assignment based inverse system of (15). Similarly, for the auxiliary system (12), if we view the component $\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})\Delta\mathbf{u}$ as the control input, then the new system:

$$\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})\Delta\mathbf{u} = \mathbf{C}\boldsymbol{\lambda} + \dot{\boldsymbol{\lambda}} \tag{18}$$

with $\boldsymbol{\lambda}$ as the input and $\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})\Delta\mathbf{u}$ as the output is the zero assignment based inverse system of (12), (18) and (12) yield a closed loop system:

$$\dot{\xi} + \mathbf{C}\xi = \dot{\boldsymbol{\lambda}} + \mathbf{C}\boldsymbol{\lambda} \tag{19}$$

The output ξ is totaly the same as the input $\boldsymbol{\lambda}$ if and only if $\xi(0) = \boldsymbol{\lambda}(0)$. So $\dot{\xi} + \mathbf{C}\xi$ or $\dot{\boldsymbol{\lambda}} + \mathbf{C}\boldsymbol{\lambda}$ can be used as a prediction of $\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})\Delta\mathbf{u}$ which is the effect of actuator saturations and can be used to compensate the effect of actuator saturations. From (9)–(12), we have

$$\mathbf{u} = \mathbf{sat}(\mathbf{u}_c) = \hat{\mathbf{G}}^{\#}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G})(-\hat{F}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{F}) + \boldsymbol{\tau} + \mathbf{u_d}) \tag{20}$$

By ignoring the term $\mathbf{u_d}$, $\mathbf{u}$ which satisfies the constraints (3) has the same form as $\mathbf{u}_{c'}$ which is obtained by dynamic inversion algorithm [11], which makes the controller be a adaptive neural network based dynamic inversion controller and the perfect tracking be available.

Remark 2. From (12) we also have that

$$\xi_i(t) = e^{-c_i t}\left(\xi(0) + \int_0^t \sum_{j=1}^m \hat{g}_{ij}(\mathbf{x}(\nu))\Delta u_j(\nu) e^{c_i \nu}\, d\nu\right)$$
$$= e^{-c_i t}\xi(0) + \int_0^t \sum_{j=1}^m \hat{g}_{ij}(\mathbf{x}(\nu))\Delta u_j(\nu) e^{-c_i(t-\nu)} d\nu$$

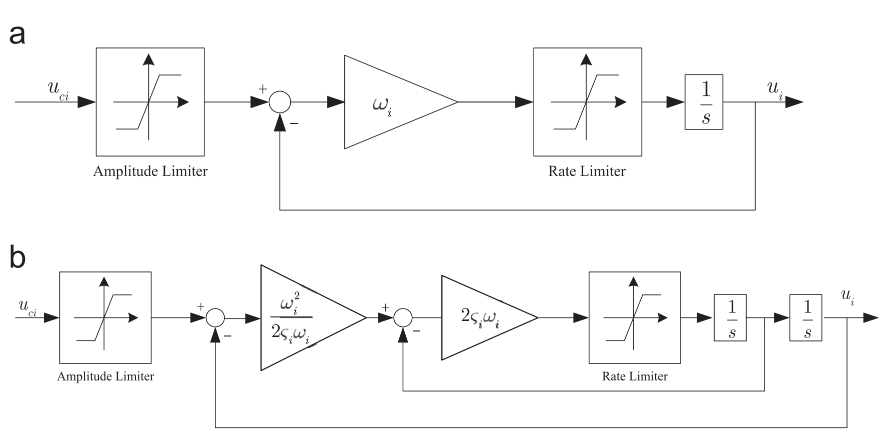

Fig. 1. Schematic for amplitude and rate limitations. (a) First-order model and (b) second-order model.

$$\le e^{-c_i t}|\xi(0)| + \int_0^t \sum_{j=1}^m |\hat{g}_{ij}(\mathbf{x}(v))|\Delta u_j(v)|)e^{-c_i(t-v)}\,dv$$

$$\le e^{-c_i t}|\xi(0)| + \frac{m}{c_i} \sup_{0\le v\le t} |\hat{g}_{ij}(\mathbf{x}(v))| \sup_{0\le v\le t} |\Delta u_j(v)| \tag{21}$$

where $\Delta u_j(v) = u_j(v) - u_{cj}(v)$. Because c_i is positive, the auxiliary system is bounded input bounded output stable.

Remark 3. Although the true value of $\mathbf{G}(\mathbf{x})$ are invertible according to the assumption, the estimate matrix $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G})$ may become singular during the adaptive process, so Moore–Penrose generalized matrix inverse [28] of $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G})$ is used.

Remark 4. In the case of known $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ and free of external disturbances, the control low

$$\mathbf{u} = \mathbf{sat}(\mathbf{u}_c) \tag{22}$$

with

$$\mathbf{u}_c = \mathbf{G}^{-1}(\mathbf{x})(-\mathbf{F}(\mathbf{x}) + \boldsymbol{\tau} + \boldsymbol{\eta}) \tag{23}$$

$$\boldsymbol{\eta} = -\mathbf{C}\xi - \dot{\xi} \tag{24}$$

$$\dot{\xi} = -\mathbf{C}\xi + \mathbf{G}(\mathbf{x})\Delta\mathbf{u} \tag{25}$$

can be applied to System (2) to obtain the asymptotically stable tracking (5).

In the following, we will specify the update laws for the RBF parameters $\mathbf{\Theta}_{f_i}$ $(i = 1, \dots, m)$, $\mathbf{\Theta}_{g_{ij}}$ $(i,j = 1, \dots, m)$ and the supervisor control $\mathbf{u_d}$, so that desired tracking performance can be achieved. Applying the control law (9) and (10) to System (2) yields

$$\boldsymbol{\tau} - \mathbf{Y}^{(\mathbf{r})} = \hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta_F}) - \mathbf{F}(\mathbf{x}) + (\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G}) - \mathbf{G}(\mathbf{x}))\mathbf{u} - \mathbf{u_d} - \mathbf{d} \tag{26}$$

Define the optimal approximation weight vectors for f_i $(i = 1, \dots, m)$, g_{ij} $(i,j = 1, \dots, m)$ as follows:

$$\mathbf{\Theta}^*_{f_i} = \arg \min_{\mathbf{\Theta}_{f_i} \in \Omega_\mathbf{F}} \left[\sup_{\mathbf{x}\in\mathbf{U}_c} |f_i(\mathbf{x}) - \hat{f}_i(\mathbf{x}|\mathbf{\Theta}_{f_i})| \right] \tag{27}$$

$$\mathbf{\Theta}^*_{g_{ij}} = \arg \min_{\mathbf{\Theta}_{g_{ij}} \in \Omega_\mathbf{G}} \left[\sup_{\mathbf{x}\in\mathbf{U}_c} |g_{ij}(\mathbf{x}) - \hat{g}_{ij}(\mathbf{x}|\mathbf{\Theta}_{g_{ij}})| \right] \tag{28}$$

where $\Omega_\mathbf{F}$, $\Omega_\mathbf{G}$, $\mathbf{U}_c$ denote the sets of suitable bounds on $\mathbf{\Theta}_{f_i}$, $\mathbf{\Theta}_{g_{ij}}$, and $\mathbf{x}$ respectively. $\mathbf{\Theta}^*_{f_i}$ $(i = 1, \dots, m)$, $\mathbf{\Theta}^*_{g_{ij}}$ $(i,j = 1, \dots, m)$ are constant vectors. The optimal approximations for $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are denoted as $\hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta}^*_F)$, $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta}^*_\mathbf{G})$ respectively. Define the minimum approximation error as

$$\mathbf{w} \overset{\text{def}}{=} \hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta}^*_\mathbf{F}) - \mathbf{F}(\mathbf{x}) + \left[\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta}^*_\mathbf{G}) - \mathbf{G}(\mathbf{x})\right]\mathbf{u} \tag{29}$$

According to universal approximation property of neural networks [19,20], the following assumption is reasonable:

Assumption 1. The minimum approximation error is square integrable, i.e.,

$$\int_0^T \mathbf{w}^T\mathbf{w}\,dt < \infty \tag{30}$$

Using the optimal approximation for $\mathbf{F}(\mathbf{x})$, $\mathbf{G}(\mathbf{x})$, the i-th subsystem of (26) can be rewritten as

$$e_i^{(r_i)} + \sum_{k=1}^{r_i} \lambda_{ik} e_i^{(k-1)} = \hat{f}_i(\mathbf{x}|\mathbf{\Theta}_{f_i}) - \hat{f}_i(\mathbf{x}|\mathbf{\Theta}^*_{f_i}) - u_{d_i} - d_i + w_i$$
$$+ \sum_{j=1}^m (\hat{g}_{ij}(\mathbf{x}|\mathbf{\Theta}_{g_{ij}}) - \hat{g}_{ij}(\mathbf{x}|\mathbf{\Theta}^*_{g_{ij}}))u_j \tag{31}$$

where u_{d_i}, d_i,w_i are the i-th element of $\mathbf{u_d}$, $\mathbf{d}$, and $\mathbf{w}$ respectively. Defining $\mathbf{e}_i = [e_i, \dots, e_i^{(r_i-1)}]^T$, $\tilde{\mathbf{\Theta}}_{f_i} = \mathbf{\Theta}_{f_i} - \mathbf{\Theta}^*_{f_i}$, $\tilde{\mathbf{\Theta}}_{g_{ij}} = \mathbf{\Theta}_{g_{ij}} - \mathbf{\Theta}^*_{g_{ij}}$, then Eq. (31) can be rewritten in the following form:

$$\dot{\mathbf{e}}_i = \mathbf{A}_i\mathbf{e}_i + \mathbf{B}_i \left(\tilde{\mathbf{\Theta}}^T_{f_i}\mathbf{\Phi}_{f_i}(\mathbf{x}) + \sum_{j=1}^m (\tilde{\mathbf{\Theta}}^T_{g_{ij}}\mathbf{\Phi}_{g_{ij}}(\mathbf{x}))u_j - u_{d_i} - d_i + w_i \right) \tag{32}$$

where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_{i1} & -\lambda_{i2} & -\lambda_{i3} & \cdots & -\lambda_{ir_i} \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

For the i-th subsystem of (26), the following theorem can be obtained.

Theorem 1. *For the i-th subsystem of (26), if we select the control law (10), and the following parameters update laws and* u_{d_i}

$$\dot{\mathbf{\Theta}}_{f_i} = -\Gamma_{f_i}\mathbf{\Phi}_{f_i}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i \tag{33}$$

$$\dot{\mathbf{\Theta}}_{g_{ij}} = -\Gamma_{g_{ij}}\mathbf{\Phi}_{g_{ij}}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i u_j \tag{34}$$

$$u_{d_i} = \frac{1}{2\rho_i^2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i \tag{35}$$

then the following H^∞ *tracking performance can be obtained*:

$$\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le \mathbf{e}_i^T(\mathbf{0})\mathbf{P}_i\mathbf{e}_i(\mathbf{0}) + \tilde{\mathbf{\Theta}}^T_{f_i}(\mathbf{0})\Gamma^{-1}_{f_i}\tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0})$$
$$+ \sum_{j=1}^m \tilde{\mathbf{\Theta}}^T_{g_{ij}}(\mathbf{0})\Gamma^{-1}_{g_{ij}}\tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0}) + \rho_i^2\int_0^T \varrho_i^2\,dt \tag{36}$$

where Γ_{f_i}, $\Gamma_{g_{ij}}$ $(j = 1, \dots, m)$ *are positive definite diagonal matrices,* ρ_i $(i = 1, \dots, m)$ *are positive parameters representing for prescribed disturbance attenuation levels,* $\varrho_i \overset{\text{def}}{=} -d_i + w_i$, $\mathbf{Q}_i \in \mathbf{R}^{m\times m}$ *is arbitrary symmetric positive definite matrices,* $\mathbf{P}_i \in \mathbf{R}^{m\times m}$ *is the symmetric positive definite solution of the following Lyapunov equation*:

$$\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i = -\mathbf{Q}_i \tag{37}$$

Proof. Define the Lyapunov function V_i for the i-th subsystem as follows:

$$V_i = \frac{1}{2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{e}_i + \frac{1}{2}\tilde{\mathbf{\Theta}}^T_{f_i}\Gamma^{-1}_{f_i}\tilde{\mathbf{\Theta}}_{f_i} + \frac{1}{2}\sum_{j=1}^m \tilde{\mathbf{\Theta}}^T_{g_{ij}}\Gamma^{-1}_{g_{ij}}\tilde{\mathbf{\Theta}}_{g_{ij}} \tag{38}$$

The time derivative of V_i is

$$\dot{V}_i = \frac{1}{2}(\dot{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\dot{\mathbf{e}}_i) + \tilde{\mathbf{\Theta}}^T_{f_i}\Gamma^{-1}_{f_i}\dot{\tilde{\mathbf{\Theta}}}_{f_i} + \sum_{j=1}^m \tilde{\mathbf{\Theta}}^T_{g_{ij}}\Gamma^{-1}_{g_{ij}}\dot{\tilde{\mathbf{\Theta}}}_{g_{ij}}$$
$$= \frac{1}{2}(\mathbf{e}_i^T\mathbf{A}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\mathbf{A}_i\mathbf{e}_i) + \tilde{\mathbf{\Theta}}^T_{f_i}\mathbf{\Phi_F}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i$$
$$+ \sum_{i=1}^m \tilde{\mathbf{\Theta}}^T_{g_{ij}}\mathbf{\Phi_G}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i u_j + \tilde{\mathbf{\Theta}}^T_{f_i}\Gamma^{-1}_{f_i}\dot{\tilde{\mathbf{\Theta}}}_{f_i} + \sum_{j=1}^m \tilde{\mathbf{\Theta}}^T_{g_{ij}}\Gamma^{-1}_{g_{ij}}\dot{\tilde{\mathbf{\Theta}}}_{g_{ij}}$$
$$+ \frac{1}{2}(-u_{d_i} + \varrho_i)(\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i)$$
$$\le -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i + \frac{1}{2}\varrho_i(\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i) - \frac{1}{2\rho^2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i$$
$$\le -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i + \frac{1}{2}\rho_i^2\varrho_i^2 - \frac{1}{2}\left(\rho_i\varrho_i - \frac{1}{\rho_i}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\right)^2$$
$$\le -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i + \frac{1}{2}\rho_i^2\varrho_i^2 \tag{39}$$

Integrating both sides of the above inequality from 0 to T yields

$$\frac{1}{2}\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le V_i(0) - V_i(T) + \frac{\rho_i^2}{2}\int_0^T \varrho_i^2\,dt \tag{40}$$

Since $V(T)>0$, the inequality (40) implies the following inequality:

$$\frac{1}{2}\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le V_i(0)+\frac{\rho_i^2}{2}\int_0^T \varrho_i^2\,dt \tag{41}$$

According to the definition of V_i, the following inequality is obtained:

$$\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le \mathbf{e}_i^T(\mathbf{0})\mathbf{P}_i\mathbf{e}_i(\mathbf{0})+\tilde{\mathbf{\Theta}}_{f_i}^T(\mathbf{0})\Gamma_{f_i}^{-1}\tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0}) + \sum_{j=1}^{m}\tilde{\mathbf{\Theta}}_{g_{ij}}^T(\mathbf{0})\Gamma_{g_{ij}}^{-1}\tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0})+\rho_i^2\int_0^T \varrho_i^2\,dt \tag{42}$$

This is (36). □

Finally, for the nonlinear system (2), the following theorem can be obtained.

Theorem 2. *For the nonlinear system* (2), *if we select the control law*

$$\mathbf{u}=\mathbf{sat}(\mathbf{u}_c) \tag{43}$$

with

$$\mathbf{u}_c=\hat{\mathbf{G}}^{\#}(\mathbf{x}|\mathbf{\Theta_G})(-\hat{F}(\mathbf{x}|\mathbf{\Theta_F})+\boldsymbol{\tau}+\boldsymbol{\eta}+\mathbf{u_d}) \tag{44}$$

$$\boldsymbol{\eta}=-\mathbf{C}\xi-\dot{\xi} \tag{45}$$

$$\dot{\xi}=-\mathbf{C}\xi+\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G})\Delta\mathbf{u} \tag{46}$$

$$\dot{\mathbf{\Theta}}_{f_i}=-\Gamma_{f_i}\mathbf{\Phi}_{f_i}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i,\quad i=1,\ldots,m \tag{47}$$

$$\dot{\mathbf{\Theta}}_{g_{ij}}=-\Gamma_{g_{ij}}\mathbf{\Phi}_{g_{ij}}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i u_j,\quad i,j=1,\ldots,m \tag{48}$$

$$u_{d_i}=\frac{1}{2\rho_i^2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i,\quad i=1,\ldots,m \tag{49}$$

then the following H^∞ tracking performance can be obtained:

$$\int_0^T \mathbf{e}^T\mathbf{Q}\mathbf{e}\,dt \le \mathbf{e}^T(\mathbf{0})\mathbf{P}\mathbf{e}(\mathbf{0})+\sum_{i=1}^{m}\tilde{\mathbf{\Theta}}_{f_i}^T(\mathbf{0})\Gamma_{f_i}^{-1}\tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0}) + \sum_{i,j=1}^{m}\tilde{\mathbf{\Theta}}_{g_{ij}}^T(\mathbf{0})\Gamma_{g_{ij}}^{-1}\tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0})+\sum_{i=1}^{m}\rho_i^2\int_0^T \varrho_i^2\,dt \tag{50}$$

where Γ_{f_i} $(i=1,\ldots,m)$, $\Gamma_{g_{ij}}$ $(i,j=1,\ldots,m)$ are positive definite diagonal matrices, ρ_i $(i=1,\ldots,m)$ are positive parameters representing for prescribed disturbance attenuation levels, $\mathbf{e}=[\mathbf{e}_1^T,\ldots,\mathbf{e}_m^T]^T$, $\varrho_i \overset{\text{def}}{=} -d_i+w_i$, $\mathbf{Q}=\mathrm{diag}(\mathbf{Q}_1,\ldots,\mathbf{Q}_m)$ and $\mathbf{Q}_i\in\mathbb{R}^{m\times m}$ $(i=1,\ldots,m)$ are arbitrary symmetric positive definite matrices, $\mathbf{P}=\mathrm{diag}\,(\mathbf{P}_1,\ldots,\mathbf{P}_m)$ and $\mathbf{P}_i$ $(i=1,\ldots,m)$ are the symmetric positive definite solutions of the following Lyapunov equations:

$$\mathbf{P}_i\mathbf{A}_i+\mathbf{A}_i^T\mathbf{P}_i=-\mathbf{Q}_i,\quad i=1,\ldots,m \tag{51}$$

A scheme of the adaptive H^∞ tracking control scheme is shown in Fig. 2.

Proof. Define the Lyapunov function $V=\sum_{i=1}^m V_i$ where V_i is defined in (38). According to the definition of V_i and Theorem 1, it is easy to obtain (50). □

Corollary 1. *The closed loop system is stable and the steady tracking errors satisfy $\lim_{t\to\infty}e_i=0, i=1,\ldots,m$, i.e., $\lim_{t\to\infty}|y_{id}(t)-y_i(t)|=0$. The bound of the transient tracking errors will be given by*

$$\|e_i\|_2^2 \le \frac{2(\mathbf{e}_i^T(\mathbf{0})\mathbf{P}_i\mathbf{e}_i(\mathbf{0})+\tilde{\mathbf{\Theta}}_{f_i}^T(\mathbf{0})\Gamma_{f_i}^{-1}\tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0}))}{\lambda_{\min}(\mathbf{Q}_i)} + \frac{2\sum_{j=1}^m\tilde{\mathbf{\Theta}}_{g_{ij}}^T(\mathbf{0})\Gamma_{g_{ij}}^{-1}\tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0})+\rho_i^2\int_0^T\varrho_i^2\,dt}{\lambda_{\min}(\mathbf{Q}_i)} \tag{52}$$

where $\lambda_{\min}(\mathbf{Q}_i)$ represents the minimum eigenvalue of matrix $\mathbf{Q}_i$.

Proof. From (39), it can be obtained that

$$\dot{V}_i \le -\tfrac{1}{2}\lambda_{\min}(\mathbf{Q}_i)\|\mathbf{e}_i\|^2+\tfrac{1}{2}\rho_i^2\varrho_i^2 \tag{53}$$

$\dot{V}_i$ is negative whenever $\|\mathbf{e}_i\| \ge \rho_i|\varrho_i|/\sqrt{\lambda_{\min}(\mathbf{Q}_i)}$. Hence the tracking error will stay in the region $\{\mathbf{e}_i \mid \|\mathbf{e}_i\| \le \rho_i|\varrho_i|/\sqrt{\lambda_{\min}(\mathbf{Q}_i)}\}$.

Obviously

$$e_i^2 \le \|\mathbf{e}_i\|^2 \le \frac{-2\dot{V}_i+\rho_i^2\varrho_i^2}{\lambda_{\min}(\mathbf{Q}_i)} \tag{54}$$

hence

$$\begin{aligned}\|e_i\|_2^2 &= \int_0^T e_i^2\,dt \le \frac{2(V_i(0)-V_i(T))+\rho_i^2\int_0^T\varrho_i^2\,dt}{\lambda_{\min}(\mathbf{Q}_i)}\\ &\le \frac{2V_i(0)+\rho_i^2\int_0^T\varrho_i^2\,dt}{\lambda_{\min}(\mathbf{Q}_i)}\\ &= \frac{2}{\lambda_{\min}(\mathbf{Q}_i)}(\mathbf{e}_i^T(\mathbf{0})\mathbf{P}_i\mathbf{e}_i(\mathbf{0})+\tilde{\mathbf{\Theta}}_{f_i}^T(\mathbf{0})\Gamma_{f_i}^{-1}\tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0}))\\ &\quad+\frac{2(\sum_{j=1}^m\tilde{\mathbf{\Theta}}_{g_{ij}}^T(\mathbf{0})\Gamma_{g_{ij}}^{-1}\tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0})+\frac{1}{2}\rho_i^2\int_0^T\varrho_i^2\,dt)}{\lambda_{\min}(\mathbf{Q}_i)}\end{aligned} \tag{55}$$

Assumption 1 implies $\int_0^T w_i^2\,dt<\infty$, then $\int_0^T\varrho_i^2\,dt=\int_0^T(w_i-d_i)^2\,dt<\infty$. Eq. (55) means $e_i\in L_2$. According to Barbalat lemma, $\lim_{t\to\infty}e_i=0$. □

Remark 5. According to Theorem 1, the i-th subsystem achieves a H^∞ tracking performance with a prescribed disturbance

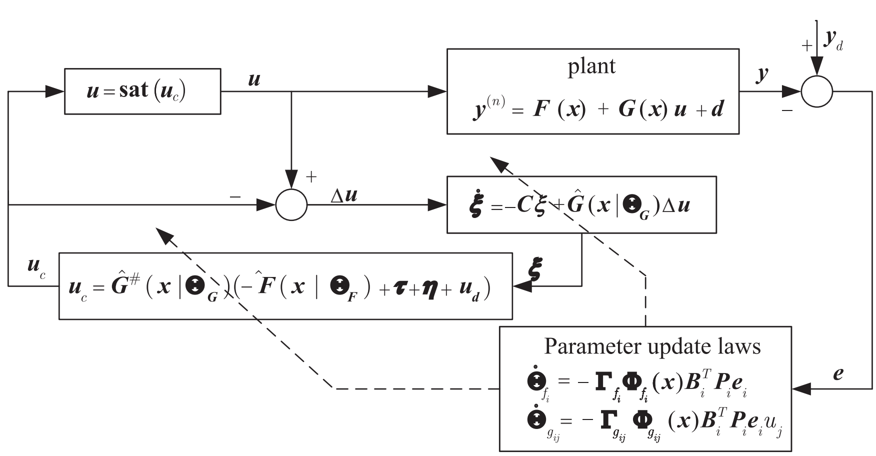

Fig. 2. The overall structure of adaptive H^∞ tracking control scheme.

attenuation level ρ_i, i.e., the L_2 gain from w_i to the tracking error e_i is equal or less than ρ_i.

Remark 6. Because the parameters $\lambda_{ij}, j=1,\ldots,r_i$ can make $\mathbf{A}_i$ be a Hurwitz stable matrix, there exists unique symmetric positive definite matrix $\mathbf{P}_i$ satisfying Lyapunov equation (37).

Remark 7. The bound for $\|y_{id}(t)-y_i(t)\|_2$ is an explicit function of the design parameters. According to Corollary 1, this bound depends on the initial estimate errors $\tilde{\boldsymbol{\Theta}}_{f_i}(0)$, $\tilde{\boldsymbol{\Theta}}_{g_{ij}}(0)$ $(j=1,\ldots,m)$. The closer the initial estimates $\boldsymbol{\Theta}_{f_i}(0)$, $\boldsymbol{\Theta}_{g_{ij}}(0)$ $(j=1,\ldots,m)$ to the true values $\boldsymbol{\Theta}^*_{f_i}(0)$, $\boldsymbol{\Theta}^*_{g_{ij}}(0)$ $(j=1,\ldots,m)$, the better the transient performance. The effects of initial estimate errors on this bound can be decreased by increasing the values of the diagonal adaptation gain matrices Γ_{f_i}, $\Gamma_{g_{ij}}$ $(j=1,\ldots,m)$ and by choosing positive definite symmetric matrix $\mathbf{Q}_i$ with larger minimum eigenvalue. On the other hand, this bound also depends on the external disturbances and neural network approximation errors. The effects of extern disturbances and neural network approximation errors on the transient performance can be reduced by decreasing ρ_i and increasing $\lambda_{\min}(\mathbf{Q}_i)$. Large ρ_i represents low disturbance attenuation level while small ρ_i represents high disturbance attenuation level.

Remark 8. If $\Delta\mathbf{u}=\mathbf{0}$ or $\Delta\mathbf{u}$ tends to zero as t tends to infinity, then $\xi_i\to 0$.

4. Numerical example

In this section, we illustrate the above methodology on the following example. We consider an affine nonlinear system with actuator amplitude and rate limitations. The dynamic model of this nonlinear system is as follows [27]:

$$\begin{aligned}
\dot{x}_1 &= -(x_1+x_2^2)+10u_1+\sin^2(x_2)u_2+0.2d_1(t)\\
\dot{x}_2 &= -x_1^2+x_1^2u_1+u_2+0.2d_2(t)\\
y_1 &= x_1,\quad y_2=x_2,\quad x_1(0)=1,\quad x_2(0)=0
\end{aligned}\tag{56}$$

where u_1, u_2 are control inputs and have the limitations $|u_i|\le 5$, $|\dot{u}_i|\le 10$, $i=1,2$, and $d_1(t)$, $d_2(t)$ are bounded random noises in the interval [0,1]. It is desired to determine control the inputs u_1, u_2 such that y_1, y_2 follow those reference trajectories defined by $y_{1d}=\sin(t)$, $y_{2d}=\cos(t)$ respectively.

Rewrite the plant (56) as

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}f_1\\ f_2\end{bmatrix}+\begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}+0.2\begin{bmatrix}d_1\\ d_2\end{bmatrix}$$

where $f_1=-(x_1+x_2^2)$, $f_2=-x_1^2$, $g_{11}=10$, $g_{12}=\sin^2(x_2)$, $g_{21}=x_1^2$, $g_{22}=1$.

According to Theorem 2, the H^∞ tracking design is given as follows.

We choose the following Gauss radial basis vector for approximating f_1, i.e.,

$$\boldsymbol{\Phi}_{f_1}(\mathbf{x})=[e^{-\|\mathbf{x}-\mathbf{c}_1\|^2/b_1^2},\ldots,e^{-\|\mathbf{x}-\mathbf{c}_{11}\|^2/b_{11}^2}]^T$$

where $\mathbf{x}=(x_1,x_2)^T$, $\mathbf{c}_i$ $(i=1,\ldots,11)$ are the center of the radial base, and are chosen as $\mathbf{c}_1=(-2,-2)^T$, $\mathbf{c}_2=(-1.6,-1.6)^T$, $\mathbf{c}_3=(-1.2,-1.2)^T$, $\mathbf{c}_4=(-0.8,-0.8)^T$, $\mathbf{c}_5=(-0.4,-0.4)^T$, $\mathbf{c}_6=(0,0)^T$, $\mathbf{c}_7=(0.4,0.4)^T$, $\mathbf{c}_8=(0.8,0.8)^T$, $\mathbf{c}_9=(1.2,1.2)^T$, $\mathbf{c}_{10}=(1.6,1.6)^T$, $\mathbf{c}_{11}=(2,2)^T$, $b_i=2$, $i=1,\ldots,11$. The radial bases for f_2, g_{11}, g_{12}, g_{21}, g_{22} are chosen the same as f_1.

The robust control term $\tau=(\tau_1,\tau_2)^T=[\dot{y}_{1d}+\lambda_{11}(y_{1d}-y_1), \dot{y}_{2d}+\lambda_{21}(y_{2d}-y_2)]^T$ and the coefficients $\lambda_{11}=5$, $\lambda_{21}=5$. Now we have $\mathbf{A}_1=[-5]_{1\times1}$, $\mathbf{A}_2=[-5]_{1\times1}$, $\mathbf{B}_1=\mathbf{B}_2=[1]_{1\times1}$, where $[\cdot]_{1\times1}$ represents a 1×1 matrix. Select $\mathbf{Q}_1=[10]_{1\times1}$ and $\mathbf{Q}_2=[10]_{1\times1}$. Solving Lyapunov equations (51), we obtain $\mathbf{P}_1=\mathbf{P}_2=[1]_{1\times1}$. We choose the parameter update gain matrices for RBF neural networks as $\Gamma_{f_1}=\Gamma_{f_2}==\Gamma_{g_{11}}=\Gamma_{g_{12}}=\Gamma_{g_{21}}=\Gamma_{g_{22}}=\mathrm{diag}(1,1,1,1,1,1,1,1,1,1,1)$ and set the parameters update laws as follows:

$$\dot{\boldsymbol{\Theta}}_{f_1}=-\Gamma_{f_1}\boldsymbol{\Phi}_{f_1}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\mathbf{e}_1,\quad \dot{\boldsymbol{\Theta}}_{f_2}=-\Gamma_{f_2}\boldsymbol{\Phi}_{f_2}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\mathbf{e}_2$$

$$\dot{\boldsymbol{\Theta}}_{g_{11}}=-\Gamma_{g_{11}}\boldsymbol{\Phi}_{g_{11}}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\mathbf{e}_1u_1,\quad \dot{\boldsymbol{\Theta}}_{g_{12}}=-\Gamma_{g_{12}}\boldsymbol{\Phi}_{g_{12}}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\mathbf{e}_1u_2$$

$$\dot{\boldsymbol{\Theta}}_{g_{21}}=-\Gamma_{g_{21}}\boldsymbol{\Phi}_{g_{21}}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\mathbf{e}_2u_1,\quad \dot{\boldsymbol{\Theta}}_{g_{22}}=-\Gamma_{g_{22}}\boldsymbol{\Phi}_{g_{22}}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\mathbf{e}_2u_2$$

where $\mathbf{e}_1=[e_1]_{1\times1}$, $\mathbf{e}_2=[e_2]_{1\times1}$, $e_1=y_{1d}-y_1$, $e_2=y_{2d}-y_2$. The initial values for $\boldsymbol{\Theta}_{f_i}(0)$ $(i=1,2)$ and $\boldsymbol{\Theta}_{g_{ij}}(0)$ $(i,j=1,2)$ are chosen as $\boldsymbol{\Theta}_{f_1}(0)=\boldsymbol{\Theta}_{f_2}(0)=\boldsymbol{\Theta}_{g_{21}}(0)=\boldsymbol{\Theta}_{g_{12}}(0)=\mathbf{0}$, $\boldsymbol{\Theta}_{g_{11}}(0)=\boldsymbol{\Theta}_{g_{22}}(0)=\mathrm{diag}(1,1,1,1,1,1,1,1,1,1,1)$.

The auxiliary system is constructed as follows:

$$\dot{\xi}_1=-c_1\xi_1+\hat{g}_{11}\Delta u_1+\hat{g}_{12}\Delta u_2,\quad \xi_1(0)=0$$

$$\dot{\xi}_2=-c_2\xi_2+\hat{g}_{21}\Delta u_1+\hat{g}_{22}\Delta u_2,\quad \xi_2(0)=0$$

where $c_1=25$, $c_2=25$, $\Delta u_1=u_1-u_{c1}$, $\Delta u_2=u_2-u_{c2}$.

We select the prescribed disturbance attenuation levels $\rho_1=\rho_2=0.5$, and the supervisory control:

$$u_{d_1}=\frac{1}{2\rho_1^2}\mathbf{e}_1^T\mathbf{P}_1\mathbf{B}_1,\quad u_{d_2}=\frac{1}{2\rho_2^2}\mathbf{e}_2^T\mathbf{P}_2\mathbf{B}_2\tag{57}$$

According to (9), we can obtain

$$\mathbf{u}_c=\begin{bmatrix}u_{c1}\\ u_{c2}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Theta}_{g_{11}}^T\boldsymbol{\Phi}_{g_{11}} & \boldsymbol{\Theta}_{g_{12}}^T\boldsymbol{\Phi}_{g_{12}}\\ \boldsymbol{\Theta}_{g_{21}}^T\boldsymbol{\Phi}_{g_{21}} & \boldsymbol{\Theta}_{g_{22}}^T\boldsymbol{\Phi}_{g_{22}}\end{bmatrix}^{\#}\left(-\begin{bmatrix}\boldsymbol{\Theta}_{f_1}^T\boldsymbol{\Phi}_{f_1}\\ \boldsymbol{\Theta}_{f_2}^T\boldsymbol{\Phi}_{f_2}\end{bmatrix}+\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}-\begin{bmatrix}25\xi_1+\dot{\xi}_1\\ 25\xi_2+\dot{\xi}_2\end{bmatrix}+\begin{bmatrix}u_{d1}\\ u_{d2}\end{bmatrix}\right)\tag{58}$$

and finally we get the dynamics of control u_1, u_2 by assuming a first-order dynamics for u_{c1}, u_{c2} as

$$\dot{u}_1=\mathrm{sat}_{10}(20.5\,\mathrm{sat}_5(u_{c1})-u_1)$$

$$\dot{u}_2=\mathrm{sat}_{10}(20.5\,\mathrm{sat}_5(u_{c2})-u_2)$$

The MATLAB solver “ode4” is used to simulate the overall control system with step size 0.01. Simulation results are presented in Figs. 3–12. Fig. 3 shows the curves of output $y_1(t)$ and its reference trajectory, meanwhile Fig. 4 shows the curves of output $y_2(t)$ and its reference trajectory. Curves in Fig. 5 describe the tracking errors for y_1 and y_2. These curves indicate that the outputs track their reference values well, and the effects of approximation error and extern disturbance on tracking errors are effectively attenuated. The control signals $u_1(t)$, $u_2(t)$ and their derivatives $\dot{u}_1(t)$, $\dot{u}_2(t)$ are given in Figs. 6 and 7. It is observed that the control signals $u_1(t)$ and $u_2(t)$ satisfy the amplitude and rate limitations. Fig. 8 shows the signals $u_{c1}(t)$, $u_{c2}(t)$. Obviously they

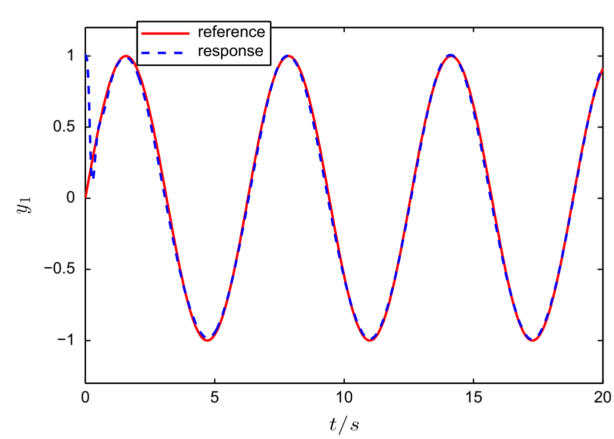

Fig. 3. The trajectory of y_1.

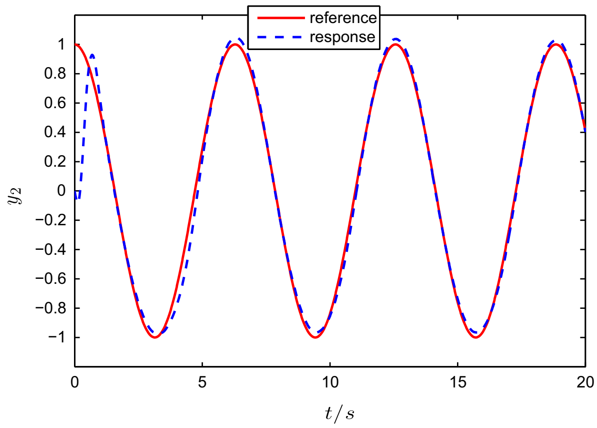

Fig. 4. The trajectory of y_2.

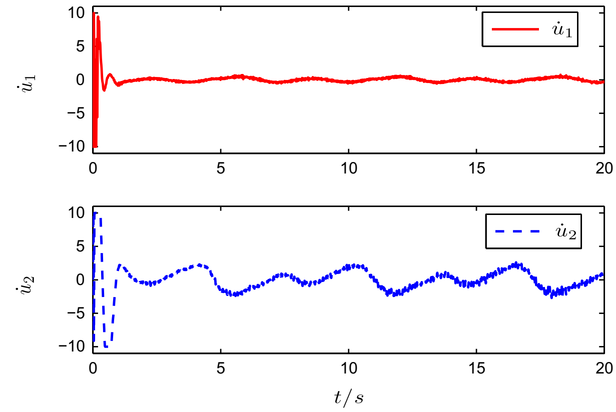

Fig. 7. $\dot{u}_1$, $\dot{u}_2$.

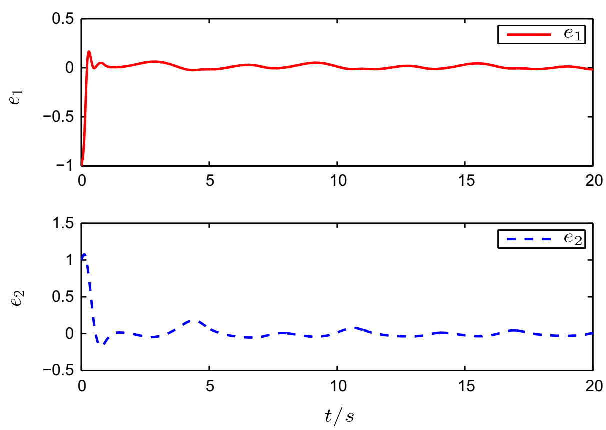

Fig. 5. The tracking errors e_1, e_2.

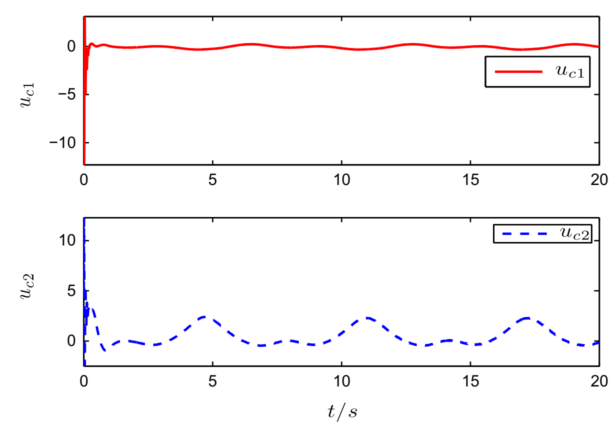

Fig. 8. u_{c1}, u_{c2}.

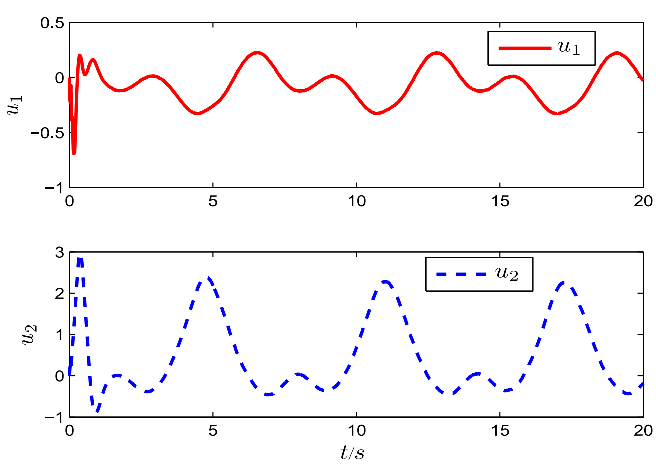

Fig. 6. u_1, u_2.

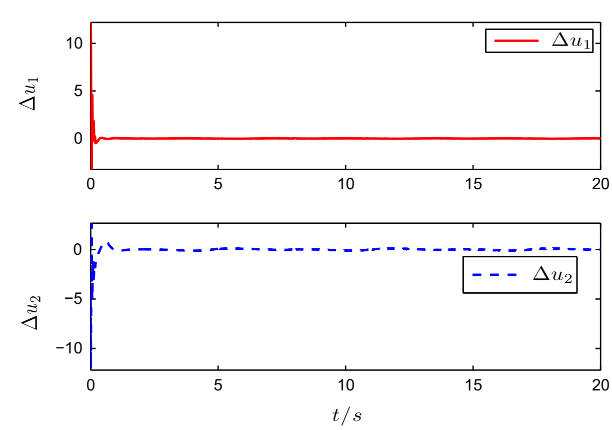

Fig. 9. Δu_1, Δu_2.

do not satisfy the control input limitations. Fig. 9 shows the signals $\Delta u_1(t)$, $\Delta u_2(t)$. These curves in Fig. 9 show that $\Delta u_1(t)$, $\Delta u_2(t)$ tend to zero soon as time goes. Fig. 10 shows the curves of ξ_1, ξ_2. From Figs. 9 and 10, we can see that ξ_1, ξ_2 also tend to zero soon as $\Delta u_1(t)$, $\Delta u_2(t)$ tend to zero.

Figs. 11 and 12 show the tracking errors e_1, e_2 at different prescribed disturbance attenuation levels $\rho_1 = \rho_2 = 1, 0.3, 0.1$. These curves indicate that under low disturbance attenuation level (ρ_i is large, e.g. $\rho_1 = \rho_2 = 1$), the H^∞ tracking performance is poor than that under higher disturbance attenuation level (ρ_i is small, i.e., $\rho_1 = \rho_2 = 0.1$).

In conclusion, the simulation results demonstrate that the adaptive controller proposed in preceding section not only can generate control signal that satisfies actuator amplitude and rate saturations but also can make sure that the closed loop system achieves H^∞ tracking performance.

5. Conclusions

In this work, the control problem for a class of uncertain nonlinear MIMO systems with actuator amplitude and rate

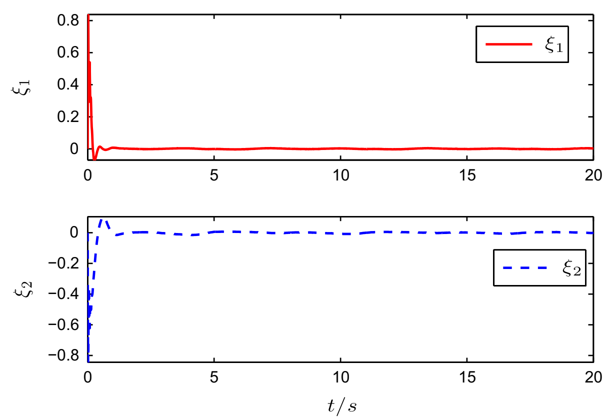

Fig. 10. ξ_1, ξ_2.

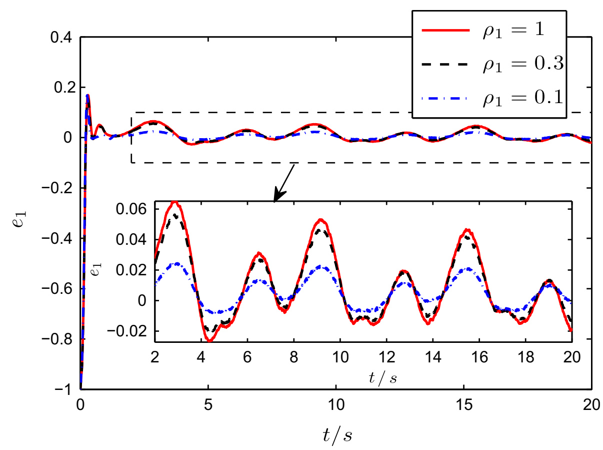

Fig. 11. The e_1 at different disturbance attenuation levels.

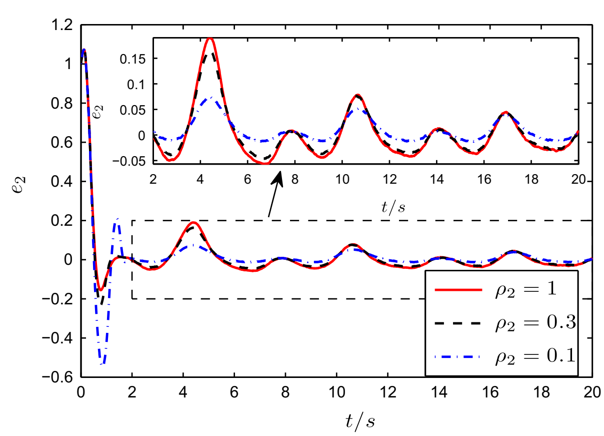

Fig. 12. The e_2 at different disturbance attenuation levels.

saturations is considered and an adaptive radial basis neural network based controller which is designed with a priori consideration of actuator saturation effects and guarantees H^∞ tracking performance is proposed. Adaptive radial basis function neural networks are used to approximate the unknown nonlinearities. An auxiliary system is constructed to compensate the effects of actuator amplitude and rate saturations. A supervisory control is designed to attenuate the effects of extern disturbance and neural network approximation errors, so that the closed loop system achieves a prescribed H^∞ tracking performance. Analysis shows that the bound of tracking error is adjustable by an explicit choice of design parameters. The proposed controller can generate control signals satisfying their constraints and guarantee a desired closed loop performance. Simulation results illustrate the effectiveness of the proposed controller.

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