Soft Comput (2013) 17:2075–2088
DOI 10.1007/s00500-013-1117-4

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Data-driven modeling and optimization of thermal comfort and energy consumption using type-2 fuzzy method

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Published online: 11 September 2013

Abstract In the research domain of intelligent buildings and smart home, modeling and optimization of the thermal comfort and energy consumption are important issues. This paper presents a type-2 fuzzy method based data-driven strategy for the modeling and optimization of thermal comfort words and energy consumption. First, we propose a methodology to convert the interval survey data on thermal comfort words to the interval type-2 fuzzy sets (IT2 FSs) which can reflect the inter-personal and intra-personal uncertainties contained in the intervals. This data-driven strategy includes three steps: survey data collection and preprocessing, ambiguity-preserved conversion of the survey intervals to their representative type-1 fuzzy sets (T1 FSs), IT2 FS modeling. Then, using the IT2 FS models of thermal comfort words as antecedent parts, an evolving type-2 fuzzy model is constructed to reflect the online observed energy consumption data. Finally, a multiobjective optimization model is presented to recommend a reasonable temperature range that can give comfortable feeling while reducing energy consumption. The proposed method can be used to realize comfortable but energy-saving environment in smart home or intelligent buildings.

Keywords Type-2 fuzzy · Data-driven method · Thermal comfort · Multiobjective optimization · Energy saving

Communicated by C. Alippi, D. Zhao and D. Liu.

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1 Introduction

Measurement or estimation of the thermal comfort index is an essential issue to realize comfortable living or working environment in smart home or intelligent buildings. In order to achieve this goal, researchers have proposed two categories of methods. The first category evaluates the thermal comfort through computing or approximating the Predicted Mean Vote (PMV) index (Fanger et al. 1970; Yun and Won 2012; Li et al. 2012a, 2013a). The other category model the thermal comfort words directly to reflect peoples' feelings on different thermal comfort words, e.g. "*cold*", "*comfort*", "*hot*", etc. Such thermal comfort words are affected by both the physical environment (air temperature, relative humidity, etc.) and the personal differences in mood, culture and other individual, organizational and social factors (Djongyang et al. 2010). They are very vague terms to be depicted, so, it is more reasonable to model them using fuzzy sets. Some studies have adopted type-1 fuzzy sets (T1 FSs) (Stephen et al. 2010) to model such words. However, different people have different feelings on such words, and even the same person at different periods has different feelings on them. The high levels of linguistic uncertainties on these words can not be sufficiently modeled by the T1 FSs. Recently, Mendel pointed out that interval type-2 fuzzy sets (IT2 FSs) are better and more reasonable than T1 FSs to model vague words when "*words mean different things to different people*" (Mendel 2001, 2012; Mendel et al. 2006).

At present, we usually use two kinds of data-driven modeling methods to construct IT2 FSs. One is the interval approach (Liu and Mendel 2008; Coupland et al. 2010; Wu et al. 2012), and the other one is the Fuzzistics approach (Mendel and Wu 2006, 2007a,b; Mendel 2007). In Li et al. (2012b) and Li et al. (2013b), we have proposed an uncertainty degree based Fuzzistic method for IT2 FS modeling

and have applied it to model the thermal comfort words. The constructed IT2 FSs in Li et al. (2012b) and Li et al. (2013b) were assumed to be symmetric, because the non-symmetric IT2 FSs have much more parameters to be determined which sometimes can not be solved by the Fuzzistics approach. However, in the modeling process, we found that it is not very reasonable to use symmetric IT2 FSs to model some thermal comfort words.

In this study, we present a new survey data-driven IT2 FS modeling method and apply it to model the thermal comfort words. We use three steps to construct the IT2 FS model: firstly, interval survey data are collected and pre-processed, secondly, the survey intervals are converted to their representative T1 FSs by preserving their ambiguity, thirdly, the constructed T1 FSs are aggregated into one IT2 FS to reflect the inter-personal uncertainties. Application results to thermal comfort words modeling demonstrate that most of the IT2 FS models are non-symmetric. This is consistent with the fact observed in our earlier study.

Modeling of energy consumption is another important topic in smart home and green buildings. Energy consumption data which contain all the buildings' operation information can be utilized to design building management and energy saving control strategies. To model the energy consumption of buildings, data-driven methods, such as the statistical method (Ansari et al. 2005), the neural network (Yokoyama et al. 2009), the support vector machines (Li et al. 2009), the hybrid optimization algorithm (Squartini et al. 2013) and the grey model (Zhou et al. 2008) are widely adopted. However, such methods focused on the yearly or monthly electricity consumption of a city or a region, at least, a whole building. In this study, we focus on the short-term load modeling of a single room, which is quite important for indoor environment control. We mainly construct the model of the hourly cooling load of the air conditioner which is used to realize indoor environmental quality control, i.e. to achieve the comfortable air temperature. Using the IT2 FS models of thermal comfort words as antecedent parts of fuzzy rules, type-2 fuzzy model is constructed for the hourly cooling load. As proved in earlier studies (Gu and Zhang 2007; Wu 2012; Fazel Zarandi and Gamasaee 2012; Nie and Tan 2008, 2012; Mendel and Wu 2013), the type-2 fuzzy model not only have the merits of conventional fuzzy models, but also can provide the capability to model high levels of uncertainties and produce better results.

The pre-mentioned data-driven work on energy consumption didn't take the issue of human comfort into account. In De Angelis et al. (2013), Favre and Peuportier (2013), Selamat et al. (2013), Malatji et al. (2013) both energy consumption and comfort are considered. In these studies, optimization algorithms are utilized to minimize the energy consumption while keeping the air temperature in an acceptable range that is manually pre-planned or given. Being different to these works, this study aims to provide the reasonable temperature range through balancing the contradiction between the thermal comfort and energy consumption. This is achieved by the multiobjective optimization methods (Deb 2009; Cheng et al. 2012; Liu and Pender 2013; Censor 1977) which can provide us a powerful tool to solve the multi-objective problems. A multiobjective optimization model is presented in this paper for the tradeoff of the comfort degree and energy consumption, and optimization result for a specific room is given. The provided method and result can be used to realize comfortable living or working environment while saving energy in smart home or intelligent buildings.

The main novelties and contributions of this study are listed as follows:

1. A novel interval survey data driven methodology is proposed to model the IT2 FSs while preserving the inter-personal and intra-personal uncertainties. And, the proposed method is applied to the modeling of thermal comfort words that can be used for living or working environment control.
2. Using the constructed IT2 FS models as antecedent parts of the type-2 fuzzy rules, an evolving type-2 fuzzy logic system based energy consumption modeling method is presented and tested through a specific room.
3. Multiobjective optimization model is constructed to provide the reasonable temperature range through balancing the thermal comfort and energy consumption. The optimized result can be used for the user-oriented control of smart home or intelligent buildings.

The rest of this paper is organized as follows: Sect. 2 briefly reviews the IT2 FSs and IT2 fuzzy logic systems. Section 3 presents the methodology for constructing IT2 FS models and applies it to model thermal comfort words. Section 4 gives the evolving IT2 fuzzy model to reflect the electricity energy consumption. Section 5 provides the multiobjective optimization model to calculate reasonable temperature range. Finally, Sect. 6 concludes the paper.

2 Type-2 fuzzy set and fuzzy logic system

2.1 Type-1 and type-2 fuzzy sets

Fuzzy set (FS) theory can gradually assess the membership of elements in a set through defining a membership function (MF) valued in the real unit interval [0, 1]. The FS with crisp MF is called type-1 (T1) FS while the one with fuzzy MF is named type-2 (T2) FS (Mendel 2001; Yang and Guan 2013; Takáč 2013).

A T1 FS, denoted as A in universe of discourse X, is commonly represented as (Mendel 2001; Zhang and Quan 2001; Yang et al. 2012; Yang and Zhang 2011)

$$A = \int_X \mu_{A(x)/x}. \tag{1}$$

where $\mu_A(x)$ is the crisp MF grade of a generic element x, $\int$ denotes union over all admissible x.

A T2 FS, denoted as $\tilde{A}$, can be characterized as (Mendel 2001; Yang and Guan 2013; Takáč 2013)

$$\tilde{A} = \int_X \mu_{\tilde{A}}(x)/x = \int_X \int_{u \in J_x} f_x(u)/u/x,\ J_x \subseteq [0, 1], \tag{2}$$

where $\mu_{\tilde{A}}(x)$ is the fuzzy MF grade of a generic element x, $\int\int$ denotes union over all admissible x and u, $f_x(u)$ is the secondary MF and J_x is the primary membership of x which is the domain of the secondary MF. Uncertainties in the primary memberships of a T2 FS $\tilde{A}$ consist of a bounded region that we call the footprint of uncertainty (FOU) (Mendel 2001; Yang and Guan 2013).

When the secondary MFs of T2 FS become interval sets, the T2 FS turns to interval type-2 (IT2) FS that can be characterized as (Mendel 2001; Yang and Guan 2013)

$$\tilde{A} = \int_{x \in X} \left[\int_{u \in J_x} 1/u \right] /x,\ J_x \subseteq [0, 1], \tag{3}$$

where the secondary grades of $\tilde{A}$ all equal 1.

IT2 FS $\tilde{A}$, which can be obtained by blurring T1 FS, can be completely described by its lower MF (LMF) $\underline{\mu}_{\tilde{A}}(x)$ and upper MF (UMF) $\overline{\mu}_{\tilde{A}}(x)$, i.e.

$$\mu_{\tilde{A}}(x) = [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]. \tag{4}$$

For IT2 FS, Representation Theorem in (Mendel et al. 2006) tells us that the FOU of an IT2 FS is the (set theory) union of all its embedded T1 FSs. In other words, the FOU of an IT2 FS is covered by the union of all its embedded T1 FSs. As a result, *We can construct an IT2 FS through the mathematical operation of the union of its embedded T1 FSs*.

In this study, we mainly use the triangular T1 FS and the trapezoidal IT2 FS as shown in Fig. 1a, b, respectively.

The MF of the triangular T1 FS can be depicted as

$$\mu_A(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, \frac{d-x}{d-b}\right\}\right\}. \tag{5}$$

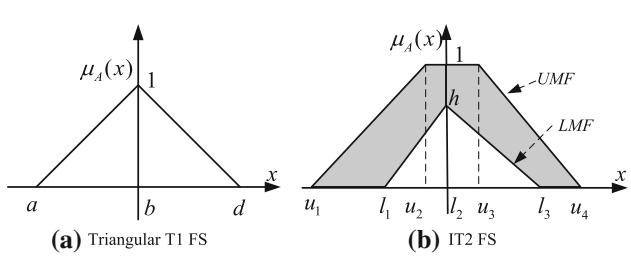


Fig. 1 The triangular T1 FS (**a**) and IT2 FS (**b**)

The LMF and UMF of the trapezoidal IT2 FS can be computed, respectively, as

$$\underline{\mu}_{\tilde{A}}(x) = \max\left\{0, \min\left\{h\frac{x-l_1}{l_2-l_1}, h\frac{l_3-x}{l_3-l_2}\right\}\right\}, \tag{6}$$

$$\overline{\mu}_{\tilde{A}}(x) = \max\left\{0, \min\left\{1, \frac{x-u_1}{u_2-u_1}, \frac{u_4-x}{u_4-u_3}\right\}\right\}, \tag{7}$$

where $l_1 \le l_2 \le l_3$, $u_1 \le u_2 \le u_3 \le u_4$ and $0 \le h \le 1$.

2.2 Interval type-2 fuzzy logic system (IT2 FLS)

Interval type-2 fuzzy sets are finding very wide applicability in rule-based fuzzy logic systems (FLSs) because they can cope with high levels of uncertainties that cannot be modeled by T1 FSs. The interval type-2 fuzzy rule-based FLSs (IT2 FLSs) have been applied in many areas, e.g. modeling, identification, prediction, control problems (Juang and Chen 2013). Such applications also have verified that IT2 FLS can produce more complex input–output mappings and better results compared with its counterpart—the T1 FLS.

The structure of the IT2 FLS is depicted in Fig. 2, and it consists of a fuzzifier, an inference engine, a rule base, a type-reducer and a defuzzifier (Mendel 2001; Yang and Guan 2013).

In this study, we adopt the following type-2 fuzzy rule base, which is the most widely-used one in modeling and control applications.

Rule k: if x_1 is $\tilde{A}_1^k$, x_2 is $\tilde{A}_2^k$, ..., x_p is $\tilde{A}_p^k$, then y_o(x) is c^k, where $k = 1, 2, \ldots, M$, $\tilde{A}_j^k$s are the antecedent IT2 FSs for the input variables, and c^ks are the consequent weights which can be viewed as the centroid of the consequent IT2 FSs.

Once a crisp input x $= (x_1, x_2, ..., x_p)$ is applied to the IT2 FLS, through the singleton fuzzifier and the type-2 inference process, the interval firing strength of Rule k can be calculated by the product operation as follows

$$F^k(\mathrm{x}) = [\underline{f}^k(\mathrm{x}), \overline{f}^k(\mathrm{x})], \tag{8}$$

where

$$\underline{f}^k(\mathrm{x}) = \prod_{j=1}^{p} \underline{\mu}_{\tilde{A}_j^k}(x_j), \tag{9}$$

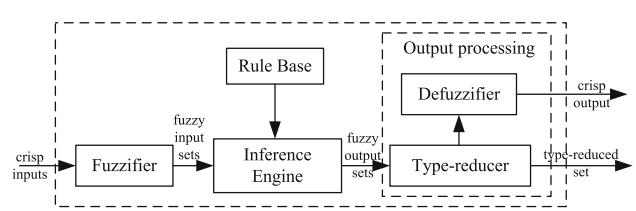


Fig. 2 The structure of the IT2 FLS

$$\overline{f}^k(\mathrm{x}) = \prod_{j=1}^{p} \overline{\mu}_{\widetilde{A}_j^k}(x_j). \tag{10}$$

To generate crisp output, the output processing including type-reduction and defuzzification is needed. There exist several different type-reduction and defuzzification methods. The IT2 FLSs using different type-reduction and defuzzification methods have different input–output mappings. In this study, we adopt the widely-used Nie–Tan (NT) method proposed by Nie and Tan (2008, 2012). With the NT method, the crisp output of the IT2 FLS can be computed as

$$\begin{aligned} y_o(\mathrm{x}) &= \frac{\sum_{k=1}^{M} c^k[\gamma \underline{f}^k(\mathrm{x}) + (1-\gamma)\overline{f}^k(\mathrm{x})]}{\sum_{k=1}^{M} \gamma \underline{f}^k(\mathrm{x}) + (1-\gamma)\overline{f}^k(\mathrm{x})} \\ &= \frac{\sum_{k=1}^{M} c^k[\gamma \prod_{j=1}^{p} \underline{\mu}_{\widetilde{A}_j^k}(x_j) + (1-\gamma)\prod_{j=1}^{p} \overline{\mu}_{\widetilde{A}_j^k}(x_j)]}{\sum_{k=1}^{M} \gamma \prod_{j=1}^{p} \underline{\mu}_{\widetilde{A}_j^k}(x_j) + (1-\gamma)\prod_{j=1}^{p} \overline{\mu}_{\widetilde{A}_j^k}(x_j)}. \end{aligned} \tag{11}$$

where γ is the inference coefficient, and $0 \le \gamma \le 1$.

This section briefly introduces the IT2 FS and IT2 FLS. Below, we will use them to model thermal comfort words and energy consumption.

3 Interval data-driven modeling of IT2 FSs with application to thermal comfort words construction

In this section, we will first show how to construct IT2 FS model using interval survey data. Also, with the proposed method, we will realize thermal comfort words modeling. The process for constructing IT2 FS model using interval data is depicted in Fig. 3. In this method, after data collection and pre-processing, we first construct the representative T1 FSs for the interval survey data, and then, the constructed T1 FSs are aggregated into one IT2 FS to model the specific word.

3.1 Construction of representative T1 FSs for survey intervals

Different methods can be used to map an interval into a T1 FS. In the interval approach (Liu and Mendel 2008; Coupland et al. 2010; Wu et al. 2012), T1 FS is constructed by equating the means and deviations of the interval and the T1 FS. In this subsection, we propose a new method to realize this objective. The proposed method equates the uncertainty measure—ambiguities of the interval and the constructed triangular T1 FS, and simultaneously minimize the distance between them.

To begin, let us give the definition of the ambiguity.

3.1.1 Ambiguity of T1 FS

There are many uncertainty measures of T1 FSs, e.g. centroid, fuzziness, cardinality and ambiguity. In this study, we consider the ambiguity of T1FSs.

The α-cut, $\alpha \in [0, 1]$, of a T1 FS A is a crisp set defined as

$$A_\alpha = \{x \in \mathbb{R} : \mu_A(x) \ge \alpha\}. \tag{12}$$

Let

$$A_L(\alpha) = inf\{x \in \mathbb{R} : \mu_A(x) \ge \alpha\}, \tag{13}$$

$$A_U(\alpha) = sup\{x \in \mathbb{R} : \mu_A(x) \ge \alpha\}. \tag{14}$$

for any $\alpha \in [0, 1]$.

The ambiguity of a T1 FS A is given by (Chanas 2001; Ban and Coroianu 2012)

$$Amb(A) = \int_0^1 \alpha[A_U(\alpha) - A_L(\alpha)]d\alpha. \tag{15}$$

For the triangular T1 FS A, the left and right end points of its α-cuts are

$$A_L(\alpha) = a + (b-a)\alpha, \tag{16}$$

$$A_U(\alpha) = d - (d-b)\alpha. \tag{17}$$

As a result, the ambiguity of the triangular T1 FS A can be derived as

$$Amb(A) = \int_0^1 \alpha[(d-a) - (d-a)\alpha]d\alpha = \frac{d-a}{6}. \tag{18}$$

Consider the following T1 FS (also called interval FS)

$$\mu_I(x) = \begin{cases} 1, & t_l \le x \le t_r \\ 0, & else \end{cases} \tag{19}$$

For this FS, the left and right end points of its α-cuts are

$$I_L(\alpha) = t_l, \tag{20}$$

$$I_U(\alpha) = t_r. \tag{21}$$

As a result, the ambiguity of this FS can be computed as

$$Amb(I) = \int_0^1 \alpha(t_r - t_l)d\alpha = \frac{t_r - t_l}{2}. \tag{22}$$

3.1.2 Construction of ambiguity-preserved T1 FS

The interval survey data can reflect the intra-uncertainties that mean the variation in one persons' understanding of the word over time. To convert the interval data to its representative triangular T1 FS, the primary features of the interval data should be maintained. In Liu and Mendel (2008), Coupland et al. (2010), Wu et al. (2012) the interval data is seen

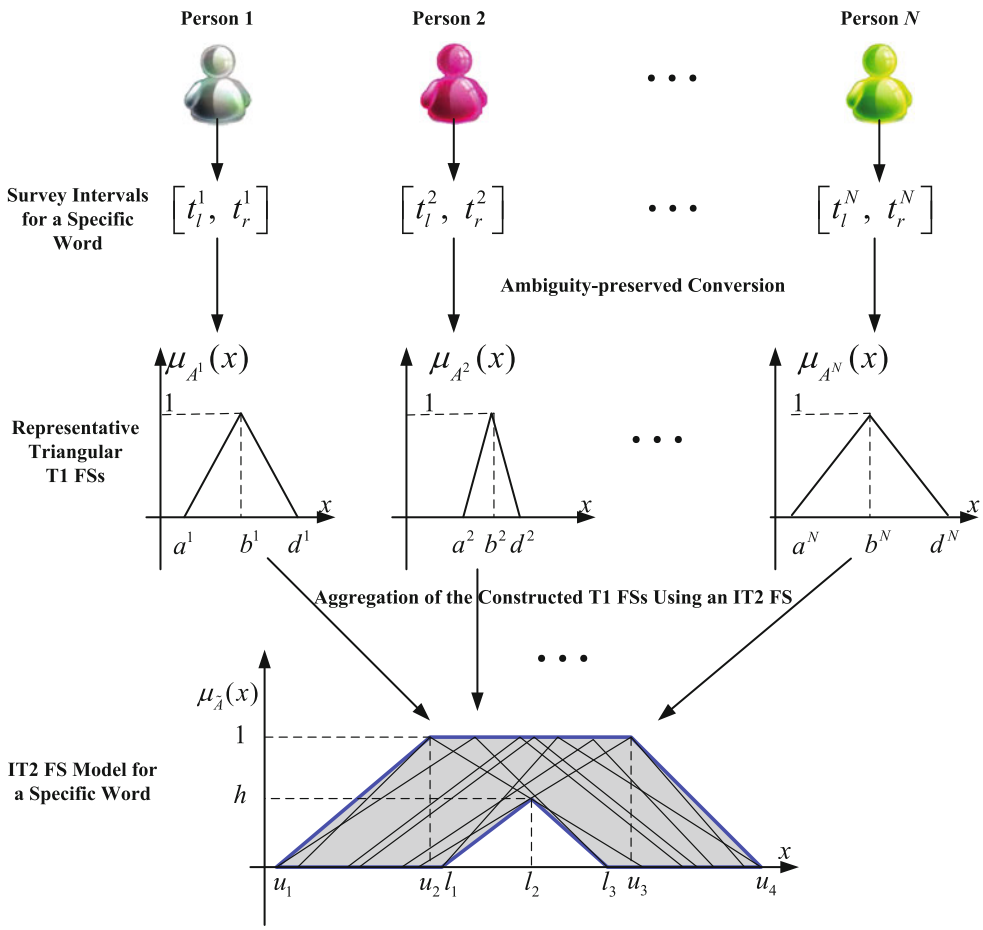


Fig. 3 The process for constructing IT2 FS model using interval data

as a uniform distribution. And, the representative T1 FS is obtained by equating the means and standard deviations of the interval data and the T1 FS. In this study, we will obtain the representative T1 FS through equating the uncertainty measures of the interval data and the T1 FS. Here, the symmetric triangular T1 FSs are adopted.

Suppose that, after data preprocessing, there exist N valid survey intervals $[t_l^i, t_r^i]$ $(i = 1, 2, \ldots, N)$. For each survey interval, as shown in Fig. 3, we build one representative T1 FS for it. The detailed process is presented below.

First, the interval data $[t_l^i, t_r^i]$ is converted to the interval FS I^i with its MF expressed in (19). In this conversion, the interval data is equivalently seen as one T1 FS, the ambiguity of which has been calculated in (22).

The next step is to convert the interval FS I^i to its representative T1 FS A^i. Suppose that the adopted symmetric triangular T1 FS is denoted as $A^i(a^i, b^i, d^i)$. In the conversion process, the center and ambiguity of the interval FS and its representative T1 FS are set to be equal, i.e.

$$b^i = \frac{t_l^i + t_r^i}{2}, \tag{23}$$

$$Amb(I^i) = Amb(A^i), \tag{24}$$

which implies that

$$\frac{d^i - a^i}{6} = \frac{t_r^i - t_l^i}{2}. \tag{25}$$

Generally, we use the following metric, which is an extension of the Euclidean distance, to measure the distance between the interval FS I^i and its representative T1 FS A^i:

$$\begin{aligned} D^2(A^i, I^i) &= \int_0^1 (A_L^i(\alpha) - I_L^i(\alpha))^2 \mathrm{d}\alpha \\ &\quad + \int_0^1 (A_U^i(\alpha) - I_U^i(\alpha))^2 \mathrm{d}\alpha \\ &= \int_0^1 ((a^i - t_l^i) + (b^i - a^i)\alpha)^2 \mathrm{d}\alpha \\ &\quad + \int_0^1 ((d^i - t_r^i) + (b^i - d^i)\alpha)^2 \mathrm{d}\alpha \end{aligned}$$

$$= (a^i - t_l^i)^2 + (a^i - t_l^i)(b^i - a^i) + \frac{(b^i - a^i)^2}{3} + (d^i - t_r^i)^2 + (d^i - t_r^i)(b^i - d^i) + \frac{(b^i - d^i)^2}{3}. \tag{26}$$

We find the nearest T1 FS A^i to be the representation of the interval data. In summary, we can solve the following optimization problem to convert the survey interval to a proper uncertainty-preserved triangular T1 FS:

$$\min_{a^i, b^i, d^i} D^2(A^i, I^i)$$
$$subject\ to\ b^i = \frac{t_l^i + t_r^i}{2}, \tag{27}$$
$$\frac{d^i - a^i}{6} = \frac{t_r^i - t_l^i}{2}.$$

Observing that $b^i = \frac{t_l^i + t_r^i}{2}$, and $d^i = a^i + 3(t_r^i - t_l^i)$, we have

$$D^2(A^i, I^i) = (a^i - t_l^i)^2 + (a^i - t_l^i)(b^i - a^i) + \frac{(b^i - a^i)^2}{3} + (a^i + 2t_r^i - 3t_l^i)^2 + (a^i + 2t_r^i - 3t_l^i)(b^i - a^i - 3t_r^i + 3t_l^i) + \frac{(b^i - a^i - 3t_r^i + 3t_l^i)^2}{3}. \tag{28}$$

As a result, the aforementioned constrained optimization problem in (27) can be transformed to the following unconstrained optimization problem

$$\min_{a^i} D^2(A^i, I^i; a^i). \tag{29}$$

By solving $\frac{\partial D^2(A^i, I^i; a^i)}{\partial a^i} = 0$, we can obtain the following solution for the triangular T1 FS $A(a^i, b^i, d^i)$:

$$a^i = \frac{t_l^i + t_r^i}{2} - \frac{3}{2}(t_r^i - t_l^i), \tag{30}$$
$$b^i = \frac{t_l^i + t_r^i}{2}, \tag{31}$$
$$d^i = \frac{t_l^i + t_r^i}{2} + \frac{3}{2}(t_r^i - t_l^i). \tag{32}$$

From above discussion, for the survey interval $[t_l^i, t_r^i]$, we can construct its representative triangular T1 FS as $A^i = A^i(2t_l^i - t_r^i, \frac{t_l^i + t_r^i}{2}, 2t_r^i - t_l^i)$. Obviously, we have $b^i - a^i = d^i - b^i$.

3.2 Aggregation of representative T1 FSs

As different people have different understandings on the same word. Therefore, these N triangular T1 FSs constructed from the survey intervals can reflect the inter-personal uncertainties that means the understanding variations between different people. Another problem is to aggregate these N triangular T1 FSs into one IT2 FS to reflect the inter-personal uncertainties in only one mathematical model.

In this study, we adopt the trapezoidal IT2 FS, whose LMF and UMF are respectively triangular and trapezoidal, to model such inter-personal uncertainties. To obtain the IT2 FS model, the LMF and UMF should be determined. As shown in Fig. 1b, the parameters of the triangular LMF are the left-end point l_1, the center l_2, the right-end point l_3, and its height h, while the four parameters of the trapezoidal UMF are u_1, u_2, u_3, u_4. Following the method proposed in Wu et al. (2012), below we will show how to derive the LMF and UMF of the IT2 FS model through aggregating the N constructed triangular T1 FSs. This process is also shown in Fig. 3. It is worth mentioning that the constructed IT2 FS should embody the N triangular T1 FSs derived from the N survey intervals.

First, let us determine the LMF of the IT2 FS model. From Fig. 3, to embody all the triangular T1 FSs, the left-end and right-end points of the LMF should be computed as

$$l_1 = \max_{i=1}^{N}\{a^i\}, \tag{33}$$
$$l_3 = \min_{i=1}^{N}\{d^i\}, \tag{34}$$

and, the vertex (l_2, h) of the LMF should be the lowest one of the $N \times N$ intersection points $(x^{i,j}, h^{i,j})$ of the lines A_i^L and A_j^R $(i, j = 1, \ldots, N)$, where

$$A_i^L(x) = \frac{x - a^i}{b^i - a^i}, \tag{35}$$
$$A_j^R(x) = \frac{d^j - x}{d^j - b^j}. \tag{36}$$

In other words, h and l_2 of the LMF can be computed as

$$h = h^{s,t} = \min_{i,j=1}^{N}\{h^{i,j}\}, \tag{37}$$
$$l_2 = x^{s,t}. \tag{38}$$

Then, the left thing is to compute the $N \times N$ intersection points $(x^{i,j}, h^{i,j})$. By settling $A_i^L(x) = A_j^R(x)$, we can derive the computation equations for the intersection points as

$$x^{i,j} = \frac{b^i d^j - a^i b^j}{b^i + d^j - a^i - b^j}, \tag{39}$$
$$h^{i,j} = \min\left\{\max\left\{\frac{x^{i,j} - a^i}{b^i - a^i}, 0\right\}, 1\right\}. \tag{40}$$

From Fig. 3, to embody all the triangular T1 FSs, the four parameters of the trapezoidal UMF should be computed as (Wu et al. 2012)

$$u_1 = \min_{i=1}^{N}\{a^i\}, \tag{41}$$

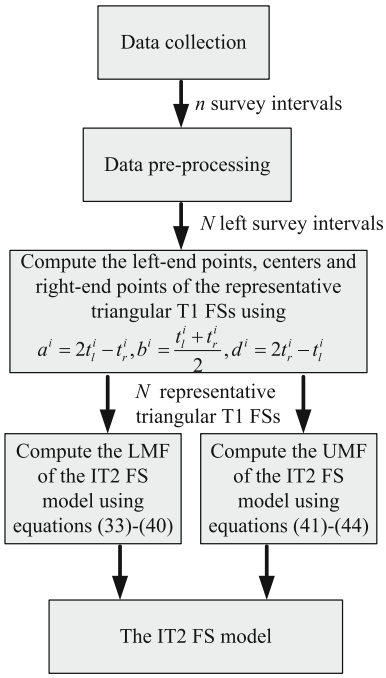


Fig. 4 The flow diagram for constructing IT2 FS model

$$u_2 = \min_{i=1}^{N}\{b^i\}, \tag{42}$$

$$u_3 = \max_{i=1}^{N}\{b^i\}, \tag{43}$$

$$u_4 = \max_{i=1}^{N}\{d^i\}. \tag{44}$$

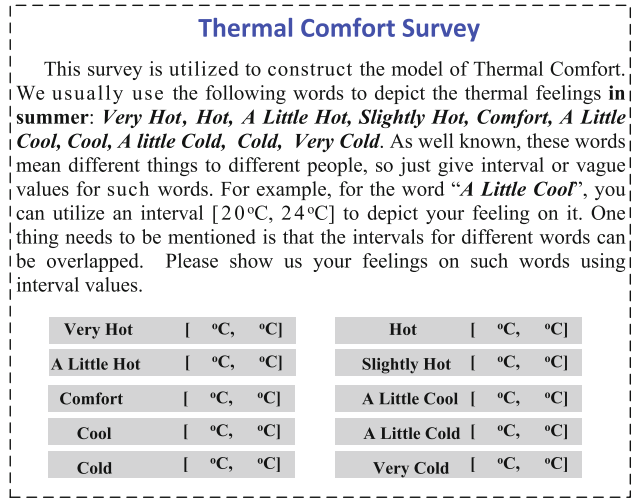
After data pre-processing, if N valid survey intervals are obtained for a word, then we can compute the Eqs. (30)–(44) to derive the corresponding IT2 FS model for such word. The flow diagram is demonstrated in Fig. 4.

3.3 Thermal comfort modeling

In this subsection, interval survey data will be collected and pre-processed, and the thermal comfort words will be modeled using IT2 FSs and the previous method.

3.3.1 The reason for choosing IT2 FSs to model thermal comfort words

The thermal comfort words are very vague terms and different people have different feelings on them. As discussed in Sect. 1, it is reasonable to model the uncertainties contained in such linguistic expressions by the FOU of the IT2 FS. Here, we take one of the thermal comfort words as an example to illustrate this judgement further. If the IT2 FS shown in Fig. 1b is utilized to model the word "*comfort*", then, for a given air temperature t °C, the comfort degree can be obtained as an interval $[\underline{\mu}_{\tilde{A}}(t), \overline{\mu}_{\tilde{A}}(t)]$. This interval can



Thermal Comfort Survey

This survey is utilized to construct the model of Thermal Comfort. We usually use the following words to depict the thermal feelings **in summer**: ***Very Hot, Hot, A Little Hot, Slightly Hot, Comfort, A Little Cool, Cool, A little Cold, Cold, Very Cold***. As well known, these words mean different things to different people, so just give interval or vague values for such words. For example, for the word "***A Little Cool***", you can utilize an interval [20°C, 24°C] to depict your feeling on it. One thing needs to be mentioned is that the intervals for different words can be overlapped. Please show us your feelings on such words using interval values.

Very Hot	[°C,	°C]		Hot	[°C,	°C]
A Little Hot	[°C,	°C]		Slightly Hot	[°C,	°C]
Comfort	[°C,	°C]		A Little Cool	[°C,	°C]
Cool	[°C,	°C]		A Little Cold	[°C,	°C]
Cold	[°C,	°C]		Very Cold	[°C,	°C]

Fig. 5 Sheet for the thermal comfort survey (Li et al. 2013b)

reflect the uncertain feelings of different people, and even of the same person at different times. If the T1 FS shown in Fig. 1a is adopted for the thermal comfort word "*comfort*", then, for the given air temperature t °C, the comfort degree is a crisp value $\mu_A(t)$ which can not model these uncertainties. Therefore, we have the conclusion that modeling thermal comfort words using IT2 FSs is more suitable than using T1 FSs.

3.3.2 Survey data acquisition and pre-processing

In summer, we usually utilize the terms—*Very Hot, Hot, A Little Hot, Slightly Hot, Comfort, A Little Cool, Cool, A little Cold, Cold, Very Cold*—to depict our feelings on the air temperature of the living or working room. For such words, different people have different feelings on them. Furthermore, even for the same person, he/she may have different feelings at different periods. Therefore, it is reasonable to ask the participants to provide their interval feelings on the thermal comfort words for our questionnaire. Following this, the sheet shown in Fig. 5 is designed to survey a group of participants to provide interval feelings on these ten thermal comfort words. One thing to be mentioned is that, for simplicity, only one physical environment factor—air temperature is taken into account in the surveys for the thermal comfort words. In this case, the relative air humidity is assumed to be normal.

The survey was conducted online through the website: http://www.sojump.com/jq/2620213.aspx last summer. In this application, we just model the thermal feelings in summer in north China. Therefore, the regional distribution of the participants was limited to north China, and our colleagues, students, relatives and friends who met this requirement were invited to present their feelings on these words through the website. The invited participants, including males and females, are from 16 years old to about 70 years old. At

Table 1 Statistics of the thermal comfort survey (original data)

Words	Number of preprocessed data	Left STD	Right STD
Very hot	41	2.54	2.48
Hot	41	2.28	2.31
A little hot	41	2.17	2.17
Slightly hot	41	2.14	2.04
Comfort	41	2.40	2.10
A little cool	41	2.70	2.54
Cool	41	2.95	2.75
A little cold	41	3.73	3.21
Cold	41	3.99	3.71
Very cold	41	4.60	4.02

Table 2 Statistics of the thermal comfort survey (pre-processed data)

Words	Number of preprocessed data	Left STD	Right STD
Very hot	21	0.75	0.80
Hot	8	0.53	0
A little hot	11	0.47	0.30
Slightly hot	11	0.52	0.87
Comfort	19	0.85	0.94
A little cool	13	0.95	0.52
Cool	11	0	0.40
A little cold	12	0.49	0.52
Cold	11	0.52	0.83
Very cold	9	0	0.33

last, 54 persons participated this survey, and we got 41 valid survey results, i.e. for each word, we got 41 intervals. The statistics of the original thermal comfort survey data is demonstrated in Table 1, where the left STD means the standard deviation of the left end points of the survey intervals, and the right STD means the standard deviation of the right end points of the survey intervals.

In order to use these intervals to construct the best and most reasonable models for the thermal comfort words, we should first do data pre-processing to delete or discard some bad, invalid or irrational data. In this paper, the data pre-processing method proposed by Wu et al. (2012) is adopted to realize our objective. This method utilizes the following three stages to preprocess data.

The first step is outlier processing (Wu et al. 2012). In this step, Box and Whisker tests are done to discard the survey intervals that do not satisfy the following constraints

$$t_l^i \in [Q_a(.25) - 1.5IQR_a, Q_a(.75) + 1.5IQR_a], \tag{45}$$

$$t_r^i \in [Q_b(.25) - 1.5IQR_b, Q_b(.75) + 1.5IQR_b], \tag{46}$$

$$L^i = t_r^i - t_l^i \in [Q_L(.25) - 1.5IQR_L, Q_L(.75) + 1.5IQR_L]. \tag{47}$$

where $Q_a(Q_b, Q_L)$ and $IQR_a(IQR_b, IQR_L)$ are the quartiles and interquartile ranges for the left (right) endpoints and interval length (Wu et al. 2012), and the test in (47) should be separately taken after the tests in (45) and (46).

After this step, the sample means and standard deviations of the remained left endpoints, right endpoints and interval lengths will be computed respectively as (m_l, σ_l), (m_r, σ_r) and (m_L, σ_L).

The second step is tolerance-limit processing. In this step, we will keep the survey intervals that satisfy

$$t_l^i \in [m_l - k\sigma_l, m_l + k\sigma_l], \tag{48}$$

$$t_r^i \in [m_l - k\sigma_l, m_l + k\sigma_l], \tag{49}$$

$$t_r^i - t_l^i \in [m_L - k\sigma_L, m_L + k\sigma_L]. \tag{50}$$

where k is the tolerance factor that can be found in (Wu et al. 2012), and the test in (50) should be separately taken after the tests in (48) and (49).

After this step, the sample means and standard deviations of the remained left endpoints, right endpoints and interval lengths will also be computed respectively as (m_l, σ_l), (m_r, σ_r) and (m_L, σ_L).

The third step is the reasonable-interval processing (Wu et al. 2012). In this step, we will only keep the survey intervals that satisfy

$$t_l^i < \xi < t_r^i \tag{51}$$

where $\xi = \frac{(m_r\sigma_l^2 - m_l\sigma_r^2)^2 \pm \sigma_l\sigma_r\sqrt{(m_l - m_r)^2 + 2(\sigma_l^2 - \sigma_r^2)\ln\frac{\sigma_l}{\sigma_r}}}{\sigma_l^2 - \sigma_r^2}$, and $\xi \in [2m_l - t_l^i, 2m_r - t_r^i]$.

Through this process, some of the n interval data are discarded and only N intervals are remained (for different words, N is different). The statistics of the pre-processed thermal comfort survey data is demonstrated in Table 2.

From Table 2, we can observe that, after data pre-processing, the remained intervals will be much more compact as the left and right standard deviations are reduced greatly. From our experiments, we found that if the original data are used to model the IT2 FSs, the resulted IT2 FSs will be too fat. The reason for this is that the original intervals scatter too widely before pre-processing.

After data collection and pre-processing, we then can use these survey intervals to model the IT2 FSs by the previously proposed method.

3.3.3 IT2 FS models for thermal comfort words

Using the proposed modeling method, we obtain the IT2 FS model for each of these ten thermal comfort words. Figure 6

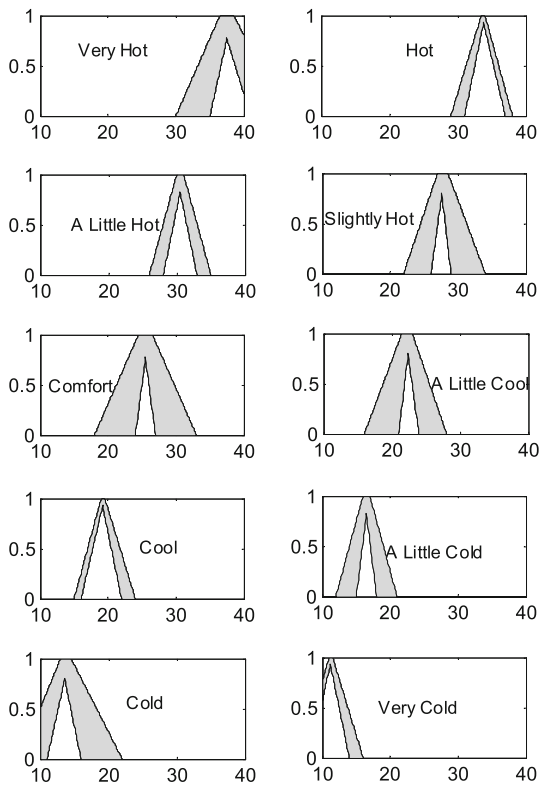


Fig. 6 The constructed IT2 FSs for thermal comfort words

depicts the constructed IT2 FSs for the ten words. And, the parameters of the LMF and UMF of the ten IT2 FSs are listed in Table 3.

From Table 2, we can observe that the standard deviations of the left and right end-points of the survey intervals are usually not equal. This means that the modeled IT2 FSs for these thermal comfort words should always be non-symmetric. Obviously, the results in Fig. 6 are consistent with this fact. From Fig. 6, we can also see that the uncertainties contained in the survey intervals can be reasonably reflected by the constructed IT2 FS models.

4 Energy consumption modeling using evolving type-2 fuzzy system

In recent years, with the rapid development of the economy and the people's living demand, the electricity energy consumption has been rising greatly. Especially, the rapid growth of the air-conditioning load has become the most important reason for the seasonal power shortages. Hence, energy consumption modeling is another important problem in intelligent buildings. The energy consumption is quite related to the operating states of the air conditioners. Using the modeled thermal comfort words, this section tries to present a novel approach to model energy consumption when different air temperatures are set up in a room.

4.1 Parameter optimization of type-2 fuzzy system based energy consumption model

In this study, we just need to use the IT2 FLS to model the energy consumption with respect to the setting air temperature t in the room. Hence, the following type-2 fuzzy rule base is considered:

Rule 1: if t is *Very Cold*, then $y_o(t)$ is c^1,

⋮

Rule 6: if t is *Comfort*, then $y_o(t)$ is c^6,

⋮

Rule 10: if t is *Very Hot*, then $y_o(t)$ is c^{10}

where $y_o(t)$ is the energy consumption with respect to the setting air temperature t.

From Sect. 2.2, the crisp output of this model is

$$y_o(t) = \frac{\sum_{k=1}^{M} c^k[\gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)]}{\sum_{k=1}^{M} \gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)}. \tag{52}$$

where $M = 10$, $\widetilde{A}^k$s represent the thermal comfort words.

Table 3 Parameters of the constructed IT2 FS models

Words	u_1	u_2	u_3	u_4	l_1	l_2	l_3	h
Very hot	30	36.5	38.5	46	35	37.5	41	0.78
Hot	29	33.5	34	38	31	33.8	37	0.93
A little hot	26	30	31	35	28	30.5	33	0.83
Slightly hot	22	27	28.5	34	26	27.6	29	0.8
Comfort	18	24.5	26.5	33	24	25.5	27	0.78
A little cool	16	21.5	23	28	21	22.4	24	0.8
Cool	15	19	19.5	24	16	19.2	22	0.93
A little cold	12	16	17	21	15	16.5	18	0.83
Cold	7	13	14.5	22	11	13.6	16	0.8
Very cold	7	11	11.5	16	8	11.2	14	0.93

Let us denote

$$\omega^k(t) = \frac{\gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)}{\sum_{k=1}^{M} \gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)}. \tag{53}$$

Then, we have

$$y_o(t) = W(t)C. \tag{54}$$

where $C = [c^1, c^2, \cdots, c^M]^{\mathrm{T}}$ is the parameter vector that needs to be optimized, and $W(t) = [\omega^1(t), \cdots, \omega^M(t)]$ is the weighting vector.

From (54), we can observe that the output $y_o(t)$ is linear with respect to the parameters in the parameter vector. Therefore, the consequent weighting parameters can be adjusted by the least square estimation algorithm.

Assume that the setting air temperatures and the observed corresponding energy consumptions are $\{(t_1, s_1), (t_2, s_2), \cdots, (t_T, s_T)\}$. For each air temperature t_i, its weighting vector can be computed as $W(t_i) = [\omega^1(t_i), \cdots, \omega^M(t_i)]$.

In order to render in a simpler form, we denote

$$\mathbf{W} = \begin{bmatrix} W(t_1) \\ \vdots \\ W(t_T) \end{bmatrix} = \begin{bmatrix} \omega^1(t_1), & \cdots, & \omega^M(t_1) \\ \vdots & \vdots & \vdots \\ \omega^1(t_T), & \cdots, & \omega^M(t_T) \end{bmatrix}, \tag{55}$$

$$S = \begin{bmatrix} s_1 \\ \vdots \\ s_T \end{bmatrix}. \tag{56}$$

The sum of the squared error for the training data set can be computed as

$$\sum_{p=1}^{T} e^{(p)} = \sum_{p=1}^{T} (y_o(t_p) - s_p)^2 = ||\mathbf{W}C - S||^2. \tag{57}$$

To minimize this equation, the least square estimation method can be adopted. Using the pseudo-inverse, the least square estimation of C can be written as

$$C^* = (\mathbf{W^T W})^{-1}\mathbf{W^T} S. \tag{58}$$

It has the possibility that $\mathbf{W^T W}$ is singular. Furthermore, since the energy consumption depends on several factors, such as the room type, the air-conditioning system, the outside temperature, the relative humidity, the energy consumption acquisition time, it is necessary or useful to make the constructed energy consumption model have on-line learning ability to deal with the changes of such factors. In other words, the constructed model should be based on observations up to the current time. In this study, to cope with the aforementioned issues, we adopt the iterative least square estimation algorithm (Ljung 1999) as the on-line learning paradigm to compute the estimation of the parameter vector C:

Table 4 Hourly cooling loads at different air temperatures (Chen et al. 2008)

Setting temperature (°C)	Cooling load (W)	Setting temperature (°C)	Cooling load (W)
16	2387.4	25	1581.9
17	2303.8	26	1486.1
18	2219.2	27	1390.0
19	2133.2	28	1288.7
20	2039.4	29	1185.4
21	1950.8	30	1082.7
22	1859.7	31	976.4
23	1769.5	32	867.8
24	1676.5		

$$C(t_{i+1}) = C(t_i) + \alpha(t_{i+1})[y_o(t_{i+1}) - W(t_{i+1})C(t_i)], \tag{59}$$

$$P(t_{i+1}) = \frac{P(t_i) - \alpha(t_{i+1})P(t_i)W^T(t_{i+1})W(t_{i+1})P(t_i)}{\lambda(t_{i+1})}, \tag{60}$$

$$\alpha(t_{i+1}) = \frac{P(t_i)W^T(t_{i+1})}{\lambda(t_{i+1}) + W(t_{i+1})P(t_i)W^T(t_{i+1})}, \tag{61}$$

in which $P(t_i)$ is the covariance matrix. The initial conditions are $C(0) = 0$ and $P(0) = \rho I$, where I is the identity matrix and ρ is a positive lager number. $0 < \lambda \leq 1$ is the forgetting factor.

With this parameter learning algorithm, learning takes place over long periods of time. As new data arrive, the model parameters can be modified, and the model has the ability to evolve over time.

4.2 Energy consumption model for a specific room

The air conditioning energy consumption in summer mainly depends on the cooling load of the air conditioner in a room. In this study, we take the cooling load to reflect the air-conditioning energy consumption index. To test the proposed model, data of the cooling load in a room is adopted from Chen et al. (2008) for simulation and shown in Table 4. When generating the simulation data, the relative humidity and the other environmental factors are assumed to be normal.

Using the simulation data in Table 4, the energy consumption model is built by the proposed method. When all the data come, the final parameters of the energy consumption model is obtained and shown in Table 5, where the best weighting factor is $\gamma = 0.4$. From Table 5, we can observe that the consequent part of the first rule is zero. The reason for this is that this rule is not fired with this training data set. For comparison, a linear regression model is also provided. And, the linear regression model is obtained as: $3930.9 - 94.7 * t$. The root-mean-square error (RMSE) of the final type-2 fuzzy model

Table 5 Final energy consumption model

Antecedent part	Consequent part
Very cold	0
Cold	2662.1
A little cold	2296.6
Cool	2043.2
A little cool	1976.2
Comfort	1496.4
Slightly hot	1321.4
A little hot	867.8
Hot	837.3
Very hot	110.4

and the linear regression model are 12.5506 and 17.6423, respectively. Obviously, the type-2 fuzzy model can present satisfactory performance and performs better than the linear regression model.

As stated previously, there exist several factors that can affect the energy consumption. However, the online observed energy consumption data can reflect these factors to some extent. Due to the learning ability of the constructed model, the model can be modified as long as these factors change.

Once the energy consumption model is built, it then can help the users to choose or find out the appropriate air temperature to save electricity consumption of this specific room.

5 Multiobjective optimization of thermal comfort and energy consumption

From our intuitive sense, comfort and energy-saving are two conflicting objectives. In this section, we will use the optimization method to show how to achieve a compromise between the human comfort and the energy consumption.

5.1 Multiobjective optimization model

For convenience, we take the average of the lower and upper membership grades of "comfort" to depict human feeling on the air temperature. The average of the word "comfort" is shown in Fig. 7. And, the comfort degree can be computed as

$$com(t) = \frac{1}{2}[\underline{\mu}_{comfort}(t) + \overline{\mu}_{comfort}(t)] = \begin{cases} 0.0769t - 1.38, & t \in [18, 24) \\ 0.3369t - 7.62, & t \in [24, 24.5) \\ 0.26t - 5.74, & t \in [24.5, 25.5) \\ -0.26t + 7.52, & t \in [25.5, 26.5) \\ -0.3369t + 9.56, & t \in [26.5, 27) \\ -0.0769t + 2.54, & t \in [27, 33] \end{cases} \tag{62}$$

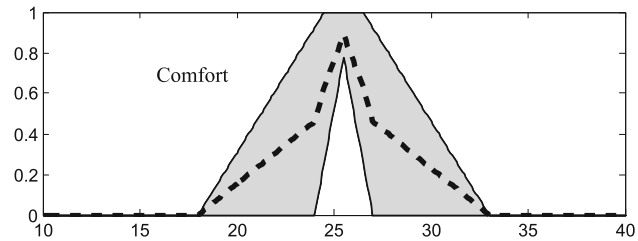


Fig. 7 The average of the word "comfort" (*dashed line*)

To obtain the most satisfactory comfort degree, we need to solve the following problem to find the best air temperature t^*:

$$t^* = \arg \max_{t \in [u_1, u_4]} com(t). \tag{63}$$

where u_1 and u_4 are respectively the left and right endpoints of the UMF of the IT2 FS model for the word "*Comfort*". In our model, $u_1 = 18$ and $u_4 = 33$.

If we do not take into account the energy consumption, then from Fig. 7, it is obvious that the best air temperature $t^* = 25.5\,°\mathrm{C}$. However, generally, in real application, we need to consider the energy consumption.

Based on the energy consumption model in Sect. 4, we can conclude that the best air temperature t^* for the energy consumption of the specific room can be computed as

$$t^* = \arg \min_{t \in [u_1, u_4]} \frac{\sum_{k=1}^{M} c^k[\gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)]}{\sum_{k=1}^{M} \gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)}. \tag{64}$$

In order to obtain the best air temperature for both comfort degree and energy consumption, we have two objectives to trade off. In other words, the best air temperature t^* needs to be achieved by solving the following optimization problem:

$$\max_{t \in [u_1, u_4]} com(t), \tag{65}$$

$$\min_{t \in [u_1, u_4]} \frac{\sum_{k=1}^{M} c^k[\gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)]}{\sum_{k=1}^{M} \gamma \underline{\mu}_{\widetilde{A}^k}(t) + (1-\gamma)\overline{\mu}_{\widetilde{A}^k}(t)}. \tag{66}$$

The relative importance of these two objectives is not generally known, and tradeoffs between these two objectives can not easily be quantified. Thus, this problem can not be transferred to and solved efficiently by the single-objective optimization and the multiobjective optimization strategy is required to solve it.

The multi-objective optimization differs greatly from the single-objective optimization. For nontrivial multiobjective problems, one can not determine a single solution which can simultaneously optimize all the objectives, i.e. there is no uniquely "best" solution (Liu and Pender 2013). Most multi-objective optimization strategies try to obtain a set of optimal solutions, known as Pareto optimal solutions (Censor 1977), to realize the trade-off between conflicting criteria. Once the

Table 6 Pareto optima for the thermal comfort and energy consumption

Pareto optima	Comfort degree	Energy consumption
25.5000	[0.7800, 1.000]	1539.6
32.9979	[0, 0.0003]	743.8
26.9110	[0.0463, 0.9368]	1398.7
26.1944	[0.4189, 1.0000]	1490.4

set of Pareto optima is obtained, we can freely choose one solution out of the possible solutions according to our experience, prior knowledge and other constraints or requirements.

5.2 Results for the specific room

For the specific room studied in the previous section, the detailed parameters or configuration of the multiobjective optimization model in (65) and (66) are as follows: $\gamma = 0.4$ and c^ks are listed in Table 5.

In this study, we adopt the GA method proposed by Deb (2009) to solve this problem. We use the *gamultiobj* function in the global optimization toolbox of Matlab 2012a to get the Pareto optimal solutions. For this specific room, a set of Pareto optima is obtained and shown in Table 6.

From Table 6, we can observe that the user can choose (25.5 °C, 27 °C) as the comfortable air temperature range of this specific room. The energy consumption model for different rooms may be different, and even for the same room, the model may still be evolving due to its learning ability. Therefore, the derived comfortable temperature range changes over time.

5.3 Summarization

The recommended temperature range can then be used to adjust the air conditioner to achieve satisfactory living or working environment. The flowchart of the whole process, from energy consumption modeling to control application, is depicted in Fig. 8.

In this scheme, the thermal comfort model which can reflect the feelings of different people should be determined firstly. This model is built according to the interval data from a general survey. We should mention that the constructed comfort model is not universal, and it is only valid in the region where the survey is conducted, e.g. the north China in this study. Using the IT2 FS comfort model as a basis, the evolving type-2 fuzzy consumption model is then created. The changes of the energy consumption with respect to the room type, the outside temperature, the relative humidity, etc, are overcome by the learning mechanism. Multiobjective optimizations are further taken to realize the tradeoff optimization between the thermal comfort degree and the energy consumption. In the optimization, Pareto optima points can help the user to choose a reasonable temperature range. For different rooms, the Pareto optima may be different, as the energy consumption data and models of different rooms are always not the same.

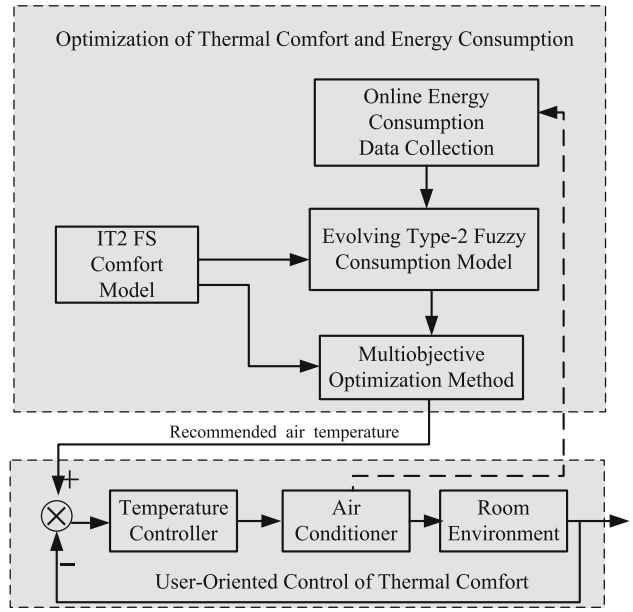


Fig. 8 The flowchart of the whole process for obtaining comfort living or working environment while saving energy

6 Conclusion

In this paper, a type-2 fuzzy method based data-driven strategy for the modeling and optimization of thermal comfort words and energy consumption is presented. We firstly construct the IT2 FS models for the thermal comfort words. The constructed models verify the fact that, when words mean different things to different people, IT2 FSs should be adopted to model the inter-personal and intra-personal uncertainties on these words. The IT2 FS models for comfort words are also utilized as antecedent parts to construct type-2 fuzzy model for predicting the hourly cooling load of air conditioner—one of the energy consumption index in a room. Our study for a specific room also shows that the recommended temperature range can be obtained to realize comfortable and energy-saving living or working environment through multiobjective optimization of the thermal comfort degree and the energy consumption. In the further study, we will utilize these models to design detailed control strategies to realize indoor environment control to provide occupants with a consistently comfortable environment while saving energy.

In this study, we have considered only one factor— air temperature for the thermal comfort words and energy con-

sumption while the relative humidity was assumed to be normal. Sometimes, the relative humidity of the room will have some impact on the thermal feeling and electricity consumption of the air conditioner. Hence, it will be more reasonable to take the relative humidity into account for modeling the thermal comfort words and energy consumption. This will also be one of our future research directions.

Acknowledgments This work is supported by National Natural Science Foundation of China (61105077, 61074149, 61273149, and 61273326), and the Excellent Young and Middle-Aged Scientist Award Grant of Shandong Province of China (BS2012DX026).

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