LMI-Based Synthesis of String-Stable Controller for Cooperative Adaptive Cruise Control

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***Abstract*— Controller synthesis is a challenging problem in cooperative adaptive cruise control (CACC). Especially the requirement of string stability makes it even harder to choose appropriate control parameters. This paper applies a time-domain definition to string stability and converts the problem to the $\mathcal{H}_\infty$ control of a time-delay system. Based on the proposed control structure, the $\mathcal{H}_\infty$ norm and stability criteria of CACC are satisfied by a set of constraints in terms of a Lyapunov-Krasovskii functional candidate. These constraints are further reduced to linear matrix inequalities so that feasible solutions can be easily and efficiently computed. Simulations on an identified model validate the performance of our method in both frequency and time domains.**

***Index Terms*— Cooperative adaptive cruise control, string stability, time-delay system, $\mathcal{H}_\infty$ control, linear matrix inequality.**

I. Introduction

TRAFFIC congestion has been a challenging task for most countries, and tremendous efforts have been made to investigate new automotive and transportation technologies [1]–[3]. Cooperative adaptive cruise control (CACC) [4], [5] as an extension of adaptive cruise control (ACC) [6]–[9], has shown great potential in reducing traffic congestion and saving fuel consumption. It connects multiple vehicles as a platoon and passes as a string. Not only measuring inter-vehicle distances and velocity gaps just like ACC, CACC also uses vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications to transmit upstream driving data to followers [10]. In this way, CACC is able to respond traffic changes in advance and further shorten inter-vehicle distances without compromising safety. If no preceding vehicles or wireless signals are available to a single vehicle, the rest ACC can still function well. Research investigations have shown that the rise of CACC market penetration can significantly increase traffic capacity and fuel-saving effect [11], [12].

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One main objective of CACC is to keep vehicle following its predecessor speed at a safe distance. A common spacing policy widely used in literature is the constant headway-time policy. The desired inter-vehicle distance is equal to the sum of two parts: a constant standstill distance and a velocity-dependent distance that is the product of current velocity and a constant headway time. To track front driving, CACC uses two sources of signals to determine own driving commands: local measurement and transmitted data. The former are relative distance and relative velocity, measured by a local detector like radar or lidar mounted at the front bumper. The latter are other vehicle driving signals like control input or acceleration, transmitted by wireless devices. In [13], authors design a linear quadratic regulator based on the whole platoon dynamics, and obtain a centralized controller taking all vehicle states as input. In [14], a decentralized optimal control is presented and $\mathcal{H}_2$-/$\mathcal{H}_\infty$-performance criteria are simultaneously considered. A linear model predictive control is used in [15] and the objective is to minimize fuel consumption under safety constraints. In [16], the problem is formulated as an l_∞-norm robust model predictive control, such that the minimum safety distance is not violated. In [17], adaptive optimal control is applied to heterogeneous platoon and the optimal controller is learned based on online data. Unfortunately, the above work pays less attention to the delay effect existing in actuator and communication, which may severely deteriorate control performance.

Another important issue for CACC is string stability (SS). It concerns the attenuation of signals propagated along vehicles. If the platoon is not string stable, any disturbance from the upstream is amplified and eventually leads to abrupt halt or collision accident in the tail. One way to check SS is to analyze signals in frequency domain. A transfer function shows flow of signals between preceding and following vehicles. If its magnitude in the frequency domain is always limited by one, the platoon is said string stable. In [18], authors consider two spacing policies: the constant spacing policy and the velocity-dependent spacing policy. They find that only the latter one is able to ensure SS in the existence of time delays, and CACC allows a shorter inter-vehicle distance than ACC. In [19], a networked control system is presented to investigate the effect of sampling frequency, zero-order-hold, and constant network delays. The SS is analyzed with the discrete-time $\mathcal{Z}$-transform. Real traffic design and testing of CACC is presented in [20].

To find satisfactory CACC, the above works tune controller parameters by trial and error, which is quite inefficient.

To overcome this limitation, some efforts have been made to synthesize string-stable CACC controllers in a computational way. Reference [21] first tries to use a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formulation to achieve reduction of spacing errors and attenuation of disturbance. The controller assumes full information on all vehicles, and is solved based on a set of linear matrix inequalities. But authors fail in considering time delays. In [22], a time-domain SS criterion is developed. The attenuation of signal flow is represented by the $\mathcal{L}_p$ norm, and a connection between time-domain signals and a frequency-domain transfer function is established. Based on that, [23] proposes an $\mathcal{H}_\infty$ optimal controller approach for $\mathcal{L}_2$ SS. Authors use a third-order Padè approximation to deal with time delays. The objective becomes finding a controller that renders the $\mathcal{H}_\infty$ norm of transformed linear model no greater than one. However, the final controller is high-order in state space, which can cause considerable difficulty in practical implementation.

When taking into account actuator and communication delays, CACC is actually a time-delay system (TDS), whose dynamics is described by delay differential equations. Researchers in the field of control theory have investigated TDS in many aspects such as stability, optimality, $\mathcal{H}_\infty$, robustness, and control synthesis. Surveys and recent development of TDS can be found in [24], [25], and successful applications are available in [26], [27]. In the literature, most time-domain or state-space analysis of TDS is based on Lyapunov-Krasovskii theorem or Lyapunov-Razumikhin theorem. The key is to find a Lyapunov-like function that satisfies certain conditions. If dynamics is linear or partly linear, constraints can be expressed in the form of linear matrix inequalities (LMIs). As a convex optimization, LMIs can be easily solved by interior-point methods, and a number of toolboxes have been developed to formulate LMIs. Motivated by that, we seek to propose an LMI-based method that can synthesize string-stable CACC controllers in a computationally efficient way.

In this paper, the CACC problem is first formulated as a state-space $\mathcal{H}_\infty$ control of a TDS. By introducing the Lyapunov-Krasovskii function, the control system is proved to achieve $\mathcal{H}_\infty$ norm and stability objectives under certain conditions. Then, these conditions are further reduced to LMIs after specifying a quadratic Lyapunov-Krasovskii function and borrowing the idea of solving a cone complementary problem. The feasible solution corresponds to the string-stable CACC controller. Compared to other works, our method can significantly reduce the difficulty of designing CACC. Simulations on an identified vehicle model validate the SS in both frequency and time domains, and a practical driving dataset further tests the performance.

The remainder of this paper is organized as follows. Section II gives the preliminary of CACC and string stability definitions. Section III presents our control structure and transforms the SS problem to the state-space $\mathcal{H}_\infty$ control. The main theorem is presented in Section IV where the string-stable CACC controller is synthesized by an LMI-based algorithm. Section V conducts simulations to validate the effectiveness. In the end we give the discussion and conclusion.

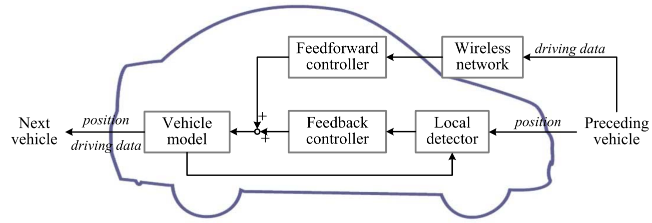

Fig. 1. CACC typical structure.

II. Preliminary

Fig. 1 shows the general configuration of a vehicle equipped with CACC. Based on the low-level closed-loop actuation of the throttle and brake system, the vehicle longitude dynamics can be expressed by a transfer function $\mathcal{G}(s)$ between vehicle position p_i and desired acceleration input u_i

$$\mathcal{G}(s) = \frac{\mathcal{P}_i(s)}{\mathcal{U}_i(s)} = \frac{1}{s^2(\tau s + 1)} e^{-l_1 s} \tag{1}$$

where s is the complex variable for Laplace transform, τ is the dynamic parameter, and l_1 is the actuator and internal delay. Subscript i is the vehicle index in platoon and $i = 1$ represents the leading vehicle. In this paper, we only investigate the homogeneous platoon, that is to say all vehicles have the same τ and l_1.

A distance detector measures the inter-vehicle distance d_i between the holder and the preceding vehicle

$$d_i(t) = p_{i-1}(t) - p_i(t) - L_{i-1}$$

where t represents current time, p_i and p_{i-1} are two vehicle positions, and L_{i-1} is the predecessor vehicle length. The desired relative distance $\bar{d}_i$ is determined by the constant headway time policy

$$\bar{d}_i(t) = r + hv_i(t). \tag{2}$$

r is called the standstill distance, h is the headway time, and v_i is the current speed of vehicle i. Based on the distance error

$$\begin{aligned} \Delta d_i(t) &= d_i(t) - \bar{d}_i(t) \\ &= p_{i-1}(t) - p_i(t) - L_{i-1} - r - hv_i(t) \end{aligned}$$

ACC adopts a feedback controller to track the predecessor driving.

When multiple vehicles form a CACC platoon, an additional wireless network is established between them to transmit front vehicle accelerations or control signals backward to the tail. These additional signals are used in a feedforward way, and are combined with the distance-error feedback to determine control commands. One common CACC topology is the follower-predecessor structure, in which each vehicle only connects to its nearest predecessor. The delay effect in wireless communication is approximated by an average l_0 with a delay module $\mathcal{L}(s) = e^{-l_0 s}$.

Definition 1 (SS in frequency domain [18]): Given a platoon of vehicles that drive in string, the transfer function of signals between the leader and the i-th follower is denoted as

$$\mathcal{SS}^*_{i,1}(s) = \frac{\mathcal{Q}_i(s)}{\mathcal{Q}_1(s)}$$

where $\mathcal{Q}_i(s)$ is the Laplace transform of signal q_i, which could be position p_i, velocity v_i, acceleration a_i, or control input u_i. If the following condition holds for all i

$$|\mathcal{SS}^*_{i,1}(j\omega)| \leq 1, \quad \forall\omega \tag{3}$$

the system is called string stable. One notes that $\mathcal{SS}^*_{i,1}(s) = \prod_{j=2}^{i} \frac{\mathcal{Q}_j(s)}{\mathcal{Q}_{j-1}(s)}$, so a conservative SS criterion is to check transfer functions between adjacent vehicles

$$|\mathcal{SS}_{i,i-1}(j\omega)| = \left|\frac{\mathcal{Q}_i(j\omega)}{\mathcal{Q}_{i-1}(j\omega)}\right| \leq 1, \quad \forall\omega. \tag{4}$$

Considering a homogeneous platoon, adjacent vehicles have transfer functions

$$\frac{\mathcal{P}_i(s)}{\mathcal{P}_{i-1}(s)} = \frac{\mathcal{V}_i(s)}{\mathcal{V}_{i-1}(s)} = \frac{\mathcal{A}_i(s)}{\mathcal{A}_{i-1}(s)} = \frac{\mathcal{U}_i(s)}{\mathcal{U}_{i-1}(s)}, \quad \forall i > 1.$$

The first two equalities directly follow Laplace transform, and the last one is due to the monotony of vehicle dynamics in (1). Distance errors for vehicles except the leading one have

$$\frac{\Delta\mathcal{D}_i(s)}{\Delta\mathcal{D}_{i-1}(s)} = \frac{\mathcal{P}_i(s)}{\mathcal{P}_{i-1}(s)} \frac{\frac{\mathcal{P}_{i-1}(s)}{\mathcal{P}_i(s)} - 1 - sh}{\frac{\mathcal{P}_{i-2}(s)}{\mathcal{P}_{i-1}(s)} - 1 - sh} = \frac{\mathcal{P}_i(s)}{\mathcal{P}_{i-1}(s)}, \quad \forall i > 2.$$

The second equality is due to the homogeneity of platoon. Based on the above analysis, we can simply analyze one signal and achieve all other string stability for homogeneous CACC. In the literature, the frequency-domain criterion is widely used to analyze CACC SS. By tuning the feedback and feedforward control parameters, the magnitude of transfer function can be limited no greater than 1, indicating signals are not amplified along vehicles and the string is stable. However, the tuning is basically conducted by trial and error, posing a degree of difficulty to the design process.

It is noteworthy that headway time h plays an important role in CACC. Generally speaking, the larger headway is chosen, the easier to maintain the platoon string stable. According to the spacing policy given in (2), the desired inter-vehicle distance is proportional to h. Longer distance gives vehicles more reaction time to follow their predecessors, but also leads to low road capacity. The best CACC is to find the lowest headway time and the corresponding controller such that the traffic throughput is increased and the string is still stable.

Another SS criterion is defined in time domain by [22]. Note that (3) or (4) is also called the $\mathcal{H}_\infty$ norm. According to the linear control theory given by [28, Theorem 5.4], the $\mathcal{H}_\infty$ norm of a transfer function is induced by the $\mathcal{L}_2$ norm on input and output. As a consequence, the condition that $\|\mathcal{SS}^*_{i,1}(j\omega)\|_{\mathcal{H}_\infty} \leq 1$ is equivalent to that the $\mathcal{L}_2$ norm between input q_1 and output q_i is no greater than 1. The time-domain SS criterion is given as follows.

Definition 2 ($\mathcal{L}_2$ string stability [22]): Suppose a platoon of vehicles drive in a string and the system is steady at the equilibrium. The response between q_1 and q_i is called $\mathcal{L}_2$-gain ≤ 1 if starting from the initial steady state, for arbitrary input $q_1(t) \in L_2[0,\infty)$, the L_2 norm of response $q_i(t)$ is bounded by $\|q_i(t)\|_{\mathcal{L}_2} \leq \|q_1(t)\|_{\mathcal{L}_2}$. If the inequality holds for all i, the CACC system is called $\mathcal{L}_2$ string stable. A conservative condition is by checking if the response between q_i and q_{i-1} has $\mathcal{L}_2$-gain ≤ 1 for all i.

With the time-domain SS definition, it is possible to synthesize CACC controllers in state space. One major difficulty in designing is the existence of time delays. In the next section, a new CACC framework is developed so that the system can be described by linear time-delay dynamics.

III. CACC Framework

According to the definitions of inter-vehicle distance and spacing policy, the time derivative of distance error has

$$\Delta\dot{d}_i(t) = \Delta v_i(t) - h a_i(t)$$

where $\Delta v_i(t) = v_{i-1}(t) - v_i(t)$ is the velocity gap between the two adjacent vehicles. Furthermore, the time derivative of Δv_i has

$$\Delta\dot{v}_i(t) = a_{i-1}(t) - a_i(t)$$

and according to the dynamics in (1), we have

$$\begin{aligned} \dot{a}_i(t) &= \frac{1}{\tau}[u_i(t-l_1) - a_i(t)] \\ \dot{a}_{i-1}(t) &= \frac{1}{\tau}[u_{i-1}(t-l_1) - a_{i-1}(t)]. \end{aligned}$$

After integrating the above equations together, the CACC system can be described by

$$\begin{bmatrix} \Delta\dot{d}_i \\ \Delta\dot{v}_i \\ \dot{a}_i \\ \dot{a}_{i-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -h & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -\frac{1}{\tau} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \Delta d_i \\ \Delta v_i \\ a_i \\ a_{i-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \\ 0 \end{bmatrix} u_i(t-l_1) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix} u_{i-1}(t-l_1).$$

Hereinafter, time variable t is omitted if one can infer according to the context. Denote control input $u = u_i$ and external disturbance $w = u_{i-1}$. The above established dynamics can now be rewritten in an interconnected form

$$\begin{cases} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + Bu(t-l_1) \\ \dot{x}_2 = A_{22}x_2 + Cw(t-l_1) \end{cases} \tag{5}$$

where the subsystem states $x_1 = [\Delta d_i, \Delta v_i, a_i]^T$ and $x_2 = a_{i-1}$, and dynamics matrices have

$$A_{11} = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix},$$
$$A_{22} = -\frac{1}{\tau}, \quad C = \frac{1}{\tau}.$$

x_1 represents the local measurement of vehicle i, and x_2 is the transmitted data from the predecessor. A linear controller is used to determine the control input u based on x_1 and x_2, and due to transmission delay, it is defined by

$$u(t) = K_1 \ x_1(t) + K_2 \ x_2(t-l_0)$$

where $K_1 = [\kappa_1, \kappa_2, \kappa_3]$ is the feedback gain and $K_2 = \kappa_4$ is the feedforward gain. If K_2 is specified to 0, the system becomes an ACC system.

According to the analysis in previous section, we focus on the SS of control input signals for homogeneous CACC. The SS of position, velocity, acceleration, and distance error can be obtained by the same CACC. Based on the conservative condition given in Definition 2, the system is string stable if (5) has $\|u(t)\|_{\mathcal{L}_2} \leq \|w(t)\|_{\mathcal{L}_2}$ for arbitrary $w(t) \in L_2[0,\infty)$ from initial $x(0) = 0$. In addition, if there is no disturbance, i.e. $w = 0$, the dynamics is desired to be asymptotically stable at zero state. Therefore, the problem now becomes synthesizing K_1 and K_2 for (5) such that it has $\mathcal{L}_2$-gain ≤ 1 with input w and output u, and is asymptotically stable at zero state when $w(t) = 0$. It is equivalent to the $\mathcal{H}_\infty$ control of a time-delay system. After inserting the controller model, the dynamics becomes

$$\begin{cases} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + BK_1x_1(t-l_1) + BK_2x_2(t-l_2) \\ \dot{x}_2 = A_{22}x_2 + C\omega(t-l_1) \end{cases} \tag{6}$$

where $l_2 = l_1 + l_0$, and the output is

$$z(t) = K_1\ x_1(t) + K_2\ x_2(t-l_0).$$

Based on the Laplace transform, the flow of signals in CACC is illustrated in Fig. 2. The controller input is composed of distance error Δd_i, velocity gap Δv_i, own acceleration a_i, and preceding acceleration a_{i-1}. The first three variables are measured by local detectors including radar/lidar, speedometer, and accelerometer, and the last variable is wireless transmitted from predecessor accelerometer. The signal flow between u_i and u_{i-1} can now be described by the transfer function

$$\mathcal{T}(s) = \frac{\mathcal{U}_i(s)}{\mathcal{U}_{i-1}(s)} = \frac{e^{-l_1 s}(e^{-l_0 s}\kappa_4 s^2 + \kappa_2 s + \kappa_1)}{\tau s^3 + (1 - e^{-l_1 s}\kappa_3)s^2 + e^{-l_1 s}(\kappa_1 h + \kappa_2)s + e^{-l_1 s}\kappa_1}. \tag{7}$$

Note that our CACC uses a linear control model in the architecture, which is quite simple compared to other structures. In [18] and [22], the actual control commands are delivered to actuator after a filter which is composed of the inverse transfer functions of spacing policy and vehicle model. In [23], authors design a high-order state-space controller, which may cause difficulty and inaccuracy in implementation. Our control model is direct and simple, but still needs to concern SS.

Without loss of generality, denote n as the state dimension of (5). Suppose the system starts from an initial zero state, i.e.

$$x_1(\theta_1) = 0, \quad -l_1 \leq \theta_1 \leq 0,$$
$$x_2(\theta_2) = 0, \quad -l_2 \leq \theta_2 \leq 0$$

and there is no disturbance before $t = 0$, i.e. $\omega(t) = 0, t < 0$. Denote x_t as the translation operator acting on the trajectory:

$$x_t = x(t+\theta), \quad -l_2 \leq \theta \leq 0$$

since $l_2 \geq l_1$. Define the continuous norm $\|\cdot\|_c$ to be

$$\|x_t\|_c = \sup_{-l_2 \leq \theta \leq 0} \|x(t+\theta)\|.$$

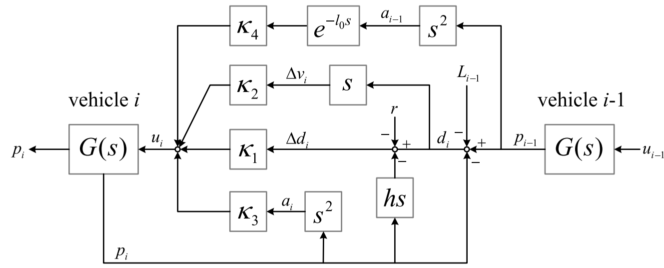

Fig. 2. CACC control structure.

$\mathcal{C}$ is a set $\mathcal{C}[-l_2, 0]$ of real valued continuous functions over $[-l_2, 0]$. The following theorem extends the time-delay $\mathcal{H}_\infty$ theory to the string stability of CACC system.

Theorem 1: Consider the system given by (6). Suppose $v_1, v_2, v_3 : \mathbb{R}^+ \to \mathbb{R}^+$ are continuously positive-definite functions. Define a continuous differentiable functional $V(x_t) : \mathbb{R}^n \times \mathcal{C} \to \mathbb{R}$ that has $V(0_t) = 0$ and

$$v_1(\|x(t)\|) \leq V(x_t) \leq v_2(\|x_t\|_c). \tag{8}$$

If for any nonzero disturbance, the time derivative of V satisfies

$$\|z(t-l_1)\|^2 - \|\omega(t-l_1)\|^2 + \dot{V}(x_t) \leq 0 \tag{9}$$

and when $\omega(t) = 0$, $\dot{V}$ has

$$\dot{V}(x_t) \leq -v_3(\|x(t)\|) \tag{10}$$

then the system has $\mathcal{L}_2$-gain ≤ 1 and is asymptotically stable when $\omega(t) = 0$.

Proof: When external disturbance exists, define the system performance as

$$J_T = \int_{t=0}^{T} \left(\|z(t)\|^2 - \|\omega(t)\|^2\right) dt.$$

It can be rewritten as

$$\begin{aligned} J_T &= \int_{t=0}^{T+l_1} \left(\|z(t-l_1)\|^2 - \|\omega(t-l_1)\|^2\right) t \\ &= \int_{t=0}^{T+l_1} \left(\|z(t-l_1)\|^2 - \|\omega(t-l_1)\|^2 + \dot{V}(x_t)\right) dt \\ &\quad -V(x_{T+l_1}) \end{aligned}$$

since we assume $x(\theta) = 0$ and $\omega(\theta) = 0$ for $-l_2 \leq \theta \leq 0$, and at $t = 0$ it has $V(x_0) = 0$. By the condition given in (9), the following inequality holds for all $T > 0$

$$\int_{t=0}^{T} \left(\|z(t)\|^2 - \|\omega(t)\|^2\right) dt \leq 0.$$

Therefore, for any $\omega \in L_2[0,\infty)$ and starting from zero initial state, the system has

$$\|z(t)\|_{\mathcal{L}_2} \leq \|\omega(t)\|_{\mathcal{L}_2}.$$

When there is no disturbance, (8) and (10) are actually conditions of Lyapunov-Krasovskii stability theorem for time-delay systems [29]. The proof is complete. ■

According to Theorem 1, the core of CACC controller synthesis is to find V and K_1, K_2 such that conditions in (8)–(10) are satisfied. In the next, we introduce a quadratic Lapunov-Krasovskii functional candidate [30] for V and use linear matrix inequality (LMI) method to search the proper K_1 and K_2.

IV. LMI-Based Synthesis of CACC Controller

LMI, as a special case of convex optimization, has been fully developed in the past few years [31]. Many problems in control theory can be expressed by LMIs. A number of LMI tools are developed to easily formulate and solve LMI problems. Inspired by the work in [30], we use LMIs to synthesize CACC controllers to ensure string stability. The theorem is reestablished to fit the specified dynamics (6) of our problem.

Theorem 2: Consider the time-delay system (6) with given time delays l_0, l_1, and $l_2 = l_0 + l_1$. For positive constants ε_1, ε_2, ε_3, ε_4, if there exist symmetric matrices $L_j > 0$, $R_j \geq 0$, $W_j > 0$, $Y_j \geq 0$, $\overline{Y}_j \geq 0$, and any appropriately dimensioned matrices V_j, M_j, $\overline{M}_j$ with $j = 1, 2$, such that the following inequalities hold

$$\begin{bmatrix} \Psi & \sqrt{l_1}\psi_2^T & \sqrt{l_2}\psi_3^T & \psi_5^T \\ * & -W_1 & 0 & 0 \\ * & * & -W_2 & 0 \\ * & * & * & -I \end{bmatrix} \leq 0 \tag{11}$$

$$\begin{bmatrix} \Omega & \sqrt{l_1}\Omega_2^T & \sqrt{l_2}\Omega_3^T & \Omega_5^T & \Omega_6^T \\ * & -W_1 & 0 & 0 & 0 \\ * & * & -W_2 & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} \leq 0 \tag{12}$$

$$\begin{bmatrix} Y_j & M_j \\ * & L_j W_j^{-1} L_j \end{bmatrix} \geq 0, \quad j = 1, 2 \tag{13}$$

$$\begin{bmatrix} \overline{Y}_j & \overline{M}_j \\ * & L_j W_j^{-1} L_j \end{bmatrix} \geq 0, \quad j = 1, 2 \tag{14}$$

where $\Psi = \psi_1 + \psi_4 + \psi_4^T + l_1\ Y_1 + l_2\ Y_2$, $\Omega = \Omega_1 + \Omega_4 + \Omega_4^T + l_1\overline{Y}_1 + l_2\overline{Y}_2$, and ψ_1 to ψ_5, Ω_1 to Ω_6, shown at the bottom of this page.

Then the controller $u(t) = K_1\ x_1(t) + K_2\ x_2(t - l_0)$ with gains $K_1 = V_1\ L_1^{-1}$ and $K_2 = V_2\ L_2^{-1}$ is an $\mathcal{H}_\infty$ controller that renders the system $\mathcal{L}_2$-gain ≤ 1 for any $\omega(t) \in L_2[0, \infty)$ and asymptotically stable when $\omega(t) = 0$.

Proof: To prove the theorem, the following quadratic Lyapunov-Krasovskii functional candidate is adopted

$$V(x_t) = \sum_{j=1}^{2} \Big(x_j^T(t) P_j x_j(t) + \int_{t-l_j}^{t} x_j^T(s) Q_j x_j(s) ds + \int_{-l_j}^{0} \int_{t+\theta}^{t} \dot{x}_j^T(s) Z_j \dot{x}_j(s) ds d\theta \Big)$$

where $P_j > 0$, $Q_j \geq 0$, and $Z_j > 0$. First consider the case when disturbance exists. From Newton-Leibniz formula, formulate the following equalities for any matrices N_j, $j = 1, 2$ with appropriate dimensions

$$0 = 2\zeta^T(t) N_j \left[x_j(t) - x_j(t - l_j) - \int_{t-l_j}^{t} \dot{x}_j(s) ds \right] \tag{15}$$

where $\zeta(t) = [x_1^T(t), x_2^T(t), x_1^T(t-l_1), x_2^T(t-l_2), \omega^T(t-l_1)]^T$. In addition, formulate equalities for any matrices $X_j \geq 0$, $j = 1, 2$

$$0 = l_j \zeta^T(t) X_j \zeta(t) - \int_{t-l_j}^{t} \zeta^T(t) X_j \zeta^T(t) ds. \tag{16}$$

Calculate the time derivative of $V(x_t)$ along the dynamics (6). After adding $\|z(t - l_1)\|^2 - \|\omega(t - l_1)\|^2$ and the right-hand

$$\psi_1 = \begin{bmatrix} A_{11}L_1 + L_1 A_{11}^T + R_1 & A_{12}L_2 & BV_1 & BV_2 & 0 \\ * & A_{22}L_2 + L_2 A_{22}^T + R_2 & 0 & 0 & C \\ * & * & -R_1 & 0 & 0 \\ * & * & * & -R_2 & 0 \\ * & * & * & * & -I \end{bmatrix}$$

$$\psi_2 = [A_{11}L_1, A_{12}L_2, BV_1, BV_2, 0]$$

$$\psi_3 = [0, A_{22}L_2, 0, 0, C]$$

$$\psi_4 = [M_1, M_2, -M_1, -M_2, 0]$$

$$\psi_5 = [0, 0, V_1, V_2, 0]$$

$$\Omega_1 = \begin{bmatrix} A_{11}L_1 + L_1 A_{11}^T + R_1 & A_{12}L_2 & BV_1 & BV_2 \\ * & A_{22}L_2 + L_2 A_{22}^T + R_2 & 0 & 0 \\ * & * & -R_1 & 0 \\ * & * & * & -R_2 \end{bmatrix}$$

$$\Omega_2 = [A_{11}L_1, A_{12}L_2, BV_1, BV_2]$$

$$\Omega_3 = [0, A_{22}L_2, 0, 0]$$

$$\Omega_4 = [\overline{M}_1, \overline{M}_2, -\overline{M}_1, -\overline{M}_2]$$

$$\Omega_5 = [\mathrm{diag}(\sqrt{\varepsilon_1}, \sqrt{\varepsilon_2}, \sqrt{\varepsilon_3})L_1, 0, 0, 0]$$

$$\Omega_6 = [0, \sqrt{\varepsilon_4}L_2, 0, 0].$$

sides of (15)–(16), we have

$$\begin{aligned}&\dot{V}(x_t) + \|z(t-l_1)\|^2 - \|\omega(t-l_1)\|^2\\ &= \zeta^T(t)\begin{bmatrix}\phi_1 + l_1\phi_2^T Z_1\phi_2 + l_2\phi_3^T Z_2\phi_3 +\\ \phi_4 + \phi_4^T + \phi_5^T\phi_5 + l_1X_1 + l_2X_2\end{bmatrix}\zeta(t)\\ &\quad -\sum_{j=1}^{2}\int_{t-l_j}^{t}\eta_j^T(t,s)\begin{bmatrix}X_j & N_j\\ * & Z_j\end{bmatrix}\eta_j(t,s)ds\end{aligned}$$

where $\eta_1(t,s) = [\zeta^T(t), \dot{x}_1^T(s)]^T$, $\eta_2(t,s) = [\zeta^T(t), \dot{x}_2^T(s)]^T$, and ϕ_1–ϕ_5, shown at the bottom of this page.

According to the analysis in Theorem 1, to obtain the $\mathcal{H}_\infty$ control, one needs to have $\dot{V}(x_t)+\|z(t-l_1)\|^2-\|\omega(t-l_1)\|^2 \le 0$, which is fulfilled if

$$\begin{bmatrix}\phi_1+\phi_4+\phi_4^T+l_1X_1+l_2X_2 & \sqrt{l_1}\phi_2^T Z_1 & \sqrt{l_2}\phi_3^T Z_2 & \phi_5^T\\ * & -Z_1 & 0 & 0\\ * & * & -Z_2 & 0\\ * & * & * & -I\end{bmatrix} \le 0 \tag{17}$$

$$\begin{bmatrix}X_j & N_j\\ * & Z_j\end{bmatrix} \ge 0. \tag{18}$$

The first inequality is obtained by the Schur complement. Define

$$\begin{aligned}\Pi &= \mathrm{diag}\left(P_1^{-1}, P_2^{-1}, P_1^{-1}, P_2^{-1}, I\right)\\ \Theta &= \mathrm{diag}\left(\Pi, Z_1^{-1}, Z_2^{-1}, I\right).\end{aligned}$$

Pre- and post-multiply the left-hand side of (17) by Θ. Pre- and post-multiply the left-hand side of (18) by $\mathrm{diag}(\Pi, P_j^{-1})$. Make the following changes in variables

$$\begin{aligned}&L_j = P_j^{-1},\quad R_j = L_jQ_jL_j,\ V_j = K_jL_j,\ Y_j = \Pi X_j\Pi,\\ &M_j = \Pi N_jL_j,\quad W_j = Z_j^{-1}.\end{aligned}$$

Then conditions in (17) and (18) are replaced by (11) and (13).

Similar for the case when $\omega(t) = 0$, introduce the following equalities with matrices $\overline{N}_j$ and $\overline{X}_j \ge 0,\ j = 1, 2$

$$0 = 2\mu^T(t)\overline{N}_j\left[x_j(t) - x_j(t-l_j) - \int_{t-l_j}^{t}\dot{x}_j(s)ds\right] \tag{19}$$

$$0 = l_j\mu^T(t)\overline{X}_j\mu(t) - \int_{t-l_j}^{t}\mu^T(t)\overline{X}_j\mu^T(t)ds \tag{20}$$

where $\mu(t) = [x_1^T(t), x_2^T(t), x_1^T(t-l_1), x_2^T(t-l_2)]^T$. Now along the non-disturbance dynamics of (6), calculate the time derivative of V and add the right-hand sides of (19)–(20) as well as $x_1^T\mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)x_1 + \varepsilon_4\ x_2^Tx_2$

$$\begin{aligned}&\dot{V}(x_t) + x_1^T\mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)x_1 + \varepsilon_4\ x_2^Tx_2\\ &= \mu^T(t)\begin{bmatrix}\lambda_1 + l_1\lambda_2^T Z_1\lambda_2 + l_2\lambda_3^T Z_2\lambda_3 + \lambda_4 +\\ \lambda_4^T + \lambda_5^T\lambda_5 + \lambda_6^T\lambda_6 + l_1\overline{X}_1 + l_2\overline{X}_2\end{bmatrix}\mu(t)\\ &\quad -\sum_{j=1}^{2}\int_{t-l_j}^{t}\rho_j^T(t,s)\begin{bmatrix}\overline{X}_j & \overline{N}_j\\ * & Z_j\end{bmatrix}\rho_j(t,s)ds\end{aligned}$$

where $\rho_1(t,s) = [\mu^T(t), \dot{x}_1^T(s)]^T$, $\rho_2(t,s) = [\mu^T(t), \dot{x}_2^T(s)]^T$, and λ_1–λ_6, shown at the bottom of this page.

If $\dot{V}(x_t)+x_1^T\mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)x_1+\varepsilon_4\,x_2^Tx_2 \le 0$, by Theorem 1 we know the dynamics is stabilizable. This can be fulfilled by the constraints

$$\begin{bmatrix}\lambda_1+\lambda_4+\lambda_4^T+l_1\overline{X}_1+l_2\overline{X}_2 & \sqrt{l_1}\lambda_2^T Z_1 & \sqrt{l_2}\lambda_3^T Z_2 & \lambda_5^T & \lambda_6^T\\ * & -Z_1 & 0 & 0 & 0\\ * & * & -Z_2 & 0 & 0\\ * & * & * & -I & 0\\ * & * & * & * & -I\end{bmatrix} \le 0 \tag{21}$$

$$\begin{bmatrix}\overline{X}_j & \overline{N}_j\\ * & Z_j\end{bmatrix} \ge 0. \tag{22}$$

Define multiplier matrices

$$\begin{aligned}\Xi &= \mathrm{diag}\left(P_1^{-1}, P_2^{-1}, P_1^{-1}, P_2^{-1}\right)\\ \Gamma &= \mathrm{diag}\left(\Xi, Z_1^{-1}, Z_2^{-1}, I, I\right).\end{aligned}$$

$$\phi_1 = \begin{bmatrix}P_1A_{11} + A_{11}^TP_1 + Q_1 & P_1A_{12} & P_1BK_1 & P_1BK_2 & 0\\ * & P_2A_{22} + A_{22}^TP_2 + Q_2 & 0 & 0 & P_2C\\ * & * & -Q_1 & 0 & 0\\ * & * & * & -Q_2 & 0\\ * & * & * & * & -I\end{bmatrix}$$

$$\phi_2 = [A_{11}, A_{12}, BK_1, BK_2, 0],\quad \phi_3 = [0, A_{22}, 0, 0, C]$$

$$\phi_4 = [N_1, N_2, -N_1, -N_2, 0],\quad \phi_5 = [0, 0, K_1, K_2, 0].$$

$$\lambda_1 = \begin{bmatrix}P_1A_{11} + A_{11}^TP_1 + Q_1 & P_1A_{12} & P_1BK_1 & P_1BK_2\\ * & P_2A_{22} + A_{22}^TP_2 + Q_2 & 0 & 0\\ * & * & -Q_1 & 0\\ * & * & * & -Q_2\end{bmatrix}$$

$$\lambda_2 = [A_{11}, A_{12}, BK_1, BK_2],\quad \lambda_3 = [0, A_{22}, 0, 0]$$

$$\lambda_4 = [\overline{N}_1, \overline{N}_2, -\overline{N}_1, -\overline{N}_2]$$

$$\lambda_5 = [\mathrm{diag}(\sqrt{\varepsilon_1}, \sqrt{\varepsilon_2}, \sqrt{\varepsilon_3}), 0, 0, 0],\quad \lambda_6 = [0, \sqrt{\varepsilon_4}, 0, 0].$$

Pre- and post-multiply (21) by Γ; pre- and post-multiply the constraint in (22) by $\text{diag}(\Xi, L_j)$; define the change variables

$$\overline{M}_j = \Xi \overline{N}_j L_j, \quad \overline{Y}_1 = \Xi \overline{X}_j \Xi.$$

Thus the conditions (21) and (22) are replaced by (12) and (14). The proof is complete. ■

The feasible solutions K_1 and K_2 define the CACC controller $u(t) = K_1 x_1(t) + K_2\, x_2(t - l_0)$ for vehicles such that the platoon is string stable. Note that conditions in (13)–(14) are not LMIs due to terms $L_1\ W_1^{-1} L_1$ and $L_2\ W_2^{-1} L_2$. Now we borrow the idea of solving a cone complementary problem in [32] to deal with that. Define new variables S_1 and S_2 such that $L_1\ W_1^{-1} L_1 \geq S_1$ and $L_2\ W_2^{-1} L_2 \geq S_2$. The conditions in (13)–(14) are fulfilled if

$$\begin{bmatrix} Y_j & M_j \\ * & S_j \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{Y}_j & \overline{M}_j \\ * & S_j \end{bmatrix} \geq 0.$$

By the Schur complement, $L_j W_j^{-1} L_j \geq S_j$ can be rewritten as

$$\begin{bmatrix} S_j^{-1} & L_j^{-1} \\ * & W_j^{-1} \end{bmatrix} \geq 0.$$

Defining new variables T_j, elements in the above inequalities can be replaced by $T_j = S_j^{-1}$, $P_j = L_j^{-1}$, $Z_j = W_j^{-1}$. The original non-convex problem (11)–(14) is now converted to a cone complementary problem with nonlinear minimization objective and LMI conditions as follows

$$\min \ \text{Trace}\left(\sum_{j=1}^{2} S_j T_j + L_j P_j + W_j Z_j \right) \tag{23}$$

$$s.t. \quad \left.\begin{array}{l} (11), (12), \text{and} \\ \begin{bmatrix} Y_j & M_j \\ * & S_j \end{bmatrix} \geq 0, \begin{bmatrix} \overline{Y}_j & \overline{M}_j \\ * & S_j \end{bmatrix} \geq 0, \\ \begin{bmatrix} T_j & P_j \\ * & Z_j \end{bmatrix} \geq 0, \begin{bmatrix} S_j & I \\ * & T_j \end{bmatrix} \geq 0, \\ \begin{bmatrix} L_j & I \\ * & P_j \end{bmatrix} \geq 0, \begin{bmatrix} W_j & I \\ * & Z_j \end{bmatrix} \geq 0. \end{array}\right\} \tag{24}$$

If the solution of the above problem is $3n$, that is $\text{Trace}\left(\sum_{j=1}^{2} S_j T_j + L_j P_j + W_j Z_j\right) = 3n$, then the controller with gains $K_1 = V_1\ L_1^{-1}$ and $K_2 = V_2\ L_2^{-1}$ are the desired CACC controller according to Theorem 2. Unfortunately, it is still difficult to find the global optimal solution to the nonlinear problem. In the literature, linearization method [25], [30] is widely used to address this issue. Based on that, an iterative algorithm is designed here to find a feasible solution that satisfies Theorem 2.

Algorithm 1: For given time delays $l_0, l_1, l_2 = l_0 + l_1$, and headway time h.

Step 1. Find a feasible set $(S_{j0}, T_{j0}, L_{j0}, P_{j0}, W_{j0}, Z_{j0})$, $j = 1, 2$ to constraints of (24), and let $k = 0$. If no feasible solution exists, then exit.

Step 2. Solve the following LMI problem for the variables $L_j, R_j, V_j, W_j, S_j, T_j, P_j, Z_j, Y_j, M_j, \overline{Y}_j, \overline{M}_j$, $j = 1, 2$

$$\min \ \text{Trace}\Big(\sum_{j=1}^{2} S_{jk} T_j + S_j T_{jk} + L_{jk} P_j + L_j P_{jk} + W_{jk} Z_j + W_j Z_{jk} \Big)$$

$$s.t. \qquad (24) \text{ is satisfied}$$

Denote the feasible solution as $S_{j,k+1} = S_j$, $T_{j,k+1} = T_j$, $L_{j,k+1} = L_j$, $P_{j,k+1} = P_j$, $W_{j,k+1} = W_j$, $Z_{j,k+1} = Z_j$, $j = 1, 2$.

Step 3. For the solution in Step 2, if constraints (13)–(14) are satisfied, the $\mathcal{H}_\infty$ controller is found and output K_1 and K_2. If such solution is not found within a specified number of iterations, say $k_{\max}$, then exit. Otherwise, let $k = k + 1$ and return to Step 2.

Since LMIs problem is a special case of convex optimization, its infeasible conditions are equivalent to infeasible conditions of convex optimization. More details on this subject are available in [33]. Back to our algorithm, if no solution is found within $k_{\max}$, it is probably because l_0 and l_1 are too large. One solution is to increase the value of h, i.e. to enlarge inter-vehicle distance, so that the following vehicle has more reaction time in response to its predecessor movement.

In the LMI problem of Algorithm 1, if we specify $K_2 = 0$ and only let K_1 be variable, it becomes an ACC controller synthesis process. The controller takes only local measurements as input and has $u(t) = K_1\ x_1(t)$.

From the proof of Theorem 2, the time derivative of Lyapunov function along dynamics trajectory without disturbance is bounded by $\dot{V}(x_t) \leq -x_1^T \text{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3) x_1 - \varepsilon_4\ x_2^T x_2$ with parameters ε_*. Any positive ε_* can ensure the string stability, but the larger parameters the inequality holds for, the better stability the system potentially has. One can tune the values to achieve their desired performance. In practice, vehicle is preferred to accurately follow its predecessor at the desired distance, so ε_1 is usually set larger than the rest ε_*.

V. Simulation Study

This section tests the performance of our method on the model that is present in [34]. The authors have used least-squares method to identify the dynamics of The Toyota Prius III Executive, and the model is expressed by (5) with parameters $\tau = 0.1$, $l_1 = 0.2$. The wireless communication delay is estimated equal to 150 ms, i.e. $l_0 = 0.15$. In literature such as [18], [20], [22], [23], it is quite common to design CACC controllers on identified models, and then test on real vehicle platoons. This process is beneficial to reduce design difficulties. With the model, we now apply our CACC structure and controller synthesis algorithm, and try to find the minimum headway h that makes the platoon string stable. It first starts from $h = 0$ and checks if Algorithm 1 can find a feasible solution within $k_{\max} = 50$ iterations. If not, add an increase $\Delta h = 0.1$ to h and repeat the algorithm for the new headway. The process continues until the minimum feasible $h_{\min}$ is found. The stability parameters choose $\varepsilon_1 = 1$ and $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 10^{-4}$ to emphasize the distance tracking performance.

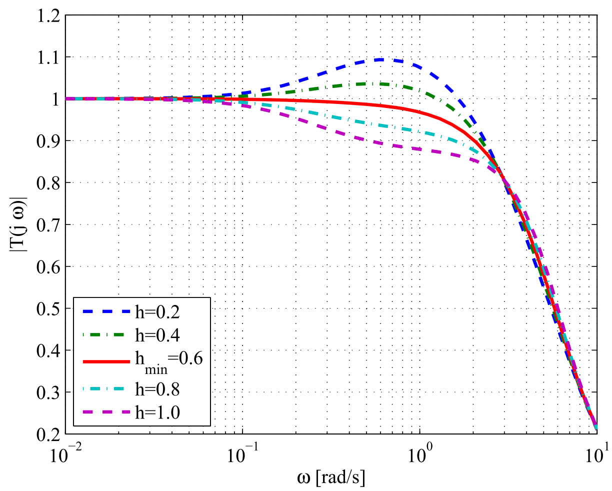

Fig. 3. Magnitude of transfer function $\mathcal{T}(s) = \frac{\mathcal{U}_i(s)}{\mathcal{U}_{i-1}(s)}$ with different headway time h.

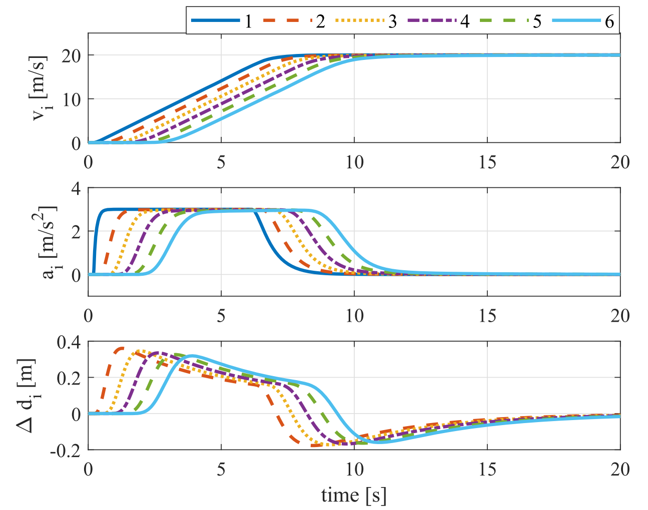

Fig. 4. Velocity, acceleration, and distance error responses of string-stable CACC platoon with $h_{\min} = 0.6$.

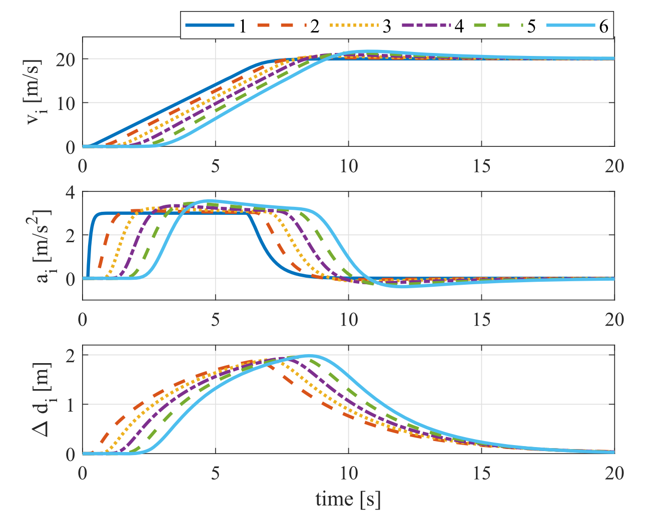

Fig. 5. Velocity, acceleration, and distance error responses of string-unstable CACC platoon with $h = 0.4$.

By the proposed method, the minimum $h_{\min}$ is searched equal to 0.6 and the output CACC controller has $K_1 = [0.5690, 2.0172, -0.2584]^T$ and $K_2 = 0.0311$. Fig. 3 gives the Bode magnitude plot of the control input transfer function according to (7). The $\mathcal{H}_\infty$ norm is less than 1, indicating the whole platoon is string stable. For comparison, $|\mathcal{T}(j\omega)|$ of other h under the same K_1 and K_2 are plotted together in the same figure. The system is no longer string stable when h is smaller than $h_{\min}$. Due to the impact of state delay and input delay, the following vehicle is not able to respond promptly to predecessor behaviors if inter-vehicle distance is very small. In [34], a manually selected CACC controller is designed for the same model and authors found that the minimum headway time to ensure string stability is $h_{\min} = 0.67$. Compared to their result, our method is able to autonomously synthesize a CACC controller with smaller headway time.

To give a time-domain illustration of string stability, we simulate a platoon of six vehicles with the synthesized CACC controller and the minimum feasible $h_{\min}$. The platoon is initially set at rest and the leader is guided by a constant acceleration until reaching speed 20 m/s. Velocity, acceleration, and distance error responses of the whole platoon are depicted in Fig. 4. For comparison, we repeat the simulation with the same setting but change the headway time to $h = 0.4$, and plot results in Fig. 5. Signals in Fig. 4 are gradually attenuated in propagation, while those in Fig. 5 are amplified. It clearly illustrates the string stability of the case $h_{\min}$ and the lack thereof for $h = 0.4$. If the platoon size is further increased, amplified signals can cause serious problems for the whole system. Vehicles at the tail are commanded by much larger acceleration or deceleration, which may eventually lead to abrupt halt or traffic accident. In practice, the CACC platoon is required to use a headway time that is larger or at least equal to $h_{\min}$.

Urban Dynamometer Driving Schedule, short for UDDS, is a driving test designed by the United States Environmental Protection Agency to represent urban driving conditions [35]. The above platoon under the CACC controller and with the minimum feasible $h_{\min}$ is tested following UUDS driving cycle. Fig. 6 presents part of speed trajectories of vehicles. Under the effect of string stability, the tailer has a smoother curve of movement than the leader. It increases comfort for passengers and reduces unnecessary acceleration that benefits fuel savings. Trajectories of distance errors and velocity gaps are depicted in Fig. 7. These signals are limited to small values and are obviously attenuated along the platoon. When more vehicles are incorporated in CACC, traffic condition can be significantly improved, and road capacity is increased with the much shorter inter-vehicle distance.

To show the synthesis capability, we now consider different pairs of delays l_0, l_1 and use the algorithm to find the corresponding minimum feasible headway times. The results are depicted in Fig. 8. The tendency is that longer delays generally require larger headway times to maintain SS. But there are also exceptions. The main reason is that the criteria

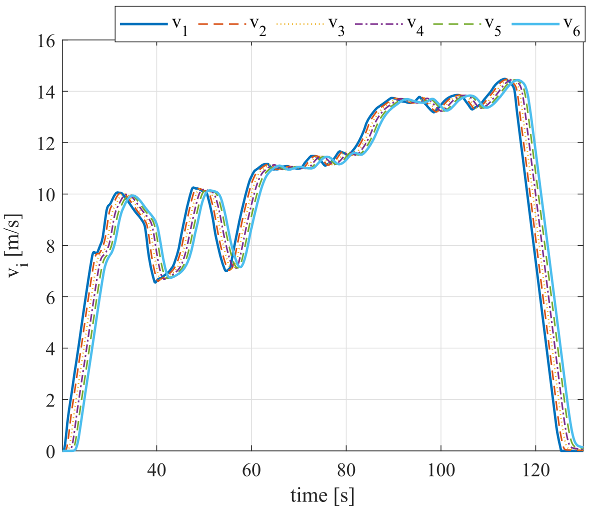

Fig. 6. Speed trajectories of vehicles following UDDS driving cycle.

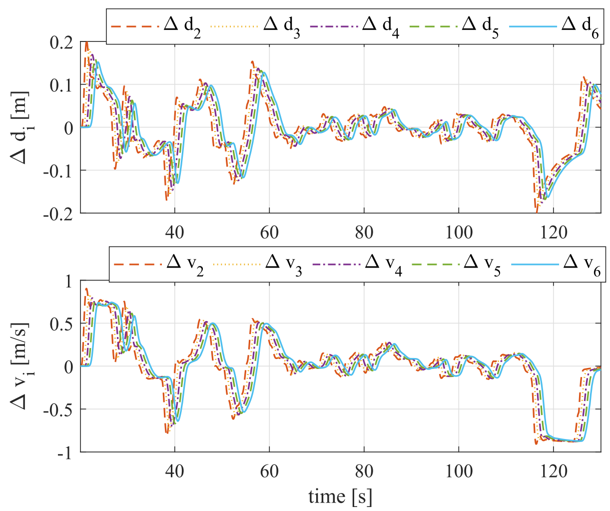

Fig. 7. Trajectories of distance errors and velocity gaps following UDDS driving cycle.

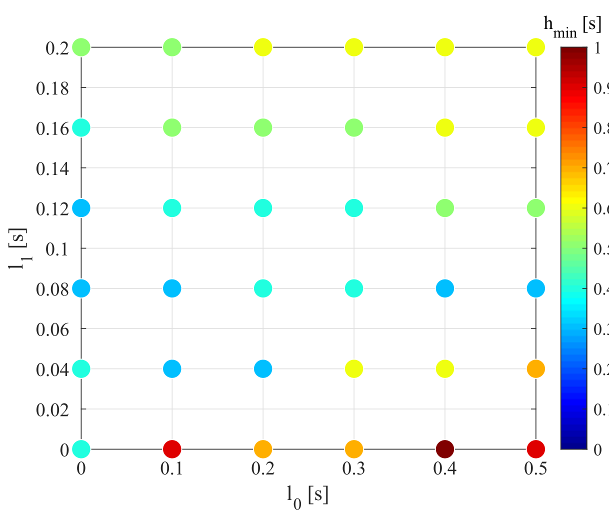

Fig. 8. Minimum feasible headway $h_{\min}$ for different delays l_0, l_1.

in Theorem 2 are only sufficient conditions. There is still space to further optimize obtained headway times and controllers. The advantage of our algorithm is to provide a convenient and computationally efficient way to design string-stable controllers.

VI. Conclusion

Most CACC controller parameters in previous literature are manually selected. In this paper we propose an LMI-based synthesis method that can autonomously search parameters for the time-delay CACC system to ensure SS. This state-space approach is theoretically conservative in comparison to frequency observation method, because of the relaxation of LMIs and the quadratic Lyapunov-Krasovskii functional candidate. But in simulations it is observed that our method achieves smaller feasible headway time than other manually designed CACC. Its computational efficiency supported by powerful LMI solvers makes it suitable for the synthesis of string-stable CACC controllers.

Note that in this paper we only test the method on an identified model, not on a real vehicle platoon. When applied to real systems, a variety of issues need to be carefully addressed, such as measurement errors, packet loss in communication, mechanical wear, and so forth. These issues may seriously deteriorate CACC performance. Besides, in many cases a group of heterogeneous vehicles with different dynamics may form a platoon and the homogeneous CACC controller is no longer applicable. Our future work is to consider these complicated issues and test CACC on a real vehicle platoon.

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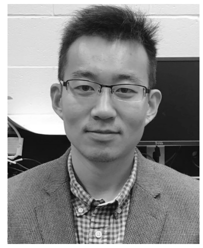

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