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Iterative Selection of GOB Poles in the Context of System Modeling

Anis Khouaja[1,2] Hassani Messaoud[2]

[1]High Institute of Applied Science and Technology, Sousse 4003, Tunisia

[2]National Engineering School of Monastir (ENIM), Monastir 5019, Tunisia

Abstract: This paper is concerned with the problem of system identification using expansions on generalized orthonormal bases (GOB). Three algorithms are proposed to optimize the poles of such a basis. The first two algorithms determine a GOB with optimal real poles while the third one determines a GOB with optimal real and complex poles. These algorithms are based on the estimation of the dominant mode associated with a residual signal obtained by iteratively filtering the output of the process to be modelled. These algorithms are iterative and based on the quadratic error between the linear process output and the GOB based model output. They present the advantage to be very simple to implement. No numerical optimization technique is needed, and in consequence there is no problem of local minima as is the case for other algorithms in the literature. The convergence of the proposed algorithms is proved by demonstrating that the modeling quadratic error between the process output and the GOB based model is decreasing at each iteration of the algorithm. The performance of the proposed pole selection algorithms are based on the quadratic error criteria and illustrated by means of simulation results.

Keywords: Generalized orthonormal bases (GOB), Laguerre functions, Kautz functions, pole estimation, modelling, identification.

1 Introduction

The use of series expansions for signal and system representation goes back to the classical work of Wiener and Lee on network synthesis during the 1930s. Recently, there has been renewed in the use of orthonormal functions, particularly Laguerre functions and Kautz functions, for system modeling. The Laguerre and Kautz functions that form a complete orthonormal set respectively on $L_2[0,+\infty]$ and $L_2([0,+\infty]\times[0,+\infty])$, can be used to represent stable transfer functions.

The representation of a system on a basis of orthogonal functions has the advantage of writing linearly with respect to its Fourier coefficients. Once the basis parameters (poles) are fixed, this gives this mode of representation interesting properties for system identification.

Any transfer function $G(z)$ of a sampled linear system can be expressed using a truncated expansion on a generalized orthonormal bases (GOB) as follows:

$$G(z)=\sum_{n=0}^{i} g_n B_n(z,\xi). \tag{1}$$

Two cases are possible:

1) The basis is characterized by a real pole. We use a set of Laguerre functions[1]

$$G(z)=\sum_{n=0}^{i} g_n L_n(z,\xi) \tag{2}$$

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where

$$L_n(z)=\frac{\sqrt{1-\xi^2}}{z-\xi}\left(\frac{1-\xi z}{z-\xi}\right)^n \tag{3}$$

ξ is called the Laguerre pole and $\{g_n\}$, $n=1,\cdots,i$, are called the Fourier coefficients.

2) The basis is characterized by two complex conjugate poles. Then, we use a set of Kautz functions[2]

$$G(z)=\sum_{n=0}^{i} g'_{2n+1}\Psi_{2n+1}(z,b,c)+g'_{2n}\Psi_{2n}(z,b,c) \tag{4}$$

where

$$\Psi_{2n+1}(z,b,c)=\frac{\sqrt{1-c^2}(z-b)z}{z^2+b(c-1)z-c}\times \left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^n \tag{5}$$

$$\Psi_{2n}(z,b,c)=\frac{\sqrt{(1-c^2)(1-b^2)}z}{z^2+b(c-1)z-c}\times \left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^n \tag{6}$$

b and c are the parameters depending on the two Kautz complex conjugate poles and g'_i are the Fourier coefficients.

In practice, truncated Laguerre or Kautz series expansion is used. For a given system, the truncation error depends on the number of used filters and the chosen Laguerre or Kautz poles. For a fixed number of filters, there exist optimal Laguerre or optimal Kautz parameters b and c that minimize the truncation error. However, the use of these orthonormal functions bases is not adequate for modeling high order systems characterized by several modes[3, 4]. For example, Heuberger et al.[5] proposed a new model by decomposing an autoregressive exogenous (ARX) model on Laguerre orthonormal bases.

Another way suitable for the several mode linear systems is to use an expansion on a generalized orthonormal bases, which allows to significantly decrease the truncation order and consequently the identification complexity[6]. When using a GOB, we have to optimize several poles. Den Brinker and Belt[7] developed an algorithm to recursively determine GOB real poles. Malti et al.[8] used the gradient algorithm and Gauss-Newton algorithm to minimize the output quadratic error. These algorithms guarantee only convergence to local minima and if the step size of the gradient algorithm is not well chosen, the convergence can be slow, oscillating or not guaranteed. Another exhaustive algorithm to determine real GOB poles was proposed in [9]. Mbarek et al.[10] have studied the topic and estimated the optimal GOB poles by solving a set of nonlinear equations. These algorithms cited above were also developed in the nonlinear case especially to optimize the Volterra kernels expansions on Laguerre or Kautz bases functions[11, 12] or GOB functions[13−16]. Other techniques were also developed in the nonlinear case as the crosscumulant based approaches[17] and the multimodel approach[18].

In this work, we are interested in the GOB poles optimization using three new algorithms. The first two algorithms concern the GOB with optimal real poles while the third one optimizes both real and complex GOB poles. These algorithms present the advantage to be very simple to implement. No numerical optimization technique is needed, and in consequence there is no problem of local minima as is the case for certain other techniques[8]. These algorithms are iterative and are composed of two steps at each iteration:

1) Determination of the dominant pole of a residual signal.

2) Generation of the residual signals using an iterative filtering.

The convergence of the proposed algorithms is proved in this work. It is based on the estimation of the dominant mode associated with a residual signal obtained by iteratively filtering the output of the process to be modeled. A quadratic error criterion is used to show the performance of the proposed pole selection algorithms.

The organization of the paper is as follows. In Section 2, the principle of linear model expansion on GOB functions is recalled and the new iterative optimization algorithms are proposed. The first two algorithms are to iteratively determine a GOB with real poles and the third one determines a GOB with real and complex poles. In Section 3, we demonstrate the convergence of the proposed GOB pole estimation algorithm. In Section 4, the performance of the proposed algorithms is illustrated by means of simulation results before concluding in Section 5.

2 GOB pole estimation

Ninness and Gustafsson[19] showed that the development of an absolutely summable function using a GOB can be written as follows:

$$G(z)=\sum_{n=0}^{i} g_n B_n(z) \tag{7}$$

with

$$B_0(z) = \frac{\sqrt{1-|\xi_0|^2}}{z-\xi_0} \tag{8}$$

$$B_n(z) = \frac{\sqrt{1-|\xi_n|^2}}{z-\xi_n}\prod_{k=0}^{n-1}\left(\frac{1-\xi_k^* z}{z-\xi_k}\right). \tag{9}$$

Such a GOB depends on several poles. In this section, we propose three algorithms for determining the real and complex poles of a GOB. All these poles are determined from the output measurements of a given system to be modeled.

2.1 GOB real poles estimation

A real pole GOB based model is caratcterized by N real poles $\{\xi_i\}_{i=1,\cdots,N}$, all in $[-1,1]$. To estimate these poles two algorithms are proposed in this section. These algorithms are iterative and estimate the N real poles of the GOB based proposed model. They are are based on a filtering process of the measured outputs. The first alogorithm uses the Fu and Dumont technique to estimate a single real Laguerre pole and the second one minimizes the quadratic error between the outputs of the system and the GOB based model.

2.1.1 Laguerre poles optimization

The bases of Laguerre functions are typically used to model linear systems with single or dominant dynamic. They are characterized by a single real pole $\xi \in [-1, 1]$, as shown in Fig. 1.

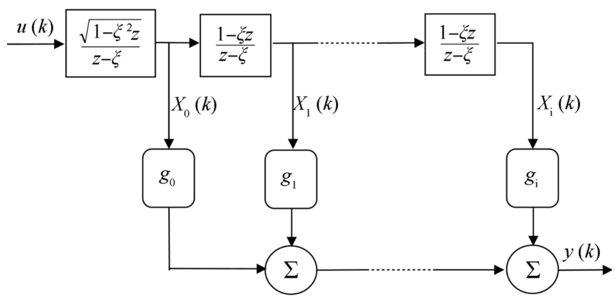

Fig. 1 Laguerre based model

Fu and Dumont′s algorithm[20, 21] allows to find analytically the optimal Laguerre pole by minimizing the following cost function:

$$J=\sum_{k=0}^{\infty} kh^2(k) \tag{10}$$

where $\{h(k)\}$ represents the impulse response of the system to be modeled. If the system is of first order, the Laguerre pole coincides with the true pole. When the system is of higher order, the Laguerre pole approaches the dominant mode of the system.

The optimal Laguerre pole is given by

$$\xi_{opt} = \frac{2M_1 - 1 - M_2}{2M_1 - 1 + \sqrt{4M_1M_2 - M_2^2 - 2M_2}} \tag{11}$$

with

$$\|h\|^2 = \sum_{k=0}^{\infty} h^2(k) \tag{12}$$

$$M_1 = \frac{1}{\|h\|^2} \sum_{k=0}^{\infty} kh^2(k) \tag{13}$$

$$M_2 = \frac{1}{\|h\|^2} \sum_{k=0}^{\infty} k\left[h(k+1) - h(k)\right]^2. \tag{14}$$

Another algorithm to analytically determine the optimal Laguerre pole was proposed in [22] It uses the system impulse response and minimizes another cost function.

2.1.2 Algorithms 1 and 2

Both proposed algorithms to determine the poles of a GOB are iterative. At each step $i(i = 1, \cdots, N)$, the procedure is composed of two substeps:

1) Determination of the dominant mode $\xi_{i,opt}$ of the residual output $\nu_{i-1}(k)$ obtained at step $(i-1)$, where $\nu_0(k)$ is:

a) The impulse response of the system to be modeled for the first algorithm.

b) The system output in response to any input in the case of the second algorithm.

2) Filtering of the residual output $\nu_{i-1}(k)$ by means of the following filter $F_i(q^{-1}) = 1 - \xi_{i,opt}q^{-1}$ to get the residual output $\nu_i(k)$.

This procedure is repeated until the residual signal becomes negligible. Once the poles of the GOB have been determined, the Fourier coefficients of the system impulse response expansion on this GOB are estimated by applying the least squares (LS) technique.

The two algorithms differ from each other in the way to determine the estimated dominant poles. With the first algorithm, the poles determination is carried out by applying the Fu and Dumont′s algorithm cited in the previous paragraph. This algorithm uses the system impulse response. It is illustrated in Fig. 2.

In the case of the second algorithm:

1) The model order is initialized to N using the system a-priori information.

2) Then, the parameter interval $[-1; 1]$ associated with stable discrete time poles is discretized to lead to P possible values $p_j\,(j = 1, \cdots, P)$. This operation is illustrated in Fig 3.

3) At the $(i+1)$-th step of the pole estimation algorithm, we first, decrement the order of the Laguerre model. Then, the optimal pole $\xi_{i+1,opt}$ is chosen as the value p_j that minimizes the quadratic error $QE(p_j)$ defined by (14) between the residual output $\nu_i(k)$ and the output $\hat{\nu}_i(k, p_j) = \sum_{\tau=1}^{L} f_i(\tau, p_j)\hat{\nu}_{i-1}(k-\tau)$ of the P^2 different Laguerre models with order $(N - i)$, each one being characterized by $\xi_i = p_j\,(j = 1, \cdots, P)$, where $f_i(\tau, p_j)$ is the inverse z-transform of $F_i(q^{-1}, p_j)$.

$$QE(p_j) = \frac{1}{L} \sum_{k=1}^{L} (\nu_i(k) - \hat{\nu}_i(k, p_j))^2 \tag{15}$$

where L represents the number of measured outputs used for the identification. Thus, we have

$$\xi_{opt} = \mathrm{Arg}\min_{p_j}(QE(p_j));\ j = 1, \cdots, P. \tag{16}$$

This second algorithm is illustrated in Fig. 3.

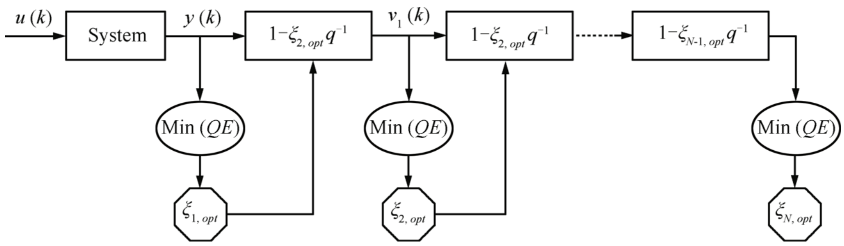

Fig. 2 Determination of the system modes by the first algorithm

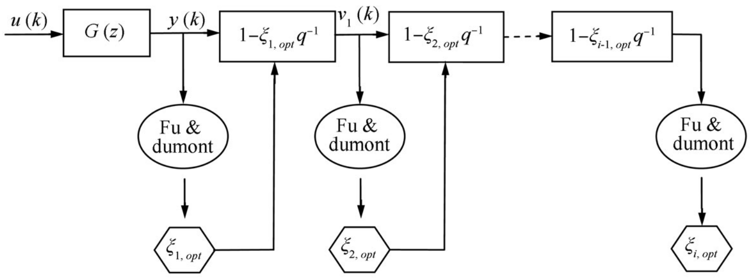

Fig. 3 Determination of the system modes by the second algorithm

2.2 GOB complex poles estimation

In the general case, a GOB based model is characterized by N real and complex poles $\{\xi_i\}_{i=1,\cdots,N}$. To estimate these poles, another algorithm is proposed in this section. This algorithm is also iterative and estimates at each iteration, either a real pole $\{\xi_i\}$ or two complex conjugate poles characterized by a couple of parameters $\{(b_i, c_i)\}$. It is based on a filtering process of the measured outputs and minimizes the quadratic error between the outputs of the system and the GOB based model.

2.2.1 Kautz complex poles optimization

The bases of Kautz functions are typically used to model linear oscillating systems. They are characterized by two complex conjugate poles ξ and ξ^* as shown in Fig. 4.

Wahlberg[2] proposed a new definition of the Kautz functions by defining a couple of parameters $(p, q) \in [-1; 1]^2$ associated with the Kautz complex poles ξ and ξ^* as follows:

$$p = \frac{\xi + \xi^*}{1 + \xi\xi^*} \tag{17}$$

$$q = -\xi\xi^*. \tag{18}$$

Kautz functions are defined by

$$\overline{\overline{\Psi}}_{2p}(z, p_j, q_m) = \frac{\sqrt{1 - q_m^2}(z - p_j)}{z^2 + p_j(q_m - 1)z - q_m} \times \left[\frac{-q_m z^2 + p_j(q_m - 1)z + 1}{z^2 + p_j(q_m - 1)z - q_m}\right]^{p-1} \tag{19}$$

$$\overline{\overline{\Psi}}_{2p+1}(z, p_j, q_m) = \frac{\sqrt{(1 - p_j^2)(1 - q_m^2)}}{z^2 + p_j(q_m - 1)z - q_m} \times \left[\frac{-q_m z^2 + p_j(q_m - 1)z + 1}{z^2 + p_j(q_m - 1)z - q_m}\right]^{p-1}. \tag{20}$$

2.2.2 Algorithm 3

Algorithm 3 can be extended to determine complex poles for GOBs using the definition of Kautz functions bases given by Wahlberg[2].

We proceed as with the second algorithm by discretizing the two-dimensional space $[-1; 1]^2$ associated with the two parameters p_j and q_m, $j, m = 1, \cdots, P$, that characterize a pair of conjugate complex poles ξ_l and ξ_l^*.

At the i-th step of the identification algorithm, the residual output $\nu_{i-1}(k)$ is filtered by

1) a first-order filter given by

$$F_i(q^{-1}, \xi_{i,opt}) = 1 - \xi_{i,opt} q^{-1} \tag{21}$$

in the case of a real pole, or

2) a second-order filter given by

$$H_i(q^{-1}, b_{i,opt}, c_{i,opt}) = 1 + b_{i,opt}(c_{i,opt} - 1)q^{-1} - c_{i,opt} q^{-2} \tag{22}$$

for conjugate complex poles, where $b_{i,opt}$ and $c_{i,opt}$ are defined in (22).

To deliver the residual output $\nu_i(k)$, the selected filter is the one which gives the least quadratic error.

At the (i+1)-th step, we first decrease by one or by two the order of the model (one if the filter used in the i-th iteration is of first order or two if it is of second order). Then, we proceed as with the second algorithm to determine either the optimal real pole $\xi_{i+1,opt}$ or the optimal parameters $b_{i+1,opt}$ and $c_{i+1,opt}$ which are chosen as the values p_j and q_m that minimize the quadratic error $QE(p_j, q_m)$ between the residual output $\nu_i(k)$ and the output $\hat{\nu}_i(k, p_j, q_m) = \sum_{\tau=1}^{L} h_i(\tau, p_j, q_m)\hat{\nu}_{i-1}(k - \tau)$ of the P^2 different Kautz models, each one being characterized by the pair of parameters $(b_{i+1}, c_{i+1}) = (p_j, q_m)$, $j, m = 1, \cdots, P$, as illustrated in Fig 5, where $h_i(\tau, p_j, q_m)$ is the inverse z-transform of $H_i(q^{-1}, p_j, q_m)$. Thus, we have

$$(b_{i+1,opt},\ c_{i+1,opt}) = \operatorname{Arg\,min}_{(p_j,q_m)}(QE(p_j, q_m))$$
$$j, m = 1, \cdots, P. \tag{23}$$

This algorithm is illustrated in Fig. 5.

3 Convergence of GOB pole estimation algorithms

All the algorithms presented in the previous section are iterative. We present here the proof of their convergence.

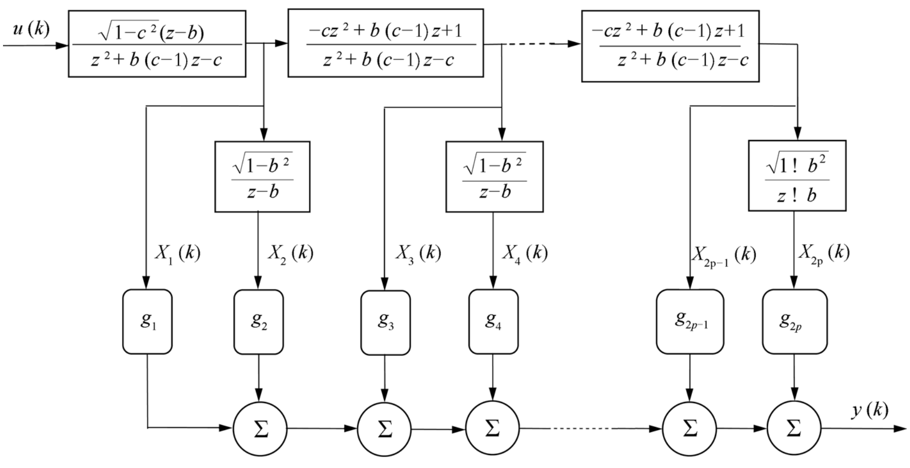

Fig. 4 Kautz based model

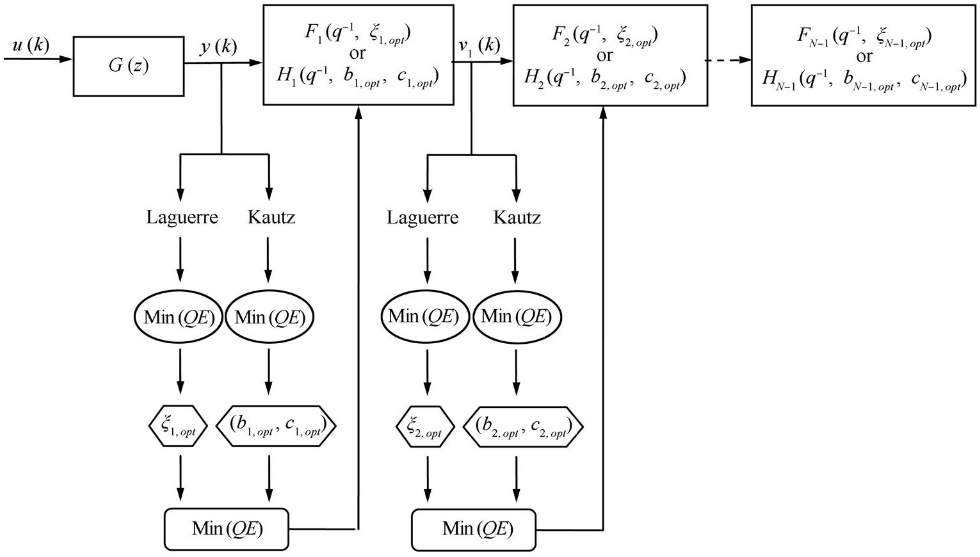

Fig. 5 Determination of the system modes – general case

The output of a GOB based model with i poles as shown in Fig 6 is written as follows:

$$\widehat{Y} = X_i G_i \tag{24}$$

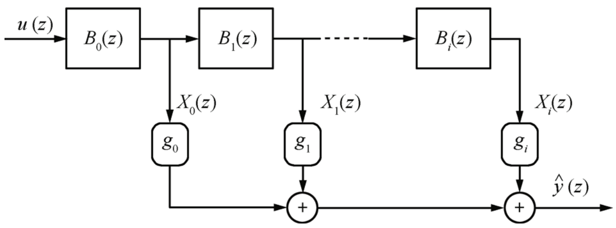

Fig. 6 Generalized orthonormal basis filter network

where

$$X_i = \begin{bmatrix} x_0(1) & \cdots & x_i(1) \\ \vdots & & \vdots \\ x_0(L) & \cdots & x_i(L) \end{bmatrix} = \begin{bmatrix} X_{i-1} & x_i \end{bmatrix} \tag{25}$$

and

$$\widehat{Y} = \begin{bmatrix} \widehat{y}(1) \\ \vdots \\ \widehat{y}(L) \end{bmatrix} \text{ and } G_i = \begin{bmatrix} g_0 \\ \vdots \\ g_i \end{bmatrix} \tag{26}$$

where $\{x_l\}_{l=0,\cdots,i}$ represent the GOB function outputs, $\{g_l\}_{l=0,\cdots,i}$ are the Fourier coefficients of the GOB expansion and L is the number of the measured outputs.

The square error between the system output and the output of the i-pole based GOB model is written as follows:

$$E_i^2 = (Y - \widehat{Y})^{\mathrm{T}}(Y - \widehat{Y}) = (Y - X_i G_i)^{\mathrm{T}}(Y - XG_i) \tag{27}$$

and the least square solution is given by

$$G_{i,opt} = \left(X_i^{\mathrm{T}} X_i\right)^{-1} X_i^{\mathrm{T}} Y. \tag{28}$$

The minimum squares error is

$$\begin{aligned} E_{\min,i}^2 = & (Y - X_i G_{i,opt})^{\mathrm{T}} (Y - X_i G_{i,opt}) = \\ & \left(Y - X_i (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} Y\right)^{\mathrm{T}} \times \\ & \left(Y - X_i (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} Y\right) = \\ & Y^{\mathrm{T}} Y - 2Y^{\mathrm{T}} X_i (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} Y + \\ & Y^{\mathrm{T}} X_i \underbrace{(X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} X_i}_{I_i} (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} Y = \\ & Y^{\mathrm{T}} Y - Y^{\mathrm{T}} X_i (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} Y. \end{aligned} \tag{29}$$

To verify the convergence of the proposed GOB pole selection algorithms, we calculate

$$\begin{aligned} E_{\min,i}^2 - E_{\min,i+1}^2 = & Y^{\mathrm{T}} \Big[X_{i+1} (X_{i+1}^{\mathrm{T}} X_{i+1})^{-1} X_{i+1}^{\mathrm{T}} - \\ & X_i (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}} \Big] Y. \end{aligned} \tag{30}$$

By defining

$$\Xi_{i+1} = \left(X_{i+1}^{\mathrm{T}} X_{i+1}\right)^{-1} \tag{31}$$

and the projection matrix

$$P_i^{\perp} = I - X_i \Xi_i X_i^{\mathrm{T}} \tag{32}$$

and using the formula of the partitioned matrix inversion[23], we can write

$$\Xi_{i+1} = \begin{bmatrix} \Xi_i + \dfrac{\Xi_i X_i^{\mathrm{T}} x_{i+1} x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \dfrac{\Xi_i X_i^{\mathrm{T}} x_{i+1}}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \\ \dfrac{x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \dfrac{1}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \end{bmatrix} \tag{33}$$

and

$$X_{i+1}(X_{i+1}^{\mathrm{T}}X_{i+1})^{-1}X_{i+1}^{\mathrm{T}} = X_{i+1}\begin{bmatrix} \Xi_i & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} X_{i+1}^{\mathrm{T}} + X_{i+1}\begin{bmatrix} \frac{\Xi_i X_i^{\mathrm{T}} x_{i+1} x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \frac{\Xi_i X_i^{\mathrm{T}} x_{i+1}}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \\ \frac{x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \frac{1}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \end{bmatrix} X_{i+1}^{\mathrm{T}} = X_i(X_i^{\mathrm{T}}X_i)^{-1}X_i^{\mathrm{T}} + X_{i+1}\begin{bmatrix} \frac{\Xi_i X_i^{\mathrm{T}} x_{i+1} x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \frac{\Xi_i X_i^{\mathrm{T}} x_{i+1}}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \\ \frac{x_{i+1}^{\mathrm{T}} X_i \Xi_i}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} & \frac{1}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} \end{bmatrix} X_{i+1}^{\mathrm{T}}. \quad (34)$$

Equation (34) becomes

$$E_{\min,i}^2 - E_{\min,i+1}^2 = \frac{1}{x_{i+1}^{\mathrm{T}} P_i^{\perp} x_{i+1}} Y^{\mathrm{T}} \times \left(X_{i+1} \begin{bmatrix} \Xi_i X_i^{\mathrm{T}} x_{i+1} \\ 1 \end{bmatrix} \right) \times \left(\begin{bmatrix} \Xi_i X_i^{\mathrm{T}} x_{i+1} \\ 1 \end{bmatrix}^{\mathrm{T}} X_{i+1}^{\mathrm{T}} \right) Y \geq 0. \quad (35)$$

So, we prove that the quadratic error obtained with a $(i+1)$-th order GOB model is smaller than the quadratic error obtained with a i-th order GOB model characterized by the first i poles. We can then conclude that the iterative algorithms for GOB pole selection converge.

4 Simulation results

Three systems are simulated in this section. The first two systems are numerical defined by their respective transfer functions $G_1(z)$ and $G_2(z)$ and the third simulated system is a 2nd order electrical linear system. The input signal $u(t)$ is a Gaussian white noise sequence with zero mean and unit variance. The signal to noise ratio (SNR) is fixed to 20 dB. The simulations results are obtained using the Monte Carlo technique with 50 different additive noise sequences.

The performance of the proposed identification algorithms is also evaluated in terms of the normalized mean square error (NMSE) as follows:

$$NMSE = \frac{1}{50}\sum_{m=1}^{50}\left(\frac{1}{L}\frac{\sum\limits_{k=1}^{L}(y(k)-\hat{y}(k))^2}{\sum\limits_{k=1}^{L}(y(k))^2}\right) \quad (36)$$

where $y(k)$ is the output of the system, $\hat{y}(k)$ is the corresponding output of the identified model based on a GOB expansion and $L = 10^3$ represents the number of output measurements used for the identification.

4.1 System 1

The first system is defined by the following transfer function $G_1(z)$. The truncation order is fixed to $i = 3$.

$$G_1(z) = 0.0017 \times \frac{z^{-1}(1+0.673z^{-1})}{(1-0.368z^{-1})(1-0.819z^{-1})(1-0.995z^{-1})}. \quad (37)$$

In Table 2, we give the corresponding QE values defined by (37) between the system output and the GOB based corresponding model output.

The performance of the two first proposed identification algorithms are compared in terms of the identified poles. In Table 1, we give for the first simulated system the true poles and the poles identified by the two algorithms, with their corresponding standard deviations σ. These poles are plotted in Fig. 7.

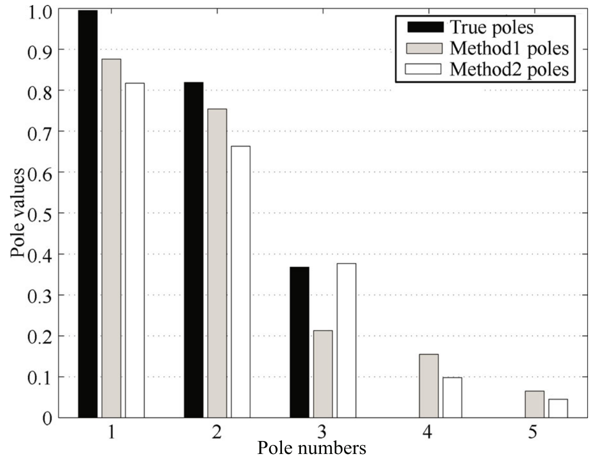

Fig. 7 True system poles compared to GOB poles identified by the two algorithms for system 1

By analyzing the results of Table 1 and Fig. 7, we can conclude that both algorithm′s estimated values are close to the true poles of the system.

Table 1 Identified GOB real poles for system 1

	True poles	Algorithm 1	Algorithm 2
ξ_1	0.995	0.876	0.817
$\sigma(\xi_1)$		2.32×10^{-3}	6.58×10^{-3}
ξ_2	0.819	0.754	0.663
$\sigma(\xi_2)$		1.75×10^{-2}	8.01×10^{-3}
ξ_3	0.368	0.213	0.377
$\sigma(\xi_3)$		2.12×10^{-2}	7.57×10^{-3}
ξ_4	0	0.155	0.098
$\sigma(\xi_4)$		1.02×10^{-2}	9.17×10^{-3}
ξ_5	0	0.065	0.045
$\sigma(\xi_5)$		1.35×10^{-2}	8.32×10^{-3}

In Table 2, we present the QE values between the system output and the GOB based models respectively identified by Algorithms 1 and 2. The two algorithms give satisfactory results.

As we can see in Table 3, Algorithm 1 is much faster than Algorithm 2. To determine one pole of the GOB, the computing time (in seconds) of the Fu and Dumont′s algorithm is shorter than that of the technique used in Algorithm 2. However, we have noticed that, with Algorithm 2 the system impulse response is not required to determine the GOB poles while it is the case with Algorithm 1. This is an advantage when the system impulse response is difficult to obtain.

Table 2 Quadratic errors obtained with different GOB expansions

	NMSE
Algorithm 1 estimated poles	7.93×10^{-3}
Algorithm 2 estimated poles	2.95×10^{-3}

Table 3 Computing times in seconds of the two algorithms

	Computing time
Algorithm 1	6.19×10^{-3}
Algorithm 2	8.22

Generally, Algorithm 1 based on the use of analytical formulae, gives satisfactory results for systems whose modes are close to each other. However, the more the system modes are distant from each other (or the system is underdamped), the less satisfactory the identification results will be, and Algorithm 2 becomes more useful and precise to determine the poles. With regards to numerical complexity, Algorithm 1 is easier to implement than Algorithm 2 that needs more calculations.

4.2 System 2

The simulated system is 5-th -order defined by its transfer function $G_2(z)$. The truncation order of the GOB based model is fixed to $i = 5$.

$$G_2(z) = \frac{b_1(z)b_2(z)}{(z-0.5)\left[a(z)\right]^2} \tag{38}$$

with

$$a(z) = z^2 - 2r\cos(\varphi)z + r^2 \tag{39}$$

$$b_1(z) = z^2 - 2r\cos(\varphi+\Delta\varphi)z + r^2 \tag{40}$$

$$b_2(z) = z^2 - 2r\cos(\varphi-\Delta\varphi)z + r^2 \tag{41}$$

where $\varphi = \dfrac{1.3\pi}{4}$, $\Delta\varphi = \dfrac{0.2\pi}{4}$, $r = 0.8$ and K is chosen such that the static gain of System 2 is equal to one.

Figs. 8 and 9 represent the quadratic error $E^2_{\min,i} - E^2_{\min,\,i+1}(j = 1, \cdots, N)$ between the residual output ν_i and respectively the Laguerre based model $\hat{y}_i$ and the Kautz based model. This leads to an optimal Laguerre pole ξ_i or an optimal (p_i, q_i) optimal Kautz parameters.

In Table 4, we give for the second simulated system, the real system parameters and the corresponding estimated real poles ξ_i and parameters p_i and q_i using the 3rd algorithm provided with their standard deviations.

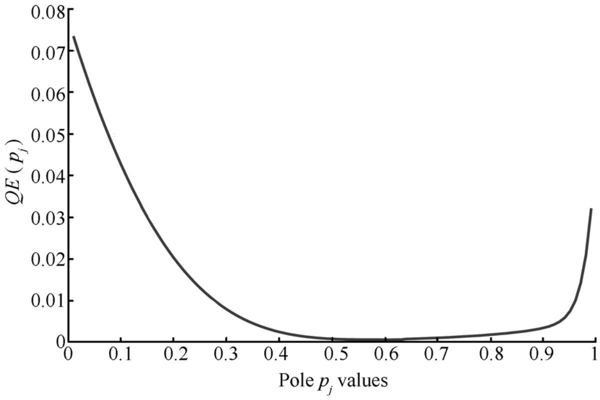

Fig. 8 Determination of $\xi_{i,opt}$ minimizing the quadratic error

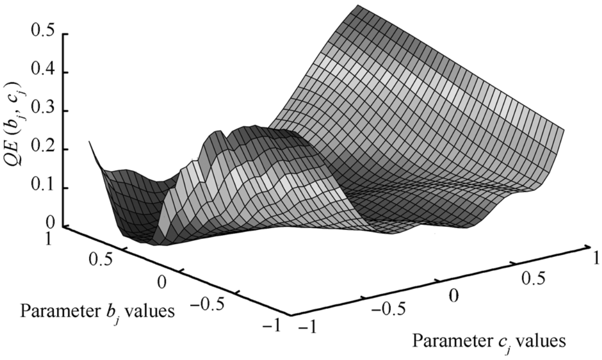

Fig. 9 Determination of $b_{i,opt}$ and $c_{j,opt}$ minimizing QE

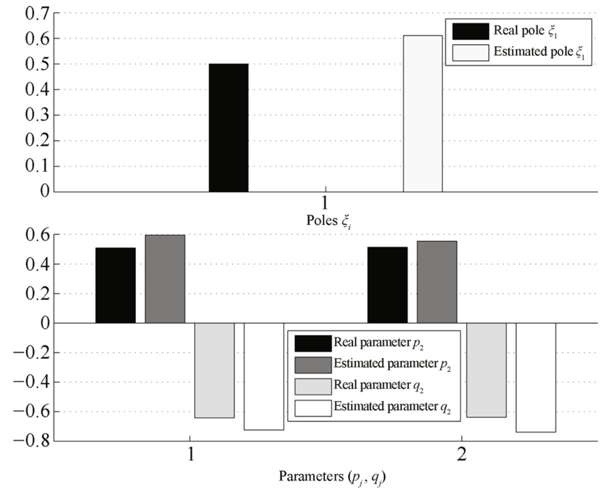

Fig. 10 True system poles and parameters compared to GOB poles and parameters for System 2

By analyzing the results of Table 4 and Fig. 10, we can conclude that Algorithm 3 estimates the poles ξ_i and the parameters (b_i, c_i) of the GOB based model that are close to those of the simulated system 2.

The evaluation of the performance of Algorithm 3 in terms of quadratic error is calculated as for the Algorithms 1 and 2. The results are satisfactory and given in Table 5.

Table 4 Identified GOB poles ξ_i or parameters (b_i, c_i) parameters for System 2

	True parameters	Estimated parameters
ξ_1	0.5	0.611
$\sigma(\xi_1)$		1.54×10^{-3}
(p_2, q_2)	$(0.507, -0.642)$	$(0.594, -0.724)$
$(\sigma(p_2), \sigma(q_2))$		$(2.12 \times 10^{-2}, 3.02 \times 10^{-2})$
(p_3, q_3)	$(0.512, -0.638)$	$(0.553, -0.739)$
$(\sigma(p_3), \sigma(q_3))$		$(2.04 \times 10^{-2}, 4.32 \times 10^{-2})$

Table 5 Normalized mean square error (NMSE) obtained with different GOB expansions

	NMSE
GOB expansion (true poles)	7.32×10^{-6}
GOB expansion (Algorithm 3 poles)	1.30×10^{-2}

4.3 System 3

Let us consider a 2nd-order electrical linear system represented by the following circuit, where

$$C_1 = 10\,\text{nF},\ C' = 0.205\,\mu\text{F and } R = R_1 = R_2 = 68\,\text{k}\Omega$$

This system is defined by the following transfer function:

$$G_c(s) = \frac{R_2}{RR_1R_2C_1C's^2 + RR_1C_1s + R_2} = \frac{K\omega_0^2}{s^2 + 2m\omega_0 s + \omega_0^2} \tag{42}$$

such as

1) $K = 1$ is the static gain,

2) $\omega_0 = \sqrt{\frac{1}{RR_1C_1C'}} = 324.8\,\text{rad.s}^{-1}$ is the natural frequency,

3) $m = \frac{1}{2R_2}\sqrt{\frac{RR_1C_1}{C'}} = 0.110\,4$ is the damping ratio.

The numerical transfer function is then equal to

$$G_c(s) = \frac{105.5 \times 10^3}{s^2 + 71.72\,s + 105.5 \times 10^3}. \tag{43}$$

Thus, the two system complex conjugate poles are

$$\xi = -35.86 + 322.81\ \text{j}$$
$$\xi^* = -35.86 - 322.81\ \text{j}$$

and the real Kautz parameters b and c defined by (17) and (18) are in Table 6.

Table 6 Identified GOB parameters (b_i, c_i) for System 3

	True parameters	Estimated parameters
(p_1, q_1)	$(-6.79\times10^{-4}, -1.05\times10^5)$	$(-7.26\times10^{-4}, -0.89\times10^5)$
$(\sigma(p_1), \sigma(q_1))$		$(3.51\times10^{-6}, 5.72\times10^2)$

Five hundred input/output observations were collected from the process at a sampling time of 0.01 s. The voltage input $u(k)$ ranges from 1 V to 3.3 V which is a pseudo-random sequence and the obtained output $y(k)$ are illustrated in Figs. 12 and 13.

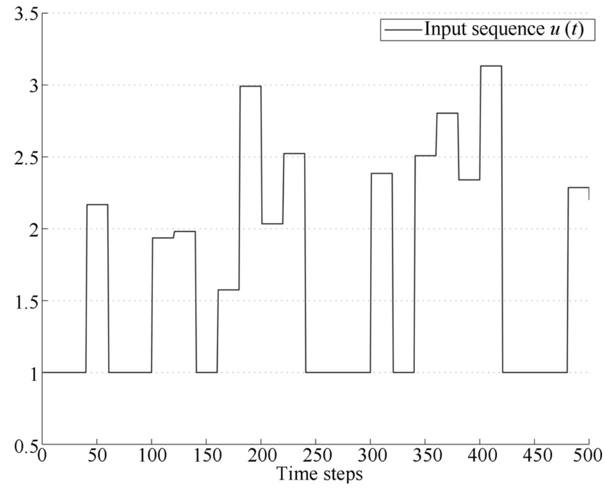

Fig. 12 Pseudo random input sequence $u(t)$

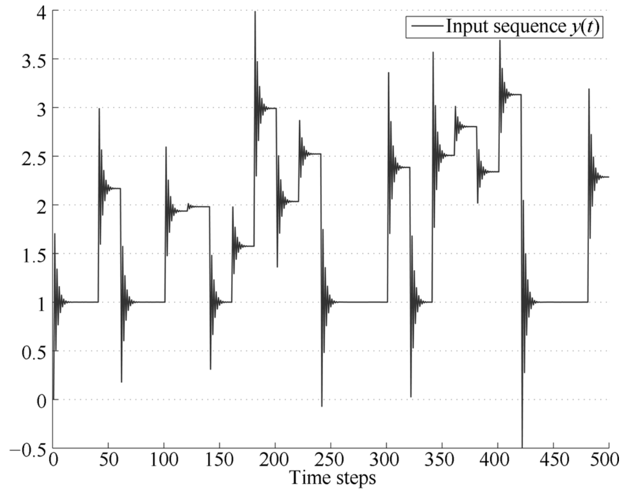

Fig. 13 The 2nd-order electrical linear system output sequence $y(t)$

To estimate the GOB based model parameters, Algorithm 3 converges to the results in Table 6 where the real system parameters are close to those estimated by Algorithm 3 with relatively small standard deviation.

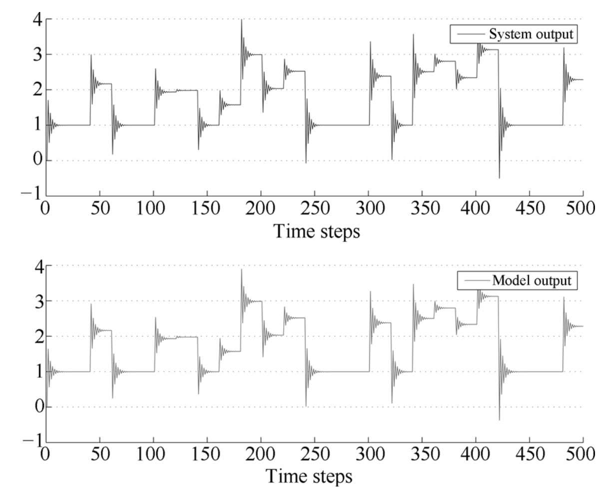

Fig. 14 System output and the GOB based model output

We illustrate in Fig. 14, the evolution of the outputs of the electrical system and the GOB based corresponding model. It resorts that the proposed GOB based model handles the process perfectly with relatively small NMSE as presented in Table 7.

Table 7 Normalized mean square error (NMSE) obtained with different GOB expansions

	NMSE
GOB expansion (True poles)	1.58×10^{-7}
GOB expansion (Algorithm 3 poles)	7.95×10^{-3}

5 Conclusions

In this paper, we have presented three algorithms for optimizing the poles of a GOB in the context of linear system modeling. The first two algorithms concern the GOB with optimal real poles. They are based on on a stage of determination of the dominant Laguerre pole of a residual signal, followed by a stage of generation of the residual signals by an iterative filtering, at each iteration.

The third algorithm is dedicated to more complex linear systems with several dynamics. It optimizes both real and complex GOB poles. It is based on on a stage of determination of the dominant Laguerre real pole or of the two complex conjugate Kautz poles of a residual signal, followed by a stage of generation of the residual signal by an iterative filtering, at each iteration.

These algorithms are iterative and based on the quadratic error between the linear process output and the GOB based model output. They present the advantage to be very simple to implement. No numerical optimization technique is needed, and in consequence there is no problem of local minima as is the case for other algorithms in the literature.

We have also analyzed the convergence of the proposed GOB pole selection algorithms by proving that the quadratic error obtained with the $(i+1)$-th order GOB based model is smaller than the quadratic error obtained with the i-th order GOB based model characterized by the first i poles.

The proposed algorithms are validated in simulations using three analytical systems. We first compare the real parameters of the numerical simulated systems with the estimated parameters of the proposed GOB based models. The performance of the algorithms in terms of mean square error is satisfactory since the quadratic error criterion between the simulated systems outputs and the optimal GOB based models outputs are relatively small in noisy conditions.

An extension of this work is the extension of the proposed GOB pole estimation algorithms to nonlinear systems.

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Anis Khouaja recieved the B. Eng. degree from the National Engineering School of Monastir (ENIM), Tunisia in 2000. He received the M. Sc. and Ph. D. degrees from the University of Nice Sophia Antipolis, France in 2001 and 2005, respectively. Currently, he is an assistant professor in Electrical Engineering Department, High Institute of Applied Science and Technology of Sousse, Tunisia. He is also with the LARATSI Laboratory of the Engineering National School of Monastir in Tunisia.

His research interests include system modeling and identification, nonlinear system theory and robust predictive control.

E-mail: anis.khouaja@issatso.rnu.tn (Corresponding author)

ORCID iD: 0000-0002-5759-2612

Hassani Messaoud is a full professor with the Electrical Engineering Department of the National School of Engineers of Monastir, Tunisia. He is also the head of the LARATSI Laboratory in the same school.

His research interests include system modeling and identification, nonlinear system theory, robust predictive control, diagnostic and digital channel equalization.

E-mail: hassani.messaoud@enim.rnu.tn