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# Stabilization for a Class of Discrete-time Switched Large-scale Systems with Parameter Uncertainties

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**Abstract:** The problem of robust stabilization for a class of discrete-time switched large-scale systems with parameter uncertainties and nonlinear interconnected terms is considered. By using state feedback and Lyapunov function technique, a decentralized switching control approach is put forward to guarantee the solutions of large-scale systems converge to the origin globally. A numerical example and a corresponding simulation result are utilized to verify the effectiveness of the presented approach.

**Keywords:** Switched large-scale systems, discrete-time, state feedback, decentralized control, switching law.

## 1 Introduction

Dynamical systems include two classes of typical systems which consist of continuous-time systems and discrete-time systems. A continuous-time system is described by an ordinary differential equation. However, a discrete-time system is described by a difference equation. If each eigenvalue of state matrix in a continuous-time linear system has a negative real part, then the linear system is Hurwitz stable. If all eigenvalues of state matrix in a discrete-time linear system locate in a unit circle in the complex plane, then the linear system is Schur stable. Stability is an important performance index in the process of system control. When all kinds of uncertainties or disturbances appear in dynamical systems, a problem on how to design suitable robust controllers for stabilizing the systems has become a challengeable task for researchers.

In the real world, a lot of systems in various engineering fields are discrete-time cases. Therefore, it is very important and valuable to investigate discrete-time dynamical systems. Hitherto, many research fruits in different aspects on discrete-time systems have been obtained respectively[1−7].

With the development of hybrid control theory, switched systems as a class of typical hybrid systems have been investigated extensively in view of their applications in some engineering fields such as aviation and spaceflight, computer control systems, underwater robots, etc. Switching control technique has become one of potential effective approaches in control theory and control engineering fields in recent two decades[8−10]. One of hot issues in research is on how to stabilize dynamical systems via an appropriate switching control approach.

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A discrete-time switched system is composed of a family of discrete-time subsystems and a piecewise constant signal that orchestrates the switching between them. The piecewise constant signal is called a switching law, which plays an important role in determining dynamical behaviors of the whole system. A discrete-time switched system with two Schur stable subsystems may be stable or unstable under a certain switching law. It is well known that the existence of a common quadratic Lyapunov function is correlative to stability of systems via arbitrary switching. It is necessary that a stable switching law will be designed to guarantee the systems stability under the condition of the nonexistence of a common quadratic Lyapunov function. Many researchers have been exploring and attempting a lot of new approaches for switched systems. In recent years, discrete-time switched systems have received a great deal of attention[11−21].

The problems of stability and stabilization for discrete-time switched systems have been investigated[11−19]. By using switched quadratic Lyapunov functions and linear matrix inequality approach, the problems of asymptotic stability and stabilization of two time scale switched systems under arbitrary switching were solved respectively[11]. A switching signal was designed to guarantee a class of switched systems exponentially stable and robust against various switching perturbations[12]. Based on average dwell time approach, the problems on stability analysis and $H_\infty$ static output feedback control of discrete-time switched systems were considered respectively[13, 14]. Essentially, design of switching signal or switching law is very important to guarantee the stability of switched systems[22−24]. Especially, switching signal design is very difficult under the particular conditions including disturbance[15], time-delay[17], Markovian probabilities[18], and uncertainties[25−28] in switched systems. Consequently, it is necessary to solve the problem of switching signal design for switched systems.

Uncertain discrete-time switched large-scale systems are composed of a family of interconnected low-dimensional discrete-time subsystems. Each subsystem is constructed by switching between several discrete-time subsystems with parameter uncertainties. However, the problem of switching control for uncertain discrete-time switched large-scale systems has not been explored so far, to the best of my knowledge.

In this paper, the problem of robust stabilization for a class of discrete-time switched large-scale systems is studied. Parameter uncertainties in the state matrix and the control matrix of the systems are norm-bounded. Interconnected terms of low-dimensional discrete-time subsystems are nonlinear and norm-bounded. Based on structural characteristics of the systems, each discrete-time subsystem cannot be stabilized by any controller. It implies that the whole discrete-time switched large-scale system cannot be stabilized by designing decentralized controllers only. However, not only decentralized controllers are designed, but also corresponding decentralized laws are designed simultaneously. A decentralized switching control approach is put forward in this paper. Based on matrix inequalities, discrete-time switched large-scale systems can be well stabilized.

So far, the problems on stabilization of an uncertain discrete-time switched system without interconnected terms or a single uncertain discrete-time large-scale system without switching have been investigated only. Compared with existing results, the contribution of the paper is that a decentralized switching control approach is derived to solve successfully a difficult problem on robust stabilization for a class of uncertain discrete-time switched large-scale systems with nonlinear interconnected terms under the condition that each discrete-time subsystem cannot be stabilized by any controller. A numerical example is utilized to illustrate the effectiveness of the decentralized switching control approach.

The organization of this paper is as follows. Section 2 describes discrete-time switched large-scale systems with parameter uncertainties. The decentralized controllers and the decentralized laws are designed to stabilize the systems in Section 3. In Section 4, a numerical example illustrates the effectiveness of the designed switching control approach. Finally, concluding remarks are given in Section 5.

**Notations.** Throughout this paper, $\mathbf{R}^n$ denotes the $n$ dimensional Euclidean space. $P^{n\times m}$ is the set of all $n\times m$ real matrices. $P>0$ denotes that $P$ is a real symmetric positive definite matrix. $P\leq 0$ denotes that $P$ is a real symmetric semi-negative definite matrix. $Q^{\mathrm{T}}$ denotes a transpose matrix of $Q$. $I$ denotes the identity matrix with appropriate dimension. Expression $B\Leftrightarrow C$ denotes $B$ is equivalent to $C$. $\lambda_{max}(A)$ denotes the maximum eigenvalue of matrix $A$. $arg\{*\}$ denotes a subscript value that satisfies the condition expression in the bracket. $||\cdot||$ denotes the Euclidean vector norm or induced matrix 2-norm.

## 2 Problem formulation and preliminaries

Consider a class of uncertain discrete-time switched large-scale systems described by

$$\begin{aligned}x_i(k+1)=&(A_{i\sigma_i}+\Delta A_{i\sigma_i})x_i(k)+\\&(B_{i\sigma_i}+\Delta B_{i\sigma_i})u_{i\sigma_i}(k)+g_i(x(k))\end{aligned}\tag{1}$$

where $x_i(k)\in\mathbf{R}^{m_i}$ and $u_{i\sigma_i}(k)\in\mathbf{R}^{r_i}$ are the state vector and control input of the $i$-th subsystem respectively, $i=1,2,\cdots,N$. $A_{i\sigma_i}\in P^{m_i\times m_i}$, $B_{i\sigma_i}\in P^{m_i\times r_i}$ are the constant matrices of appropriate dimensions. $\Delta A_{i\sigma_i}\in P^{m_i\times m_i}$ and $\Delta B_{i\sigma_i}\in P^{m_i\times r_i}$ are the parameter uncertainties. $\sigma_i=j\in S_i$ is a switching law or switching signal of the $i$-th subsystem. In fact, $\sigma_i$ is a piecewise constant function. $S_i=\{1,2,\cdots,k_i\}$ is a set with positive integers. $g_i(x(k))\in\mathbf{R}^{m_i}$ are nonlinear interconnected terms. $x(k)=[x_1(k)\ x_2(k)\ \cdots\ x_N(k)]^{\mathrm{T}}\in\mathbf{R}^q$ is the state vector of discrete-time switched large-scale systems, where $q=\sum_{i=1}^N m_i$.

The parameter uncertainties are norm-bounded and are assumed to be of the following form

$$[\Delta A_{i\sigma_i}\ \Delta B_{i\sigma_i}]=D_{i\sigma_i}F_{i\sigma_i}(k)[M_{i\sigma_i}\ N_{i\sigma_i}]\tag{2}$$

where $D_{i\sigma_i},M_{i\sigma_i},N_{i\sigma_i}$ are known constant real matrices of appropriate dimensions and $F_{i\sigma_i}(k)$ are unknown matrices that satisfy

$$F_{i\sigma_i}^{\mathrm{T}}(k)F_{i\sigma_i}(k)\leq I.\tag{3}$$

The nonlinear interconnected terms are norm-bounded and satisfy

$$g_i(x(k))\leq\gamma_i||x(k)||\tag{4}$$

where $\gamma_i$ is a positive real number.

By employing linear state feedback, decentralized controllers are designed as

$$u_{i\sigma_i}(k)=K_{i\sigma_i}x_i(k).\tag{5}$$

Uncertain discrete-time switched large-scale systems (1) result in the closed-loop systems

$$\begin{aligned}x_i(k+1)=&(A_{i\sigma_i}+B_{i\sigma_i}K_{i\sigma_i}+\Delta A_{i\sigma_i}+\\&\Delta B_{i\sigma_i}K_{i\sigma_i})x_i(k)+g_i(x(k)).\end{aligned}\tag{6}$$

In this paper, the significance and value of research embodies that the closed-loop systems (6) are unstable for any $K_{i\sigma_i}$. It implies that uncertain discrete-time switched large-scale systems (1) cannot be stabilized by any controller. A new switching control approach of the systems will be put forward under the above restricted condition to guarantee the closed-loop systems (6) be stable. In order to obtain main results of this paper, two important results are employed firstly.

**Lemma 1.**[29] Let $M,N,Q$ be constant matrices of appropriate dimensions, $Q=Q^{\mathrm{T}}>0$, then for any scalar $\varepsilon>0$, we have

$$M^{\mathrm{T}}N+N^{\mathrm{T}}M\leq\varepsilon M^{\mathrm{T}}Q^{-1}M+\varepsilon^{-1}N^{\mathrm{T}}QN\tag{7}$$

**Lemma 2.**[30] Suppose $A, D, E$ are the given matrices. If $P$ is a positive definite matrix and $\delta > 0$ is a scalar, such that $\delta^{-1}I - D^{\mathrm{T}}PD > 0$, then

$$(A+DFE)^{\mathrm{T}}P(A+DFE) \leq A^{\mathrm{T}}(P^{-1}-\delta DD^{\mathrm{T}})^{-1}A + \delta^{-1}E^{\mathrm{T}}E \tag{8}$$

holds for the arbitrary norm-bounded time-varying uncertainty $F$ with $F^{\mathrm{T}}F \leq I$.

## 3 Main results

Firstly, a class of uncertain discrete-time non-switched large-scale systems is considered

$$x_i(k+1) = (A_i + \Delta A_i)x_i(k) + (B_i+\Delta B_i)u_i(k) + g_i(x(k)). \tag{9}$$

The parameter uncertainties satisfy the norm-bounded condition

$$[\Delta A_i \ \Delta B_i] = D_i F_i(k)[M_i \ N_i] \tag{10}$$

where $D_i, M_i, N_i$ are known constant real matrices of appropriate dimensions and $F_i(k)$ are unknown matrices that satisfy

$$F_i^{\mathrm{T}}(k)F_i(k) \leq I. \tag{11}$$

The nonlinear interconnected terms are norm-bounded and satisfy the above condition (4). By employing linear state feedback, decentralized controllers of the systems (9) are designed as

$$u_i(k) = K_i x_i(k). \tag{12}$$

Uncertain discrete-time non-switched large-scale systems (9) result in the closed-loop systems

$$x_i(k+1) = (A_i + B_iK_i + \Delta A_i + \Delta B_iK_i)x_i(k) + g_i(x(k)). \tag{13}$$

**Theorem 1.** The closed-loop systems (13) are asymptotically stable via linear state feedback (12), if there exist symmetric positive definite matrices $P_i > 0$ and constant real matrices $K_i$ and positive real scalars $\varepsilon > 0$ and $\delta > 0$, such that the following matrix inequalities hold

$$(1+\varepsilon)[\delta^{-1}(M_i+N_iK_i)^{\mathrm{T}}(M_i+N_iK_i) + (A_i+B_iK_i)^{\mathrm{T}}(P_i^{-1}-\delta D_iD_i^{\mathrm{T}})^{-1}(A_i+B_iK_i)] - P_i + (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{\max}(P_l)I_i < 0. \tag{14}$$

**Proof.** By Lemma 1, the following linear matrix inequality (LMI) holds.

$$A^{\mathrm{T}}PB + B^{\mathrm{T}}PA \leq \alpha A^{\mathrm{T}}PP^{-1}PA + \alpha^{-1}B^{\mathrm{T}}PB = \alpha A^{\mathrm{T}}PA + \alpha^{-1}B^{\mathrm{T}}PB \tag{15}$$

where $P = P^{\mathrm{T}} > 0$ is a symmetric positive definite matrix and $\alpha > 0$ is a positive scalar.

Considering a Lyapunov function

$$V(x(k)) = V_1(x_1(k)) + V_2(x_2(k)) + \cdots + V_N(x_N(k)) = \sum_{i=1}^{N} V_i(x_i(k)) = \sum_{i=1}^{N} x_i^{\mathrm{T}}(k)P_i(x_i(k)) \tag{16}$$

where $P_i > 0, V_i(x_i(k)) = x_i^{\mathrm{T}}(k)P_i(x_i(k)), i = 1, 2, \cdots, N$.

Along the trajectories of systems (13), the difference of $V(x(k))$ is given by

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)) = \sum_{i=1}^{N}[V_i(x_i(k+1)) - V_i(x_i(k))] = \sum_{i=1}^{N}[x_i^{\mathrm{T}}(k+1)P_ix_i(k+1) - x_i^{\mathrm{T}}(k)P_ix_i(k)]. \tag{17}$$

Equation (17) can be rewritten as

$$\Delta V(x(k)) = \sum_{i=1}^{N}\{[(A_i+B_iK_i+\Delta A_i + \Delta B_iK_i)x_i(k) + g_i(x(k))]^{\mathrm{T}} \times P_i[(A_i+B_iK_i+\Delta A_i+\Delta B_iK_i)x_i(k)+g_i(x(k))] - x_i^{\mathrm{T}}(k)P_ix_i(k)\}. \tag{18}$$

Further, it is obtained

$$[(\bar{A}_ix_i(k) + g_i(x(k))]^{\mathrm{T}}P_i[(\bar{A}_ix_i(k)+g_i(x(k))] = x_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_ix_i(k) + x_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_ig_i(x(k)) + g_i^{\mathrm{T}}(x(k))P_i\bar{A}_ix_i(k) + g_i^{\mathrm{T}}(x(k))P_ig_i(x(k)) \tag{19}$$

where $\bar{A}_i = A_i + B_iK_i + \Delta A_i + \Delta B_iK_i$.

From Lemma 1, the following matrix inequality holds

$$x_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_ig_i(x(k)) + g_i^{\mathrm{T}}(x(k))P_i\bar{A}_ix_i(k) \leq \varepsilon x_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_ix_i(k) + \varepsilon^{-1}g_i^{\mathrm{T}}(x(k))P_ig_i(x(k)) \tag{20}$$

where $\varepsilon > 0$ is a positive scalar.

According to expression (4), it is obtained

$$g_i^{\mathrm{T}}(x(k))P_ig_i(x(k)) \leq ||g_i(x(k))||^2 \times ||P_i|| \leq \gamma_i^2\lambda_{max}(P_i)||x(k)||^2. \tag{21}$$

Therefore,

$$\Delta V(x(k)) \leq \sum_{i=1}^{N} x_i^{\mathrm{T}}(k)[(1+\varepsilon)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i - P_i + (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{max}(P_l)I_i]x_i(k). \tag{22}$$

From (10), it is obtained

$$\begin{aligned}\bar{A}_i = A_i + B_iK_i + \Delta A_i + \Delta B_iK_i = \\ A_i + B_iK_i + D_iF_i(M_i + N_iK_i) = \\ \hat{A}_i + D_iF_iE_i\end{aligned}$$

where $\hat{A}_i = A_i + B_iK_i, E_i = M_i + N_iK_i$.

It is obtained

$$\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i = (\hat{A}_i + D_iF_iE_i)^{\mathrm{T}}P_i(\hat{A}_i + D_iF_iE_i).$$

By Lemma 2, the following inequality holds

$$\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i \le \delta^{-1}E_i^{\mathrm{T}}E_i + \hat{A}_i^{\mathrm{T}}(P_i^{-1} - \delta D_iD_i^{\mathrm{T}})^{-1}\hat{A}_i. \tag{23}$$

Hence, it is obtained

$$\begin{aligned}\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i \le \delta^{-1}(M_i + N_iK_i)^{\mathrm{T}}(M_i + N_iK_i)+ \\ (A_i + B_iK_i)^{\mathrm{T}}(P_i^{-1} - \delta D_iD_i^{\mathrm{T}})^{-1}(A_i + B_iK_i)\end{aligned} \tag{24}$$

where $\delta > 0,\ P_i^{-1} - \delta D_iD_i^{\mathrm{T}} > 0$.

Combine (22) with (24)

$$\begin{aligned}\Delta V(x(k)) \le \sum_{i=1}^{N} x_i^{\mathrm{T}}(k)\{(1+\varepsilon)[\delta^{-1}(M_i + N_iK_i)^{\mathrm{T}}\times \\ (M_i + N_iK_i) + (A_i + B_iK_i)^{\mathrm{T}}(P_i^{-1} - \delta D_iD_i^{\mathrm{T}})^{-1}\times \\ (A_i + B_iK_i)] - P_i + (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{max}(P_l)I_i\}x_i(k).\end{aligned}$$

It is obtained

$$\Delta V(x(k)) < 0.$$

The closed-loop systems (13) are asymptotically stable via linear state feedback (12). □

**Theorem 2.** For given positive scalars $\alpha_{ij} > 0$ that satisfy $\sum_{j=1}^{k_i}\alpha_{ij} = 1,\ i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, k_i$, the closed-loop systems (6) are asymptotically stable via switching control laws (26), if there exist symmetric positive definite matrices $P_i > 0$, constant real matrices $K_{ij}$ and positive real scalars $\varepsilon > 0$ and $\delta > 0$, such that the following matrix inequalities hold:

$$\begin{aligned}(1+\varepsilon)\sum_{j=1}^{k_i}\alpha_{ij}[\delta^{-1}(M_{ij} + N_{ij}K_{ij})^{\mathrm{T}}(M_{ij} + N_{ij}K_{ij})+ \\ (A_{ij} + B_{ij}K_{ij})^{\mathrm{T}}(P_i^{-1} - \delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij} + B_{ij}K_{ij})]- \\ P_i + (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{\max}(P_l)I_i < 0.\end{aligned} \tag{25}$$

The decentralized switching control laws are designed as

$$\begin{aligned}\sigma_i = j = \arg\{\min x_i^{\mathrm{T}}(k)[\delta^{-1}(M_{ij} + N_{ij}K_{ij})^{\mathrm{T}}\times \\ (M_{ij} + N_{ij}K_{ij}) + (A_{ij} + B_{ij}K_{ij})^{\mathrm{T}}\times \\ (P_i^{-1} - \delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij} + B_{ij}K_{ij})]x_i(k)\}.\end{aligned} \tag{26}$$

**Proof.** Considering Lyapunov function (16), along the trajectories of systems (6), the difference of $V(x(k))$ is given by (17).

According to equation (17) and switching control laws (26), it is obtained

$$\begin{aligned}\Delta V(x(k)) = \sum_{i=1}^{N}\{[(A_{i\sigma_i} + \Delta A_{i\sigma_i})x_i(k) + (B_{i\sigma_i}+ \\ \Delta B_{i\sigma_i})u_{i\sigma_i}(k) + g_i(x(k))]^{\mathrm{T}}P_i[(A_{i\sigma_i} + \Delta A_{i\sigma_i})x_i(k)+ \\ (B_{i\sigma_i} + \Delta B_{i\sigma_i})u_{i\sigma_i}(k) + g_i(x(k))] - x_i^{\mathrm{T}}(k)P_ix_i(k)\} = \\ \sum_{i=1}^{N}\{[(A_{ij} + \Delta A_{ij})x_i(k) + (B_{ij} + \Delta B_{ij})u_{ij}(k)+ \\ g_i(x(k))]^{\mathrm{T}}P_i[(A_{ij} + \Delta A_{ij})x_i(k)+ \\ (B_{ij} + \Delta B_{ij})u_{ij}(k) + g_i(x(k))] - x_i^{\mathrm{T}}(k)P_ix_i(k)\}.\end{aligned} \tag{27}$$

Further, it is obtained

$$\begin{aligned}[(\bar{A}_{ij}x_i(k) + g_i(x(k))]^{\mathrm{T}}P_i[(\bar{A}_{ij}x_i(k) + g_i(x(k))] = \\ x_i^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_i\bar{A}_{ij}x_i(k) + x_i^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_ig_i(x(k))+ \\ g_i^{\mathrm{T}}(x(k))P_i\bar{A}_{ij}x_i(k) + g_i^{\mathrm{T}}(x(k))P_ig_i(x(k))\end{aligned} \tag{28}$$

where $\bar{A}_{ij} = A_{ij} + B_{ij}K_{ij} + \Delta A_{ij} + \Delta B_{ij}K_{ij}$.

By Lemma 1, the following matrix inequality holds

$$\begin{aligned}x_i^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_ig_i(x(k)) + g_i^{\mathrm{T}}(x(k))P_i\bar{A}_{ij}x_i(k) \le \\ \varepsilon x_i^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_i\bar{A}_{ij}x_i(k) + \varepsilon^{-1}g_i^{\mathrm{T}}(x(k))P_ig_i(x(k)).\end{aligned} \tag{29}$$

where $\varepsilon > 0$.

According to expression (21), it is obtained

$$\begin{aligned}\Delta V(x(k)) \le \sum_{i=1}^{N} x_i^{\mathrm{T}}(k)[(1+\varepsilon)\bar{A}_{ij}^{\mathrm{T}}P_i\bar{A}_{ij} - P_i+ \\ (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{max}(P_l)I_i]x_i(k).\end{aligned} \tag{30}$$

By inequality (24), the following inequality holds

$$\begin{aligned}\bar{A}_{ij}^{\mathrm{T}}P_i\bar{A}_{ij} \le \delta^{-1}(M_{ij} + N_{ij}K_{ij})^{\mathrm{T}}(M_{ij} + N_{ij}K_{ij})+ \\ (A_{ij} + B_{ij}K_{ij})^{\mathrm{T}}(P_i^{-1} - \delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij} + B_{ij}K_{ij}).\end{aligned} \tag{31}$$

Combining (30) with (31),

$$\begin{aligned}\Delta V(x(k)) \le \sum_{i=1}^{N} x_i^{\mathrm{T}}(k)\{(1+\varepsilon)[\delta^{-1}(M_{ij} + N_{ij}K_{ij})^{\mathrm{T}}\times \\ (M_{ij} + N_{ij}K_{ij}) + (A_{ij} + B_{ij}K_{ij})^{\mathrm{T}}(P_i^{-1}- \\ \delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij} + B_{ij}K_{ij})] - P_i+ \\ (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{\max}(P_l)I_i\}x_i(k).\end{aligned}$$

Design decentralized switching control laws

$$\begin{aligned}
&\sigma_i = j = \arg\{\min x_i^{\mathrm{T}}(k)[(1+\varepsilon)(\delta^{-1}(M_{ij}+N_{ij}K_{ij})^{\mathrm{T}}\times \\
&\quad (M_{ij}+N_{ij}K_{ij}) + (A_{ij}+B_{ij}K_{ij})^{\mathrm{T}}\times \\
&\quad (P_i^{-1}-\delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij}+B_{ij}K_{ij}))- \\
&\quad P_i + (\varepsilon^{-1}+1)\sum_{l=1}^{N}\gamma_l^2\lambda_{\max}(P_l)I_i]x_i(k)\} \Leftrightarrow \\
&\quad \sigma_i = j = \arg\{\min x_i^{\mathrm{T}}(k)[\delta^{-1}(M_{ij}+N_{ij}K_{ij})^{\mathrm{T}}\times \\
&\quad (M_{ij}+N_{ij}K_{ij}) + (A_{ij}+B_{ij}K_{ij})^{\mathrm{T}}\times \\
&\quad (P_i^{-1}-\delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij}+B_{ij}K_{ij})]x_i(k)\}.
\end{aligned}$$

It is obtained

$$\Delta V(x(k)) < 0.$$

The closed-loop systems (6) are asymptotically stable via switching control laws (26). □

## 4 A numerical example and its simulation

Consider a class of uncertain discrete-time switched large-scale systems with two low-dimensional subsystems given by

$$\begin{aligned}
&x_1(k+1) = (A_{1\sigma_1}+\Delta A_{1\sigma_1})x_1(k) + (B_{1\sigma_1}+ \\
&\quad \Delta B_{1\sigma_1})u_{1\sigma_1}(k) + g_1(x(k)) \\
&x_2(k+1) = (A_{2\sigma_2}+\Delta A_{2\sigma_2})x_2(k) + (B_{2\sigma_2}+ \\
&\quad \Delta B_{2\sigma_2})u_{2\sigma_2}(k) + g_2(x(k))
\end{aligned} \tag{32}$$

where

$$x_1(k) = [x_{11}(k)\ x_{12}(k)]^{\mathrm{T}}$$

$$x_2(k) = [x_{21}(k)\ x_{22}(k)\ x_{23}(k)]^{\mathrm{T}}$$

$$x(k) = [x_1(k)\ x_2(k)]^{\mathrm{T}}$$

$$A_{11} = \begin{bmatrix} 6 & 0 \\ 0 & 1.3 \end{bmatrix},\ A_{12} = \begin{bmatrix} 1.2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta A_{11} = D_{11}F_{11}(k)M_{11} = -10^{-3}\times\begin{bmatrix} \sin(k) & 0 \\ 0 & 4\cos(k) \end{bmatrix}$$

$$\Delta A_{12} = D_{12}F_{12}(k)M_{12} =$$

$$10^{-3}\times\begin{bmatrix} 18\cos(k) & 0 \\ 0 & -4\sin(k) \end{bmatrix}$$

$$\Delta B_{11} = D_{11}F_{11}(k)N_{11} = -10^{-3}\times\begin{bmatrix} \sin(k) \\ 0 \end{bmatrix}$$

$$\Delta B_{12} = D_{12}F_{12}(k)N_{12} = 10^{-3}\times\begin{bmatrix} 0 \\ 8\sin(k) \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\ D_{12} = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$F_{11}(k) = \begin{bmatrix} 0.1\sin(k) & 0 \\ 0 & -0.2\cos(k) \end{bmatrix}$$

$$F_{12}(k) = \begin{bmatrix} -0.3\cos(k) & 0 \\ 0 & -0.2\sin(k) \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ M_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix}$$

$$N_{11} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},\ N_{12} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.1 \end{bmatrix},\ A_{22} = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ B_{23} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta A_{21} = D_{21}F_{21}(k)M_{21} =$$

$$-10^{-3}\times\begin{bmatrix} 2\sin(k) & 0 & 0 \\ 0 & 6\sin(k) & 0 \\ 0 & 0 & 12\cos(k) \end{bmatrix}$$

$$\Delta A_{22} = D_{22}F_{22}(k)M_{22} =$$

$$-10^{-3}\times\begin{bmatrix} 2\cos(k) & 0 & 0 \\ 0 & 4\cos(k) & 0 \\ 0 & 0 & 9\cos(k) \end{bmatrix}$$

$$\Delta A_{23} = D_{23}F_{23}(k)M_{23} =$$

$$10^{-3}\times\begin{bmatrix}20\cos(k)&0&0\\0&9\sin(k)&0\\0&0&0\end{bmatrix}$$

$$\Delta B_{21}=D_{21}F_{21}(k)N_{21}=10^{-3}\times\begin{bmatrix}4\sin(k)\\0\\0\end{bmatrix}$$

$$\Delta B_{22}=D_{22}F_{22}(k)N_{22}=10^{-3}\times\begin{bmatrix}0\\2\cos(k)\\0\end{bmatrix}$$

$$\Delta B_{23}=D_{23}F_{23}(k)N_{23}=-10^{-3}\times\begin{bmatrix}0\\0\\12\sin(k)\end{bmatrix}$$

$$D_{21}=\begin{bmatrix}0.2&0&0\\0&0.1&0\\0&0&0.3\end{bmatrix}$$

$$D_{22}=\begin{bmatrix}-0.1&0&0\\0&0.2&0\\0&0&0.1\end{bmatrix}$$

$$D_{23}=\begin{bmatrix}0.5&0&0\\0&0.3&0\\0&0&0.4\end{bmatrix}$$

$$F_{21}(k)=\begin{bmatrix}0.1\sin(k)&0&0\\0&-0.2\sin(k)&0\\0&0&0.2\cos(k)\end{bmatrix}$$

$$F_{22}(k)=\begin{bmatrix}0.2\cos(k)&0&0\\0&0.1\cos(k)&0\\0&0&0.3\cos(k)\end{bmatrix}$$

$$F_{23}(k)=\begin{bmatrix}0.2\cos(k)&0&0\\0&0.3\sin(k)&0\\0&0&0.3\sin(k)\end{bmatrix}$$

$$M_{21}=\begin{bmatrix}-0.1&0&0\\0&0.3&0\\0&0&-0.2\end{bmatrix}$$

$$M_{22}=\begin{bmatrix}0.1&0&0\\0&-0.2&0\\0&0&-0.3\end{bmatrix}$$

$$M_{23}=\begin{bmatrix}0.2&0&0\\0&0.1&0\\0&0&0\end{bmatrix}$$

$$N_{21}=\begin{bmatrix}0.2\\0\\0\end{bmatrix},\ N_{22}=\begin{bmatrix}0\\0.1\\0\end{bmatrix},\ N_{23}=\begin{bmatrix}0\\0\\-0.1\end{bmatrix}$$

$$g_1(x(k))=0.1\times\begin{bmatrix}\sqrt{x_2^{\mathrm{T}}(k)x_2(k)}\\\sqrt{x_1^{\mathrm{T}}(k)x_1(k)}\end{bmatrix}$$

$$g_2(x(k))=0.2\times\begin{bmatrix}\dfrac{\sqrt{2x_1^{\mathrm{T}}(k)x_1(k)}}{2}\\\dfrac{\sqrt{2x_1^{\mathrm{T}}(k)x_1(k)}}{2}\\\sqrt{x_2^{\mathrm{T}}(k)x_2(k)}\end{bmatrix}.$$

It is easy to verify that each subsystem of the systems (32) cannot be stabilized by any controller. However, by utilizing the switching control approach presented in this paper, the whole large-scale system can be stabilized effectively.

Firstly, the parameters are given as

$$\alpha_{11}=\alpha_{12}=0.5$$
$$\alpha_{21}=\alpha_{22}=\alpha_{23}=\frac{1}{3}$$
$$\gamma_1=0.1,\gamma_2=0.2.$$

The solutions to the matrix inequalities (25) are found.

$$\varepsilon=0.1,\ \delta=1$$

$$P_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ P_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$K_{11}=\begin{bmatrix}-6&0\end{bmatrix},\ K_{12}=\begin{bmatrix}0&-3\end{bmatrix}$$

$$K_{21}=\begin{bmatrix}-3&0&0\end{bmatrix},\ K_{22}=\begin{bmatrix}0&-4&0\end{bmatrix}$$

$$K_{23}=\begin{bmatrix}0&0&-5\end{bmatrix}.$$

Take the initial condition

$$x_0=[20\ -20\ -10\ 25\ 10]^{\mathrm{T}},\ 1\leq k\leq 16.$$

Let

$$S_{ij}=\delta^{-1}(M_{ij}+N_{ij}K_{ij})^{\mathrm{T}}(M_{ij}+N_{ij}K_{ij})+\\(A_{ij}+B_{ij}K_{ij})^{\mathrm{T}}(P_i^{-1}-\delta D_{ij}D_{ij}^{\mathrm{T}})^{-1}(A_{ij}+B_{ij}K_{ij}).$$

By calculating, it is obtained

$$S_{11}=\begin{bmatrix}0.250\,0&0\\0&1.770\,4\end{bmatrix}$$

$$S_{12} = \begin{bmatrix} 1.622\,4 & 0 \\ 0 & 0.490\,0 \end{bmatrix}$$

$$Y_1 = S_{11} - S_{12} = \begin{bmatrix} -1.372\,4 & 0 \\ 0 & 1.280\,4 \end{bmatrix}$$

$$S_{21} = \begin{bmatrix} 0.490\,0 & 0 & 0 \\ 0 & 1.544\,5 & 0 \\ 0 & 0 & 1.369\,7 \end{bmatrix}$$

$$S_{22} = \begin{bmatrix} 1.232\,2 & 0 & 0 \\ 0 & 0.360\,0 & 0 \\ 0 & 0 & 1.544\,5 \end{bmatrix}$$

$$S_{23} = \begin{bmatrix} 1.960\,0 & 0 & 0 \\ 0 & 1.339\,7 & 0 \\ 0 & 0 & 0.250\,0 \end{bmatrix}$$

$$Y_2 = S_{21} - S_{22} = \begin{bmatrix} -0.742\,2 & 0 & 0 \\ 0 & 1.184\,5 & 0 \\ 0 & 0 & -0.174\,9 \end{bmatrix}$$

$$Y_3 = S_{21} - S_{23} = \begin{bmatrix} -1.470\,0 & 0 & 0 \\ 0 & 0.204\,9 & 0 \\ 0 & 0 & 1.119\,7 \end{bmatrix}$$

$$Y_4 = S_{22} - S_{23} = \begin{bmatrix} -0.727\,8 & 0 & 0 \\ 0 & -0.979\,7 & 0 \\ 0 & 0 & 1.294\,5 \end{bmatrix}.$$

Decentralized switching controllers are

$$u_{11}(k) = K_{11}x_1(k) = -6x_{11}(k)$$

$$u_{12}(k) = K_{12}x_1(k) = -3x_{12}(k)$$

$$u_{21}(k) = K_{21}x_2(k) = -3x_{21}(k)$$

$$u_{22}(k) = K_{22}x_2(k) = -4x_{22}(k)$$

$$u_{23}(k) = K_{23}x_2(k) = -5x_{23}(k).$$

And decentralized switching laws are as

$$\sigma_1 = \begin{cases} 1, & x_1^{\mathrm{T}}(k)Y_1x_1(k) < 0 \\ 2, & \text{otherwise} \end{cases}$$

$$\sigma_2 = \begin{cases} 1, & x_2^{\mathrm{T}}(k)Y_2x_2(k) < 0 \;\&\; x_2^{\mathrm{T}}(k)Y_3x_2(k) < 0 \\ 2, & x_2^{\mathrm{T}}(k)Y_2x_2(k) > 0 \;\&\; x_2^{\mathrm{T}}(k)Y_4x_2(k) < 0 \\ 3, & \text{otherwise.} \end{cases}$$

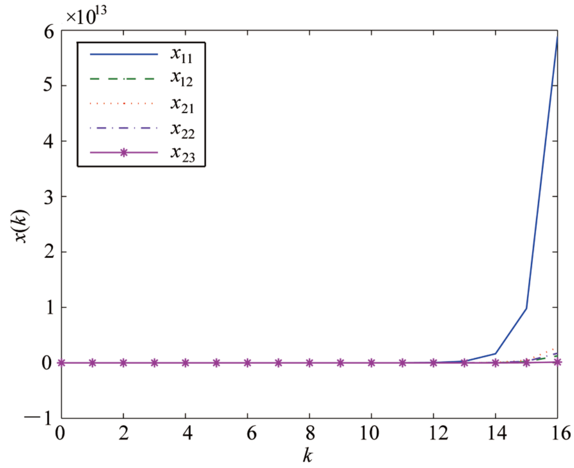


Fig. 1 State response of original large-scale systems with $A_{11}$ and $A_{21}$

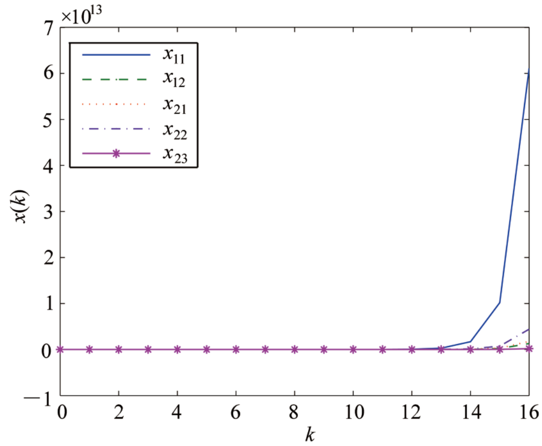


Fig. 2 State response of original large-scale systems with $A_{11}$ and $A_{22}$

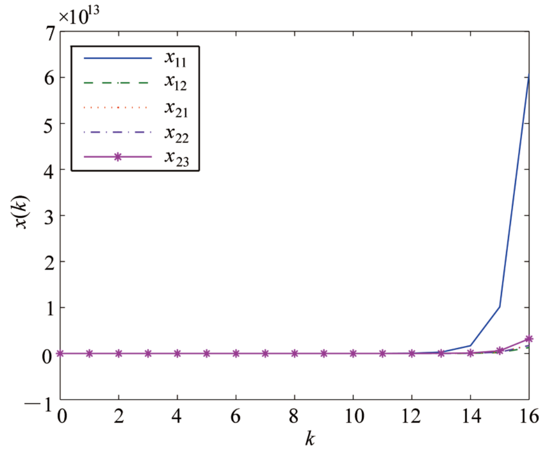


Fig. 3 State response of original large-scale systems with $A_{11}$ and $A_{23}$

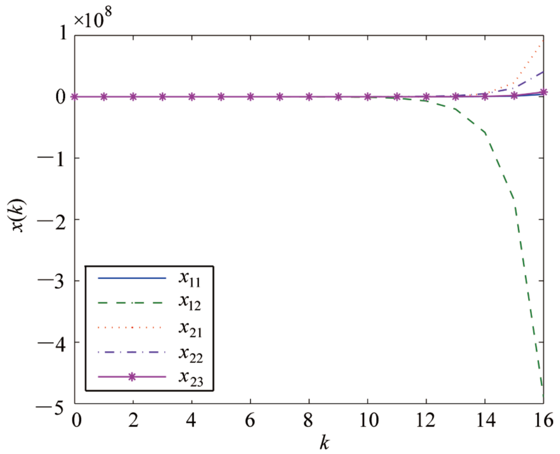


Fig. 4 State response of original large-scale systems with $A_{12}$ and $A_{21}$

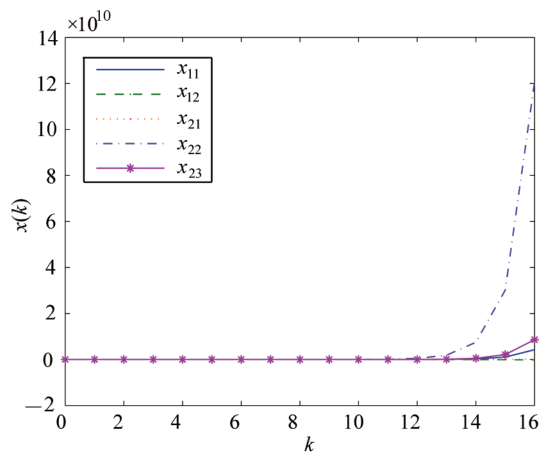


Fig. 5 State response of original large-scale systems with $A_{12}$ and $A_{22}$

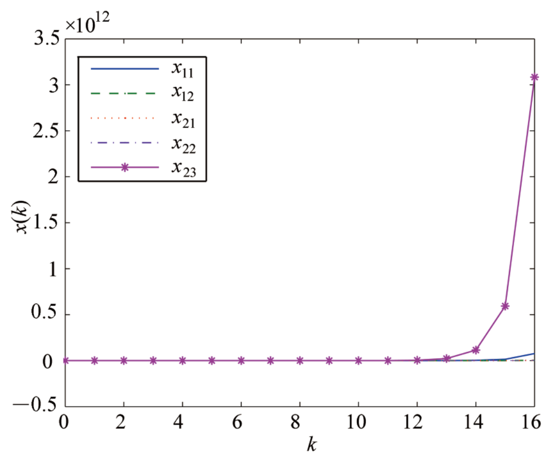


Fig. 6 State response of original large-scale systems with $A_{12}$ and $A_{23}$

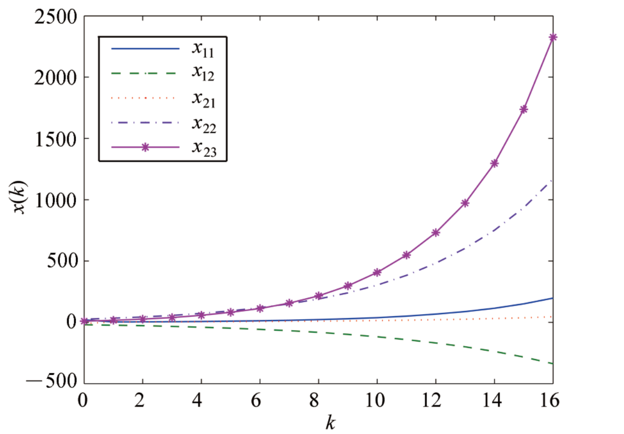


Fig. 7 State response of controlled large-scale systems with $A_{11}$ and $A_{21}$

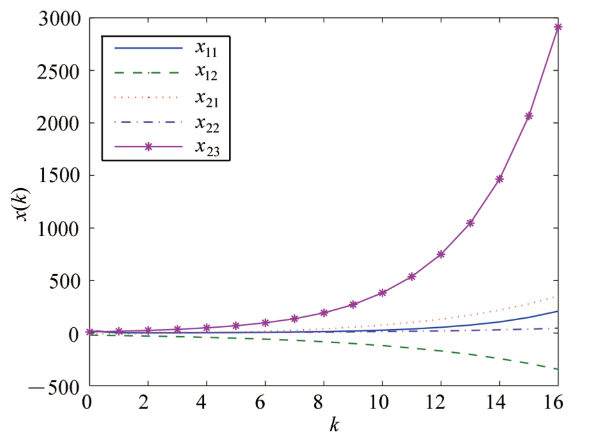


Fig. 8 State response of controlled large-scale systems with $A_{11}$ and $A_{22}$

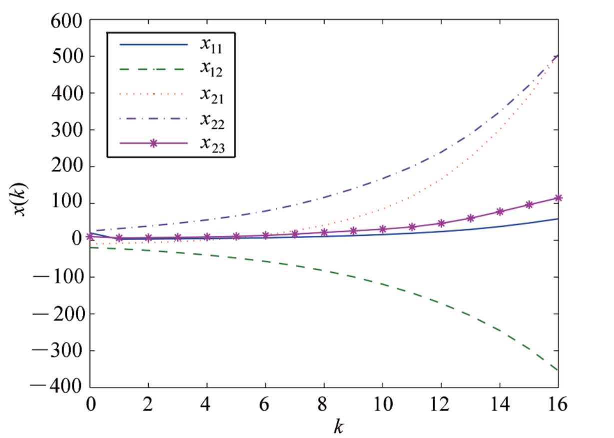


Fig. 9 State response of controlled large-scale systems with $A_{11}$ and $A_{23}$

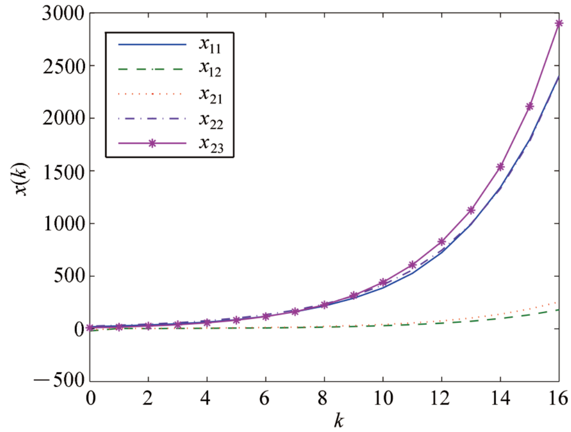


Fig. 10 State response of controlled large-scale systems with $A_{12}$ and $A_{21}$

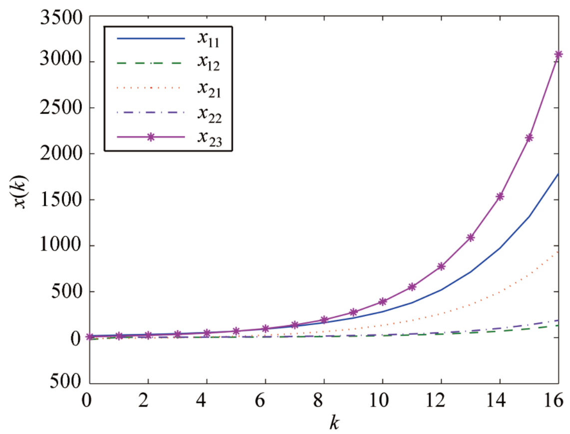


Fig. 11 State response of controlled large-scale systems with $A_{12}$ and $A_{22}$

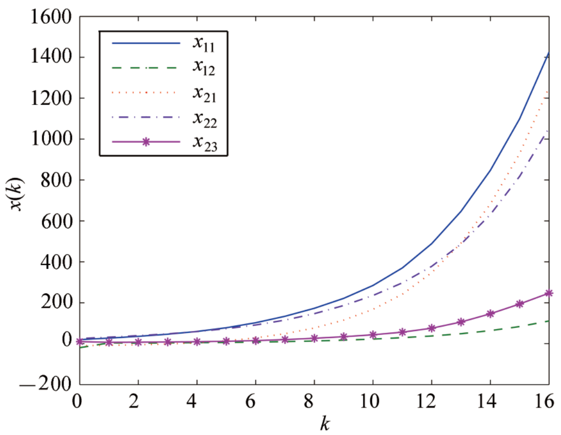


Fig. 12 State response of controlled large-scale systems with $A_{12}$ and $A_{23}$

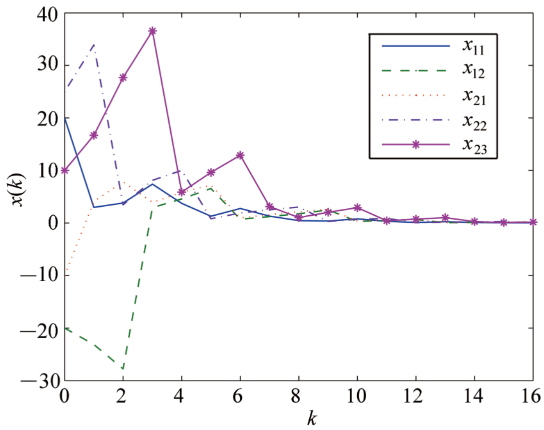


Fig. 13 Stable state response of discrete-time large-scale systems via switching

In order to show better the effectiveness of switching control approach presented in this paper, three results are compared by several numerical simulations in three different cases.

**Case 1.** The original large-scale systems are unstable when all decentralized controllers $u_{i\sigma_i}(k) = 0$ (see Figs. 1–6).

**Case 2.** The controlled large-scale systems without switching are also unstable when all decentralized controllers $u_{i\sigma_i}(k) = K_{i\sigma_i}x_i(k)$ are active (see Figs. 7–12).

**Case 3.** The controlled large-scale systems with switching are asymptotically stable when all decentralized controllers $u_{i\sigma_i}(k) = K_{i\sigma_i}x_i(k)$ are active (see Fig. 13).

Therefore, the class of discrete-time large-scale systems with parameter uncertainties can be stabilized robustly via a combinational control approach. In other words, when decentralized state feedback controllers and decentralized switching laws are active simultaneously, the closed-loop large-scale systems can achieve stability.

## 5 Conclusions

The problem of robust stabilization for a class of discrete-time switched large-scale systems with parameter uncertainties is studied. Based on matrix inequalities, a sufficient condition of stabilization for the systems is obtained. Robust state feedback decentralized controllers and corresponding decentralized laws are designed to stabilize the systems. A numerical example and its simulation illustrate the validity of switching control approach derived in this paper.

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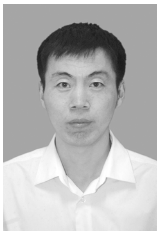

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