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A Linear Quadratic Controller Design Incorporating a Parametric Sensitivity Constraint

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Abstract: The purpose of this paper is to propose a synthesis method of parametric sensitivity constrained linear quadratic (SCLQ) controller for an uncertain linear time invariant (LTI) system. System sensitivity to parameter variation is handled through an additional quadratic trajectory parametric sensitivity term in the standard LQ criterion to be minimized. The main purpose here is to find a suboptimal linear quadratic control taking explicitly into account the parametric uncertainties. The paper main contribution is threefold: 1) A descriptor system approach is used to show that the underlying singular linear-quadratic optimal control problem leads to a non-standard Riccati equation. 2) A solution to the proposed control problem is then given based on a connection to the so-called Lur′e matrix equations. 3) A synthesis method of multiple parametric SCLQ controllers is proposed to cover the whole parametric uncertainty while degrading as less as possible the intrinsic robustness properties of each local linear quadratic controller. Some examples are presented in order to illustrate the effectiveness of the approach.

Keywords: Linear quadratic control, parametric uncertainties, trajectory sensitivity, non-standard Riccati equation, Lur′e matrix equations, linear time invariant (LMI), particle swarm optimization.

1 Introduction

Finding a parametric sensitivity constrained linear quadratic controller by including a quadratic trajectory sensitivity to the standard quadratic functional cost is still of major importance from a practical point of view. Previous papers of this author[1, 2] have given a new insight on such a control problem taking explicitly into account the parametric uncertainties of an uncertain linear time invariant (LTI) system. Following the same lines the current paper gives a revised proof.

Since the seminal works of Kreindler[3] or Newmann[4], the system sensitivity to parameter variations was handled, in the literature, in various ways through criterion sensitivity, closed-loop eigenvalues sensitivity or trajectory sensitivity measures (see e. g. [3−6] and references therein). The sensitivity to parameter variation remains a relevant control design criterion as attested by the large number of references on the subject see, e. g. [7−9]. In particular, finding a parametric sensitivity constrained linear quadratic controller by including quadratic trajectory sensitivity terms to the standard quadratic criterion is still a meaningful control problem to the design engineers. Moreover, when the bounds on parameter deviation are not a priori known, it is still of interest to be able to reduce the potential performance degradation that results from uncertain parameter deviation (with respect to some nominal values)[10].

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Furthermore, such a control problem can pave the way to a potentially new parametric sensitivity constrained H_2 control design due to the well-known superposition principle. Even if many attempts have been carried out in the literature in order to solve this problem in the H_2 context (see e. g., [11−13]), the existing methods are still either computationally unwieldy or suffer from an augmentation of the controller order. Hence, we believe that reconsidering the singular linear quadratic control problem underlying the proposed parametric sensitivity constrained linear quadratic control problem — henceforth denoted sensitivity constrained linear quadratic (SCLQ) in the sequel — may lead to a promising new solution that is computationally tractable.

In a first step, as a contribution of this paper we show that the SCLQ problem leads to a singular infinite-horizon LQ optimal control problem (i.e., where the matrix, conventionally denoted by R, weighting the input in the cost function is only positive semi-definite). Thus, relying on a descriptor system approach[14, 15], the link between the SCLQ control problem and a non-standard Riccati equation — with a pseudo-inverse of the weighting matrix R instead of its inverse — is explicitly investigated as a second contribution of this paper. It should be noted that the non-standard Riccati equation is known in the literature as a generalized Riccati equation (see [16] for some of its properties and the paper [17] for the connection between this generalized Riccati equation and the solution of the singular LQ optimal control problem).

Then, a new solution for the SCLQ problem is proposed based on a Lur′e matrix equations formulation of the underlying non-standard Riccati equation.

If the SCLQ controller reduces the trajectory sensitivity it has, however, to retain the advantages of the linear optimal control. Particularly, the robustness margins degradation and the extent of the parametric area on which acts the sensitivity reduction are directly linked to the choice of the weighting matrices.

Moreover, dividing the entire parametric uncertainty into small local subsets[18], in relation with the level of parametric sensitivity has not yet received adequate attention in the literature. To overcome these difficulties, we focus, in the last part of the current paper, on how to design a parsimonious partition of the uncertainty simultaneously with a set of SCLQ controllers in order to improve the total insensitivity to parametric variations while preserving, as far as possible, the classical robustness margins of the standard linear quadratic controllers.

This paper is organized as follows. Section 2 states the SCLQ control problem. It also recalls some existing attempts to solve such a control problem. Section 3 is devoted to the link between the SCLQ control problem and a nonstandard Riccati equation. A Lur′e matrix equations based solution is also presented in this section. Section 4 is devoted to the multiple SCLQ control problem formulation and the particle swarm optimization (PSO) based algorithm dedicated to solving this problem.

Furthermore, some examples are presented in Section 5 in order to demonstrate the applicability of the proposed approach. Finally, some concluding remarks take place in Section 6.

Notations. Hereafter $\otimes$ denotes the Kronecker product of matrices. $\circ$ is the element wise multiplication of vectors. The matrix I_n is the identity matrix of dimension $n \times n$. diag$(\cdot)$ is a block-diagonal matrix formed from the arguments. M_g (resp. M_ϕ, M_r and M_m) denotes the gain margin (resp. the phase margin, the delay margin and the modulus margin) of a given LTI system.

Finally, M^+ denotes the Moore-Penrose pseudo-inverse of the matrix M.

2 SCLQ control problem

2.1 Problem formulation

Consider the uncertain linear system given by

$$\dot{x} = A(\theta)x + B(\theta)u, \; x(0) = x_0 \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$ are matrix functions of a time-invariant parameter vector $\theta = [\theta_1, \cdots, \theta_q] \in \mathbf{R}^q$.

In this paper, we focus on a parameter dependence such that $A(\theta), B(\theta)$ are matrix functions with all entries of class $C^{n_\theta}, n_\theta \geq 1$. The system given by (1) is assumed to be controllable. Let us also define the trajectory sensitivity $x_\theta = \frac{\partial x}{\partial \theta}$ due to parametric deviation from a nominal value $\theta = \theta^0$. To simplify the presentation only the first-order derivative term will be considered in the sequel although the results easily extend to the higher-order derivative case.

Problem 1. Standard SCLQ control problem

SCLQ control problem consists of finding a control law u that minimizes a modified linear quadratic cost functional including a quadratic trajectory sensitivity given by

$$J_{SC} = \int_0^\infty x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru + x_\theta^{\mathrm{T}}Q_\theta x_\theta \mathrm{d}t \tag{2}$$

with Q, Q_θ are positive semi-definite matrices and R is a positive definite matrix. The trajectory sensitivity function, when differentiating the state space equations of (1) with respect to θ, taking into account a small deviation from the nominal value is described by the following state equation

$$\begin{aligned}&\dot{x}_\theta = A_\theta x + (I_q \otimes A)x_\theta + B_\theta u + (I_q \otimes B)u_\theta\\ &x_\theta(0) = 0\end{aligned} \tag{3}$$

with $A_\theta = \frac{\partial A(\theta)}{\partial \theta}_{|\theta=\theta^0}$, $B_\theta = \frac{\partial B(\theta)}{\partial \theta}_{|\theta=\theta^0}$ and $A = A(\theta^0), u_\theta = \frac{\partial u}{\partial \theta}$.

2.2 SCLQ problem as a structure constrained LQ control problem

As mentioned in the introduction, various scholars have attempted to solve the SCLQ problem. Fleming and Newmann proposed in [5], by augmenting the state vector x with the sensitivity vector x_θ, a full state-feedback control law of the form

$$u = Kx + Fx_\theta. \tag{4}$$

Hence, to implement such a control law the trajectory sensitivity vector x_θ has to be simulated. Partially differentiating (4) with respect to θ gives

$$u_\theta = Kx_\theta + F\frac{\partial x_\theta}{\partial \theta} \tag{5}$$

and substituting (4) and (5) into (3) leads to

$$\begin{aligned}\dot{x}_\theta = &(A_\theta + B_\theta K)x + (I_q \otimes B)F(\tfrac{\partial x_\theta}{\partial \theta}) + \\ &((I_q \otimes A) + (I_q \otimes B)K + B_\theta F)x_\theta.\end{aligned} \tag{6}$$

Thus, Fleming and Newmann proposed to neglect the second-derivative term $\frac{\partial x_\theta}{\partial \theta}$ in order to implement the control law (5). Unfortunately, neither the optimality nor the robustness of the resulting dynamic state-feedback control, due to this approximation, was discussed. In fact, note that the real control law in this case is of the form

$$\begin{aligned}u = &(F(sI_{nq} - (I_q \otimes A) + (I_q \otimes B)K + B_\theta F)^{-1}\\ &(A_\theta + B_\theta K) + K)x.\end{aligned} \tag{7}$$

Authors in [6, 19] were the first to propose to constrain the control law (4) as follows:

$$u = \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} x^{\mathrm{T}} & x_\theta^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{8}$$

so as the SCLQ problem can be formulated as an optimal structure constrained LQ problem that consists of finding a state feedback gain $K \in \mathbf{R}^{n\times m}$ such as the structured control law given by

$$\begin{bmatrix} u^{\mathrm{T}} & u_\theta^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = (I_{q+1} \otimes K)\begin{bmatrix} x^{\mathrm{T}} & x_\theta^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{9}$$

minimizes an approximated objective function of the form

$$\begin{aligned} &J=\int_0^\infty \left(\bar{x}^{\mathrm{T}}\bar{Q}\bar{x}+\bar{u}^{\mathrm{T}}\bar{R}\bar{u}\right)\mathrm{d}t\\ &\bar{x}^{\mathrm{T}}=\left[\begin{array}{cc} x^{\mathrm{T}} & x_\theta^{\mathrm{T}}\end{array}\right],\ \bar{u}^{\mathrm{T}}=\left[\begin{array}{cc} u^{\mathrm{T}} & u_\theta^{\mathrm{T}}\end{array}\right]\\ &\bar{R}=\mathrm{diag}\left(R,\varepsilon I_m\right),\ \bar{Q}=\mathrm{diag}\left(Q,\, Q_\theta\right),\ 0\prec\varepsilon\ll 1 \end{aligned} \tag{10}$$

under the following constraints

$$\begin{aligned} &\dot{\bar{x}}=\bar{A}\bar{x}+\bar{B}\bar{u},\ \bar{x}_0=\left[\begin{array}{cc} x_0 & 0\end{array}\right]^{\mathrm{T}}\\ &\bar{A}=\left[\begin{array}{cc} A & 0\\ A_\theta & (I_q\otimes A)\end{array}\right],\ \bar{B}=\left[\begin{array}{cc} B & 0\\ B_\theta & (I_q\otimes B)\end{array}\right]. \end{aligned} \tag{11}$$

Note that the cost functional J is a standard LQ cost function since matrix $\bar{R}>0$. This problem can obviously be formulated as an optimization problem of a linear objective under body mass index (BMI) constraints. The underlying optimization control problem is hard to handle owing to the type of structure imposed by (9). One can note that the difficulty is accentuated in the case where higher-order trajectory sensitivity is considered.

3 A new solution to the SCLQ control problem

The purpose of this section is to investigate the singular linear quadratic control problem underlying the SCLQ problem associated to the cost functional (2). Hence, characterizing the solution of this singular LQ problem will lead to a new formulation of the SCLQ problem where the structure constraint (9) is simplified. Thereby, based on a connection to the Lur′e matrix equations, an alternative solution to the SCLQ problem will be proposed. Let us consider the singular LQ problem (SCLQ) that consists of finding a control law minimizing an objective function of the form

$$\begin{aligned} &J_s=\int_0^\infty \left(\bar{x}^{\mathrm{T}}\bar{Q}\bar{x}+\bar{u}^{\mathrm{T}}\bar{R}\bar{u}\right)\mathrm{d}t\\ &\bar{R}=\mathrm{diag}\left(R,0_m\right),\ \bar{Q}=\mathrm{diag}\left(Q,Q_\theta\right). \end{aligned} \tag{12}$$

The first contribution of this paper is to characterize the solution of this singular LQ problem paving the way to a new solution to the SCLQ problem.

Theorem 1. Let the symmetric matrix $X^*>0$ be the maximal solution of the following nonstandard Riccati equation

$$\bar{A}^{\mathrm{T}}X+X\bar{A}-X\bar{B}\bar{R}^{+}\bar{B}^{\mathrm{T}}X+\bar{Q}=0 \tag{13}$$

where $X\in\mathbf{R}^{n(q+1)\times n(q+1)}, X>0$ is the unknown matrix, then all solutions of the infinite-horizon SCLQ problem are given by

$$\bar{u}^*\left(t\right)=-\bar{R}^{+}\bar{B}^{\mathrm{T}}X^*\bar{x}\left(t\right)+V^{\mathrm{T}}\tilde{u}\left(t\right) \tag{14}$$

where $\tilde{u}\in L_2$ is an arbitrary function and $V=\left[\begin{array}{cc} 0 & I_m\end{array}\right]$.

Moreover, the optimal cost is given by $J_s^*=\bar{x}_0^{\mathrm{T}}X^*\bar{x}_0$.

Proof. Consider the following augmented descriptor system defined by

$$(\Sigma)\left\{\begin{array}{l} E_a\left[\begin{array}{c}\dot{\bar{x}}\\ \dot{\xi}\end{array}\right]=A_a\left[\begin{array}{c}\bar{x}\\ \xi\end{array}\right]+\left(E_a\bar{x}_0\right)w+B_a\bar{u}\\ z=C_a\left[\begin{array}{c}\bar{x}\\ \xi\end{array}\right]+Dw+D_a\bar{u}\end{array}\right. \tag{15}$$

with

$$\begin{aligned} &E_a=\left[\begin{array}{cc} I_{n(q+1)} & 0\\ 0 & 0_m\end{array}\right],A_a=\left[\begin{array}{cc}\bar{A} & 0\\ 0 & -I\end{array}\right],\ B_a=\left[\begin{array}{c}\bar{B}\\ V\end{array}\right]\\ &C_a=\left[\begin{array}{c}\mathrm{diag}\left(\bar{Q}^{\frac{1}{2}},0_m\right)\\ V\end{array}\right],\ D_a=\left[\begin{array}{c} 0\\ \mathrm{diag}\left(R^{\frac{1}{2}},-I_m\right)\end{array}\right]\\ &D=0\end{aligned}$$

and w is a virtual exogenous input.

One can easily verify that

$$\begin{aligned} &\int_0^\infty z^{\mathrm{T}}z\mathrm{d}t=\\ &\int_0^\infty\left(\begin{array}{ccc}\bar{x}^{\mathrm{T}}\bar{Q}^{\frac{1}{2}} & u^{\mathrm{T}}R^{\frac{1}{2}} & 0\end{array}\right)\left(\begin{array}{c}\bar{Q}^{\frac{1}{2}}\bar{x}\\ R^{\frac{1}{2}}u\\ 0\end{array}\right)\mathrm{d}t=\\ &\int_0^\infty \bar{x}^{\mathrm{T}}\bar{Q}\bar{x}+u^{\mathrm{T}}Ru\mathrm{d}t=\\ &\int_0^\infty \bar{x}^{\mathrm{T}}\bar{Q}\bar{x}+\bar{u}^{\mathrm{T}}\bar{R}\bar{u}\mathrm{d}t=J_s.\end{aligned}$$

According to Lemma 10 in [15], the descriptor system (Σ) given by (15) verifies the following sufficient conditions $\left[\begin{array}{cc}\bar{x}_0^{\mathrm{T}} & 0\end{array}\right]Ker\left(E_a^{\mathrm{T}}\right)=\{0\}$ and $D=0$ under which the static gain matrix

$$K^*=-\left(D_a^{\mathrm{T}}D_a\right)^{-1}\left(D_a^{\mathrm{T}}C_a+B_a^{\mathrm{T}}P\right). \tag{16}$$

Minimizes J_s with the matrix P as a stabilizing solution for the following generalized algebraic Riccati equation (GARE)

$$\begin{aligned} &E_a^{\mathrm{T}}P=P^{\mathrm{T}}E_a\\ &A_a^{\mathrm{T}}P+P^{\mathrm{T}}A_a+C_a^{\mathrm{T}}C_a-\left(C_a^{\mathrm{T}}D_a+P^{\mathrm{T}}B_a\right)\\ &\qquad\left(D_a^{\mathrm{T}}D_a\right)^{-1}\left(D_a^{\mathrm{T}}C_a+B_a^{\mathrm{T}}P\right)=0.\end{aligned} \tag{17}$$

Note that the sufficient solvability conditions for the GARE (17) (see [20]) obviously hold for the system (Σ) given by (15). Let us now partition the matrix P as follows:

$$P=\left[\begin{array}{cc} X & P_1\\ P_2 & P_3\end{array}\right],\ P_3\in\mathbf{R}^m.$$

Thus, the first equation of (17) leads to $X=X^{\mathrm{T}}, P_1=0$ and the second one leads to the following equalities

$$\begin{aligned} &\bar{A}^{\mathrm{T}}X+X\bar{A}+\bar{Q}-\left(X\bar{B}+P_2^{\mathrm{T}}V\right)\\ &\qquad\left(\mathrm{diag}\left(R,-I\right)\right)^{-1}\left(X\bar{B}+P_2^{\mathrm{T}}V\right)^{\mathrm{T}}=0\end{aligned} \tag{18}$$

$$P_2+\left(I+P_3^{\mathrm{T}}\right)V\left(\mathrm{diag}\left(R,-I\right)\right)^{-1}\left(X\bar{B}+P_2^{\mathrm{T}}V\right)^{\mathrm{T}}=0 \tag{19}$$

$$P_3+P_3^{\mathrm{T}}+I+\left(I+P_3^{\mathrm{T}}\right)V(\operatorname{diag}(R,-I))^{-1} V^{\mathrm{T}}(I+P_3)=0. \tag{20}$$

From equation (20), it follows that

$$P_3=0. \tag{21}$$

Hence, substituting (21) into (19) we have

$$V\bar{B}^{\mathrm{T}}X=0. \tag{22}$$

Since the following equality holds

$$\left(X\bar{B}\right)(\operatorname{diag}(R,I))^{-1}\left(X\bar{B}\right)^{\mathrm{T}}= \left(X\bar{B}\right)(\operatorname{diag}(R,I))^{-1}\left(X\bar{B}\right)^{\mathrm{T}}+\left(X\bar{B}\right)V^{\mathrm{T}}V\left(X\bar{B}\right)^{\mathrm{T}}$$

equation (18) leads to (13). Furthermore, according to (16) the optimal state-feedback gain is given by

$$K^*=-\begin{bmatrix} R^{-1} & 0\\ 0 & -I\end{bmatrix}\begin{bmatrix}\bar{B}^{\mathrm{T}}X^* & V^{\mathrm{T}}\end{bmatrix}.$$

Thus, using (22) we have

$$\bar{u}(t)=K^*\begin{bmatrix}\bar{x}\\ \xi\end{bmatrix}= \left[\left(\begin{bmatrix}-R^{-1} & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0\\ V\end{bmatrix}\right)\bar{B}^{\mathrm{T}}X^* \quad V^{\mathrm{T}}\right]\begin{bmatrix}\bar{x}\\ \xi\end{bmatrix}= \begin{bmatrix}-\bar{R}^{+}\bar{B}^{\mathrm{T}}X^* & V^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\bar{x}\\ \xi\end{bmatrix}=-\bar{R}^{+}\bar{B}^{\mathrm{T}}X^*\bar{x}+V^{\mathrm{T}}\xi.$$

Moreover, on account of the structure of the matrix $\bar{R}$, a control law parameterized as in (14) is obviously optimal. Finally, according to Theorem 11 in [15], the optimal cost in this case is given by $J_s^*=\bar{x}_0^{\mathrm{T}}E_a^{\mathrm{T}}PE_a\bar{x}_0=\bar{x}_0^{\mathrm{T}}X^*\bar{x}_0$. □

Remark 1. Theorem 1 allows characterizing all optimal solutions of the singular LQ problem underlying the exact SCLQ control problem. Nevertheless, neither the non-standard Riccati (13) nor the GARE (17) allow to solve numerically the SCLQ problem because of the structure constraint (8). The following result provides the second contribution of this paper, which is a new linear time invariant (LMI) formulation of a suboptimal SCLQ control problem relying on the Lur′e matrix equations that were first introduced by Lur′e in [21] (interested readers can refer, for instance, to [22] and references therein).

Theorem 2. Suppose that the following matrix equations are solved for $X\in\mathbf{R}^{n(q+1)\times n(q+1)}$ and $K_0\in\mathbf{R}^{m\times n}$

$$\begin{cases}\bar{A}^{\mathrm{T}}X+X\bar{A}+\bar{Q}=\operatorname{diag}\left(K_0^{\mathrm{T}}K_0,0_{qn}\right)\\ X\bar{B}=\operatorname{diag}\left(K_0^{\mathrm{T}}R^{\frac{1}{2}},0_{qn\times m}\right)\\ X=X^{\mathrm{T}}>0\end{cases} \tag{23}$$

then $u^*=-R^{-\frac{1}{2}}K_0x$ is an optimal solution for the SCLQ problem.

Proof. According to Theorem 1, the SCLQ problem admits $\bar{u}^*$ given by (14) as an optimal solution with $X^*>0$ the maximal solution of (13). Thus, X^* is also a maximal solution for the following Lur′e equations

$$\begin{aligned}&\bar{A}^{\mathrm{T}}X+X\bar{A}+\bar{Q}=K_1^{\mathrm{T}}K_1\\ &X\bar{B}=K_1^{\mathrm{T}}L_1\\ &\bar{R}=L_1^{\mathrm{T}}L_1\end{aligned} \tag{24}$$

where $(K_1,L_1)\in\mathbf{R}^{p\times(q+1)n}\times\mathbf{R}^{p\times m}$ with $p\le m$ as small as possible. Moreover, since $\bar{R}=\operatorname{diag}(R,0_m), R>0$ we have $L_1=\begin{bmatrix}R^{\frac{1}{2}} & 0\end{bmatrix}$.

The solution of (24) implies that $K_1\bar{x}+L_1\bar{u}=K_1\bar{x}+R^{\frac{1}{2}}u=0$. At this stage, it is worth noting that a sufficient condition for obtaining a structured state-feedback gain of the form (8) is $K_1=\begin{bmatrix}K_0 & 0\end{bmatrix}$. Hence, it is easy to see that (24) reduces to (23). $u^*=-R^{-\frac{1}{2}}K_0x$ is an optimal solution for the SCLQ problem since the choice $\tilde{u}=-\left(I_{q+1}\otimes\left(R^{-\frac{1}{2}}K_0\right)\right)x_\theta$ can be made without loss of the optimality according to the result in Theorem 1. □

Remark 2. Equation (23) may be regarded as a necessary and sufficient condition for the existence of a structure constrained solution to the optimal SCLQ problem.

If the equations (23) do not admit a solution it is possible to find a suboptimal SCLQ controller. In fact, suppose that the symmetric matrix X^* is a maximal solution for the Lur′e matrix equations (24) (i.e., $X^*\ge X$ for all solutions X of (24)). According to the LMI formulation of the LQ problem (with a reversed inequality) first introduced in [23] and recalled in [24] (page 115), solving the following optimization problem with a linear objective under LMI/LME (LME stands for linear matrix equality here) constraints

$$\begin{aligned}&\max_{X,K_0}\bar{x}_0^{\mathrm{T}}X^*\bar{x}_0\\ &\bar{A}^{\mathrm{T}}X+X\bar{A}+\bar{Q}\ge\operatorname{diag}\left(K_0^{\mathrm{T}}K_0,0_{qn}\right)\\ &X\bar{B}=\operatorname{diag}\left(K_0^{\mathrm{T}}R^{\frac{1}{2}},0_{qn\times m}\right)\\ &X=X^{\mathrm{T}}>0\end{aligned} \tag{25}$$

leads to a suboptimal SCLQ controller of the form $u^*=-R^{\frac{1}{2}}K_0x$.

Remark 3. The Lur′e equations allow formulating a suboptimal SCLQ problem as an optimization problem of linear objective under LMI/LME constraints while taking explicitly into account the structure constraint (8).

4 Multiple SCLQ controllers design

4.1 Parsimonious tuning parameters

In order to have a reduced number of tuning parameters we use, hereafter, the well-known finite time controllability Gramian and some additional sensitivity reduction parameters associated to each parameter $\theta_i, i\in\{1,\cdots,q\}$. Thus we propose the following weighting matrices:

$$\begin{aligned}&R=T_c\int_0^{\mathrm{T}_c}\left(\mathrm{e}^{At}B\right)\left(\mathrm{e}^{At}B\right)^T\mathrm{dt},\quad \mathrm{T_c}\in\mathbf{R}^{+*}\\ &Q=I_n\\ &Q_\theta=\operatorname{diag}(\sigma_1,\cdots,\sigma_q)\otimes I_n,\quad \sigma_i\in\mathbf{R}^{+*}.\end{aligned} \tag{26}$$

The set of tuning parameters $\sigma_i \in \mathbf{R}, i \in \{1,\cdots,q\}$ have a direct effect on the sensitivity reduction from a nominal value $\theta = \theta_0$ and the extent of the parametric area on which it acts.

4.2 Problem formulation

Assume that N balls of predetermined, sufficiently small, radiuses δ_{θ^j} centred on $\theta = \theta^j$, $j \in \{1,\cdots,N\}$ and given by

$$\Theta^{\theta^j}\left(\delta_{\theta^j}, \mathbf{R}^q\right) = \left\{\theta \in \frac{\mathbf{R}^q}{\|\theta - \theta^j\|_2} \leq \delta_{\theta^j}\right\} \tag{27}$$

are uniformly distributed in the search space (i.e., parameter set). As an initialization step, it is supposed that

$$0 < \delta_{\theta_j} < \left(\frac{\varepsilon}{2}\right)\left\|\theta^j - \theta^{j+1}\right\|_2, \quad \forall j \in \{1,\cdots,N\}$$

for some $0 < \varepsilon < 1$. For each nominal value, namely $\theta = \theta^j$, an SCLQ controller, minimizing a criterion J^j of the form (12), is synthesized by means of the following set of tuning parameters: $T_c, \sigma^j_{i\in\{1,\cdots,q\}} \in \mathbf{R}$ where T_c is fixed and $\sigma^j_{i\in\{1,\cdots,q\}} = \delta_{\theta^j}$.

Problem 2. Multiple SCLQ controllers design

The multiple SCLQ control problem consists of determining a set of radiuses δ_{θ^j} and tuning parameters $\sigma^j_i, i \in \{1,\cdots,q\}$, $j \in \{1,\cdots,N\}$ solution of the following optimization problem

$$\begin{aligned}
&\max_{\delta^j>0,\sigma^j_i, i\in\{1,\cdots,q\}, j\in\{1,\cdots,N\}} \sum_{j=1}^{N} \delta_{\theta^j}\\
&\quad \max_j M^j_g < \bar{M}_g\\
&\quad \max_j M^j_\phi < \bar{M}_\phi\\
&\quad \max_j M^j_r < \bar{M}_r\\
&\quad \max_j M^j_m < \bar{M}_m\\
&\quad \sigma_{m^j_i} \geq \alpha\delta_{\theta^j}
\end{aligned} \tag{28}$$

with $\bar{M}_g$, $\bar{M}_\phi$, $\bar{M}_r$ and $\bar{M}_m$ are some predetermined upper bounds on different margins, $\alpha > 1$, $\sigma_{m^j_i} = \min_i \sigma^j_i$ and M^j_g, M^j_ϕ, M^j_r, M^j_m are the margins obtained with the SCLQ controllers minimizing criterions J^j and synthesized by means of $T_c, \sigma^j_{i\in\{1,\cdots,q\}}$.

Remark 4. The number of decision variables is $N(1+q)$.

Remark 5. The proposed margin constraints in Problem 2 can be considered entirely or in part. In fact, in some cases only the constraints on M_ϕ, M_r are needed since M_g and M_m are slightly degraded.

Remark 6. To reduce the number of decision variables it is possible to substitute the last constraint in (28) by an equality constraint of the form $\forall i \in \{1,\cdots,q\}, \sigma^j_i = \alpha\delta_{\theta^j}$ with some given $\alpha > 1$. In other words, it is possible to limit the decision variables to the radiuses of the N small l_2 balls given by (27).

Remark 7. The parameter α allows reducing or expanding the space search for the parametric sensitivity reduction tuning parameters and from thence it has a direct impact on the global computation time.

One can note that maximizing the criterion $\sum_{j=1}^{N}\delta_{\theta^j}$ implies a maximization of the sensitivity tuning parameters $\sigma^j_i, i \in \{1,\cdots,q\}$ which is, indeed, needed for the sensitivity reduction. In opposition, it is also the margin constraints in (28). Moreover, the sample generation problem which consists of generating real vector samples $\theta^j \in \mathbf{R}^q$, uniformly distributed in the search space Θ, can be reduced to multiple random vector generation for which the technique borrowed from [25] can be used. In fact, the algorithm used in this note, for this purpose, is borrowed from the randomized algorithms control toolbox (RATC) (http://ract.sourceforge.net).

4.3 A PSO based algorithm

Firstly introduced by Eberhart and Kennedy[26], PSO is inspired by the social behavior, for instance, of bird flocking or fish schooling. Let us consider the following optimization problem:

$$\min_{x\in\Lambda} \quad f(x) \tag{29}$$

particles are moving in the search space Λ. Each particle has its own position (x^k_p: position of particle p at iteration k) and velocity (v^k_p: velocity of particle p at iteration k). It can remember where it has found its best position ($b^k_p = \arg\min(f(x)), x \in \{b^{k-1}_p, x^k_p\}$: best position found by particle p until iteration). Each particle has also some "co-particles" ($V(x^k_p) \subset \{1,2,\cdots,P\}$ set of "co-particles" of particle p at iteration). $g^k_p = \arg\min f(x)$, $x \in \{b^k_i, i \in V(x^k_p)\}$ denotes the best position found by the co-particles of particle p until iteration k.

The particles move in Λ according to the following transition equations:

$$\begin{aligned}
v^{k+1}_p &= c_0.v^k_p + c_1 \circ (b^k_p - x^k_p) + c_2 \circ (g^k_p - x^k_p)\\
x^{k+1}_p &= x^k_p + v^{k+1}_p.
\end{aligned} \tag{30}$$

In this equation c_0 is the inertia factor and c_1 (respectively c_2) is a random number in $[0, \bar{c}_1]$ (respectively in $[0, \bar{c}_2]$). To guarantee the convergence of the PSO algorithm the choice of parameters (c_0, c_1, c_2) is central[27]. It is well-known that, in case of a large number of decision variables, the PSO algorithm may suffer from undesirable convergence to local minima. This can be the case when dealing with the multiple SCLQ control synthesis. To overcome this difficulty, some recent PSO modified versions have been proposed. The underlying idea is to modify the rules (30) so as to bring a random movement towards the best particle. Particularly, a step is considered as a success if the best value found by the particles is improved and a failure otherwise. Then the number of consecutive successes and failures is used in the modified transition rule.

Following the lines of the algorithm presented in [28], the new transition rules are reconsidered as

$$\begin{aligned}
v^{k+1}_p &= c_0.v^k_p + (g^k_p - x^k_p) + \rho^k(1 - 2r_{[0,1]})\\
x^{k+1}_p &= x^k_p + v^{k+1}_p
\end{aligned} \tag{31}$$

where $r_{[0,1]}$ denotes a random vector in [0,1]. The value of ρ^k is updated at each iteration according to the following equation:

$$\rho^{k+1} = \begin{cases} 2\rho^k, & \text{if } nb\_success > s_c \\ 0.5\rho^k, & \text{if } nb\_failure > f_c \\ \rho^k, & \text{otherwise} \end{cases} \qquad (32)$$

where $nb\_success$ is the number of consecutive successes and $nb\_failure$ the number of consecutive failures. s_c and f_c are some additional tuning parameters.

The following algorithm *sketches* in few lines the proposed method for solving Problem 2.

Algorithm 1.

Step 0. Fix T_c, $\alpha > 1$, $\varepsilon < 1$, N and a maximum iteration number $\bar{k}$.

Step 1. Initialization

Generate N real vector samples $\theta^j \in \mathbf{R}^q$, uniformly distributed in Θ. Choose randomly N parameters δ_{θ^j} such that

$$0 < \delta_{\theta^j} < \frac{\varepsilon}{2}\left\|\theta^j - \theta^{j+1}\right\|_2.$$

Fix $\sigma^j_{i\in\{1,\cdots,q\}} = \delta_{\theta^j} j \in \{1,\cdots,N\}$.

Step 2. PSO optimization

Associate the particle positions in the perturbed PSO algorithm, with the transition rule given by (31), to

$$x_p = \left[\delta^1,\cdots,\delta^N,\sigma^1_1,\cdots,\sigma^N_1,\cdots,\sigma^1_q,\cdots,\sigma^N_q\right].$$

Optimize (28) until a stopping criterion is verified or $\bar{k}$ is reached.

Step 3. Uncertainty set covering test

Generate randomly N^{2q} points in Θ. If all these points belong to $\bigcup_j \Theta^{\theta^j}$ stop. Else set $N = N+1$ and go to Step 1.

Remark 8. Note that when the local regions overlap one can use the SCLQ controller associated to the ball with the closest center to the considered point.

5 Numerical examples

5.1 Example 1

Let us consider the system with a rational parametric dependence given by

$$\dot{x} = \begin{bmatrix} \theta^2 & \dfrac{-0.1}{\theta} \\ -1 & -\theta \end{bmatrix} x + \begin{bmatrix} 2\theta \\ 1 \end{bmatrix} u$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad |\theta - \theta_0| \le 0.5 \qquad (33)$$

where $\theta_0 = 1$. The standard LQ controller designed, according to (26), for $T_c = 1.5$ leads to

$$u^* = [-1.128\,5 \quad 0.064\,8]\, x. \qquad (34)$$

Solving the LMI/LME problem with $\sigma_1 = 10$ (note that LMI/LME programming was done using YALMIP parser[29] and solved with SeDuMi solver[30]) leads to the following suboptimal SLQ controller

$$u^* = [-3.885\,5 \quad 1.470\,3]\, x. \qquad (35)$$

The effect of the sensitivity reduction on the closed-loop system trajectory is easily noticeable when comparing the two controllers (LQ and SCLQ) for $\theta = \theta_0$ and $\theta = 0.5$ (see Fig. 1). The gain and phase margins are somehow preserved (i.e., gain margins are ∞ for two controllers and phase margins slightly decrease from $M_\phi = 61.12°$ to $M_\phi = 53.42°$. In contrast, the delay margin is markedly reduced from $M_r = 0.55$ s to $M_r = 0.19$ s.

5.2 Example 2

Consider the second order system with a polynomial parametric dependence given by

$$\dot{x} = \begin{bmatrix} \theta^2 & 0 \\ 1 & -\theta \end{bmatrix} x + \begin{bmatrix} 1 \\ 2\theta \end{bmatrix} u, \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad (36)$$

where $\theta_0 = 1$. The standard LQ design, for $Q = I_2, R = 1$ leads to

$$u^* = \begin{bmatrix} -2.899\,6 & -0.167\,6 \end{bmatrix} x \qquad (37)$$

when $\theta = \theta_0$. The Lur'e equations (23) for $Q = Q_\theta = I_2, R = 1$ do not admit a solution. Solving the LMI/LME problem (25) leads to the following suboptimal SCLQ controller

$$u^* = \begin{bmatrix} -4.022\,7 & 0.052\,3 \end{bmatrix} x. \qquad (38)$$

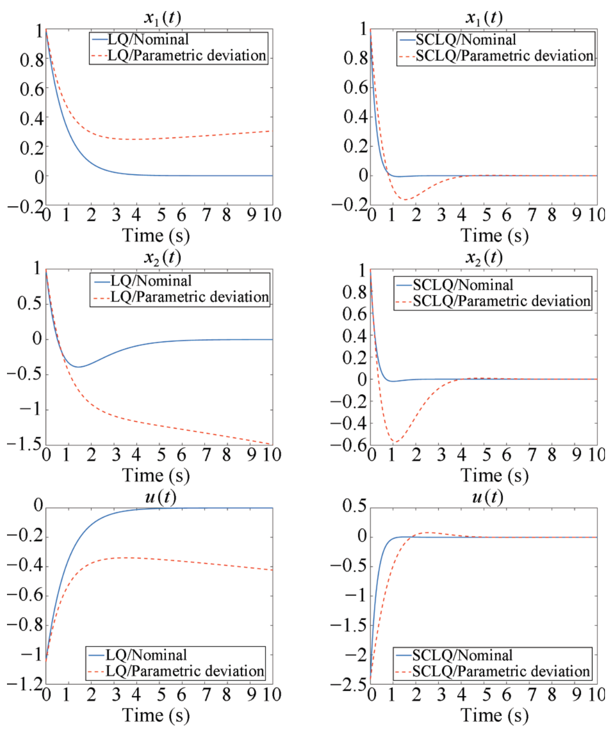

Fig. 1 Closed-loop behavior with the SCLQ controller vs. the standard LQ controller for Example 1

Note that all LMI/LME programming was done using YALMIP parser[29] and solved with SeDuMi solver[30].

Comparing the two controllers for $\theta = \theta_0$ and $\theta = 1.35$ (see Fig. 2) it is easy to see that the closed-loop system trajectory deviates less from the nominal trajectory for the suboptimal SCLQ controller given by (38).

5.3 Example 3

Let us consider now the second order system with a rational parametric dependence given by

$$\dot{x} = \begin{bmatrix} \theta^3 - 1 & 0 \\ 1 & \dfrac{-1}{\theta} \end{bmatrix} x + \begin{bmatrix} 1 \\ 2\theta \end{bmatrix} u,\ x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{39}$$

where $\theta_0 = 1$. The optimal control law (standard LQ problem), for $Q = I_2, R = 1$, is found to be

$$u^* = -\begin{bmatrix} 0.857\,2 & 0.557\,1 \end{bmatrix} x \tag{40}$$

when $\theta = \theta_0$. Here, we have made the choice of considering a SCLQ problem with a first and a second-derivative of the trajectory sensitivity such as

$$J_{SC} = \int_0^\infty x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u + x_\theta^{\mathrm{T}} Q_\theta x_\theta + x_{\theta\theta}^{\mathrm{T}} Q_{\theta\theta} x_{\theta\theta} \mathrm{d}t \tag{41}$$

with $x_{\theta\theta} = \frac{\partial x_\theta}{\partial \theta}$. The objective is to find a structured state-feedback gain of the form

$$u = \begin{bmatrix} K & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_\theta \\ x_{\theta\theta} \end{bmatrix} \tag{42}$$

minimizing an objective function of the form

$$\begin{gathered} J = \int_0^\infty \left(\bar{x}^{\mathrm{T}} \bar{Q} \bar{x} + \bar{u}^{\mathrm{T}} \bar{R} \bar{u} \right) \mathrm{d}t \\ \bar{x}^{\mathrm{T}} = \begin{bmatrix} x^{\mathrm{T}} & x_\theta^{\mathrm{T}} & x_{\theta\theta}^{\mathrm{T}} \end{bmatrix},\ \bar{u}^{\mathrm{T}} = \begin{bmatrix} u^{\mathrm{T}} & u_\theta^{\mathrm{T}} & u_{\theta\theta}^{\mathrm{T}} \end{bmatrix} \\ \bar{R} = \mathrm{diag}\,(R, 0_{2m}),\ \ \bar{Q} = \mathrm{diag}\,(Q, Q_\theta,\ Q_{\theta\theta}) \end{gathered} \tag{43}$$

under the following constraints

$$\begin{gathered} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u},\quad \bar{x}_0^{\mathrm{T}} = \begin{bmatrix} x_0^{\mathrm{T}} & 0_{1\times 2n} \end{bmatrix} \\ \bar{A} = \begin{bmatrix} A & 0 & 0 \\ A_\theta & (I_q \otimes A) & 0 \\ A_{\theta\theta} & 2A_\theta & (I_q \otimes A) \end{bmatrix},\ A_{\theta\theta} = \frac{\partial A_\theta}{\partial \theta} \\ \bar{B} = \begin{bmatrix} B & 0 & 0 \\ B_\theta & (I_q \otimes B) & 0 \\ B_{\theta\theta} & 2B_\theta & (I_q \otimes B) \end{bmatrix},\ B_{\theta\theta} = \frac{\partial B_\theta}{\partial \theta}. \end{gathered} \tag{44}$$

Solving the associated LMI/LME problem, with $Q = I_2, Q_\theta = Q_{\theta\theta} = 0.1 I_2, R = 1$, leads to the following suboptimal SCLQ controller

$$u^* = \begin{bmatrix} -1.595\,0 & 0.009\,8 \end{bmatrix} x. \tag{45}$$

Fig. 3 shows a performance comparison of the two controllers for $\theta = \theta_0$ and $\theta = 1.2$. Clearly, the standard LQ controller leads to instability when $\theta = 1.2$ while closed-loop with the SCLQ controller is not only stable but deviates slightly from the nominal value $\theta = \theta_0$ case.

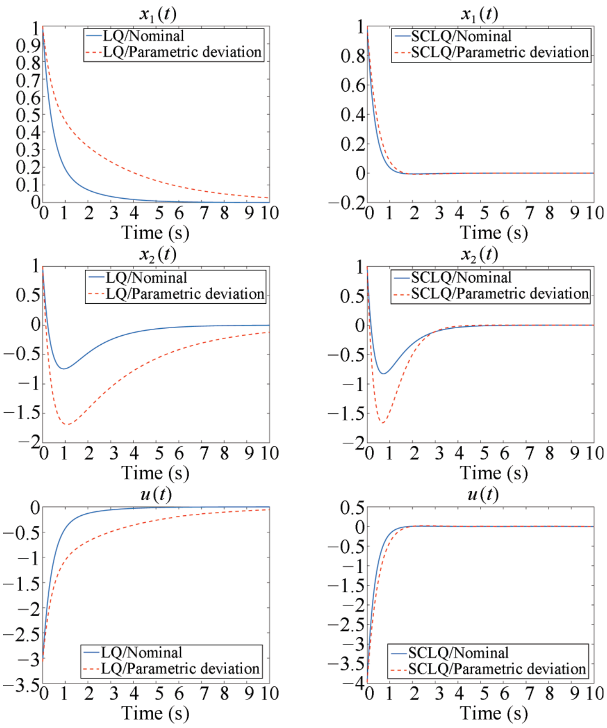

Fig. 2 Performances obtained with the LQ controller (1st column) and SCLQ controller (2nd column) for Example 2

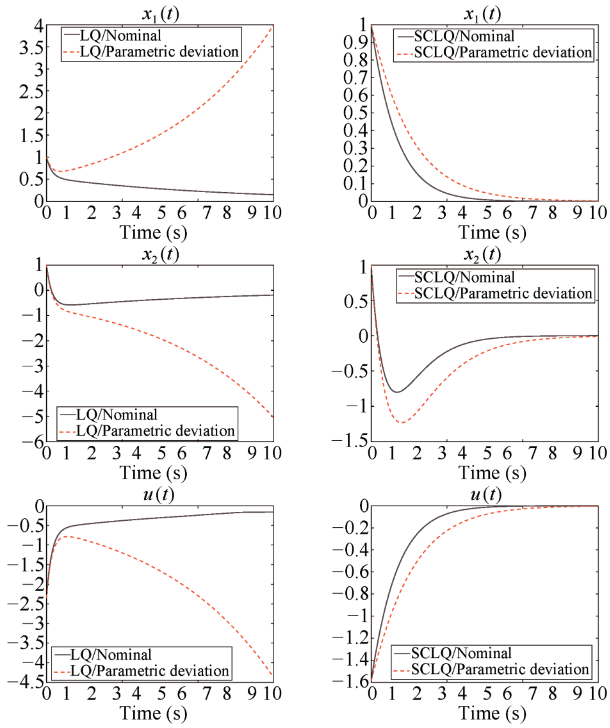

Fig. 3 Performances obtained with the LQ controller (1st column) and SCLQ controller (2nd column) for Example 3

5.4 Example 4

The following example, dealing with a robust vehicle dynamics control as considered in [31], shows the applicability of the proposed method for solving Problem 2. In this example the lateral velocity (V_y) and the yaw velocity (ψ) of a vehicle have to be controlled through two control inputs, namely the yaw moment (C_z), obtained by differential braking, and the rear steering (α). The vehicle must stay near to the desired trajectory even in the presence of some disturbance efforts acting on it and represented by a lateral force and a yaw moment. The well-known "bicycle model" parameterized by the road friction parameter and the vehicle longitudinal velocity, is considered. A normalization of the problem is used in order to have two uncertain parameters henceforth denoted θ_1 and θ_2 such that

$$\Theta\left(1, \mathbf{R}^2\right) = \left\{ \begin{array}{l} \theta = \begin{pmatrix} \theta_1 & \theta_2 \end{pmatrix}^{\mathrm{T}} \in \dfrac{\mathbf{R}^2}{\|\theta - \theta_0\|_2 \leq 1} \\ \theta_0 = \begin{pmatrix} 1 & 1 \end{pmatrix}^{\mathrm{T}} \end{array} \right\}.$$

At Step 0 of Algorithm 1, the following data is fixed $T_c = 1$, $\alpha = 10$, $\varepsilon = 0.1$ and $N = 10$. This leads, according to Step 2, to the distribution of the centers illustrated by Fig. 4.

With radiuses: $\begin{bmatrix} \delta_{\theta^1} & \cdots & \delta_{\theta^{10}} \end{bmatrix} = 10^{-2}$ [4.34 3.98 4.80 5.32 2.81 5.47 5.10 3.28 6.68 8.28]. Step 3 is carried out with the following upper bounds $\bar{M}_g = 8\,\mathrm{dB}$, $\bar{M}_\phi = 45°$, $\bar{M}_r = 1\,\mathrm{s}$ and $\bar{M}_m = 0.5$.

After 6 iterations of Algorithm 1 the entire uncertainty set was substantially covered by 16 disks of different radiuses as shown in Fig. 5.

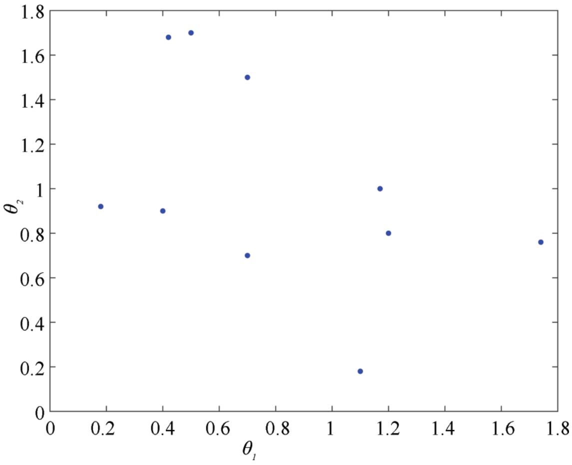

Fig. 4 10 disk centers uniformly distributed in $\Theta\left(1, \mathbf{R}^2\right)$

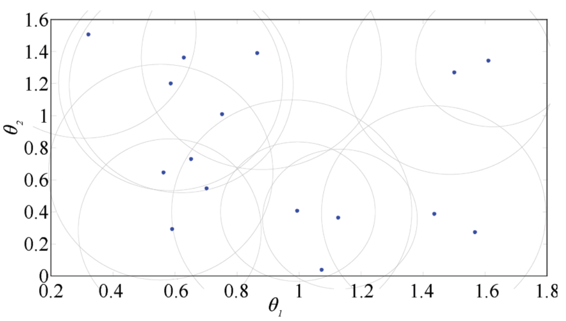

Fig. 5 10 disk centers uniformly distributed in $\Theta\left(1, \mathbf{R}^2\right)$

Therefore, 16 SCLQ controllers are designed which practically cover the entire uncertainty set.

The SCLQ controller (Fig. 6) designed for the associated local region clearly improves the parametric robustness in comparison with the standard LQ controller (Fig. 7).

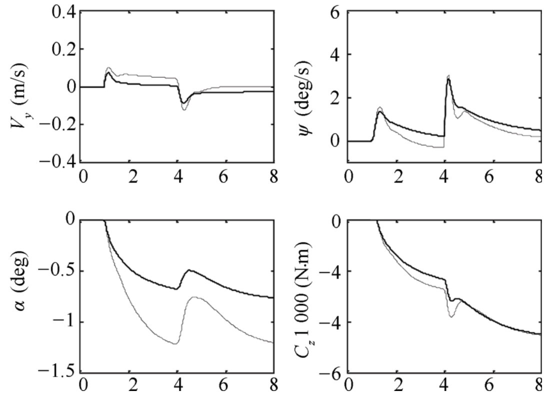

Fig. 6 Closed-loop behavior with the SCLQ controller

If these examples show an undeniable effect of the parametric sensitivity reduction they also point out clearly the contribution of the proposed algorithm to the important question of the weighting matrices choice (see for instance [32]).

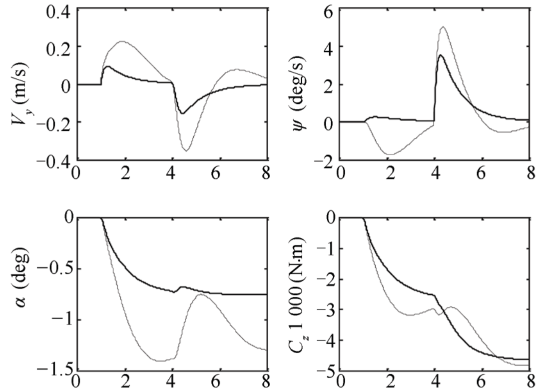

Fig. 7 Closed-loop behavior with the LQ controller

6 Concluding remarks

This paper shows that reconsidering the singular linear quadratic control problem underlying the parametric SCLQ control design problem leads to a new formulation and, by the same token, to a necessary and sufficient condition of the optimal SCLQ controller existence. A suboptimal parametric SCLQ controller is then obtained by means of a computationally tractable optimization problem under some LMI/LME constraints.

Furthermore, a new synthesis method for multiple parametric SCLQ controllers is proposed. These controllers are designed to cover the entire parametric uncertainty set while degrading as less as possible the intrinsic robustness properties of each local linear quadratic controller. An adequate PSO based algorithm was presented to find the best distribution of the local design regions simultaneously with the set of the sensitivity reduction tuning parameters.

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