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New LMI Conditions for Reduced-order Observer of Lipschitz Discrete-time Systems: Numerical and Experimental Results

Noussaiba Gasmi[1,2] Assem Thabet[2] Mohamed Aoun[2]

[1]Research Center for Automatic Control of Nancy, CRAN-CNRS UMR 7039, University of Lorraine, Cosnes et Romain 54400, France

[2]Modeling, Analysis and Control of Systems (MACS) Laboratory, National Engineering School of Gabes, University of Gabes, Gabes 6072, Tunisia

Abstract: The objective of this paper is to propose a reduced-order observer for a class of Lipschitz nonlinear discrete-time systems. The conditions that guarantee the existence of this observer are presented in the form of linear matrix inequalities (LMIs). To handle the Lipschitz nonlinearities, the Lipschitz condition and the Young′s relation are adequately operated to add more degrees of freedom to the proposed LMI. Necessary and sufficient conditions for the existence of the unbiased reduced-order observer are given. An extension to H_∞ performance analysis is considered in order to deal with H_∞ asymptotic stability of the estimation error in the presence of disturbances that affect the state of the system. To highlight the effectiveness of the proposed design methodology, three numerical examples are considered. Then, high performances are shown through real time implementation using the ARDUINO MEGA 2560 device.

Keywords: Reduced-order observer, discrete-time systems, Lipschitz systems, H_∞, ARDUINO MEGA 2560 device.

1 Introduction and preliminaries

1.1 Introduction

Observer and control design for linear and nonlinear systems has attracted the attention of several researchers over the past two decades[1–9]. Generally, the size of the output vector is smaller than that of the state vector for several reasons (technical implementation, cost, etc.). Therefore, at a given time t, the state can not be deduced algebraically from the output measurements. The purpose of an observer is to give an estimate of the current value of the state as a function of the system inputs and outputs. Particular attention was given to the reduced-order observer since it allows the estimation of only the unavailable components of the state[10, 11]. Until today, the majority of existing works on reduced-order observer design deals only with continuous linear systems[12–14]. On the other hand, there are not enough works in the discrete case[15]. Returning to reality, it is almost impossible to find a system without a nonlinear part. Furthermore, many physical systems satisfy the Lipschitz condition, therefore it is the most used for the synthesis of nonlinear observers. Some works on state observer design for this class of systems were recently developed in [16–18].

In this context, the main contribution of this paper is to deal with reduced-order observers for both discrete-time and Lipschitz nonlinear systems. A useful decomposition (into two sub-functions) and reformulation of the Lipschitz property allow us to combine the results of Lipschitz systems and unknown inputs to synthesize a reduced-order observer for this class of nonlinear systems. Then, thanks to a judicious use of Young′s relation, additional degrees of freedom are included in the linear matrix inequality (LMI) constraints. Indeed, the asymptotic stability of the estimation error is guaranteed. Afterwards, the obtained result will be extended to the case of nonlinear systems in the presence of disturbances with bounded energy.

This paper is organized as follows: Section 2 presents some preliminaries, the nonlinear discrete-time system and the considered observer. The conditions of unbiasedness of the estimated error are considered and the reduced-order observer design for Lipschitz discrete-time systems is detailed in Section 3. In Section 4, an extension to H_∞ performance analysis is presented. Section 5 is devoted to emphasizing the effectiveness of the proposed design methodology through three numerical examples.

Notation. The following notation will be used throughout this paper:

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1) $e_m(i) = \left[\underbrace{0, \cdots, 0, \overbrace{1}^{i\text{-th}}, 0, \cdots, 0}_{m-\text{components}} \right]^{\mathrm{T}} \in \mathbf{R}^m,\ m \geq 1,$ is a vector of the canonical basis of $\mathbf{R}^m$;

2) X^{T} is the transposed matrix of X;

3) X is a square matrix. The notation $X > 0$ ($X < 0$) means that X is positive definite (negative definite);

4) In a matrix, the notation $(*)$ is used for the blocks induced by symmetry.

1.2 Preliminaries

This section is devoted to presenting some preliminaries that are useful for developing the proposed design methodology.

Definition 1.[19] Consider the following two vectors:

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbf{R}^n \quad \text{and} \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \in \mathbf{R}^n \ .$$

For all $i = 0, \cdots, n$, an auxiliary vector $X^{Y_i} \in \mathbf{R}^n$ corresponding to X and Y is defined as

$$\begin{cases} X^{Y_i} = \begin{bmatrix} y_1 \\ \vdots \\ y_i \\ x_{i+1} \\ \vdots \\ x_n \end{bmatrix}, \quad \text{for} \quad i = 1, \cdots, n \\ X^{Y_0} = X. \end{cases} \tag{1}$$

Lemma 1. Consider a function h: $\mathbf{R}^n \to \mathbf{R}^n$. Then, the following expressions are equivalent[19]:

1) h is globally Lipschitz with respect to its argument, i.e.,

$$\| h(X) - h(Y) \| \leq \alpha_h \| X - Y \|,\ \forall X, Y \in \mathbf{R}^n. \tag{2}$$

2) For all $i, j = 1, \cdots, n$, there exist functions

$$\mathfrak{h}_{ij}\colon\ \mathbf{R}^n \times \mathbf{R}^n \longrightarrow \mathbf{R} \tag{3}$$

and constants $\underline{\alpha}_{\mathfrak{h}_{ij}}$ and $\overline{\alpha}_{\mathfrak{h}_{ij}}$, so that $\forall\ X, Y \in \mathbf{R}^n$

$$h(X) - h(Y) = \sum_{i,j=1}^{n,n} \mathfrak{h}_{ij} \mathcal{H}_{ij} (X - Y) \tag{4}$$

and

$$\underline{\alpha}_{\mathfrak{h}_{ij}} \leq \mathfrak{h}_{ij} \leq \overline{\alpha}_{\mathfrak{h}_{ij}} \tag{5}$$

where
$\mathfrak{h}_{ij} \triangleq \mathfrak{h}_{ij}(X^{Y_{j-1}}, X^{Y_j})$ and $\mathcal{H}_{ij} = e_n(i) e_n^{\mathrm{T}}(j)$.

Lemma 2. Reformulation of Young′s lemma[20]. Given two matrices X and Y of appropriate dimensions, then the following inequality holds for any symmetric positive definite matrix S of appropriate dimension:

$$X^{\mathrm{T}} Y + Y^{\mathrm{T}} X \leq \frac{1}{2} [X + SY]^{\mathrm{T}} S^{-1} [X + SY]. \tag{6}$$

This lemma will be one of the tools for the main contributions of this paper. It allows us to provide less restrictive and conservative LMI conditions for the considered class of nonlinear systems.

2 Problem statement

As previously stated, we address the problem of the design of a reduced-order observer for a class of nonlinear discrete-time systems. In the following, we present the system description, the structure of the observer and a useful method of decomposing the nonlinear part.

2.1 System description

Let us consider a nonlinear discrete-time system described by

$$x_{k+1} = Ax_k + Bu_k + h(x_k, u_k) \tag{7a}$$

$$y_k = Cx_k \tag{7b}$$

$$z_k = Lx_k \tag{7c}$$

where $x_k \in \mathbf{R}^n$, $u_k \in \mathbf{R}^m$ and $y_k \in \mathbf{R}^p$ denote respectively the state, the input and the output vectors. $z_k \in \mathbf{R}^r$ is the vector to be estimated where $r \leq n$. $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{p\times n}$ and $L \in \mathbf{R}^{r\times n}$ are constant matrices of adequate dimensions. $h : \mathbf{R}^n \times \mathbf{R}^m \longrightarrow \mathbf{R}^n$ is a real nonlinear vector.

Without loss of generality, it is assumed that

1) rank(C)= p;

2) rank(L)= r;

3) rank $\begin{bmatrix} C \\ L \end{bmatrix}$ =($p + r \leq n$).

2.2 Structure of the reduced-order observer

For system (7), let us consider the following state observer

$$\begin{cases} \chi_{k+1} = E\chi_k + Jy_k + Hh_1 \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix}, u_k \right) + HBu_k \\ \hat{z}_k = \chi_k + Gy_k \end{cases} \tag{8}$$

where $\chi_k \in \mathbf{R}^r$ and $\hat{z}_k \in \mathbf{R}^r$ is the estimate of z_k. The matrices $E \in \mathbf{R}^{r\times r}$, $J \in \mathbf{R}^{r\times p}$, $H \in \mathbf{R}^{r\times n}$ and $G \in \mathbf{R}^{r\times p}$ are to be determined such that $\hat{z}_k$ converge asymptotically to z_k.

An important step to get the solution lies in the decomposition of the nonlinear function $h(x_k, u_k)$ into two portions: one portion will be considered as a Lipschitz function with respect to z_k and the other portion will be treated as an unknown input.

2.3 Decomposition of the nonlinear function

Referring to [18], the nonlinear vector $h(x_k, u_k)$ is decomposed as

$$h(x_k, u_k) = h_1(\varrho_k, u_k) + Dh_2(x_k, u_k) \tag{9}$$

where $\varrho_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} C \\ L \end{bmatrix} x_k \in \mathbf{R}^{p+r}$, $D \in \mathbf{R}^{n\times s}$ and $\mathrm{rank}(D) = s$ with $0 \le s \le n$.

The nonlinear functions $h_1(\varrho_k, u_k)$ and $h_2(x_k, u_k)$ can be obtained from the following decomposition procedure.

The vector ϱ_k can be expressed as

$$\varrho_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} C \\ L \end{bmatrix} x_k = \begin{bmatrix} T & 0 \end{bmatrix} K x_k = \begin{bmatrix} T & 0 \end{bmatrix} v_k \tag{10}$$

where $v_k = Kx_k \in \mathbf{R}^n$, matrix T is non-singular and K is a unitary matrix.

Then, v_k can be partitioned as

$$v_k = \begin{bmatrix} v_{1k} \\ v_{2k} \end{bmatrix} \tag{11}$$

where $v_{1k} \in \mathbf{R}^{(p+r)}$ and $v_{2k} \in \mathbf{R}^{n-(p+r)}$.

So, ϱ_k and $h(x_k, u_k)$ can be written as

$$\varrho_k = Tv_{1k} \tag{12}$$

$$h(x_k, u_k) = h\left(K^{\mathrm{T}} v_k, u_k\right). \tag{13}$$

Decomposing the right-hand side of (13), we get (based on works of [18–20]):

$$h\left(K^{\mathrm{T}} v_k, u_k\right) = h_1(\varrho_k, u_k) + \tilde{h}(v_k, u_k) \tag{14}$$

where

$$h_1(\varrho_k, u_k) = h\left(K^{\mathrm{T}} \begin{bmatrix} T^{-1}\varrho_k \\ 0 \end{bmatrix}, 0\right) + h(0, u) \tag{15}$$

$$\tilde{h}(x_k, u_k) = h\left(K^{\mathrm{T}} v_k, u_k\right) - h_1(\varrho_k, u_k) = h(x_k, u_k) - h_1\left(\begin{bmatrix} C \\ L \end{bmatrix} x_k, u_k\right). \tag{16}$$

Moreover, $\tilde{h}(x_k, u_k)$ can be expressed as

$$\tilde{h}(x_k, u_k) = Dh_2(x_k, u_k) \tag{17}$$

where $h_2(x_k, u_k) \in \mathbf{R}^s$ is treated as unknown input vector and $0 \le s \le n$ is the number of independent unknown inputs. Without loss of generality, matrix $D \in \mathbf{R}^{n\times s}$ is a full-column rank.

Otherwise, the following rank decomposition can be applied to the matrix D:

$$Dh_2(x_k, u_k) = \tilde{D}\bar{D}h_2(x_k, u_k) \tag{18}$$

where $\tilde{D}$ is a full-column rank matrix and $\bar{h}_2(x_k, u_k) = \bar{D}h_2(x_k, u_k)$ can be considered as a new unknown input vector. Thanks to the decomposition of $h(x_k, u_k)$ in the form of (9), we can combine both the results of Lipschitz nonlinear systems and unknown inputs to synthesize a reduced-order observer for this class of nonlinear systems.

3 New reduced-order observer design methodology

3.1 Necessary and sufficient conditions

To ensure the convergence of the estimation error and the existence of the reduced-order observer, the necessary and sufficient conditions are determined in this section.

Let $H \in \mathbf{R}^{r\times n}$ be a full-row rank matrix. We define the error vectors $\varepsilon_k \in \mathbf{R}^r$ and $e_k \in \mathbf{R}^r$ as follows:

$$\varepsilon_k = \chi_k - Hx_k \tag{19a}$$

$$e_k = \hat{z}_k - z_k. \tag{19b}$$

Then, we introduce the following corollary:

Corollary 1. The estimation error e_k converges asymptotically to zero for the decomposition of the nonlinearity as in (9) for any x_0, $\hat{z}_0$, u_k and all possible set of the nonlinear function $h(x_k, u_k) \in \mathbf{R}^n$ if and only if

1) ε_{k+1}, as defined in the following, converges asymptotically to zero;

$$\begin{aligned} \varepsilon_{k+1} = {} & E\varepsilon_k + (EH + JC - HA)x_k + \\ & H\big(h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)\big) - \\ & HDh_2(x_k, u_k) \end{aligned} \tag{20}$$

with $\varrho_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$ and $\hat{\varrho}_k = \begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix}$.

2) The following equations are satisfied

$$EH + JC - HA = 0, \quad E \text{ is Hurwitz} \tag{21a}$$

$$HD = 0 \tag{21b}$$

$$H + GC - L = 0. \tag{21c}$$

Proof. From (7a) and (9), the error vector ε_{k+1} can be expressed as

$$\begin{aligned}\varepsilon_{k+1} = \chi_{k+1} - Hx_{k+1} = E\chi_k + \\ (JC - HA)x_k - HDh_2(x_k, u_k) + \\ H\big(h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)\big).\end{aligned} \tag{22}$$

From (19a), we have $\chi_k = \varepsilon_k + Hx_k$. Then we obtain:

$$\begin{aligned}\varepsilon_{k+1} = E\varepsilon_k + (EH + JC - HA)x_k - HDh_2(x_k, u_k) + \\ H\big(h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)\big).\end{aligned} \tag{23}$$

From (19b), we can obtain:

$$e_k = \varepsilon_k + (H + GC - L)x_k. \tag{24}$$

The unbiasedness of the filter is achieved if and only if (21a) – (21c). This ends the proof of Corollary 1. □

By substituting (21c) into (21a) and (21b), we obtain the following equations:

$$EL = LA - \begin{bmatrix} G & J - EG \end{bmatrix} \begin{bmatrix} CA \\ C \end{bmatrix} \tag{25}$$

$$GCD = LD. \tag{26}$$

Post-multiplying both sides of (25) by the following full-row rank matrix:

$$\begin{bmatrix} O_1 & O_2 \end{bmatrix} = \begin{bmatrix} L^+ & I_n - L^+L \end{bmatrix} \tag{27}$$

where L^+ denotes the generalized matrix inverse of L. This yields the following two equations:

$$E = LAO_1 - [GJ - EG] \begin{bmatrix} CAO_1 \\ CO_1 \end{bmatrix} \tag{28}$$

$$LAO_2 = [GJ - EG] \begin{bmatrix} CAO_2 \\ CO_2 \end{bmatrix}. \tag{29}$$

The augmented matrix equation resulting from (26) and (29) can be expressed as

$$[GJ - EG] \begin{bmatrix} CAO_2 & CD \\ CO_2 & 0 \end{bmatrix} = [LAO_2 LD]. \tag{30}$$

The equality (30) can be rewritten as

$$\Sigma \Omega = \Pi \tag{31}$$

with

$$\Sigma = \begin{bmatrix} G & J - EG \end{bmatrix} \tag{32a}$$

$$\Omega = \begin{bmatrix} CAO_2 & CD \\ CO_2 & 0 \end{bmatrix} \tag{32b}$$

$$\Pi = \begin{bmatrix} LAO_2 & LD \end{bmatrix}. \tag{32c}$$

Now, let us introduce Corollary 2.

Corollary 2. There exists matrices G, J and E such that (31) is satisfied if and only if

$$\operatorname{rank} \begin{bmatrix} CA & CD \\ C & 0 \\ LA & LD \\ L & 0 \end{bmatrix} = \operatorname{rank} \begin{bmatrix} CA & CD \\ C & 0 \\ L & 0 \end{bmatrix}. \tag{33}$$

Proof. Equation (31) has a solution if and only if

$$\operatorname{rank} \begin{bmatrix} \Omega \\ \Pi \end{bmatrix} = \operatorname{rank} \tag{34}$$

i.e.,

$$\operatorname{rank} \begin{bmatrix} CAO_2 & CD \\ CO_2 & 0 \\ LAO_2 & LD \end{bmatrix} = \operatorname{rank} \begin{bmatrix} CAO_2 & CD \\ CO_2 & 0 \end{bmatrix}. \tag{35}$$

Then, by post-multiplying both sides of (35) by the full-row rank matrix:

$$\begin{bmatrix} O_1 & O_2 & 0 \\ 0 & 0 & I \end{bmatrix},$$

it is easy to verify that (35) is equivalent to (33). This ends the proof of Corollary 2. □

Therefore, if the equality (33) is satisfied, then there exists a general solution of (31) given by

$$\Sigma = \Pi \Omega^+ + Z(I - \Omega \Omega^+) \tag{36}$$

where $\Omega^+ = (\Omega^{\mathrm{T}} \Omega)^{-1} \Omega^{\mathrm{T}}$ is the generalized inverse of Ω and Z is an arbitrary matrix that will be determined using the LMI approach.

From Σ, we can determine the matrices G, E, H and J:

$$G = \Sigma \begin{bmatrix} I \\ 0 \end{bmatrix} = G_1 - ZG_2 \tag{37}$$

with

$$G_1 = \Pi \Omega^+ \begin{bmatrix} I \\ 0 \end{bmatrix}$$
$$G_2 = (\Omega \Omega^+ - I) \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$$E = LAO_1 - \Sigma \begin{bmatrix} CAO_1 \\ CO_1 \end{bmatrix} = E_1 - ZE_2 \tag{38}$$

with

$$lE_1 = LAO_1 - \Pi \Omega^+ \begin{bmatrix} CAO_1 \\ CO_1 \end{bmatrix}$$

$$E_2 = (I - \Omega \Omega^+) \begin{bmatrix} CAO_1 \\ CO_1 \end{bmatrix}$$

$$H = L - GC = H_1 - ZH_2 \tag{39}$$

with

$$H_1 = L - \Pi\Omega^+ \begin{bmatrix} C \\ 0 \end{bmatrix}$$

$$H_2 = (I - \Omega\Omega^+) \begin{bmatrix} C \\ 0 \end{bmatrix}$$

and

$$J - EG = \Sigma \begin{bmatrix} 0 \\ I \end{bmatrix} = \Pi\Omega^+ \begin{bmatrix} 0 \\ I \end{bmatrix} + Z(I - \Omega\Omega^+) \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{40}$$

3.2 New LMI based reduced-order observer

Since the unbiasedness conditions given in the previous paragraph are satisfied, the dynamic of the estimation error can be written as follows:

$$\varepsilon_{k+1} = E\varepsilon_k + H\Delta h_{1k} \tag{41}$$

with $\Delta h_{1k} = h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)$.

Or $h_1(\varrho_k, u_k)$ is globally Lipschitz, then from Lemma 1, there exist functions $\mathfrak{h}_{ij}$, constant $\underline{\alpha}_{\mathfrak{h}_{ij}}$ and constant $\overline{\alpha}_{\mathfrak{h}_{ij}}$, such that

$$h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k) = \sum_{i,j=1}^{n,p+r} \mathfrak{h}_{ij}\mathcal{H}_{ij}(\hat{\varrho}_k - \varrho_k) \tag{42}$$

with $\underline{\alpha}_{\mathfrak{h}_{ij}} \leq \mathfrak{h}_{ij} \leq \overline{\alpha}_{\mathfrak{h}_{ij}}$, $\mathfrak{h}_{ij} \triangleq \mathfrak{h}_{ij}(\hat{\varrho}^{\varrho_{j-1}}, \hat{\varrho}^{\varrho_j})$ and $\mathcal{H}_{ij} = e_n(i)e_{p+r}^T(j)$.

We can take $\bar{\mathfrak{h}}_{ij} = \overline{\alpha}_{\mathfrak{h}_{ij}} - \underline{\alpha}_{\mathfrak{h}_{ij}}$ and then we can assume $\underline{\alpha}_{\mathfrak{h}_{ij}} = 0$ without loss of generality.

Now, let us introduce Corollary 3.

Corollary 3. The reduced-order observer design problem corresponding to the system (7) and the observer (8) is solvable if there exist matrices R, $S_{ij} = S_{ij}^{\rm T} > 0$ and $P = P^{\rm T} > 0$ of appropriate dimensions such that the following LMI is feasible:

$$\begin{bmatrix} \begin{bmatrix} -P & E_1^{\rm T}P + E_2^{\rm T}R \\ (*) & -P \end{bmatrix} & [\Upsilon_1 \ \cdots \ \Upsilon_n] \\ (*) & -\Xi\mathcal{S} \end{bmatrix} < 0 \tag{43}$$

with

$$E_1 = LAL^+ - \Pi\Omega^+ \begin{bmatrix} CAL^+ \\ CL^+ \end{bmatrix}$$

$$E_2 = (I - \Omega\Omega^+) \begin{bmatrix} CAL^+ \\ CL^+ \end{bmatrix}$$

$$H_1 = L - \Pi\Omega^+ \begin{bmatrix} C \\ 0 \end{bmatrix}$$

$$H_2 = (I - \Omega\Omega^+) \begin{bmatrix} C \\ 0 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} CA(I_n - L^+L) & CD \\ C(I_n - L^+L) & 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} LA(I_n - L^+L) & FD \end{bmatrix}$$

$$\Upsilon_i = [\mathcal{B}_{i1} \quad \cdots \quad \mathcal{B}_{ir}]$$

$$\mathcal{B}_i = \begin{bmatrix} S_{ij}\mathcal{H}_{ij} \begin{bmatrix} 0_{p\times r} \\ I_r \end{bmatrix} & H_1^{\rm T}P - H_2^{\rm T}R \end{bmatrix}^{\rm T}$$

$$\Xi = \text{block} - \text{diag}(\ _1, \cdots, \ _n)$$

$$\Xi_i = \text{block} - \text{diag}\left(\frac{2}{\bar{\mathfrak{h}}_{i1}}I_r, \cdots, \frac{2}{\bar{\mathfrak{h}}_{in}}I_r\right)$$

$$\mathcal{S} = \text{block} - \text{diag}(\mathcal{S}_1, \cdots, \mathcal{S}_n)$$

$$\mathcal{S}_i = \text{block} - \text{diag}((\mathcal{S}_1, \cdots, (\mathcal{S}_r)$$

when the LMI (43) is feasible, the matrix Z is given by $Z = P^{-1}R^{\rm T}$.

Proof. Let us consider the classic quadratic Lyapunov function:

$$V_k = \varepsilon_k^{\rm T} P \varepsilon_k \tag{44}$$

where $P = P^{\rm T} > 0$.

The variation $\Delta V = V_{k+1} - V_k$ of this Lyapunov function is given by

$$\Delta V = \varepsilon_k^{\rm T}\left(\mathcal{G}P\mathcal{G}^{\rm T} - P\right)\varepsilon_k \tag{45}$$

with $\mathcal{G} = \left(E + H\sum_{i,j=1}^{n,p+r}\mathfrak{h}_{ij}\mathcal{H}_{ij}\begin{bmatrix} 0_{p\times r} \\ I_r \end{bmatrix}\right)^{\rm T}$.

We can deduce that $\Delta V < 0$ for all $\varepsilon_k \neq 0$, if the following inequality holds:

$$\mathcal{G}P\mathcal{G}^{\rm T} - P < 0. \tag{46}$$

Using Schur′s Lemma, the inequality (46) becomes equivalent to

$$\begin{bmatrix} -P & \mathcal{G}P \\ (*) & -P \end{bmatrix} < 0. \tag{47}$$

Then, the inequality (47) can be rewritten as the following:

$$\begin{bmatrix} -PE^{\rm T}P & \\ (*) & -P \end{bmatrix} + \sum_{i,j=1}^{n,p+r} \mathfrak{h}_{ij} \underbrace{\begin{bmatrix} 0 \\ PH \end{bmatrix}}_{M^{\rm T}} \underbrace{\begin{bmatrix} \mathcal{H}_{ij}\begin{bmatrix} 0_{p\times r} \\ I_r \end{bmatrix} & 0 \end{bmatrix}}_{N_{ij}} + N_{ij}^{\rm T}M < 0. \tag{48}$$

Hence, by applying the reformulation of Young's relation (6), we obtain

$$M^{\mathrm{T}} N_{ij} + N_{ij}^{\mathrm{T}} M \le \frac{1}{2} \left[M + S_{ij} N_{ij}\right]^{\mathrm{T}} S_{ij}^{-1} \left[M + S_{ij} N_{ij}\right]. \tag{49}$$

From (5), inequality (48) is satisfied if

$$\begin{bmatrix} -PN^{\mathrm{T}}P & \\ (*) & -P \end{bmatrix} + \sum_{i,j=1}^{n,p+r} \left[M + S_{ij} N_{ij}\right]^{\mathrm{T}} \frac{\bar{\mathfrak{h}}_{ij}}{2} S_{ij}^{-1} \left[M + S_{ij} N_{ij}\right] < 0. \tag{50}$$

Then, using Schur's lemma and the notation $R = Z^{\mathrm{T}} P$, inequality (50) becomes equivalent to (43). □

Now, we can define the next algorithm to synthesise the proposed reduced-order observer.

Design algorithm 1.

1) Decompose the nonlinear function $h(x_k, u_k)$ in (7a) according to (9) and find D and $h_1(\varrho_k, u_k)$;

2) Check if the condition given in Corollary 2 is satisfied. If yes, continue. If not, a reduced-order observer does not exist;

3) Solve LMI (43) to get the matrices P and R;

4) Get Z from Corollary 3;

5) Get filter matrices from (37) to (40).

4 Extension to H_∞ filtering design

Let us consider the same system described by (7) with added noise on the state equation:

$$x_{k+1} = Ax_k + Bu_k + h(x_k, u_k) + F\omega_k \tag{51a}$$

$$y_k = Cx_k \tag{52b}$$

$$z_k = Lx_k \tag{53c}$$

where $x_k \in \mathbf{R}^n$, $u_k \in \mathbf{R}^m$, $y_k \in \mathbf{R}^p$ and $\omega_k \in \mathbf{R}^q$ denote respectively the state, the input, the output and the bounded disturbance vectors. $z_k \in \mathbf{R}^r$ is the vector to be estimated where $r \le n$. $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $F \in \mathbf{R}^{n\times q}$, $C \in \mathbf{R}^{p\times n}$ and $L \in \mathbf{R}^{r\times n}$ are constant matrices of adequate dimensions. $h : \mathbf{R}^n \times \mathbf{R^m} \to \mathbf{R^n}$ is a real nonlinear vector. The same assumptions given in Section 2 will be reconsidered in this section.

Now, we consider the following state observer:

$$\begin{cases} \chi_{k+1} = E\chi_k + Jy_k + Hh_1\left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix}, u_k\right) + HBu_k \\ \hat{z}_k = \chi_k + Gy_k \end{cases} \tag{52}$$

where $\chi_k \in \mathbf{R}^r$, $\hat{z}_k \in \mathbf{R}^r$ is the estimate of z_k. The matrices E, J, H and M are to be determined such that $\hat{z}_k$ converge asymptotically to z_k.

Corollary 4. The estimation error e_k converges asymptotically to zero for the decomposition of the nonlinearity as in (9) for any x_0, z_0, u_k, ω_k and all possible set of the nonlinear function $h(x_k, u_k) \in \mathbf{R}^n$ if and only if

1) ε_{k+1}, as defined in the following, converges asymptotically to zero;

$$\begin{aligned} \varepsilon_{k+1} = E\varepsilon_k + (EH + JC - HA)x_k + \\ H\big(h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)\big) - \\ HDh_2(x_k, u_k) - HF\omega_k \end{aligned} \tag{53}$$

with $\varrho_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$ and $\hat{\varrho}_k = \begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix}$.

2) The following equations are satisfied:

$$EH + JC - HA = 0,\ E \text{ is Hurwitz} \tag{54a}$$

$$HD = 0 \tag{54b}$$

$$H + GC - L = 0. \tag{54c}$$

Proof. According to (19a) and (19b), it is easy to prove the results given by Corollary 4. □

If Corollary 4 is verified, then the dynamic of the estimation error can be written as follows:

$$\varepsilon_{k+1} = E\varepsilon_k + H\Delta h_{1k} - HF\omega_k \tag{55}$$

with $\Delta h_{1k} = h_1(\hat{\varrho}_k, u_k) - h_1(\varrho_k, u_k)$.

The aim is to find the reduced-order H_∞ observer parameters where ε_k converges H_∞ asymptotically toward zero, i.e., we must search for the parameters that satisfy the following condition:

$$\| \varepsilon_k \|_{l_2} \le \lambda \| \omega_k \|_{l_2}, \quad \text{for} \quad \varepsilon(0) = 0 \tag{56}$$

with a disturbance attenuation level $\lambda > 0$ that will be minimized.

So, it is sufficient to find a Lyapunov function V_k so that

$$\Delta V + \varepsilon_k^{\mathrm{T}} \varepsilon_k - \lambda^2 \omega_k^{\mathrm{T}} \omega_k < 0. \tag{57}$$

Corollary 5. For a disturbance attenuation level $\lambda > 0$, the reduced-order H_∞ observer design problem corresponding to the system (51) and the observer (52) is solvable if there exist matrices R, $S_{ij} = S_{ij}^{\mathrm{T}} > 0$ and $P = P^{\mathrm{T}} > 0$ of appropriate dimensions such that the following LMI is feasible

$$\begin{aligned} &\min \lambda \\ &s.t. \begin{bmatrix} (1,1) & [\Upsilon_1 \cdots \Upsilon_n] \\ (*) & -\Xi\mathcal{S} \end{bmatrix} < 0 \end{aligned} \tag{58}$$

with

$$(1,1)=\begin{bmatrix}-P+I & 0 & -E_1^{\mathrm{T}}P+E_2^{\mathrm{T}}R\\ (*) & -\lambda^2 I & F^{\mathrm{T}}(H_1^{\mathrm{T}}P-H_2^{\mathrm{T}}R)\\ (*) & (*) & -P\end{bmatrix}$$

$$E_1=LAL^{+}-\Pi\varOmega^{+}\begin{bmatrix}CAL^{+}\\ CL^{+}\end{bmatrix}$$

$$E_2=(I-\varOmega\varOmega^{+})\begin{bmatrix}CAL^{+}\\ CL^{+}\end{bmatrix}$$

$$H_1=L-\Pi\varOmega^{+}\begin{bmatrix}C\\ 0\end{bmatrix}$$

$$H_2=(I-\varOmega\varOmega^{+})\begin{bmatrix}C\\ 0\end{bmatrix}$$

$$\varOmega=\begin{bmatrix}CA(I_n-L^{+}L) & CD\\ C(I_n-L^{+}L) & 0\end{bmatrix}$$

$$\Pi=\begin{bmatrix}LA(I_n-L^{+}L) & LD\end{bmatrix}$$

$$\varUpsilon_i=[\mathcal{B}_{i1}\quad\cdots\quad\mathcal{B}_{ir}]$$

$$\mathcal{B}_i=\left[-S_{ij}\mathcal{H}_{ij}\begin{bmatrix}0_{p\times r}\\ I_r\end{bmatrix}\quad 0\quad -(H_1^{\mathrm{T}}P-H_2^{\mathrm{T}}R)\right]^{\mathrm{T}}$$

$$\Xi=\mathrm{block}-\mathrm{diag}(\ _1,\cdots,\ _\mathrm{n})$$

$$\Xi_i=\mathrm{block}-\mathrm{diag}\left(\frac{2}{\underline{\alpha}_{h_{i1}}}I_r,\cdots,\frac{2}{\underline{\alpha}_{\mathfrak{h}_{in}}}I_r\right)$$

$$\mathcal{S}=\mathrm{block}-\mathrm{diag}(\mathcal{S}_1,\cdots,\mathcal{S}_n)$$

$$\mathcal{S}_i=\mathrm{block}-\mathrm{diag}(\mathcal{S}_1,\cdots,\mathcal{S}_r).$$

When the LMI (43) is feasible, the matrix Z is given by $Z=P^{-1}R^{\mathrm{T}}$.

Proof. To demonstrate this corollary, it is sufficient to choose the standard form of the Lyapunov function $V_k=\varepsilon_k^{\mathrm{T}}P\varepsilon_k$, where $P=P^{\mathrm{T}}>0$. We deduce the LMI (58) by choosing $M=\begin{bmatrix}0 & 0 & H^{\mathrm{T}}P\end{bmatrix}$, $N_{ij}=\begin{bmatrix}\mathcal{H}_{ij}\times\begin{bmatrix}0_{p\times r}\\ I_r\end{bmatrix} & 0 & 0\end{bmatrix}$ and using the convexity principle as in Corollary 3. □

Now, we can define the following algorithm to synthesize the proposed reduced-order H_∞ observer.

Design algorithm 2.

1) Decompose the nonlinear function $h(x_k,u_k)$ in (7a) according to (9) and find D and $h_1(\varrho_k,u_k)$;

2) Check if the condition given in Corollary 2 is satisfied. If yes, continue. If not, a reduced-order observer does not exist;

3) Solve LMI (58) to get the matrices P and R;

4) Get Z from the Corollary 5;

5) Get filter matrices from (37) to (40).

5 Simulation results

In this section, three numerical examples are considered to illustrate the effectiveness of the proposed design methodology.

5.1 Example 1

Consider a nonlinear system as described by (7) with

$$A=I_7+T_eA_c$$

$$A_c=\begin{bmatrix}-1&0&0&1&0&0&0\\ 2&0&1&-1&1&0&0\\ 0&3&0&0&1&1&0\\ 0&0&0&-3&0&1&1\\ 0&0&0&0&1&0&-1\\ 1&0&0&0&0&-1&0\\ 0&1&0&0&1&0&-2\end{bmatrix}$$

$$B=[0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]^{\mathrm{T}}$$

$$C=\begin{bmatrix}1&0&0&0&0&0&0\\ 1&1&0&0&0&0&0\\ -1&0&1&0&0&0&0\end{bmatrix}$$

$$L=[0\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0]$$

$$h(x_k)=T_e\begin{bmatrix}4x_{4k}x_{7k}\\ 0\\ 0.1x_{4k}x_{7k}\\ 0.45\sin^2(x_{4k}+x_{5k})\\ 0\\ 0\\ x_{4k}x_{7k}\end{bmatrix}$$

where $T_e=0.1\,\mathrm{s}$ is the sample time.

The nonlinear function h can be decomposed according to (9) as the following:

$$h_1(\varrho_k)=\begin{bmatrix}0\\ 0\\ 0\\ 0.45\sin^2(x_{4k}+x_{5k})\\ 0\\ 0\\ 0\end{bmatrix},D=\begin{bmatrix}4T_e\\ 0\\ 0.1T_e\\ 0\\ 0\\ 0\\ T_e\end{bmatrix}$$

and $h_2(x_k)=x_{4k}x_{7k}$.

$h_1(\varrho_k)$ is a Lipschitz function and satisfies the condition (42) with $\overline{\alpha}_{\mathfrak{h}_{44}}=0.45T_e$ and $\underline{\alpha}_{\mathfrak{h}_{44}}=0$. $h_2(x_k)$ is considered as an unknown input.

It is clear that the conditions given in Corollary 1 and

2 are satisfied. Therefore, the design of the reduced-order observer for this system can be studied. By solving the LMIs (43) and from (37) to (40), we obtain:

$$G = [-0.512\,5 \quad 1.487\,5 \quad 1.000\,0]$$

$$E = 0.851\,3$$

$$H = [0.025\,0 \quad -1.487\,5 \quad -1.000\,0 \quad 1.000\,0 \quad 1.000\,0 \quad 0 \quad 0]$$

$$J = [-0.072\,5 \quad -0.521\,3 \quad -0.297\,5]\,.$$

The initial conditions for the system and for the observers have been chosen as: $z_0 = 1$ and $\hat{z}_0 = -0.5$. Fig. 1 presents the trajectory of z_k and its estimate.

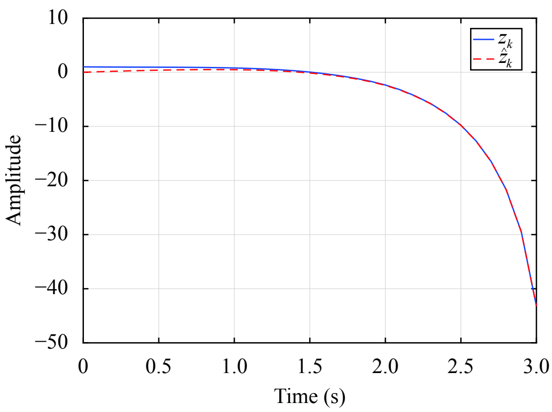

Fig. 1 Response of z_k and its estimate

As shown in Fig. 1, the state is accurately estimated using the proposed design method.

5.2 Example 2

Let us consider the system described in (7) with

$$A = I_4 + T_e A_c$$

$$A_c = \begin{bmatrix} -10.283\,1 & 1.234\,1 & 2.319\,2 & -1.486\,9 \\ -48.823\,7 & -2.396\,8 & 0.438\,9 & 2.194\,3 \\ 1.002\,3 & -1.910\,2 & -20.348\,6 & 0.398\,1 \\ -3.342\,1 & 0.304\,1 & 5.902\,3 & -3.460\,9 \end{bmatrix}$$

$$B = T_e \begin{bmatrix} 0.587\,1 & 0.236\,1 \\ 1.358\,8 & -2.327\,2 \\ -0.619\,1 & 1.255\,2 \\ 0.382\,1 & 1.465\,2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$h(x_k) = T_e \begin{bmatrix} x_{2k}x_{3k} \\ 0 \\ 3x_{2k}x_{3k} \\ 0.2\sin(x_{4k}) \end{bmatrix}$$

where $T_e = 0.01$s is the sample time.

Then the nonlinear function h can be decomposed according to (9) as the following:

$$h_1(\varrho_k) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2T_e\sin(x_{4k}) \end{bmatrix}, \ D = \begin{bmatrix} T_e \\ 0 \\ 3T_e \\ 0 \end{bmatrix}$$

and $h_2(x_k) = x_{2k}x_{3k}$.

$h_1(\varrho_k)$ is a Lipschitz function and satisfies the condition (42) with $\overline{\alpha}_{\mathfrak{h}_{45}} = 0.2T_e$ and $\underline{\alpha}_{\mathfrak{h}_{45}} = -0.2T_e$. Then we can assume that $\underline{\alpha}_{\mathfrak{h}_{45}} = 0$ and $\bar{\mathfrak{h}}_{45} = 0.4T_e$.

$h_2(x_k)$ is considered as an unknown input.

It is clear that the conditions given in Corollaries 1 and 2 are satisfied. Therefore, the design of the reduced-order observer for this system can be studied. By solving the LMIs (43) and from (37) to (40), we obtain:

$$G = \begin{bmatrix} -0.645\,7 & -7.061\,5 & 0.548\,6 \\ -11.553\,8 & 1.915\,9 & 3.851\,3 \end{bmatrix}$$

$$E = 10^{-3} \begin{bmatrix} 0.026\,2 & -0.028\,4 \\ 0.006\,9 & 0.283\,0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1.645\,7 & 7.061\,5 & -0.548\,6 & -0.548\,6 \\ 11.553\,8 & -1.915\,9 & -3.851\,3 & -2.851\,3 \end{bmatrix}$$

$$J = \begin{bmatrix} -1.958\,1 & 6.921\,1 & -0.400\,3 \\ 11.354\,5 & -1.662\,0 & -2.975\,2 \end{bmatrix}.$$

The initial conditions for the system and for the observers have been chosen as: $z_0 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^{\mathrm{T}}$ and $\hat{z}_0 = \begin{bmatrix} -0.5 & -0.5 \end{bmatrix}^{\mathrm{T}}$. Figs. 2 and 3 present the trajectory of z_k and its estimate.

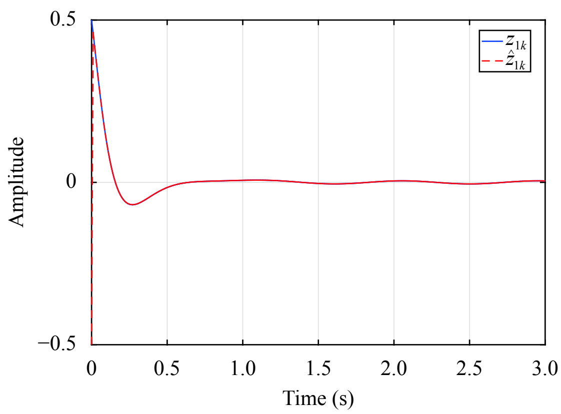

Fig. 2 Response of z_{1k} and its estimate

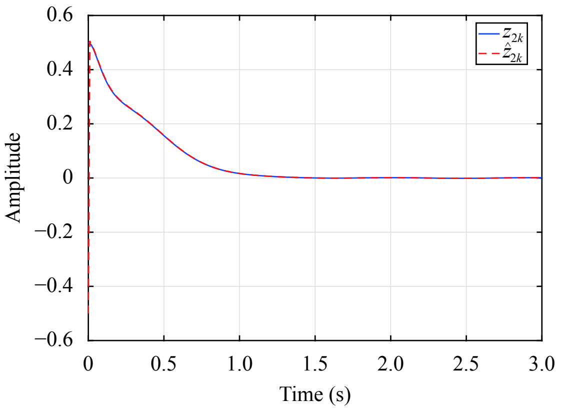

Fig. 3 Response of z_{2k} and its estimate

As shown in Figs. 2 and 3, the state is accurately estimated using the proposed design method.

5.3 Example 3

We reconsider Example 2 with the presence of a disturbance. Let $F = \begin{bmatrix} 0 & 0.01 & 0 & 0.01 \end{bmatrix}^{\rm T}$. By solving the LMIs (58) and from (37) to (40), we obtain

$$G = \begin{bmatrix} -0.650\,7 & -7.084\,9 & 0.550\,2 \\ -11.160\,9 & 3.605\,1 & 3.720\,3 \end{bmatrix}$$

$$E = 10^{-3} \begin{bmatrix} 0.011\,8 & 0.007\,4 \\ 0.013\,8 & 0.229\,2 \end{bmatrix}$$

$$H = \begin{bmatrix} 1.650\,7 & 7.084\,9 & -0.550\,2 & -0.550\,2 \\ 11.160\,9 & -3.605\,1 & -3.720\,3 & -2.720\,3 \end{bmatrix}$$

$$J = \begin{bmatrix} -1.965\,4 & 6.944\,2 & -0.401\,3 \\ 11.824\,4 & -3.317\,4 & -2.880\,0 \end{bmatrix}.$$

In the following, real time implementation using an ARDUINO MEGA 2560 device is considered. The diagram illustrating the implementation is given in Fig. 4.

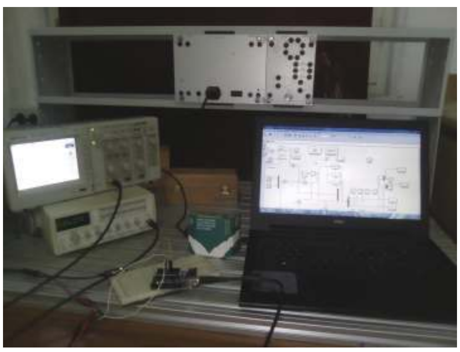

Fig. 4 Real time implementation

5.3.1 ARDUINO I/O interface mode

The first mode consists in using the ARDUINO card as an I/O interface with Matlab Simulink. After loading the firmware "adioserv.pde" into the Arduino card, we install the Arduino I/O library to Simulink Libraries.

In this phase of implementation, a noise was added to the state of the system. The added signal is a sinusoidal signal with variable frequencies (between 42 Hz and 680 Hz).

Figs. 5 and 6 present the trajectory of z_k and its estimate.

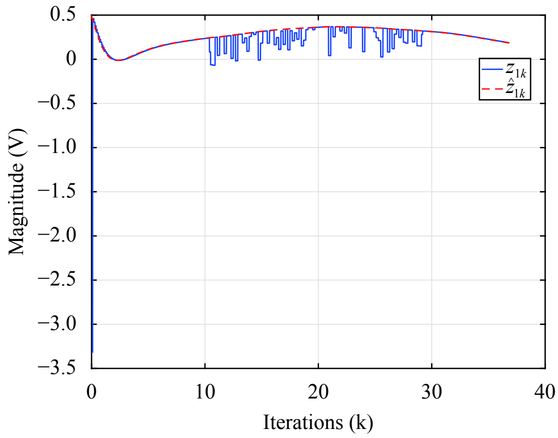

Fig. 5 Response of z_{1k} and its estimate

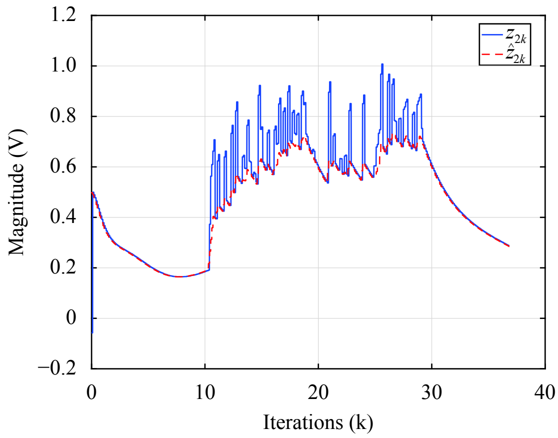

Fig. 6 Response of z_{2k} and its estimate

As shown in Figs. 5 and 6, the states are accurately estimated.

5.3.2 ARDUINO target interface mode

In this mode of programming, the Arduino card becomes a target of the Simulink code compiled with the tool "Run on target hardware". The Arduino kit operates completely in an autonomous way. It can also be managed online via the USB port of the PC (external mode enable).

In this second phase of implementation, the added noise is sinusoidal signals with variable frequencies (between 14 Hz and 1 k Hz, 2 k Hz).

The reconstruction of the output signals is provided by sending the desired data to the pulse-width modulation (PWM) outputs. These PWM outputs are then connected to low pass filters (with $R = 4.2\,\mathrm{K}\Omega$ and $C = 30\,\mu\mathrm{F}$).

We present, in Figs. 7 and 8, the real z_1, z_2 and its estimates $\hat{z}_1$, $\hat{z}_2$, respectively.

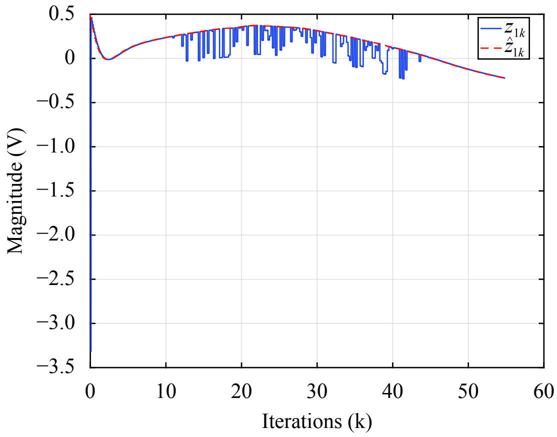

Fig. 7 Response of z_{1k} and its estimate

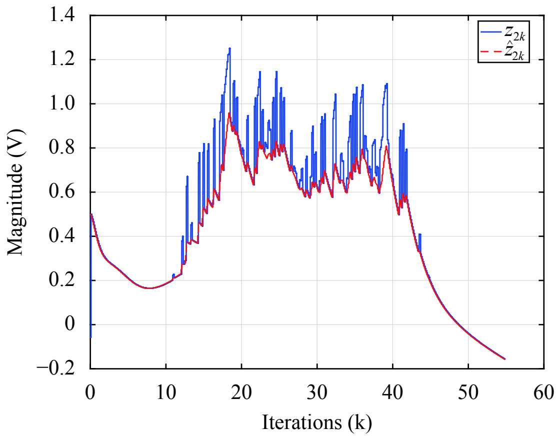

Fig. 8 Response of z_{2k} and its estimate

As shown in Figs. 7 and 8, the states are very accurately estimated.

6 Conclusions

In this paper, a new design methodology of reduced-order observers for Lipschitz nonlinear discrete-time systems is proposed. Once the necessary and sufficient conditions for the existence of the unbiased reduced-order observer are satisfied, and with reformulation of both the Lipschitz condition and the Young's inequality, new LMI conditions are given. Then, an extension to the H_∞ filtering synthesis is provided in order to guarantee the asymptotic stability of the estimation error in the presence of disturbances. Numerical examples are given to show the effectiveness of the proposed design methodology.

In future work, we plan to generalize the proposed reduced-order filter to develop observer-based control laws for nonlinear systems in the presence of uncertain parameters.

References

[1] R. Rajamani. Observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, vol. 43, no. 3, pp. 397–401, 1998. DOI: 10.1109/9.661604.

[2] M. Boutayeb, C. Aubry. A strong tracking extended Kalman observer for nonlinear discrete-time systems. *IEEE Transactions on Automatic Control*, vol. 44, no. 8, pp. 1550–1556, 1999. DOI: 10.1109/9.780419.

[3] M. Abbaszadeh, H. J. Marquez. Nonlinear observer design for one-sided Lipschitz systems. In *Proceedings of American Control Conference*, Baltimore, USA, pp. 5284–5289, 2010. DOI: 10.1109/ACC.2010.553071.

[4] C. Y. Wang, Z. Y. Zuo, Z. L. Lin, Z. T. Ding. Consensus control of a class of Lipschitz nonlinear systems with input delay. *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 11, pp. 2730–2738, 2015. DOI: 10.1109/TCSI.2015.2479046.

[5] D. Z. Wang, F. Song, W. Zhang. State estimation for discrete-time systems with generalized Lipschitz nonlinear dynamics. *Advances in Difference Equations*, vol. 2015, no. 1, Article number 307, 2015. DOI: 10.1186/s13662-015-0645-x.

[6] K. Hfaiedh, K. Dahech, T. Damak. A sliding mode observer for uncertain nonlinear systems based on multiple models approach. *International Journal of Automation and Computing*, vol. 14, no. 2, pp. 202–212, 2017. DOI: 10.1007/s11633-016-0970-x.

[7] J. A. Isaza, H. A. Botero, H. Alvarez. State estimation using non-uniform and delayed information: A review. *International Journal of Automation and Computing*, vol. 15, no. 2, pp. 125–141, 2018. DOI: 10.1007/s11633-017-1106-7.

[8] A. H. D. Markazi, M. Maadani, S. H. Zabihifar, N. Doost-Mohammadi. Adaptive fuzzy sliding mode control of under-actuated nonlinear systems. *International Journal of Automation and Computing*, vol. 15, no. 3, pp. 364–376, 2018. DOI: 10.1007/s11633-017-1108-5.

[9] K. P. Niamsup, V. N. Phat. Robust finite-time H_∞ control of linear time-varying delay systems with bounded control via Riccati equations. *International Journal of Automation and Computing*, vol. 15, no. 3, pp. 355–363, 2018. DOI: 10.1007/s11633-016-1018-y.

[10] H. Trinh. Linear functional state observer for time-delay systems. *International Journal of Control*, vol. 72, no. 18, pp. 1642–1658, 1999. DOI: 10.1080/002071799219986.

[11] S. Halabi, H. S. Ali, H. Rafaralahy, M. Zasadzinski. H_∞ functional filtering for stochastic bilinear systems with multiplicative noises. *Automatica*, vol. 45, no. 4, pp. 1038–1045, 2009. DOI: 10.1016/j.automatica.2008.11.027.

[12] M. Darouach. On the functional observers for linear descriptor systems. *Systems & Control Letters*, vol. 61, no. 3, pp. 427–434, 2012. DOI: 10.1016/j.sysconle.2012.01.006.

[13] H. S. Ali, M. Darouach, C. Delattre, Y. Becis-Aubry. Functional observers design for two interconnected linear systems via LMI. *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 14348–14352, 2011. DOI: 10.3182/20110828-6-IT-1002.03664.

[14] M. Darouach. Existence and design of functional observers for linear systems. *IEEE Transactions on Automatic Control*, vol. 45, no. 5, pp. 940–943, 2000. DOI: 10.1109/9.855556.

[15] M. Darouach, F. Amato, M. Alma. Functional observers design for descriptor systems via LMI: Continuous and discrete-time cases. *Automatica*, vol. 86, pp. 216–219, 2017. DOI: 10.1016/j.automatica.2017.08.016.

[16] A. Alessandri. Design of observers for Lipschitz nonlinear systems using LMI. In *Proceedings of IFAC Symposium on Nonlinear Control Systems*, Stuttgart, Germany, 2004.

[17] T. Fernando, S. Alahakoon, N. Nikraz. Sliding mode functional observers for nonlinear systems. In *Proceedings of IEEE Region 10 and the 3rd International Conference on Industrial and Information Systems*, Kharagpur, India, Article number 540, 2008. DOI: 10.1109/ICIINFS.2008.4798498.

[18] P. S. Teh, H. Trinh. Design of unknown input functional observers for nonlinear systems with application to fault diagnosis. *Journal of Process Control*, vol. 23, no. 8, pp. 1169–1184, 2013. DOI: 10.1016/j.jprocont.2013.06.013.

[19] A. Zemouche, R. Rajamani, B. Boulkroune, H. Rafaralahy, M. Zasadzinski. Convex optimization based dual gain observer design for Lipschitz nonlinear systems. In *Proceedings of American Control Conference*, Boston, USA, pp. 125–130, 2016. DOI: DOI: 10.1109/ACC.2016.7524903.

[20] A. Zemouche, R. Rajamani, B. Boulkroune, H. Rafaralahy, M. Zasadzinski. H_∞ circle criterion observer design for Lipschitz nonlinear systems with enhanced LMI conditions. In *Proceedings of American Control Conference*, Boston, USA, pp. 131–136, 2016. DOI: 10.1109/ACC.2016.7524904.

Noussaiba Gasmi received the degree in electrical and automatic engineering from National Engineering School of Gabes, Tunisia in 2014. Since 2015, she is a Ph. D. degree candidate in automatic control at Research Center for Automatic Control of Nancy, University of Lorraine, France, and the MACS Laboratory (Modeling, Analysis and Control of Systems), National Engineering School of Gabes, University of Gabes, Tunisia.

Her research interests include identification, state estimation and robust control of dynamical systems.

E-mail: noussaiba.gasmi@univ-lorraine.fr (Corresponding author)

ORCID iD: 0000-0002-2993-9834

Assem Thabet received the B. Eng. degree in electrical and automatic engineering from National Engineering School of Gabes, Tunisia in 2006, and the M. Eng. and Ph. D. degrees in automatic control from the University of Gabes, Tunisia in 2008 and 2012, respectively. Since 2012, he is an associate professor with the University of Gabes, Tunisia, and a member of the MACS Laboratory (Modeling, Analysis and Control of Systems) of National Engineering School of Gabes, Tunisia.

His research interests include identification, state estimation and control of dynamical systems.

E-mail: assem.thabet@yahoo.fr

Mohamed Aoun received the Ph. D. degree in automatic control from University of Bordeaux, France in 2005. He is currently an associate professor in automatic control, electrical engineering, and computer engineering at the National Engineering School of Gabes, Tunisia, and a member of its MACS Laboratory (modeling, analysis and control of systems).

His research interests include automatic control, system identification, robust state estimation, diagnosis and fractional differentiation.

E-mail: mohamed.aoun@gmail.com