International Journal of Automation and Computing 15(3), June 2018, 335-344
DOI: 10.1007/s11633-017-1073-z

Recursive Bayesian Algorithm for Identification of Systems with Non-uniformly Sampled Input Data

Shao-Xue Jing[1, 2] Tian-Hong Pan[1] Zheng-Ming Li[1]
[1]School of Electrical Information and Engineering, Jiangsu University, Zhenjiang 212013, China
[2]Department of Electrical Engineering, Huaian College of Information and Technology, Huaian 223003, China

Abstract: To identify systems with non-uniformly sampled input data, a recursive Bayesian identification algorithm with covariance resetting is proposed. Using estimated noise transfer function as a dynamic filter, the system with colored noise is transformed into the system with white noise. In order to improve estimates, the estimated noise variance is employed as a weighting factor in the algorithm. Meanwhile, a modified covariance resetting method is also integrated in the proposed algorithm to increase the convergence rate. A numerical example and an industrial example validate the proposed algorithm.

Keywords: Parameter estimation, discrete time systems, Gaussian noise, Bayesian algorithm, covariance resetting.

1 Introduction

For decades, many efforts have been devoted to system identification, and lots of results have been reported[1−3]. This paper considers the identification of non-uniformly sampled data (NUSD) systems, whose sampling intervals for the input and/or output channels are non-equidistant in time[4]. NUSD systems can be found in many industrial processes, such as induction motors, continuous stirred tank heater systems, filter bank transceivers, transmultiplexers, bioreactors, etc[5−9]. The identification of NUSD systems has attracted much attention[10−15].

To identify systems with colored noise, many techniques, such as bias compensation[16−18] and data-filtering[19, 20], have been used. Owing to their easy implementation and high efficiency, filter based algorithms have been reported extensively[21−23]. Using the estimated noise transfer function, Xie et al.[24] proposed a filter based least square algorithm. For the identification of multi-rate NUSD systems, Liu et al.[25] gave an auxiliary model based recursive least squares algorithm, and analyzed the convergence properties of the proposed algorithm. For identification of continuous time systems with NUSD, Goodwin and Cea[26] formulated the problem in the context of nonlinear filtering, and applied minimum distortion filtering to identify these systems. Based on the Kalman filtering principle, Wang et al.[27] derived the state filtering algorithm by minimizing the estimation error covariance matrix, and further calculated the state estimates of the original systems by using the inverse transformation. To obtain the optimal state estimate for multi-rate dynamic system with NUSD, Yan et al.[28] proposed a modified Kalman filter.

Although the algorithms mentioned above work well, there are still some problems. For example, in the filter based algorithms, the data filter was used to alleviate the computational burden of the identification algorithms, but the noise variance was not integrated into the algorithms to improve the estimates. In the Kalman filter based algorithms, the Kalman filter was used to estimate the unknown states and parameters of the state-space models. However the means and variances of the process noise and the output noise must be known a priori. In order to achieve high-accuracy estimates of the time-domain models with less computation, a filter based recursive Bayesian (RB) algorithm is proposed in this paper. To improve the convergence rate, a novel covariance resetting (CR) method is employed into the proposed algorithm.

This paper is organized as follows. The NUSD model is introduced in Section 2, and the algorithm is derived in Section 3. After the convergence analysis in Section 4, a numeric example and an industrial application are given in Section 5. At last, main conclusions are given in Section 6.

2 Problem description

Consider the following system with non-uniformly sampled input data, as depicted in Fig. 1.

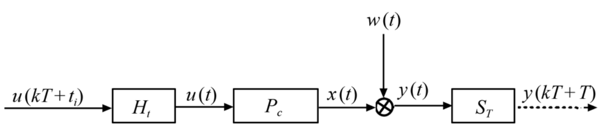

Fig. 1 Systems with non-uniformly sampled input data

Research Article
Manuscript received March 16, 2016; accepted November 8, 2016; published online May 4, 2017
This work was supported by National Natural Science Foundation of China (Nos. 61273142 and 51477070), the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), Foundation for Six Talents by Jiangsu Province and Graduate Scientific Innovation Projects of Jiangsu University (No. KYXX_0003).
Recommended by Associate Editor Ding-Li Yu

The system includes a zero-order hold H_τ, a linear continuous-time invariant process P_c and a sampler S_T. $u(t)$, $x(t)$ and $y(t)$ are the input, noise-free output and output with additive noise respectively, $w(t)$ is an autoregressive noise. $u(kT+t_i)$ and $y(kT+T)$ are the discrete-time input and output, where T is output sampling period. Irregular input sampling intervals $\tau_i (i=0,1,\cdots,r)$ satisfy $\tau_0=0$ and $T=\sum_{i=0}^{r}\tau_i$. So the equivalent formulation in Fig. 1 using lifting technique[4, 24] is as

$$y(t)=x(t)+w(t) \tag{1}$$

with

$$\begin{cases} u_i(k)=u(kT+\sum\limits_{j=0}^{i-1}\tau_j) \\ x(t)=\sum\limits_{i=1}^{r}\dfrac{B_i(z^{-1})}{A(z^{-1})}u_i(k) \\ w(t)=\dfrac{1}{C(z^{-1})}v(t) \\ A(z^{-1})=1+a_1z^{-1}+\cdots+a_{n_a}z^{-n_a} \\ B_1(z^{-1})=b_{1,0}+b_{1,1}z^{-1}+\cdots+b_{1,n_b}z^{-n_b} \\ B_i(z^{-1})=b_{i,1}z^{-1}+\cdots+b_{i,n_b}z^{-n_b}, i=2,3,\cdots,r \\ C(z^{-1})=1+c_1z^{-1}+c_2z^{-2}+\cdots+c_{n_c}z^{-n_c} \end{cases} \tag{2}$$

where n_a, n_b, n_c and r are assumed to be known and $v(t)$ is a Gaussian white noise ($v(t)\sim N(0,\sigma_v^2)$).

The parameter vector θ is defined as

$$\theta=\left[\theta_s^{\rm T},\theta_n^{\rm T}\right]^{\rm T}\in\mathbf{R}^{n\times 1} \tag{3}$$

where

$$\begin{cases} \theta_s=[a_1,a_2,\cdots,a_{n_a},b_{1,0},b_{1,1},\cdots,b_{1,n_b},b_{2,1},b_{2,2},\cdots, \\ \quad b_{2,n_b},\cdots,b_{r,1},b_{r,2},\cdots,b_{r,n_b}]^{\rm T}\in\mathbf{R}^{n_s\times 1} \\ \theta_n=[c_1,c_2,\cdots,c_{n_c}]^{\rm T}\in\mathbf{R}^{n_c\times 1} \\ n_s=n_a+rn_b+1 \\ n=n_s+n_c. \end{cases}$$

The information vector φ is defined as

$$\varphi(k)=\left[\varphi_s^{\rm T}(k),\varphi_n^{\rm T}(k)\right]^{\rm T}\in\mathbf{R}^{n\times 1}$$

where

$$\begin{cases} \varphi_s=[-x(k-1),-x(k-2),\cdots,-x(k-n_a),u_1(k), \\ \quad u_1(k-1),\cdots,u_1(k-n_b),u_2(k-1),u_2(k-2), \\ \quad u_2(k-n_b),\cdots,u_r(k-1),u_r(k-2),\cdots, \\ \quad u_r(k-n_b)]^{\rm T}\in\mathbf{R}^{n_s\times 1} \\ \varphi_n=[w(k-1),w(k-2),\cdots,w(k-n_c)]^{\rm T}\in\mathbf{R}^{n_c\times 1}. \end{cases}$$

At $t=kT$, (1) can be rewritten as

$$y(k)=\varphi_s^{\rm T}(k)\theta_s+w(k) \tag{4}$$

or

$$y(k)=\varphi^{\rm T}(k)\theta+v(k). \tag{5}$$

Then, the challenge is to estimate the parameter vector only using collection $D^{(k)}=\{u(i),y(i)\}_{i=1}^{k}$.

3 Filter based recursive Bayesian algorithm with modified covariance resetting

3.1 Recursive Bayesian algorithm

Based on the observations, the recursive Bayesian estimate for (5) is obtained by maximizing the following posterior probability density function (PDF) of parameters:

$$\hat{\theta}=\arg\max_{\theta}p(\theta|D^{(k)}). \tag{6}$$

Using Bayesian theory, the posterior PDF of parameters is expressed as

$$p(\theta|D^{(k)})=\frac{p(y(k)|\theta,D^{(k-1)})p(\theta|D^{(k-1)})}{p(y(k)|D^{(k-1)})} \tag{7}$$

where $p(y(k)|\theta,D^{(k-1)})$is the prior PDF of $y(k)$ given θ and $D^{(k-1)}$.

It can be seen that $p(\theta|D^{(k)})$ cannot be calculated by (7) because both $p(y(k)|\theta,D^{(k-1)})$ and $p(\theta|D^{(k-1)})$ are unknown. A feasible assumption is that $p(\theta|D^{(k-1)})$ satisfies a Gaussian distribution with mean $\hat{\theta}(k-1)$ and variance $P(k-1)$, i.e.,

$$p\left(\theta|D^{(k-1)}\right)= \eta\exp\left\{-\frac{1}{2}(\theta-\hat{\theta}(k-1))^{\rm T}P^{-1}(k-1)(\theta-\hat{\theta}(k-1))\right\} \tag{8}$$

where $\eta=\frac{1}{(2\pi)^{\frac{n}{2}}|P(k-1)|^{\frac{1}{2}}}$, and n is the dimension of parameter vector θ.

Considering (5) and $v(k)\tilde{}N(0,\sigma_v^2)$, $y(k)$ satisfies $y(k)\tilde{}N(\varphi^{\rm T}(k)\theta,\sigma_v^2)$, then

$$p(y(k)|\theta,D^{(k-1)})= \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_v}\exp\left\{-\frac{1}{2}\left(\frac{y(k)-\varphi^{\rm T}(k)\hat{\theta}(k-1)}{\sigma_v}\right)^2\right\}. \tag{9}$$

Thus, $p(\theta|D^{(k)})$ is formulated as

$$p(\theta|D^{(k)})=\frac{p(y(k)|\theta,D^{(k-1)})p(\theta|D^{(k-1)})}{p(y(k)\,|D^{(k-1)})}= \delta\exp\left\{-\frac{1}{2}\left(\frac{y(k)-\varphi^{\rm T}(k)\hat{\theta}(k-1)}{\sigma_v}\right)^2- \frac{1}{2}\left(\theta-\hat{\theta}(k-1)\right)^{\rm T}P^{-1}(k-1)\left(\theta-\hat{\theta}(k-1)\right)\right\} \tag{10}$$

where $\delta=\frac{\eta}{p(y(k)|D^{(k-1)})(2\pi)^{\frac{1}{2}}\sigma_v}$ is irrelevant to θ.

It is well known that maximizing the PDF of θ and maximizing the logarithmic PDF of θ is equivalent. So let

$$\frac{\partial\log p(\theta|D^{(k)})}{\partial\theta}\Big|_{\theta=\hat{\theta}(k)}=0$$

then

$$(\theta - \hat{\theta}(k))^{\mathrm{T}} P^{-1}(k)(\theta - \hat{\theta}(k)) = 0 \tag{11}$$

where

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + \dfrac{1}{\sigma_v^2} P(k)\varphi(k)[y(k) - \varphi^{\mathrm{T}}(k)\hat{\theta}(k-1)] \\ P^{-1}(k) = P^{-1}(k-1) + \dfrac{1}{\sigma_v^2}\varphi(k)\varphi^{\mathrm{T}}(k). \end{cases} \tag{12}$$

According to the matrix inversion theorem, $P^{-1}(k)$ can be written as

$$P(k) = \left[I - \frac{P(k-1)\varphi(k)\varphi^{\mathrm{T}}(k)}{\sigma_v^2 + \varphi^{\mathrm{T}}(k)P(k-1)\varphi(k)} \right] P(k-1). \tag{13}$$

Set $L(k) = \frac{1}{\sigma_v^2} P(k)\varphi(k)$, the recursive Bayesian algorithm is summarized as

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + L(k)[y(k) - \varphi^{\mathrm{T}}(k)\hat{\theta}(k-1)] \\ L(k) = \dfrac{P(k-1)\varphi(k)}{\sigma_v^2 + \varphi^{\mathrm{T}}(k)P(k-1)\varphi(k)} \\ P(k) = [I - L(k)\varphi^{\mathrm{T}}(k)]P(k-1). \end{cases} \tag{14}$$

There are two problems in the RB algorithm:

1) The $x(k-i)$ in $\varphi(k)$ is unknown, $w(k-i)$ and $v(k-i)$ are unmeasurable.

2) The σ_v^2 is unknown.

A good solution to issue 1) is to adopt the auxiliary model principle[1−2], i.e.,

$$\begin{cases} x(k-i) \hat{=} \hat{x}(k-i) \\ w(k-i) \hat{=} \hat{w}(k-i) \\ v(k-i) \hat{=} \hat{v}(k-i). \end{cases}$$

In other words, the unknown variables are replaced by their estimates respectively, and this replacement does not affect the convergence of the algorithm[29].

Define

$$\hat{\varphi}(k) = \begin{bmatrix} \hat{\varphi}_s(k) \\ \hat{\varphi}_n(k) \end{bmatrix} \in \mathbf{R}^{n \times 1} \tag{15}$$

where

$$\begin{cases} \hat{\varphi}_s(k) = [-\hat{x}(k-1), -\hat{x}(k-2), \cdots, -\hat{x}(k-n_a), u_1(k), \\ \qquad u_1(k-1), \cdots, u_1(k-n_b), u_2(k-1), u_2(k-2), \\ \qquad \cdots, u_2(k-n_b), \cdots, u_r(k-1), u_r(k-2), \cdots, \\ \qquad u_r(k-n_b)]^{\mathrm{T}} \in \mathbf{R}^{n_s \times 1} \\ \hat{\varphi}_n(k) = [-\hat{w}(k-1), -\hat{w}(k-2), \cdots, \\ \qquad -\hat{w}(k-n_c)]^{\mathrm{T}} \in \mathbf{R}^{n_c \times 1}. \end{cases} \tag{16}$$

Considering (4) and (5), $\hat{x}(k)$, $\hat{w}(k)$ and $\hat{v}(t)$ are calculated as

$$\begin{cases} \hat{x}(k) = \hat{\varphi}_s^{\mathrm{T}}(k)\hat{\theta}_s(k) \\ \hat{w}(k) = y(k) - \hat{x}(k) \\ \hat{v}(k) = y(k) - \hat{\varphi}^{\mathrm{T}}(k)\hat{\theta}(k). \end{cases}$$

The variance of the noise is estimated simultaneously[30, 31]:

$$\hat{\sigma}_v^2(k) = \frac{1}{k-n}\left[(k-n-1)\hat{\sigma}_v^2(k-1) + \hat{v}^2(k)\right]. \tag{17}$$

In (17), $\hat{v}^2(k)$ is unknown when θ is being estimated, so $\hat{\sigma}_v^2(k)$ cannot be calculated directly, and thus it is substituted by $\hat{\sigma}_v^2(k-1)$. Then, the recursive Bayesian algorithm for (5) becomes

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + L(k)[y(k) - \hat{\varphi}^{\mathrm{T}}(k)\hat{\theta}(k-1)] \\ L(k) = \dfrac{P(k-1)\hat{\varphi}^{\mathrm{T}}(k)}{\hat{\sigma}_v^2(k-1) + \hat{\varphi}^{\mathrm{T}}(k)P(k-1)\hat{\varphi}(k)} \\ P(k) = [I_n - L(k)\hat{\varphi}^{\mathrm{T}}(k)]P(k-1). \end{cases} \tag{18}$$

It can be seen that the RB algorithm is a weighted least squares (WLS) algorithm with time-variant weighting factor $\frac{1}{\hat{\sigma}_v^2(k-1)}$. Similar to traditional WLS algorithm, this weighting does not affect the convergence of the algorithm[32].

Another form of estimate for θ is as[14]

$$\hat{\theta}(L) = \left[\sum_{i=1}^{L} \frac{1}{\sigma_v^2(i-1)} \hat{\varphi}(i)\hat{\varphi}^{\mathrm{T}}(i) \right]^{-1} \sum_{i=1}^{L} \frac{1}{\sigma_v^2(i-1)} \hat{\varphi}(i)y(i). \tag{19}$$

It is supposed the inverse $[\cdot]^{-1}$ in (19) exists, and L is the number of the samples.

Using (4) and (19),

$$\begin{aligned} &\left[\sum_{i=1}^{L} \frac{1}{\sigma_v^2(i-1)} \hat{\varphi}(i)\hat{\varphi}^{\mathrm{T}}(i) \right] \left[\hat{\theta}(L) - \theta_0 \right] = \\ &\left[\sum_{i=1}^{L} \frac{1}{\sigma_v^2(i-1)} \hat{\varphi}(i)y(i) - \right. \\ &\left. \sum_{i=1}^{L} \frac{1}{\sigma_v^2(i-1)} \hat{\varphi}(i)(y(i) - v(i)) \right] = \\ &\left[\sum_{i=1}^{L} \hat{\varphi}(i)v(i) \right] \end{aligned} \tag{20}$$

where θ_0 is the true value of parameter θ.

By the assumption of stationary ergodicity, i.e., $\lim\limits_{L\to\infty} \frac{1}{L}\sum_{k=1}^{L} \hat{\varphi}(k)v(k) = \mathrm{E}\left[\hat{\varphi}(k)v(k)\right]$ with probability one, where $\mathrm{E}(\cdot)$ is the expectation operator. So

$$\lim_{L\to\infty} \left\{ \hat{\theta}(L) - \theta_0 \right\} = C^{-1}\mathrm{E}\left(\hat{\varphi}(k)v(k)\right) \tag{21}$$

where

$$C = \lim_{L\to\infty} \left\{ \frac{1}{L} \sum_{k=1}^{L} \varphi(k)\varphi^{\mathrm{T}}(k) \right\}.$$

The RB estimate obtained by (19) is unbiased when $\mathrm{E}\left(\hat{\varphi}(k)v(k)\right) = 0$. In other words, unbiased estimate can be obtained when $\hat{\varphi}(k)$ is uncorrelated with $v(k)$. If $v(k)$ is a white noise, the condition is satisfied. Though the algorithm in (18) can give unbiased estimate for (5) when $v(k)$ is a white noise, the computational cost of (18) is heavy with $4n^2$ flops.

The RB algorithm for (4) is as

$$\begin{cases} \hat{\theta}_s(k) = \hat{\theta}_s(k-1) + L_s(k)[y(k) - \hat{\varphi}_s^{\rm T}(k)\hat{\theta}_s(k-1)] \\ L_s(k) = \dfrac{P_s(k-1)\hat{\varphi}_s^{\rm T}(k)}{\hat{\sigma}_v^2(k-1) + \hat{\varphi}_s^{\rm T}(k)P_s(k-1)\hat{\varphi}_s^{\rm T}(k)} \\ P_s(k) = [I_{n_s} - L_s(k)\hat{\varphi}_s^{\rm T}(k)]P_s(k-1). \end{cases} \quad (22)$$

So

$$\lim_{L\to\infty}\left\{\hat{\theta}_s(L) - \theta_{s0}\right\} = C_s^{-1}{\rm E}\left(\hat{\varphi}_s(k)w(k)\right) \quad (23)$$

where $C_s = \lim\limits_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\hat{\varphi}_s(k)\hat{\varphi}_s^{\rm T}(k)\right\}$ and θ_{s0} is the true value of θ_s.

It can be seen from (23) that the estimate $\hat{\theta}_s(L)$ obtained by (22) is biased because of the colored noise $w(k)$.

3.2 Filter based recursive Bayesian algorithm

In order to obtain unbiased estimate with less computation, filter technique is employed to whiten the colored noise. Multiply both sides of (4) by $C(z^{-1})$

$$y_f(k) = \varphi_f^{\rm T}(k)\theta_s + v(k) \quad (24)$$

where

$$\begin{cases} y_f(k) = C(z^{-1})y(k) \\ \varphi_f(k) = C(z^{-1})\varphi_s(k). \end{cases} \quad (25)$$

Applying the RB algorithm to (24) gives the filter based RB (F-RB) algorithm:

$$\begin{cases} \hat{\theta}_s(k) = \hat{\theta}_s(k-1) + L_s(k)\left[y_f(k) - \hat{\varphi}_f^{\rm T}(k)\hat{\theta}_s(k-1)\right] \\ L_s(k) = \dfrac{P_s(k-1)\hat{\varphi}_f(k)}{\hat{\sigma}_v^2(k-1) + \hat{\varphi}_f^{\rm T}(k)P_s(k-1)\hat{\varphi}_f(k)} \\ P_s(k) = [I_{n_s} - L_s(k)\hat{\varphi}_f^{\rm T}(k)]P_s(k-1) \end{cases} \quad (26)$$

where $\hat{\sigma}_v^2(k-1)$ is the estimate of σ_v^2 at $(k-1)T$. For the noise $w(k) = \frac{1}{C(z^{-1})}v(k)$, the RB algorithm is used again to estimate the coefficients of $C(z^{-1})$:

$$\begin{cases} \hat{\theta}_n(k) = \hat{\theta}_n(k-1) + L_n(k)[\hat{w}(k) - \hat{\varphi}_n^{\rm T}(k)\hat{\theta}_n(k-1)] \\ L_n(k) = \dfrac{P_n(k-1)\hat{\varphi}_n(k)}{\hat{\sigma}_v^2(k-1) + \hat{\varphi}_n^{\rm T}(k)P_n(k-1)\hat{\varphi}_n(k)} \\ P_n(k) = [I_{n_c} - L_n(k)\hat{\varphi}_n^{\rm T}(k)]P_n(k-1). \end{cases} \quad (27)$$

where

$$\begin{cases} \hat{\theta}_n(k) = [\hat{c}_1(k), \hat{c}_2(k), \cdots, \hat{c}_{n_c}(k)]^{\rm T} \in {\bf R}^{n_c\times 1} \\ \hat{\theta}_n(k-1) = [\hat{c}_1(k-1), \hat{c}_2(k-1), \cdots, \hat{c}_{n_c}(k-1)]^{\rm T} \\ \hat{\varphi}_n = [\hat{w}(k-1), \hat{w}(k-2), \cdots, \hat{w}(k-n_c)]^{\rm T} \in {\bf R}^{n_c\times 1}. \end{cases}$$

The $C(z^{-1})$ in (25) at $t = kT$ is unknown. A feasible way is to replace it by its estimate at $t = (k-1)T$, i.e., $\hat{C}(k-1, z^{-1})$. So (25) is rewritten as

$$\begin{cases} \hat{y}_f(k) = \hat{C}(k-1, z^{-1})y(k) = \\ \qquad y(k) + \hat{c}_1(k-1)y(k-1) + \cdots + \\ \qquad \hat{c}_{n_c}(k-1)y(k-n_c) \\ \hat{\varphi}_f(k) = \hat{C}(k-1, z^{-1})\hat{\varphi}_s(k) = \\ \qquad \hat{\varphi}_s(k) + \hat{c}_1(k-1)\hat{\varphi}_s(k-1) + \cdots + \\ \qquad \hat{c}_{n_c}(k-1)\hat{\varphi}_s(k-n_c). \end{cases} \quad (28)$$

It is easy to find that if the colored noise is not an autoregressive noise, what we need to change is the form of the filter. In other words, this algorithm can be used to identify systems with autoregressive, moving average or autoregressive moving average noise.

3.3 F-RB algorithm with modified covariance resetting

For faster convergence, the covariance matrix resetting technique is employed. Although there are many resetting techniques, the aims of these operations are to obtain good tracking of time-varying parameters, so the diagonal elements of the covariance matrix after resetting tend to be constant, rather than zero. This resetting algorithm improves the estimator′s sensitivity to the noise and leads to frequent jitters of estimates inevitably. In order to increase the speed of convergence, a modified covariance resetting method for time-invariant systems is proposed.

1) For filtered system:

$$P_s(k) = \begin{cases} \lambda_{s,k}I, & 0 < \Delta_s(k) < \dfrac{n_s}{n}\gamma \\ P_s(k), & \Delta_s(k) \geq \dfrac{n_s}{n}\gamma \end{cases} \quad (29)$$

with

$$\begin{cases} \Delta_s(k) = {\rm tr}(P_s(k-1)) - {\rm tr}(P_s(k)) \\ 0 < \lambda_{\min}(P_s(k)) \leq \lambda_{s,k} \leq \lambda_{\max}(P_s(k)) \end{cases}$$

where γ is a positive number specified by user, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of corresponding matrices respectively, ${\rm tr}(\cdot)$ denotes the trace of a matrix.

2) For noise,

$$P_n(k) = \begin{cases} \lambda_{n,k}I, & 0 < \Delta_n(k) < \dfrac{n_c}{n}\gamma \\ P_n(k), & \Delta_n(k) \geq \dfrac{n_c}{n}\gamma \end{cases} \quad (30)$$

with

$$\begin{cases} \Delta_n(k) = {\rm tr}(P_n(k-1)) - {\rm tr}(P_n(k)) \\ 0 < \lambda_{\min}(P_n(k)) \leq \lambda_{n,k} \leq \lambda_{\max}(P_n(k)). \end{cases}$$

The F-RB with CR (F-RB-CR) algorithm is summarized as follows and its flowchart is shown in Fig. 2.

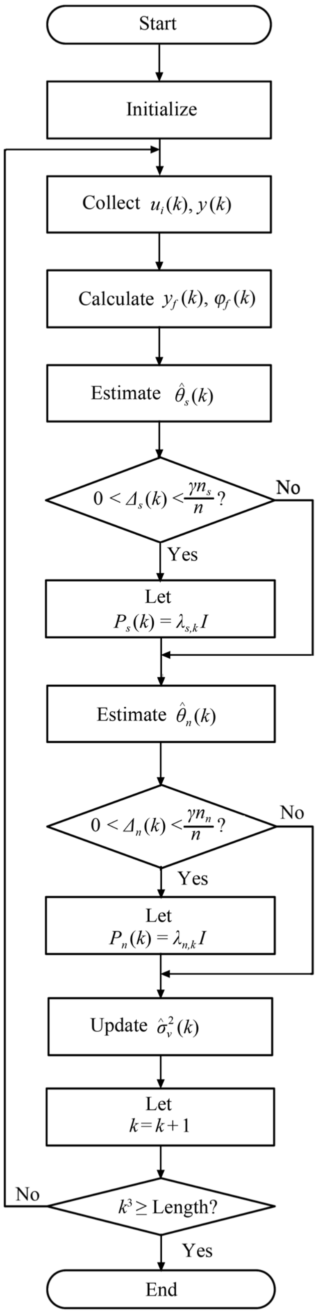

Fig. 2 Flowchart of the F-RB-CR algorithm

Step 1. Set $u_i(k) = 0$, $y(k) = 0$, $\hat{x}(k) = 0$, $\hat{w}(k) = 0$, $\hat{v}(k) = 0$, $\hat{\sigma}_v^2(k) = 1$ for $k \leq 0$. Set $P_s(0) = p_0 I_{n_s}$, $\theta_s(0) = \frac{1_{n_s}}{p_0}$; $P_n(0) = p_0 I_{n_c}$, $\theta_n(0) = \frac{1_{n_c}}{p_0}$; where $p_0 = 10^6$; $1_{n_s} \in \mathbf{R}^{n_s \times 1}$, $1_{n_c} \in \mathbf{R}^{n_c \times 1}$, whose elements are all 1. Set $k = 1$.

Step 2. Collect $u_i(k)$ and $y(k)$

Step 3. Calculate $\hat{y}_f(k)$ and $\hat{\varphi}_f(k)$ by (28)

Step 4. Estimate $\hat{\theta}_s(k)$ by (26)

Step 5. Reset $P_s(k)$ by (29) if needed

Step 6. Estimate $\hat{\theta}_n(k)$ by (27)

Step 7. Reset $P_n(k)$ by (30) if needed

Step 8. Calculate $\hat{\sigma}_v^2(k)$ by (17)

Step 9. Let $k = k + 1$, if $k < L$, go to Step 2, else save the estimate and terminate.

4 Convergence analysis

Equation (26)−(28) are converted into the following equivalent forms:

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + \dfrac{1}{\hat{\sigma}_v^2(k-1)} \bar{P}(k)\bar{\varphi}(k)[y(k) - \bar{\varphi}^{\mathrm{T}}(k)\hat{\theta}(k-1)] \\ \bar{P}^{-1}(k) = \bar{P}^{-1}(k-1) + \dfrac{1}{\hat{\sigma}_v^2(k-1)} \bar{\varphi}(k)\bar{\varphi}^{\mathrm{T}}(k) \end{cases} \tag{31}$$

where

$$\begin{cases} \bar{P}(k) = \begin{bmatrix} P_s(k) & 0 \\ 0 & P_n(k) \end{bmatrix} \\ \bar{\varphi}(k) = \begin{bmatrix} \varphi_s(k) \\ \varphi_n(k) \end{bmatrix}. \end{cases}$$

The matrices $P_s(k)$ and $P_n(k)$ are symmetric and positive definitive, so $\lambda_{\min}(P_s(k)) \leq \lambda_{s,k} \leq \lambda_{\max}(P_s(k))$ and $\lambda_{\min}(P_n(k)) \leq \lambda_{n,k} \leq \lambda_{\max}(P_n(k))$, where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of the corresponding matrices. It is well known that the CR operation does not affect the convergence tendency of the estimator[33], but only has effect on the convergence rate. So Theorem 1 can be gotten.

Theorem 1. Consider algorithm (31) subject to a white noise sequence $\{v(k)|v(k) \sim N(0, \sigma_v^2)\}$. And assume that $\hat{\sigma}_v^2(k)$ is bounded, there exist positive α, β, which make the following excitation:

$$\alpha E_n \leq \frac{1}{k} \sum_{j=1}^{k} \bar{\varphi}(j)\bar{\varphi}^{\mathrm{T}}(j) \leq \beta E_n \tag{32}$$

where $E_n = \mathrm{blockdiag}\,[E_{n_s}, I_{n_n}] \in \mathbf{R}^{n \times n}$ is a block diagonal matrix, $E_{n_s} \in \mathbf{R}^{n_s \times n_s}$ is a symmetric and positive definite matrix, and I_{n_n} is an identity matrix. Then,

$$\lim_{k \to \infty} \hat{\theta}(k) = \theta_0 \tag{33}$$

where θ_0 is the true value.

Proof. Subtract θ_0 from both sides of first equation of (31) and replace $y(k)$ by $(\varphi^{\mathrm{T}}(k)\theta_0 + v(k))$:

$$\begin{aligned} \tilde{\theta}(k) = & \tilde{\theta}(k-1) + \bar{P}(k)S(k) + \bar{P}(k)R(k) - \\ & \bar{P}(k)K(k) \end{aligned} \tag{34}$$

where

$$\begin{cases} \tilde{\theta}(k) = \hat{\theta}(k) - \theta_0 \\ \tilde{\theta}(k-1) = \hat{\theta}(k-1) - \theta_0 \\ S(k) = \dfrac{1}{\hat{\sigma}_v^2(k-1)} \bar{\varphi}(k)[\varphi(k) - \bar{\varphi}(k)]^{\mathrm{T}} \theta_0 \\ R(k) = \dfrac{1}{\hat{\sigma}_v^2(k-1)} \bar{\varphi}(k) v(k) \\ K(k) = \dfrac{1}{\hat{\sigma}_v^2(k-1)} \bar{\varphi}(k)\bar{\varphi}^{\mathrm{T}}(k)\tilde{\theta}(k-1). \end{cases}$$

Multiply (34) by $\bar{P}^{-1}(k)$, and consider second equation of (31):

$$\bar{P}^{-1}(k)\tilde{\theta}(k) = \bar{P}^{-1}(k-1)\tilde{\theta}(k-1) + S(k) + R(k) = \bar{P}^{-1}(0)\tilde{\theta}(0) + \sum_{j=1}^{k} S(j) + \sum_{j=1}^{k} R(j). \quad (35)$$

Multiply (35) by $\bar{P}(k)$,

$$\tilde{\theta}(k) = \bar{P}(k)\bar{P}^{-1}(0)\tilde{\theta}(0) + \bar{P}(k)\sum_{j=1}^{k} S(j) + \bar{P}(k)\sum_{j=1}^{k} R(j). \quad (36)$$

Consider $\lim\limits_{k\to\infty} \bar{P}(k) = 0$, and $\bar{P}^{-1}(0)\tilde{\theta}(0)$ is a constant determined by the user, then $\lim\limits_{k\to\infty} \bar{P}(k)\bar{P}^{-1}(0)\tilde{\theta}(0) = 0$. Assume that $[\varphi(k) - \bar{\varphi}(k)]$ is bounded, consider $\lim\limits_{k\to\infty} \bar{P}(k) = 0$ and (31), then

$$\lim_{k\to\infty} \bar{P}(k)\sum_{j=1}^{k} S(j) = \lim_{k\to\infty} \bar{P}(k)\sum_{j=1}^{k} \frac{1}{\hat{\sigma}_v^2(j-1)}\bar{\varphi}(j)[\varphi(j) - \bar{\varphi}(j)]^{\rm T}\theta_0 = 0$$

where $\{\bar{\varphi}(k)\}$ and $\{v(k)\}$ are irrelevant, so

$$\lim_{k\to\infty} \bar{P}(k)\sum_{j=1}^{k} R(j) = \lim_{x\to\infty} \bar{P}(k)\sum_{j=1}^{k} \frac{1}{\hat{\sigma}_v^2(j-1)}\bar{\varphi}(j)v(j) = 0.$$

Three terms of the right side of (36) tend to be 0 when $k \to +\infty$. That is to say: $\lim\limits_{k\to\infty} \tilde{\theta}(k) = 0$, and then (32) follows. □

5 Example

5.1 A numerical example

Consider the following system with non-uniformly sampled input data:

$$y(k) = \frac{B_1(z^{-1})}{A(z^{-1})}u_1(k) + \frac{B_2(z^{-1})}{A(z^{-1})}u_2(k) + \frac{1}{C(z^{-1})}v(k)$$

where

$$\begin{cases} A(z^{-1}) = 1 - 0.580\,3z^{-1} + 0.829\,0z^{-2} \\ B_1(z^{-1}) = 0.496\,7z^{-1} + 0.102\,3z^{-2} \\ B_2(z^{-1}) = 0.581\,0z^{-1} + 0.273\,4z^{-2} \\ C(z^{-1}) = 1 - 0.286\,5z^{-1}. \end{cases}$$

Using the proposed F-RB-CR algorithm, the result is shown in Tables 1−2, where the variance of the noise is 0.4^2 and 0.6^2 respectively. The estimation error are shown in Fig. 3. The results using the F-RLS algorithm[11] are shown in Tables 3 and 4 and taken for comparison. The estimates of the proposed algorithm versus k are shown in Fig. 4. Here, the estimation error is defined as $\delta = \frac{\|\hat{\theta}(k)-\theta_0\|}{\|\theta_0\|} \times 100\%$.

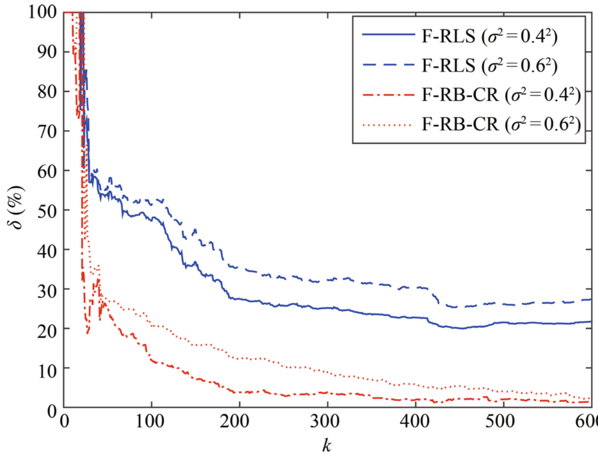

Fig. 3 Estimation errors versus k using the F-RLS and F-RB-CR

It can be seen that:

1) The estimation errors of the two algorithms become small when k is increasing.

2) The accuracy of the F-RB-CR is higher than the F-RLS (see, e.g., Fig. 3).

3) The estimation errors of the two algorithms fall rapidly in the initial stage of identification, and then descend slowly. And the curve of the proposed algorithm is steeper than that of the competitor when $k < 100$, which means the proposed algorithm converges quickly than the compared algorithm.

Table 1 Results using the F-RB-CR ($\sigma_v^2 = 0.4^2$)

k	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}	c_1	$\delta(\%)$
100	−0.513 5	0.749 7	0.528 8	0.203 9	0.549 6	0.237 6	−0.321 8	12.008 5
200	−0.563 1	0.809 2	0.502 6	0.127 6	0.556 6	0.251 2	−0.273 8	3.832 3
300	−0.562 0	0.833 7	0.507 1	0.118 7	0.560 0	0.246 8	−0.255 5	3.999 9
400	−0.579 2	0.844 5	0.492 1	0.110 4	0.585 3	0.256 8	−0.280 0	1.934 2
500	−0.589 2	0.833 3	0.498 1	0.093 5	0.574 4	0.264 9	−0.264 9	2.070 1
600	−0.585 3	0.838 6	0.500 0	0.092 0	0.579 5	0.261 4	−0.285 4	1.466 1
True values	−0.580 3	0.829 0	0.496 7	0.102 3	0.581 0	0.273 4	−0.286 5	

Table 2 Results using the F-RB-CR ($\sigma_v^2 = 0.6^2$)

k	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}	c_1	δ (%)
100	−0.411 8	0.651 0	0.552 6	0.193 7	0.600 5	0.300 5	−0.239 4	20.536 2
200	−0.487 3	0.715 2	0.504 9	0.149 3	0.594 5	0.294 8	−0.236 3	12.342 7
300	−0.496 7	0.770 7	0.500 0	0.139 1	0.594 9	0.280 1	−0.238 5	8.975 0
400	−0.549 3	0.790 3	0.478 5	0.127 3	0.614 9	0.286 0	−0.255 8	5.647 8
500	−0.589 7	0.808 6	0.487 4	0.117 2	0.595 7	0.291 8	−0.246 7	4.074 4
600	−0.573 0	0.827 4	0.491 2	0.112 5	0.598 6	0.279 2	−0.267 3	2.254 9
True values	−0.580 3	0.829 0	0.496 7	0.102 3	0.581 0	0.273 4	−0.286 5	

Table 3 Results using the F-RLS ($\sigma_v^2 = 0.4^2$)

k	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}	c_1	δ(%)
100	−0.442 8	0.524 1	0.261 9	0.028 2	0.568 3	0.613 3	−0.615 8	47.263 6
200	−0.571 5	0.731 5	0.318 8	−0.013 8	0.609 9	0.474 4	−0.482 1	27.494 2
300	−0.579 7	0.763 0	0.317 6	−0.034 1	0.593 6	0.446 2	−0.453 4	25.219 6
400	−0.602 7	0.770 7	0.321 4	−0.011 9	0.609 2	0.422 4	−0.431 8	22.737 8
500	−0.625 1	0.847 1	0.341 6	−0.056 6	0.613 1	0.389 2	−0.412 9	21.494 7
600	−0.611 1	0.829 9	0.334	−0.055 2	0.612 5	0.395 8	−0.411 3	21.719 8
True values	−0.580 3	0.829 0	0.496 7	0.102 3	0.581 0	0.273 4	−0.286 5	

Table 4 Results using the F-RLS ($\sigma_v^2 = 0.6^2$)

k	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}	c_1	δ(%)
100	−0.480 5	0.512 7	0.188 7	−0.022 3	0.637 6	0.699 3	−0.533 0	51.214 9
200	−0.563 3	0.679 9	0.223 4	−0.010 1	0.648 2	0.575 3	−0.416 0	35.385 0
300	−0.553 6	0.724 1	0.232 4	0.024 9	0.616 0	0.565 6	−0.391 9	32.303 9
400	−0.564 6	0.734 3	0.238 4	0.072 5	0.630 2	0.548 5	−0.377 0	30.308 1
500	−0.579 4	0.831 3	0.278 3	0.069 6	0.623 9	0.526 0	−0.370 2	26.147 0
600	−0.558 5	0.816 4	0.265 6	0.073 5	0.624 3	0.536 4	−0.372 7	27.405 8
True values	−0.580 3	0.829 0	0.496 7	0.102 3	0.581 0	0.273 4	−0.286 5	

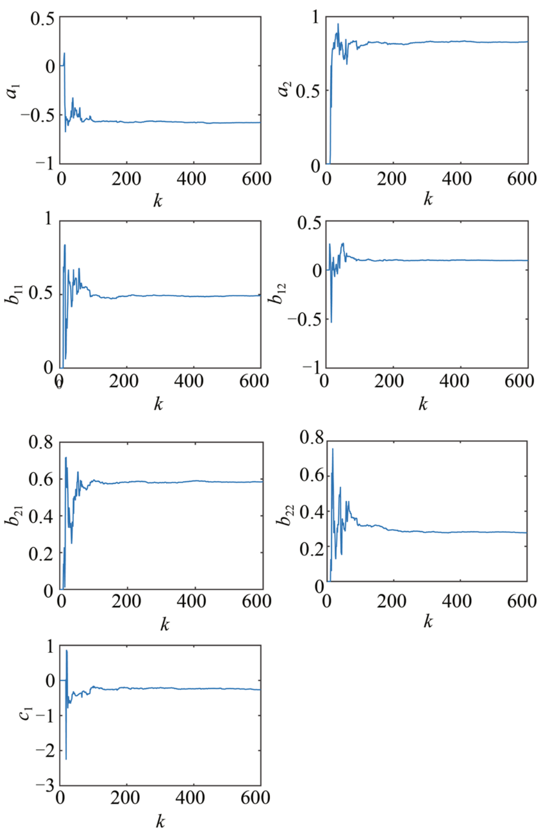

Fig. 4 Estimates versus k using the F-RB-CR ($\sigma_v^2 = 0.4^2$)

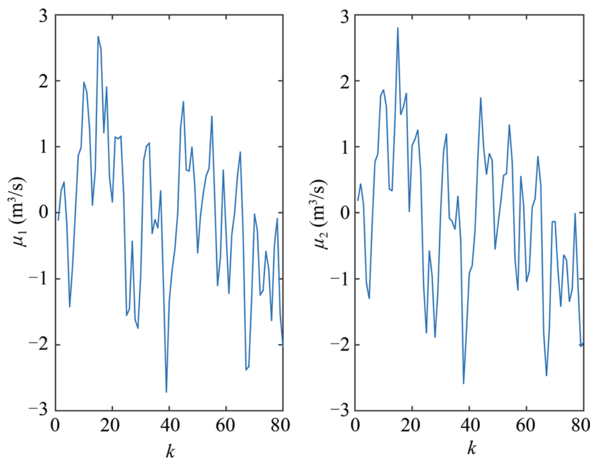

Fig. 5 Input data

5.2 A gas furnace

The gas furnace was considered firstly by Box and Jenkins, in which air and methane were combined to form a mixture of gases containing CO_2 (carbon dioxide)[34]. The air feed was kept constant, but the methane feed rate could be varied, and the resulting CO_2 concentration in the off-gases was measured. The input gas rate ($u(k)$) is selected as input variable and the concentration of the output CO_2, i.e, ($y(k)$), as the output variable.

the concentration of the output CO_2, i.e, $y(k)$, as the output variable

The original data were modified to NUSD form: the output sampling period is 27 s, and irregular input sampling intervals are $\tau_1 = 9$ and $\tau_2 = 18$ s. 80 samples were adopted, among which first 60 were used for modeling and the next 20 were used to validate the model. These samples are depicted in Figs. 5 and 6.

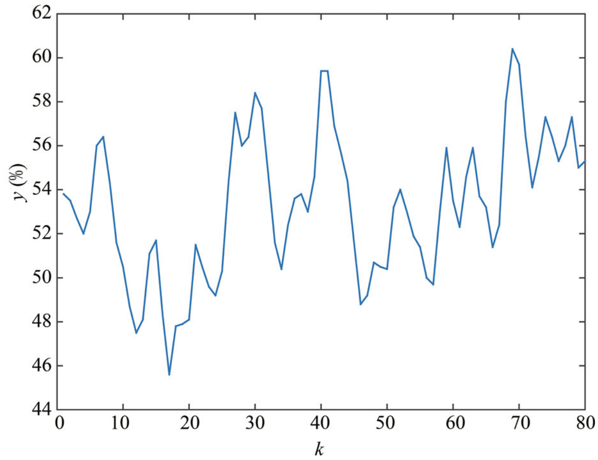

Fig. 6 Output data

The selected model structure is of the form

$$y_1(k) = \frac{B_1(z^{-1})}{A(z^{-1})}u_{11}(k) + \frac{B_2(z^{-1})}{A(z^{-1})}u_{22}(k) + \frac{1}{C(z^{-1})}v(k)$$

with

$$\begin{cases} A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \\ B_1(z^{-1}) = b_{1,1} z^{-1} + b_{1,2} z^{-2} \\ B_2(z^{-1}) = b_{2,1} z^{-1} + b_{2,2} z^{-2} \\ C(z^{-1}) = 1 + c_1 z^{-1} \\ u_{11}(k) = u_1(k) - \dfrac{1}{L}\displaystyle\sum_{i=1}^{L} u_1(i) \\ u_{22}(k) = u_2(k) - \dfrac{1}{L}\displaystyle\sum_{i=1}^{L} u_2(i) \\ y_1(k) = y(k) - \dfrac{1}{L}\displaystyle\sum_{i=1}^{L} y(i). \end{cases}$$

The estimate obtained by the proposed algorithm is as

$$[a_1, a_2, c_1] = [-0.2049, 0.0208, -0.9484]$$
$$[b_{1,1}, b_{1,2}, b_{2,1}, b_{2,2}] = [-0.8775, -0.7103, 0.1755, -0.8891].$$

The mean square error is 0.061 6. The estimate versus k is shown in Fig. 7.

The comparison between the observed outputs and predicted outputs obtained by using the later 20 inputs and estimated model is shown in Fig. 8.

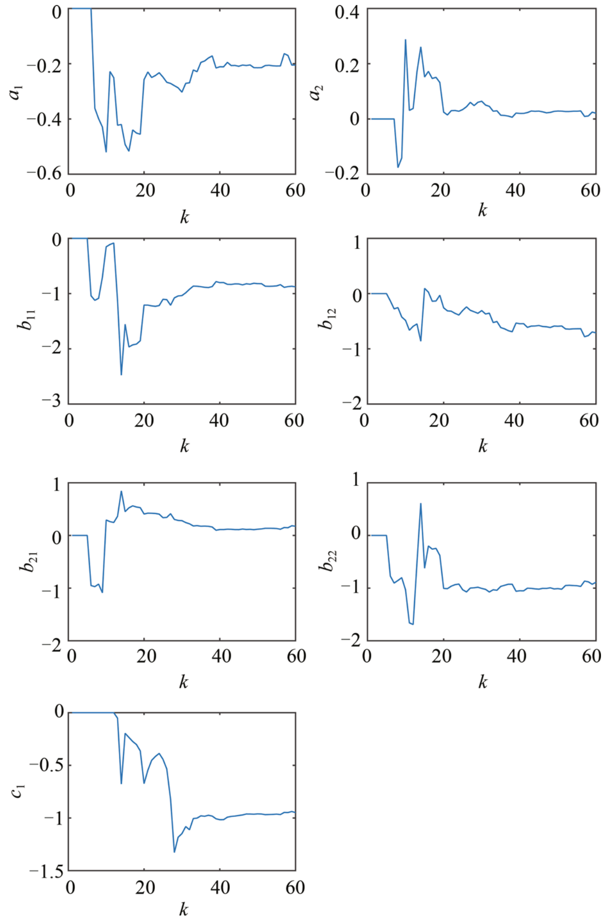

Fig. 7 Estimates of the gas furnace versus k

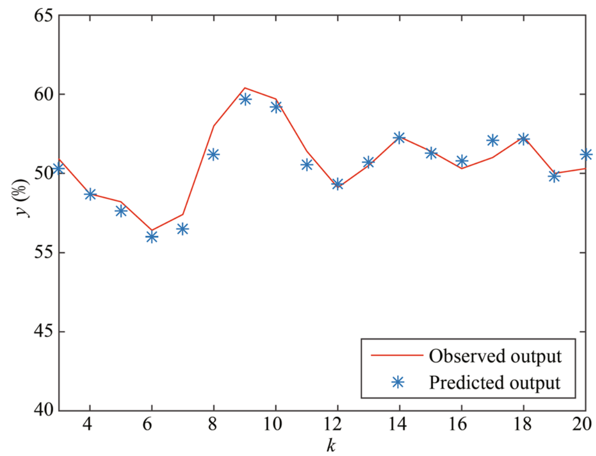

Fig. 8 Comparison of the observed output and the predicted output

6 Conclusions

For the identification of systems with non-uniformly sampled input data, a filter based recursive Bayesian algorithm with covariance resetting is presented. In this algorithm, estimated noise transform function is used to whiten the colored noise, and then the recursive Bayesian algorithm is taken to estimate the parameter vector. A modified covariance resetting is also integrated into the algorithm for faster convergence. A numeric example and a gas furnace data were used to validate the algorithm. The proposed method can be extended to identify other linear or linear-in-parameters systems with colored noise.

References

[1] F. Ding. *System IdentificationNew Theory and Methods*, Beijing, China: Science Press, pp. 117–131, 2013. (in Chinese)

[2] F. Ding. *System IdentificationPerformances Analysis for Identification Methods*, Beijing, China: Science Press, pp. 1–12, 2014. (in Chinese)

[3] F. Ding. *System IdentificationMulti-Innovation Identification Theory and Methods*, Beijing, China: Science Press, pp. 1–29, 2016. (in Chinese)

[4] Y. J. Liu, F. Ding, Y. Shi. Least squares estimation for a class of non-uniformly sampled systems based on the hierarchical identification principle. *Circuits, Systems, and Signal Processing*, vol. 31, no. 6, pp. 1985–2000, 2012.

[5] S. Wang, V. Dinavahi, J. Xiao. Multi-rate real-time model-based parameter estimation and state identification for induction motors. *IET Electric Power Applications*, vol. 7, no. 1, pp. 77–86, 2013.

[6] W. H. Li, S. L. Shah, D. Y. Xiao. Kalman filters in non-uniformly sampled multirate systems: For FDI and beyond. *Automatica*, vol. 44, no. 1, pp. 199–208, 2008.

[7] Y. Shi, T. W. Chen. 2-norm-based iterative design of filter-bank transceivers: A control perspective. *Journal of Control Science and Engineering*, vol. 2008, Article number 143085, 2008.

[8] Y. Shi, F. Ding, T. W. Chen. 2-norm based recursive design of transmultiplexers with designable filter length. *Circuits, Systems and Signal Processing*, vol. 25, no. 4, pp. 447–462, 2006.

[9] R. D. Gudi, S. L. Shah, M. R. Gray. Adaptive multirate state and parameter estimation strategies with application to a bioreactor. *AIChE Journal*, vol. 41, no. 11, pp. 2451–2464, 1995.

[10] D. Q. Wang. Least squares-based recursive and iterative estimation for output error moving average systems using data filtering. *IET Control Theory & Application*, vol. 5, no. 14, pp. 1648–1657, 2011.

[11] F. W. Chen, H. Garnier, M. Gilson, J. C. Agüero, B. I. Godoy. Identification of continuous-time transfer function models from non-uniformly sampled data in presence of colored noise. In *Proceedings of the 19th IFAC World Congress*, IFAC, Cape Town, South Africa, pp. 10379–10384, 2014.

[12] L. Xie, Y. J. Liu, H. Z. Yang, F. Ding. Modelling and identification for non-uniformly periodically sampled-data systems. *IET Control Theory & Applications*, vol. 4, no. 5, pp. 784–794, 2010.

[13] F. Ding, L. Qiu, T. W. Chen. Reconstruction of continuous-time systems from their non-uniformly sampled discrete-time systems. *Automatica*, vol. 45, no. 2, pp. 324–332, 2009.

[14] J. Ding, J. X. Lin. Modified subspace identification for periodically non-uniformly sampled systems by using the lifting technique. *Circuits, Systems, and Signal Processing*, vol. 33, no. 5, pp. 1439–1449, 2014.

[15] F. Ding, G. J. Liu, X. P. Liu. Partially coupled stochastic gradient identification methods for non-uniformly sampled systems. *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1976–1981, 2010.

[16] Y. Zhang, Z. Zhao, G. M. Cui. Auxiliary model method for transfer function estimation from noisy input and output data. *Applied Mathematical Modelling*, vol. 39, no. 15, pp. 4257–4265, 2015.

[17] Y. Zhang. Unbiased identification of a class of multi-input single-output systems with correlated disturbances using bias compensation methods. *Mathematical and Computer Modelling*, vol. 53, no. 9–10, pp. 1810–1819, 2011.

[18] Y. Zhang, G. M. Cui. Bias compensation methods for stochastic systems with colored noise. *Applied Mathematical Modelling*, vol. 35, no. 4, pp. 1709–1716, 2011.

[19] K. H. Yang, W. S. Yu, X. Q. Ji. Rotation estimation for mobile robot based on single-axis gyroscope and monocular camera. *International Journal of Automation and Computing*, vol. 9, no. 3, pp. 292–298, 2012.

[20] D. K. Arif, Widodo, Salmah, E. Apriliani. Construction of the Kalman filter algorithm on the model reduction. *International Journal of Control and Automation*, vol. 7, no. 9, pp. 257–270, 2014.

[21] Y. J. Wang, F. Ding. Novel data filtering based parameter identification for multiple-input multiple-output systems using the auxiliary model. *Automatica*, vol. 71, pp. 308–313, 2016.

[22] Y. J. Wang, F. Ding. Filtering-based iterative identification for multivariable systems. *IET Control Theory & Applications*, vol. 10, no. 8, pp. 894–902, 2016.

[23] Y. J. Wang, F. Ding. The auxiliary model based hierarchical gradient algorithms and convergence analysis using the filtering technique. *Signal Processing*, vol. 128, pp. 212–221, 2016.

[24] L. Xie, H. Z. Yang, F. Ding. Recursive least squares parameter estimation for non-uniformly sampled systems based on the data filtering. *Mathematical and Computer Modelling*, vol. 54, no. 1–2, pp. 315–324, 2011.

[25] Y. J. Liu, L. Xie, F. Ding. An auxiliary model based on a recursive least-squares parameter estimation algorithm for non-uniformly sampled multirate systems. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 223, no. 4, pp. 445–454, 2009.

[26] G. C. Goodwin, M. G. Cea. Application of minimum distortion filtering to identification of linear systems having non-uniform sampling period. *System Identification, Environmental Modelling, and Control System Design*, L. P. Wang, H. Garnier, Eds., London, UK: Springer, pp. 97–114, 2012.

[27] J. H. Wang, H. X. Jiang, F. Ding. Filtering of non-uniformly multirate sampled-data systems using the lifting technique. In *Proceedings of IEEE International Conference on Automation and Logistics*, IEEE, Ji′nan, China, pp. 46–50, 2007.

[28] L. P. Yan, B. Xiao, Y. Q. Xia, M. Y. Fu. State estimation for a kind of non-uniform sampling dynamic system. *International Journal of Systems Science*, vol. 44, no. 10, pp. 1913–1924, 2013.

[29] F. Ding, G. J. Liu, X. P. Liu. Parameter estimation with scarce measurements. *Automatica*, vol. 47, no. 8, pp. 1646–1655, 2011.

[30] J. Mikleš, M. Fikar. *Process Modelling, Identification, and Control*, Berlin, Germany: Springer, pp. 221–251, 2007.

[31] D. Y. Xiao. *Theory of System Identification with Applications*, Beijing, China: Tsinghua University Press, pp. 95–134, 2014. (in Chinese)

[32] L. Ljung. *System Identification: Theory for the User*, 2nd ed., Englewood Cliffs, USA: Prentice Hall PTR, pp. 197–245, 1999.

[33] G. C. Goodwin, K. S. Sin. *Adaptive Filtering Prediction and Control*, New York, USA: Dover Publications Inc., pp. 121–150, 2009.

[34] G. E. P. Box, G. M. Jenkins, G. C. Reinsel. *Time Series Analysis: Forecasting and Control*, New Jersey, USA: John Wiley & Sons, pp. 439–472, 2011.

Shao-Xue Jing received the B. Sc. degree from Anhui Agriculture University and the M. Sc. degree from Gansu University of Technology, China in 1997 and 2000, respectively. And he received the Ph. D. degree in control theory and control engineering from Shanghai Jiao Tong University, China in 2007. Now he has been a professor in School of Electrical and Information Engineering, Jiangsu University, China.

His research interests include multiple model approach and its application, machine learning, virtual metrology, predictive control and run-to-run control theory and practice, system identification.

E-mail: jingsx3@outlook.com
ORCID ID: 0000-0002-8912-8611

Tian-Hong Pan received the B. Sc. degree from Anhui Agriculture University, the M. Sc. degree from Gansu University of Technology, China in 1997 and 2000, respectively. And he received the Ph. D. degree in control theory and control engineering from Shanghai Jiao Tong University, China in 2007. Now he has been a professor in School of Electrical and Information Engineering, Jiangsu University, China.

His research interests include multiple model approach and its application, machine learning, virtual metrology, predictive control and run-to-run control theory and practice, system identification.

E-mail: thpan@ujs.edu.cn (Corresponding author)
ORCID ID: 0000-0002-0993-3937

Zheng-Ming Li received the B. Sc. degree from the Zhenjiang Institute of Agricultural Machinery, China in 1982, and the M. Sc. degree in control theory and control engineering from the Department of Automation, Xi′an Jiao Tong University, China in 1987. He has been a professor in the School of Electrical Information and Engineering, Jiangsu University, China.

His research interests include modeling of complex processes and wireless sensor network.

E-mail: lzming@ujs.edu.cn