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Adaptive Tracking Control of Mobile Manipulators with Affine Constraints and Under-actuated Joints

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Abstract: Adaptive motion/force tracking control is considered for a class of mobile manipulators with affine constraints and under-actuated joints in the presence of uncertainties in this paper. Dynamic equation of mobile manipulator is transformed into a controllable form based on dynamic coupling technique. In view of the asymptotic tracking idea and adaptive theory, adaptive controllers are proposed to achieve the desired control objective. Detailed simulation results confirm the validity of the control strategy.

Keywords: Tracking control, affine constraints, mobile manipulators, under-actuated joints, dynamic coupling.

1 Introduction

Due to the higher performances possessed by mobile manipulator, such as much larger work space, the better kinematic flexibility beyond that of the traditional one, considerable efforts[1−4] have been made to guarantee stability and robustness for it. However, control design for this mechanical system is still a challenging problem owing to complex and strongly coupled dynamics of the mobile platform and the robotic arm.

The motion and force tracking control for mobile manipulators have been systematically investigated in literatures[5−9] by state-feedback, output-feedback and neural network, etc. However, most researches have been done to investigate mobile manipulators with the full-actuated joints. The under-actuation in any joint of mobile manipulators may occur in the full-actuated manipulators, and the effective control of under-actuated robotic system could enhance the fault-tolerance if the actuator fails. For these reasons, great efforts[10−12] have been made to design controllers for the under-actuated mobile manipulators.

A new mobile manipulator shown in Fig. 1, is made up of a multi-link manipulator with under-actuated joints and a boat, and is subjected to affine constraints[13−16]. The traditional control methods are hardly applicable to such mechanical systems. Therefore, investigating tracking control problems for such mobile manipulator has theoretical and practical meanings. Considering the mentioned problems, this paper considers the tracking control of the affine constraint mobile manipulators with under-actuated joints, and addresses mathematical modeling, algorithm design and theory analysis for practical mechanical systems. Using the technique of dynamic coupling, dynamic equation of mobile manipulator is transformed into a controllable form. By constructing an appropriate upper bound parameter, adaptive control design becomes much easier, and only one parameter updating law is needed. Hence, the dynamic order of adaptive controller is reduced to be minimum.

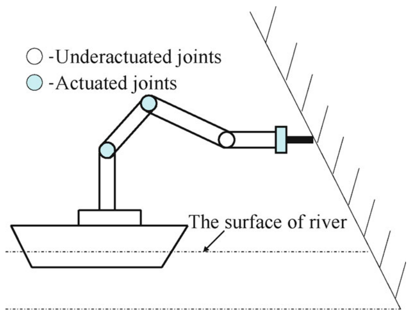

Fig. 1 Manipulators mounted on a boat

2 System description

2.1 Dynamic model

Consider an n-DOF mobile manipulator shown in Fig. 1. It consists of a mobile boat and a multi-link manipulator. According to the kinematic analysis of the boat on a running river[5], the affine constraints can be written as

$$J_a(q_b)\dot{q}_b = A(q_b) \tag{1}$$

where $q_b \in \mathbf{R}^3 = [q_{b_1}, q_{b_2}, q_{b_3}]^{\mathrm{T}}$ is the coordinates of the boat, $J_a(q_b) = [\cos q_{b_3}, -\sin q_{b_3}, 0]$ and $A(q_b) = C(q_{b_2})\cos q_{b_3}$. The affine constraint forces are given by

$$f_a = J_a^{\mathrm{T}}(q_b) l_a \tag{2}$$

where $l_a \in \mathbf{R}^m$ is a Lagrangian multiplier corresponding to m affine constraints.

The manipulator is a series-chain multi-link manipulator, $q_a \in \mathbf{R}^{n_a}$ and $q_u \in \mathbf{R}^{n_u}$ are the coordinates of the active

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and under-actuated joints of the manipulator, respectively. For convenience, let $q = [q_b^{\mathrm{T}}, q_a^{\mathrm{T}}, q_u^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^n$ be the vector of generalized coordinates of the whole system.

According to Euler-lagrangian formulation, after considering the affine constraints, dynamic equations of the mobile manipulator are described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F = B(q)\tau + f \tag{3}$$

with

$$M(q) = \begin{bmatrix} M_b & M_{ba} & M_{bu} \\ M_{ba} & M_a & M_{au} \\ M_{bu} & M_{au} & M_u \end{bmatrix}, \; B(q)\tau = \begin{bmatrix} B_b\tau_b \\ \tau_a \\ 0 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} C_b & C_{ba} & C_{bu} \\ C_{ba} & C_a & C_{au} \\ C_{bu} & C_{au} & C_u \end{bmatrix}, \; f = \begin{bmatrix} f_a \\ 0 \\ 0 \end{bmatrix}$$

where M_b, M_a and M_u describe the inertia matrices for the mobile boat and the active links and the passive links, respectively. M_{ba}, M_{bu} M_{au} are the coupling inertia matrices of the boat, the active links and the passive links, respectively. C_b, C_a and C_u denote the Centripetal and Coriolis torques for the boat, the active links and the passive links. C_{ba}, C_{bu} and C_{au} are the coupling Centripetal and Coriolis torques of the boat, the active links and the passive links, respectively. $G(q) = [G_b, G_a, G_u]^{\mathrm{T}}$ is the gravitational torque vector. B_b, as input transformation matrix of the boat, is assumed to be known because it is a function of fixed geometry of the system. τ_b and τ_a are the control input vectors for the mobile boat and the active links, $F = [F_b, F_a, F_u]^{\mathrm{T}}$ denotes the external forces.

2.2 State transformation

For the affine constraints (1), according to our previous results [13, 14], there exists a known full-rank matrix $S(q_b) \in \mathbf{R}^{n_b \times (n_b - m)}$ satisfying

$$\dot{q}_b = S(q_b)\dot{z} + \eta(q_b) \tag{4}$$

where z corresponds to the internal state variable of q_b, $S(q_b)$ and $\eta(q_b)$ satisfying $J_a(q_b)S(q_b) = 0$ and $J_a(q_b)\eta(q_b) = A(q_b)$, respectively. As mentioned by Wang et al.[17], the internal states $z(q)$ and $\dot{z}(q)$ possess practical physical meanings and $z(q)$ can be considered as $(n-m)$ "output equations" of original system. Substituting (4) into (3) gives

$$MH\ddot{\xi} + (M\dot{H} + CH)\dot{\xi} + G + F = \tau + J^{\mathrm{T}}l_a \tag{5}$$

with

$$\xi = \begin{bmatrix} z \\ q_a \\ q_u \end{bmatrix}, \; H = \begin{bmatrix} S & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \; F = \begin{bmatrix} F_b \\ F_a \\ F_u \end{bmatrix}$$

$$G = \begin{bmatrix} M_b\dot{\eta} + C_b\eta + G_b \\ M_{ba}\dot{\eta} + C_{ba}\eta + G_a \\ M_{bu}\dot{\eta} + C_{bu}\eta + G_u \end{bmatrix}, \; \tau = \begin{bmatrix} B_b\tau_b \\ \tau_a \\ 0 \end{bmatrix}. \tag{6}$$

If considering the control input τ in the form

$$\tau = \hat{\tau} + J^{\mathrm{T}}\tau_0 \tag{7}$$

and pre-multiplying $H^{\mathrm{T}}(q)$ on both sides of (5), and noting $J(q)H(q) = 0$, one can obtain

$$\bar{M}\ddot{\xi} + \bar{C}\dot{\xi} + \bar{G} + \bar{F} = \bar{\tau} \tag{8}$$

where $\bar{M} = H^{\mathrm{T}}MH$ is symmetric and positive definite, $\bar{C} = H^{\mathrm{T}}(M\dot{H} + CH)$, $\bar{G} = H^{\mathrm{T}}G$, $\bar{F} = H^{\mathrm{T}}F$, $\bar{\tau} = H^{\mathrm{T}}\hat{\tau}$, and the force multipliers can be obtained by (5)

$$l_a = J^*\left((M\dot{H} + CH)\dot{\xi} + G + F - \hat{\tau}\right) - \tau_0 \tag{9}$$

where $J^* = \left(JM^{-1}J^{\mathrm{T}}\right)^{-1}JM^{-1}$.

2.3 Dynamic coupling

The state variable of mobile manipulators is partitioned in quantities related to the active joints, the passive joints, and the remaining joints as ξ_1, ξ_3, and ξ_2, respectively, such that the dimension of ξ_1 and ξ_3 are equal. According to these partitions, we have the partition structure for (8) as

$$\bar{M}(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$\bar{C}(q,\dot{q})\xi = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} C_{11}\dot{\xi}_1 + C_{12}\dot{\xi}_2 + C_{13}\dot{\xi}_3 \\ C_{21}\dot{\xi}_1 + C_{22}\dot{\xi}_2 + C_{23}\dot{\xi}_3 \\ C_{31}\dot{\xi}_1 + C_{32}\dot{\xi}_2 + C_{33}\dot{\xi}_3 \end{bmatrix}$$

$$\bar{G}(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{bmatrix}. \tag{10}$$

In order to make ξ_3 controllable, we assume that matrices M_{13} and M_{31} are not equal to 0 and M_{11}^{-1} exists. Considering the new partition in (10), after some manipulations, one obtains the following dynamic equations

$$M_{11}\ddot{\xi}_1 + M_{12}\ddot{\xi}_2 + M_{13}\ddot{\xi}_3 + C_1 + G_1 + F_1 = \tau_1 \tag{11}$$

$$\Gamma_{11}\ddot{\xi}_2 + \Gamma_{12}\ddot{\xi}_3 + \Upsilon_1 + \Pi_1 = \tau_2 - M_{21}M_{11}^{-1}\tau_1 \tag{12}$$

$$\Gamma_{21}\ddot{\xi}_2 + \Gamma_{22}\ddot{\xi}_3 + \Upsilon_2 + \Pi_2 = -M_{31}M_{11}^{-1}\tau_1 \tag{13}$$

where

$$\Gamma_{11} = M_{22} - M_{21}M_{11}^{-1}M_{12}$$
$$\Gamma_{12} = M_{23} - M_{21}M_{11}^{-1}M_{13}$$
$$\Gamma_{21} = M_{32} - M_{31}M_{11}^{-1}M_{12}$$
$$\Gamma_{22} = M_{33} - M_{31}M_{11}^{-1}M_{13}$$
$$\Upsilon_1 = (C_{22} - M_{21}M_{11}^{-1}C_{12})\dot{\xi}_2 + (C_{22} - M_{21}M_{11}^{-1}C_{13})\dot{\xi}_3$$
$$\Upsilon_2 = (C_{32} - M_{31}M_{11}^{-1}C_{12})\dot{\xi}_2 + (C_{33} - M_{31}M_{11}^{-1}C_{13})\dot{\xi}_3$$
$$\Pi_1 = (C_{21} - M_{21}M_{11}^{-1}C_{11})\dot{\xi}_1 + G_2 + F_2 - M_{21}M_{11}^{-1}(G_1 + F_1)$$
$$\Pi_2 = (C_{31} - M_{31}M_{11}^{-1}C_{11})\dot{\xi}_1 + G_3 + F_3 - M_{31}M_{11}^{-1}(G_1 + F_1).$$

Let $y = [\xi_3^{\mathrm{T}}, \xi_2^{\mathrm{T}}]^{\mathrm{T}}$, we can rewrite (12) and (13) as

$$M_1(\xi)\ddot{y} + C_1(\xi,\dot{\xi})\dot{y} + G_1 + F_1 = B_1u \tag{14}$$

where

$$C_1 = \begin{bmatrix} C_{33} - M_{31}M_{11}^{-1}C_{13} & C_{32} - M_{31}M_{11}^{-1}C_{12} \\ C_{23} - M_{21}M_{11}^{-1}C_{13} & C_{22} - M_{21}M_{11}^{-1}C_{12} \end{bmatrix}$$

$$M_1 = \begin{bmatrix} \Gamma_{22} & \Gamma_{21} \\ \Gamma_{12} & \Gamma_{11} \end{bmatrix}, G_1 = \begin{bmatrix} \Upsilon_2 \\ \Upsilon_1 \end{bmatrix}, F_1 = \begin{bmatrix} \Pi_2 \\ \Pi_1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -M_{31}M_{11}^{-1} & 0 \\ -M_{21}M_{11}^{-1} & I \end{bmatrix}, u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}.$$

It should be noted that dynamic (14) possesses considerable properties which are listed as follows:

Property 1. The matrix M_1 is symmetric and positive definite.

Property 2. One can decompose $C_1 = \hat{C}_1 + \tilde{C}_1$ such that the matrix $\dot{M}_1 - 2\hat{C}_1$ is skew symmetric.

In practice, by virtue of structural complexity of the mobile manipulator and pay-load variation from task to task, the inertia parameters of the system are often unknown. Hence, we suppose matrix functions M_1, C_1, G_1 and F_1 in system (14) are unknown due to uncertain parameters. B_1 is known because input transformation matrix B_b and $S(q_b)$ are all known.

3 Control design and stability analysis

3.1 ξ_2 and ξ_3 subsystems control

The control objective is specified as: the designed controllers ensure that the tracking errors $e_y = y - y_d = [\xi_3^{\mathrm{T}} - \xi_{3d}^{\mathrm{T}}, \xi_2 - \xi_{2d}^{\mathrm{T}}]^{\mathrm{T}}$ remain within a small neighborhood of 0, i.e.,

$$\|\xi_i - \xi_{id}\| \leq \varepsilon_i$$

where $\|\cdot\|$ denotes the Euclidean norm, $i = 2, 3$, ε_i are arbitrarily small constants, $l - l_d$ and ξ are bounded.

Lemma 1[17]**.** There exist time varying positive functions $\delta(t)$ and $\iota(t)$ converging to 0 as $t \to \infty$, and they satisfy

$$\int_0^\infty \delta(t)\mathrm{d}t = \nu_1 < +\infty, \quad \int_0^\infty \iota(t)\mathrm{d}t = \nu_2 < +\infty$$

where ν_1 and ν_2 are known non-negative constants.

Lemma 2 [7]**.** $\forall x \geq 0$ and $\forall \alpha \geq 1$, one has $\ln\big(\cosh(x)\big) + \alpha \geq x$.

To this end, the following milder assumptions should be imposed on the unknown matrix functions of system (14):

Assumption 1. There exist some unknown finite positive constants $\sigma_i > 0 (i = 1, \cdots, 8)$, such that

$$\begin{aligned} &\|M_1 - M_\Delta\| \leq \sigma_1 \\ &\|\hat{C}_1 - \hat{C}_\Delta\| \leq \sigma_2 + \sigma_3\|\dot{\xi}\| \\ &\|\tilde{C}_1 - \tilde{C}_\Delta\| \leq \sigma_4 + \sigma_5\|\dot{\xi}\| \\ &\|G_1 - G_\Delta\| \leq \sigma_6 + \sigma_7\|\dot{\xi}\| \\ &\|F_1 - F_\Delta\| \leq \sigma_8 \end{aligned}$$

where M_Δ, $\hat{C}_\Delta$, $\tilde{C}_\Delta$, G_Δ and F_Δ, as nominal parameter matrices, are known exactly.

Assumption 2. The desired reference trajectory ξ_d is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired $l_d(t)$ is bounded and uniformly continuous.

In order to reduce the dynamic order of the designed controller or the number of adaptive updating laws, we choose $\Theta = \max\{\sigma_i\}(i = 1, \cdots, 8)$. Next, the following filtered tracking errors are introduced:

$$\begin{cases} \dot{y}_r = \dot{y}_d - K_y e_y \\ r \ = \dot{e}_y + K_y e_y \\ e_l = l_h - l_{hd} \end{cases} \tag{15}$$

where $K_y = \mathrm{diag}\{K_{yj}\}$ is positive diagonal matrix.

Consider the following adaptive control laws given by

$$\begin{cases} B_1 u = -Kr + M_\Delta \ddot{y}_r + \tilde{C}_\Delta r + C_\Delta \dot{y}_r + G_\Delta + F_\Delta - \\ \qquad \dfrac{r\hat{\Theta}(t)\Gamma}{\|r\|\hat{\Theta}(t)\Gamma + \iota(t)}\Big(\ln\big(\cosh(\hat{\Theta}(t)\Gamma)\big) + \alpha\Big) \\ \tau_0 \ = -l_{hd} + Pe_\lambda \end{cases} \tag{16}$$

with

$$\dot{\hat{\Theta}}(t) = -\delta(t)\hat{\Theta}(t) + \gamma\|r\|\Gamma, \quad \hat{\Theta}(0) > 0 \tag{17}$$

where K is positive definite, α is an arbitrary positive constant and must satisfy $\alpha \geq 1$, $\hat{\Theta}(t) \in \mathbf{R}$ is the parameter estimation of Θ and $\Gamma = \|\ddot{y}_r\| + \|r\| + \|\dot{\xi}\| \cdot \|r\| + \|\dot{y}_r\| + \|\dot{\xi}\| \cdot \|\dot{y}_r\| + \|\dot{\xi}\| + 2$, $\delta(t)$ and $\iota(t)$ are defined in Lemma 1, without loss of generality, let $\iota(t) = \delta(t) = \frac{1}{(1+t)^2}$.

Lemma 3. For the ξ_2-subsystem (12) and ξ_3-subsystem (13), considering controllers (16) and (27), then the output tracking errors $\xi_2 - \xi_{2d}$ and $\xi_3 - \xi_{3d}$ asymptotically converge to 0 and e_l, τ and all the other signals in the closed-loop system are bounded.

Proof. In view of (15), system dynamics (14) can be rewritten as

$$M_1\dot{r} = B_1 u - M_1\ddot{y}_r - C_1 r - C_1\dot{y}_r - G_1 - F_1. \tag{18}$$

Choose a continuously differentiable, positive definite and radially unbounded function

$$V_1 = \frac{1}{2}r^{\mathrm{T}}M_1 r + \frac{1}{2\gamma}\tilde{\Theta}^2 \tag{19}$$

where $\tilde{\Theta}(t) = \Theta - \hat{\Theta}(t)$ represents the parameter estimation error. Taking the time derivative of V_1 and substituting

(16) and (17) into it, one has

$$\begin{aligned}\dot{V}_1 = &r^{\mathrm{T}}M_1\dot{r} + \frac{1}{2}r^{\mathrm{T}}\dot{M}_1 r - \frac{1}{\gamma}\tilde{\Theta}\dot{\hat{\Theta}} \leq \\ &- r^{\mathrm{T}}Kr + \hat{\Theta}\|r\|\Gamma - \frac{\|r\|^2\hat{\Theta}^2(t)\Gamma^2}{\|r\|\hat{\Theta}(t)\Gamma + \iota(t)} + \\ &\tilde{\Theta}\|r\|\Gamma + \tilde{\Theta}\Big(\frac{\delta(t)}{\gamma}\hat{\Theta} - \|r\|\Gamma\Big) \leq \\ &- r^{\mathrm{T}}Kr + \iota(t) - \frac{\delta(t)}{\gamma}\left(\hat{\Theta}(t) - \frac{1}{2}\Theta\right)^2 + \frac{\Theta^2\delta(t)}{4\gamma} \leq \\ &- \lambda_{\min}(K)\|r\|^2 + \iota(t) + \frac{\Theta^2\delta(t)}{4\gamma}. \end{aligned} \tag{20}$$

Obviously, we arrive at $\dot{V}_1 \leq -oV + \iota(t) + \epsilon\delta(t)$. $\iota(t)$ and $\delta(t)$ are bounded time varying positive functions converging to 0 as $t \to \infty$ and o is an appropriate constant. Therefore, there exist $t > T$ and $\iota(t) + \frac{\Theta^2\delta(t)}{4\gamma} < \varepsilon$, when $\|r\| \geq \sqrt{\frac{\varepsilon}{\lambda_{\min}(K)}}$, $\dot{V}_1 \leq 0$. Hence, it implies that r converges to a set containing the origin as $t \to \infty$.

On the other hand, integrating both sides of (20) gives

$$V_1(t) \leq V_1(0) - \int_0^t \lambda_{\min}(K)\|r(\varsigma)\|^2 \mathrm{d}\varsigma + \frac{\Theta^2}{4\gamma}\nu_1 + \nu_2 < \infty.$$

Hence, V_1 is bounded, which implies $r \in L_\infty^{n-k-m}$ and $\hat{\Theta}(t) \in L_\infty^{n-k-m}$. $\int_0^t r^{\mathrm{T}}Kr\mathrm{d}r = V_1(0) - V_1(t) + \frac{\Theta^2}{4\gamma}\nu_1 + \nu_2 \in L_\infty$ can be used to show that $r \in L_2^{n-k-m}$. $r = \dot{e}_\xi + K_\xi e_\xi$ means $\dot{e}_\xi, e_\xi \in L_\infty^{n-k-m} \bigcap L_2^{n-k-m}$. ξ, $\dot{\xi}$, $\dot{\xi}_r$, $\ddot{\xi}_r \in L_\infty^{n-k-m}$ can be further concluded from (15). Therefore, all the signals on the right-hand side of (17) are bounded. one can deduce that $\dot{r}$ and $\ddot{\xi}$ are bounded. Therefore, by the well-known Barbalat Lemma[18], we immediately get $\lim_{t\to\infty} r = 0$ and $\lim_{t\to\infty} e_\xi = 0$. Consequently, one has $\lim_{t\to\infty} \dot{e}_\xi = 0$.

Finally, substituting the control (16) into dynamic (19) yields

$$(I + P)e_l = J^*\left((M\dot{H} + CH)\dot{\xi} + G + F - \hat{\tau}\right). \tag{21}$$

Since all the signals on the right-hand side of (21) are bounded, $(I + P)e_l$ is also bounded. Hence, the size of e_l can be adjusted by choosing the proper gain matrix P. □

3.2 Stability analysis of ξ_1 subsystem

Finally, for system (11) – (13) under control laws (16) – (17), apparently, the ξ_1-subsystem (11) can be rewritten as

$$\dot{\chi} = g(\phi, \chi, \varphi) \tag{22}$$

where $\chi = [\xi_1^{\mathrm{T}}, \dot{\xi}_1^{\mathrm{T}}]^{\mathrm{T}}$, $\phi = [r^{\mathrm{T}}, \dot{r}^{\mathrm{T}}]^{\mathrm{T}}$, $\varphi = [\tau_1^{\mathrm{T}}, \tau_2^{\mathrm{T}}]^{\mathrm{T}}$. To analyse the stability of ξ_1-subsystems, we need the following assumption:

Assumption 3. There exist Lipschitz positive constants L_i, $i = 1, \cdots, 4$ such that

$$\|C_1 + G_1 + F_1\| \leq L_1\|\omega\| + L_2 \tag{23}$$

$$\|\Upsilon_2 + \Pi_2\| \leq L_3\|\omega\| + L_4. \tag{24}$$

Moreover, from the stability analysis of ξ_2 and ξ_3 subsystems, ω converges to a small neighborhood of $\omega_d = [\xi_{2d}^{\mathrm{T}}, \dot{\xi}_{2d}^{\mathrm{T}}, \xi_{3d}^{\mathrm{T}}, \dot{\xi}_{3d}^{\mathrm{T}}]^{\mathrm{T}}$, i.e., $\|\omega - \omega_d\| \leq \epsilon_1$, it is easy to obtain $\|\omega\| \leq \|\omega_d\| + \epsilon_1$, and similarly, $\|[\ddot{\xi}_2^{\mathrm{T}}, \ddot{\xi}_3^{\mathrm{T}}]^{\mathrm{T}}\| \leq \|[\ddot{\xi}_{2d}^{\mathrm{T}}, \ddot{\xi}_{3d}^{\mathrm{T}}]^{\mathrm{T}}\| + \epsilon_2$, where ϵ_1 and ϵ_2 are small bounded errors.

Lemma 4. The ξ_1-subsystem (11) is stable.

Proof. One can choose the following Lyapunov function candidate as

$$V_2 = V_1 + \ln(\cosh(\dot{\xi}_1)). \tag{25}$$

Differentiating (25) and substituting (13) into it will give

$$\begin{aligned}\dot{V}_2 = &\dot{V}_1 + \tanh(\dot{\xi}_1)\ddot{\xi}_1 = \\ &\dot{V}_1 + \tanh(\dot{\xi}_1)\Big(- M_{31}^{-1}(\Gamma_{21}\ddot{\xi}_2 + \Gamma_{22}\ddot{\xi}_3 + \Upsilon_2 + \Pi_2) - \\ &M_{11}^{-1}(M_{12}\ddot{\xi}_2 + M_{13}\ddot{\xi}_3 + C_1 + G_1 + F_1)\Big) = \\ &\dot{V}_1 + \tanh(\dot{\xi}_1)(-M_{31}^{-1}(\Upsilon_2 + \Pi_2) - \tanh(\dot{\xi}_1)M_{11}^{-1}C_1 - \\ &\begin{bmatrix} \tanh(\dot{\xi}_1)M_{31}^{-1}\Gamma_{21} + \tanh(\dot{\xi}_1)M_{11}^{-1}M_{12} \\ \tanh(\dot{\xi}_1)M_{31}^{-1}\Gamma_{22} + \tanh(\dot{\xi}_1)M_{11}^{-1}M_{13} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \ddot{\xi}_2 \\ \ddot{\xi}_3 \end{bmatrix}.\end{aligned}$$

$\|\tanh(\dot{\xi}_1)\| \leq 1$ and the boundedness of M_{12}, M_{13}, M_{11}^{-1} and M_{31}^{-1} mean that there exist bounded constants ϖ_1, $i = 1, 2, 3$, such that

$$\left\| \begin{bmatrix} \tanh(\dot{\xi}_1)M_{31}^{-1}\Gamma_{21} + \tanh(\dot{\xi}_1)M_{11}^{-1}M_{12} \\ \tanh(\dot{\xi}_1)M_{31}^{-1}\Gamma_{22} + \tanh(\dot{\xi}_1)M_{11}^{-1}M_{13} \end{bmatrix}^{\mathrm{T}} \right\| \leq \varpi_1$$

$$\|M_{11}^{-1}\| \leq \varpi_2, \ \|M_{31}^{-1}\| \leq \varpi_3.$$

With these in mind and using Assumption 3, we finally get

$$\begin{aligned}\dot{V}_2 \leq &-\lambda_{\min}(K)\|r\|^2 + \iota(t) + \frac{\Theta^2\delta(t)}{4\gamma} + \varpi_3(L_3(\|\omega_d\| + \epsilon_1) + \\ &L_4) + \varpi_2(L_1(\|\omega_d\| + \epsilon_1) + L_2) + \\ &\varpi_1(\|[\ddot{\xi}_{2d}^{\mathrm{T}}, \ddot{\xi}_{3d}^{\mathrm{T}}]^{\mathrm{T}}\| + \epsilon_2).\end{aligned} \tag{26}$$

Let $p = \varpi_3(L_3(\|\omega_d\| + \epsilon_1) + L_4) + \varpi_2(L_1(\|\omega_d\| + \epsilon_1) + L_2) + \varpi_1(\|[\ddot{\xi}_{2d}^{\mathrm{T}}, \ddot{\xi}_{3d}^{\mathrm{T}}]^{\mathrm{T}}\| + \epsilon_2)$, when $\|r\| \geq \sqrt{\frac{2(\varepsilon+p)}{(\lambda_{\min}(K))}}$, one has $\dot{V}_2 \leq 0$. Furthermore, r can be arbitrarily small by choosing a proper K. Therefore, the ξ_1-subsystem (11) is stable. □

Theorem 1. Consider the mobile manipulators described by (3) with affine constraints (1). Using the adaptive control laws (16) and (17), the following are guaranteed:

1) The output tracking errors e_y and $\dot{e}_y$ of ξ_2 and ξ_3 subsystems converge to 0 as $t \to \infty$.

2) The ξ_1-subsystem (11) is stable and e_l, τ in the closed-loop system are bounded for all $t \geq 0$.

4 Simulation example

In this section, computer simulation is conducted to examine the performance of the tracking controller for the mobile manipulator as shown in Fig. 2. According to the Euler-Lagrangian equations, the following standard form can be obtained.

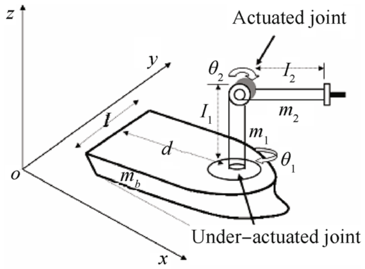

Fig. 2 2-DOF manipulator mounted on boat

$$q_b = [q_{b_1},\, q_{b_2},\, q_{b_3}]^{\rm T},\ [q_u,\, q_a]^{\rm T} = [\theta_1,\, \theta_2]^{\rm T}$$

$$M_b = \begin{bmatrix} m_{b12} & 0 & m_{12} d\cos q_{b_3} \\ 0 & m_{b12} & -m_{12} d\sin q_{b_3} \\ m_{12} d\cos q_{b_3} & -m_{12} d\sin q_{b_3} & I_{b12} + m_{12} d^2 \end{bmatrix}$$

$$M_m = \begin{bmatrix} I_{12} & 0 \\ 0 & I_2 \end{bmatrix}, M_{mb} = \begin{bmatrix} 0 & 0 & I_{12} \\ 0 & 0 & 0 \end{bmatrix}, M_{bm} = M_{mb}^{\rm T}$$

$$C_b = \begin{bmatrix} 0 & 0 & -m_{12}\dot q_{b_3} d\sin q_{b_3} \\ 0 & 0 & -m_{12}\dot q_{b_3} d\cos q_{b_3} \\ 0 & 0 & 0 \end{bmatrix}, C_{mb} = 0, C_{bm}^{\rm T} = C_{mb}$$

$$G_b = 0, G_m = [0,\, m_2 g l_2 \sin\theta_2]^{\rm T}, F_b = [f_{b_1},\, f_{b_2},\, f_{b_3}]^{\rm T}$$

$$B_b = \begin{bmatrix} -\cos q_{b_3} & -\cos q_{b_3} \\ \sin q_{b_3} & -\sin q_{b_3} \\ -\frac{l}{2} & \frac{l}{2} \end{bmatrix}, \quad \tau_b = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, \quad \tau_a = \tau_1.$$

where $m_{b12} = m_b + m_1 + m_2$, $m_{12} = m_1 + m_2$, $I_{b12} = I_b + I_1 + I_2$, $I_{12} = I_1 + I_2$. The parameters used in the simulation are $m_b = 500\,{\rm kg}$, $m_1 = 50\,{\rm kg}$, $l = l_1 = l_2 = 4\,{\rm m}$, $d = 6\,{\rm m}$ $I_b = 50\,{\rm kg}\cdot{\rm m}^2$, $I_1 = 10\,{\rm kg}\cdot{\rm m}^2$, $f_{b_i} = \bar b\sin t$ and $f_{m_i} = \bar m\cos t$ with unknown $\bar b$ and $\bar m$, $C(q_{b_2}) = q_{b_2}$. Because of the second operating arm with varied pay-load, one assumes that m_2 and I_2 are unknown. The system is subject to the affine constraint:

$$\dot q_{b_1}\cos q_{b_3} - \dot q_{b_2}\sin q_{b_3} = C(q_{b_2})\cos q_{b_3}.$$

We select

$$S(q_b) = \begin{bmatrix} \tan q_{b_3} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \eta(q_b) = \begin{bmatrix} q_{b_2} \\ 0 \\ 0 \end{bmatrix}.$$

Considering the above transformation, the whole system dynamics are converted as

$$\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & m_{23} & 0 \\ 0 & m_{32} & m_{33} & 0 \\ 0 & 0 & 0 & m_{44} \end{bmatrix}\ddot\zeta + \begin{bmatrix} c_{11} & c_{12} & 0 & 0 \\ c_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\dot\zeta +$$

$$\begin{bmatrix} g_1 \\ g_2 \\ 0 \\ g_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ u_3 \end{bmatrix}.$$

where $\zeta = [z_1, z_2, q_u, q_a]^{\rm T}$, $m_{11} = \frac{550+m_2}{\cos^2 q_{b_3}}$, $m_{22} = 1860 + 36m_2 + I_2$, $m_{23} = 10 + I_2$ $m_{32} = 10 + I_2$, $m_{33} = 10 + I_2$, $m_{44} = I_2$, $c_{11} = \frac{(550+m_2)\sin q_{b_3}\dot q_{b_3}}{\cos^3 q_{b_3}}$, $c_{12} = -\frac{6(50+m_2)}{\cos^2 q_{b_3}}\dot q_{b_3}$, $c_{21} = \frac{6(50+m_2)}{\cos q_{b_3}}\dot q_{b_3}$, $g_1 = (550 + m_2)\tan q_{b_3}\dot z_1$, $g_2 = 6(50 + m_2)\cos q_{b_3}\dot z_1$, $g_4 = 4m_2 g\sin\theta_2$, $f_1 = \tan q_{b_3} f_{b_1} + f_{b_2}$, $f_2 = f_{b_3}$, $f_3 = f_{m_1}$, $f_4 = f_{m_2}$, $u_1 = -2\sin q_{b_3}\tau_r$, $u_2 = -2\tau_r + 2\tau_l$, $u_3 = \tau_1$.

Then the above dynamic equation can be further transformed into the following dynamics:

$$\begin{bmatrix} m_{33} - \frac{m_{32}m_{23}}{m_{22}} & 0 & 0 \\ 0 & m_{11} & 0 \\ 0 & 0 & m_{44} \end{bmatrix}\ddot{\bar\zeta} + \begin{bmatrix} 0 & -\frac{m_{32}c_{11}}{m_{22}} & 0 \\ 0 & c_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot{\bar\zeta} +$$

$$\begin{bmatrix} \frac{-m_{32}g_2}{m_{22}} \\ g_1 + c_{12}\dot z_2 \\ g_4 \end{bmatrix} + \begin{bmatrix} f_3 - \frac{m_{32}f_2}{m_{22}} \\ f_1 \\ f_4 \end{bmatrix} = \begin{bmatrix} \frac{-m_{32}}{m_{22}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\bar u.$$

$$\ddot z_2 = \frac{1}{m_{22}}(u_2 - c_{21}\dot z_1 - g_2 - f_2 - m_{23}\ddot q_u)$$

with $\bar\zeta = [q_u, z_1, q_a]^{\rm T}$, $\bar u = [u_2, u_1, u_3,]^{\rm T}$. Given the desired trajectory $y_d = [q_{ud},\, z_{1d},\, q_{ad}]^{\rm T} = [\frac{\pi}{4},\, \sin t + \cos t,\, \frac{\pi}{4}(1 - 0.5\sin t)]^{\rm T}$, $l_{hd} = 10\,{\rm N}$. The control objective is to determine an adaptive controller so that the trajectory $y = [q_u,\, z_1,\, q_a]^{\rm T}$ and $\dot y$ follow y_d and $\dot y_d$, respectively, and z_2-subsystem is stable and l is bounded. We get the actual controller

$$B_1 u = \begin{bmatrix} m^* & 0 & 0 \\ 0 & m_{11} & 0 \\ 0 & 0 & m_{44} \end{bmatrix}\ddot y_r + \begin{bmatrix} 0 & \frac{-m_{32}c_{11}}{m_{22}} & 0 \\ 0 & c_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot y_r +$$

$$\begin{bmatrix} \frac{-m_{32}g_2}{m_{22}} \\ g_1 + c_{12}\dot z_2 \\ g_4 \end{bmatrix} + \frac{r\hat\Theta(t)\Gamma}{\|r\|\hat\Theta(t)\Gamma + \iota(t)}(\ln(*) + 2) - r$$

with

$$\begin{cases} \dot{\hat\Theta} = -\delta(t)\hat\Theta(t) + \|r\|\Gamma,\ \hat\Theta(0) = 2 \\ \tau_0 = -10 + P(l_h - 10) \end{cases}$$

where $m^* = \frac{m_{33}m_{22} - m_{32}m_{23}}{m_{22}}$, $* = (\cosh(\hat\Theta(t)\Gamma)) + 2)$, $r = [\dot q_u + q_u - \frac{\pi}{4},\, \dot z_1 + z_1 - 2\cos t,\, \dot q_a + q_a + \frac{\pi}{8}\cos t + \frac{\pi}{8}\sin t - \frac{\pi}{4}]^{\rm T}$, $\ddot y_r = [-\dot q_u,\, -\dot z_1 - 2\sin t,\, -\dot q_a + \frac{\pi}{8}\sin t - \frac{\pi}{8}\cos t]^{\rm T}$, $\dot y_r = [-q_u + \frac{\pi}{4},\, -z_1 + 2\cos t,\, -q_a - \frac{\pi}{8}\sin t - \frac{\pi}{8}\cos t + \frac{\pi}{4}]^{\rm T}$, $\iota(t) = \delta(t) = \frac{1}{(1+t)^2}$ and $\Gamma = \|\ddot y_r\| + \|r\| + \|\dot\xi\|\cdot\|r\| + \|\dot y_r\| + \|\dot\xi\|\cdot\|\dot y_r\| + \|\dot\xi\| + 2.$

The position state performances of q_u, z_1 and q_a are illustrated in Figs. 3–5 and the velocity tracking results of $\dot{q}_u$, $\dot{z}_1$ and $\dot{q}_a$ are presented in Figs. 6–8. The stability of z_2-subsystem is shown in Fig. 9. The force tracking error of $l_a - l_{ad}$ becomes arbitrarily small and the parameter updating laws $\hat{\Theta}$ are bounded as shown in Figs. 10 and 11. The input torques are all bounded as shown in Fig. 12.

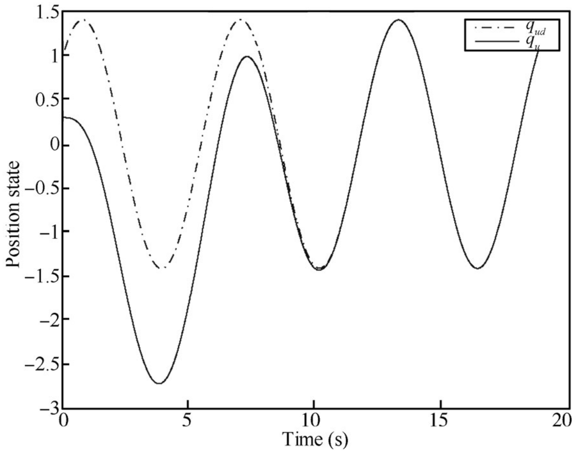

Fig. 3 Trajectories of q_u, q_{ud}

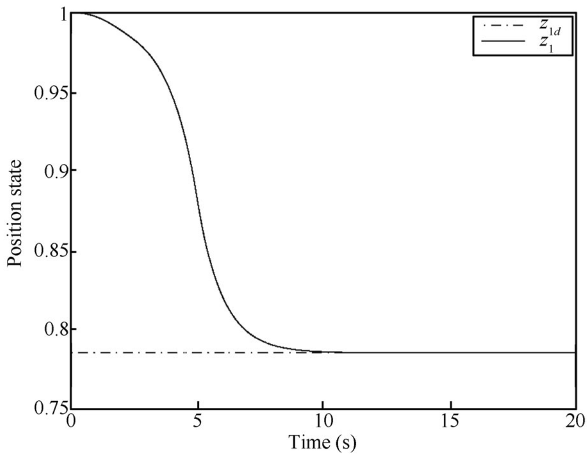

Fig. 4 Trajectories of z_1, z_{1d}

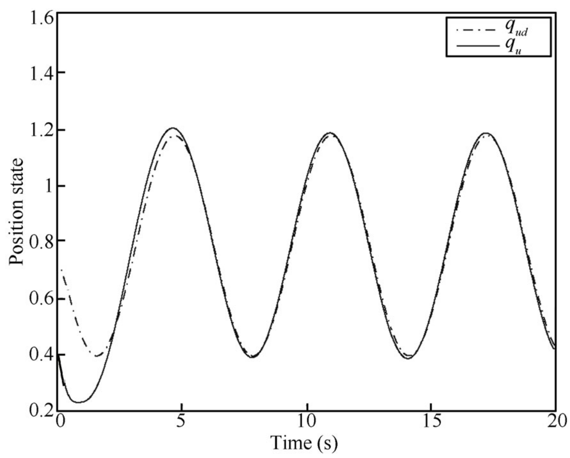

Fig. 5 Trajectories of q_a, q_{ad}

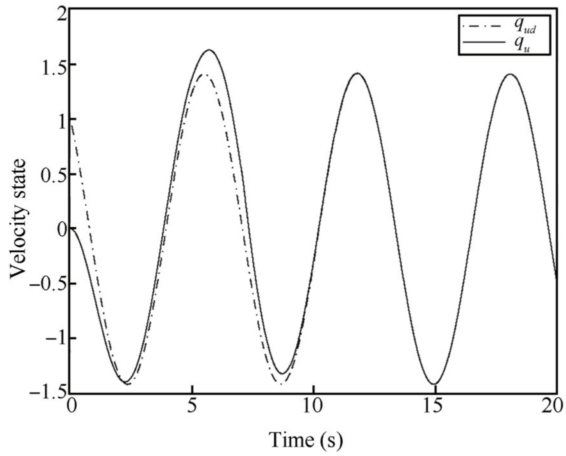

Fig. 6 Trajectories of $\dot{q}_u$

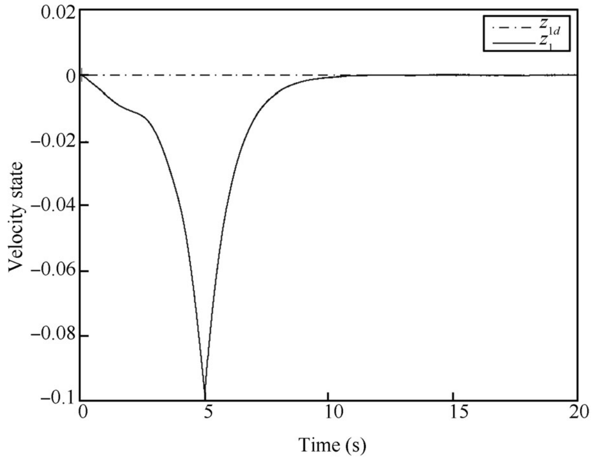

Fig. 7 Trajectories of $\dot{z}_1$, $\dot{z}_{1d}$

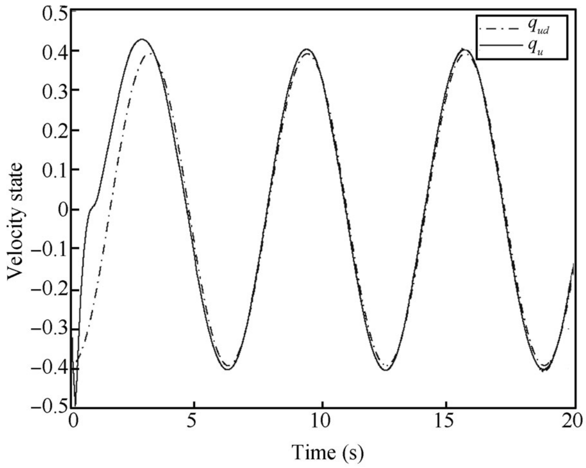

Fig. 8 Trajectories of $\dot{q}_a$, $\dot{q}_{ad}$

5 Conclusions

This paper discusses tracking control of the uncertain affine constraints mobile manipulator with under-actuated joints, and mainly discusses mathematical modeling, algorithm design and theory analysis for it. An adaptive tracking controller is proposed. Practical simulation is presented to illustrate the effectiveness of the control strategy.

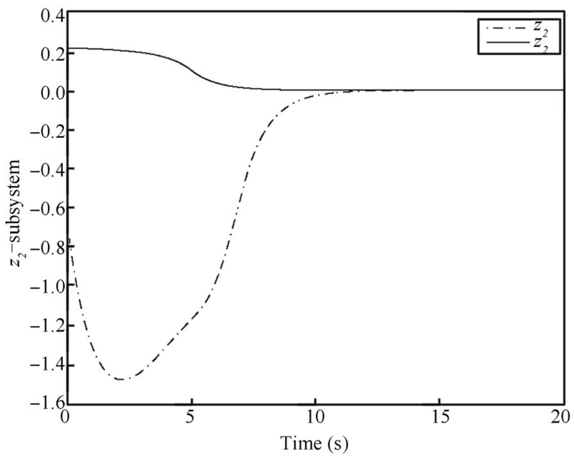

Fig. 9 Trajectories of z_2, $\dot{z}_2$

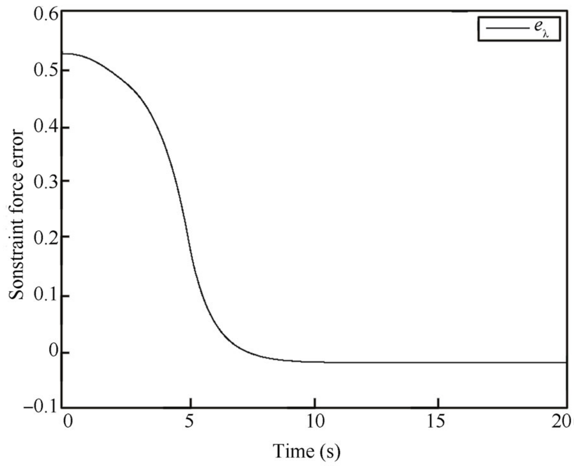

Fig. 10 Trajectory of $\lambda_a - \lambda_{ad}$

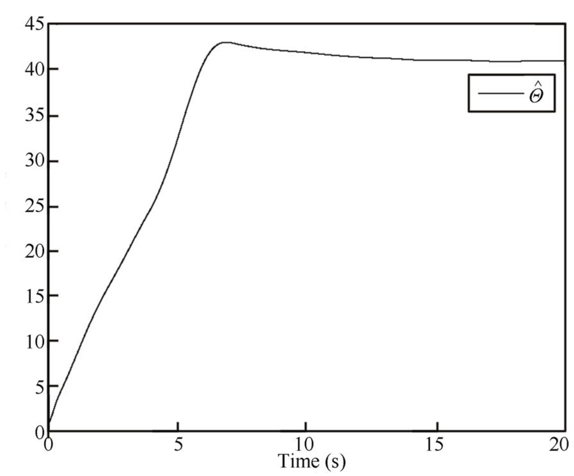

Fig. 11 Trajectory of $\hat{\Theta}$

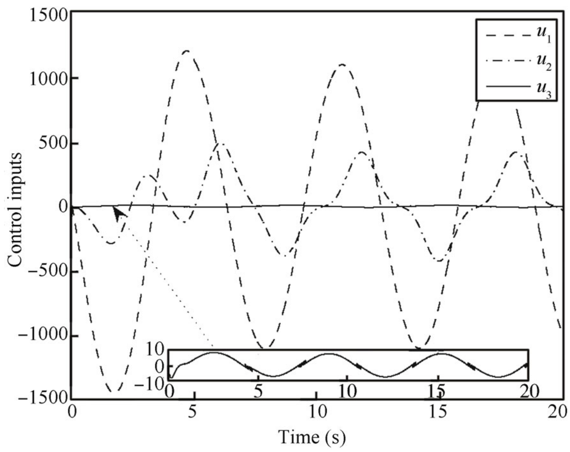

Fig. 12 Trajectories of inputs

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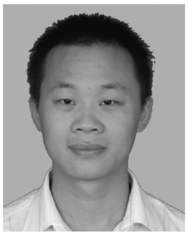

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