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Event-triggered Control of Positive Switched Systems with Actuator Saturation and Time-delay

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Abstract: This paper investigates the event-triggered control of positive switched systems with randomly occurring actuator saturation and time-delay, where the actuator saturation and time-delay obey different Bernoulli distributions. First, an event-triggering condition is constructed based on a 1-norm inequality. Under the presented event-triggering scheme, an interval estimation method is utilized to deal with the error term of the systems. Using a co-positive Lyapunov functional, the event-triggered controller and the cone attraction domain gain matrices are designed via matrix decomposition techniques. The positivity and stability of the resulting closed-loop systems are reached by guaranteeing the positivity of the lower bound of the systems and the stability of the upper bound of the systems, respectively. The proposed approach is developed for interval and polytopic uncertain systems, respectively. Finally, two examples are provided to illustrate the effectiveness of the theoretical findings.

Keywords: Positive switched systems, event-triggered control, randomly occurring actuator saturation, linear programming, time-delay.

1 Introduction

In past decades, the research of positive systems has received increasing attention in the field of control theory and applications[1–5]. Many practical systems can be modeled by positive systems such as chemical engineering, ecology, network employing, water systems, etc. As an important class of hybrid systems, switched systems[6, 7] are more powerful for modeling practical systems than other ones[8, 9]. Positive switched systems consist of a finite number of positive subsystems and a switching logic[10]. Liu and Dang[11] constructed a sequence of functions for positive switched systems and these functions can serve as an upper bound of the system trajectories starting from a particular region. Co-positive Lyapunov functions[12–14] are often used to investigate the stability of positive switched systems, which are simpler than quadratic Lyapunov functions. An upper bound to the L_1-induced norm of switched linear positive systems was computed under dwell time constraints in [15]. The optimal control of positive switched systems was explored to minimize a positive linear combination of the state variable in [16]. Fornasini and Valcher[17] established some necessary and sufficient conditions to deal with the stabilizability of discrete-time positive switched systems.

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By observing the above literature on positive switched systems, we find that most of the existing controllers are based on a time-triggered strategy. This control strategy requires the update of the control law of systems at each time instant, which increases the cost of controller design. As the first attempt, Dorf et al.[18] presented a novel control scheme based on an event-triggered sample to save resources. A new approach for event-based state-feedback control was proposed by selecting the appropriate threshold value of the event generator in [19]. Postoyan et al.[20] constructed a framework for the event-triggered control of nonlinear systems by means of hybrid systems tools. The event-trigger based adaptive control for uncertain nonlinear systems was investigated using the switching threshold strategy in [21]. Static state-feedback controllers and dynamical output-based controllers were designed for general systems in [22]. To avoid zero behavior, a lower bound of the inter-event time was explicitly presented for the event-triggered control of switched systems in [23]. Using a piecewise Lyapunov functional method, the decentralized event-triggered H_∞ control issue of switched systems with delay and exogenous disturbance was considered in [24]. To obtain the largest domain of attraction, the feedback gain and the event-triggering strategy were designed for positive systems subject to input saturation in [25]. Liu et al.[26] introduced a linear approach to deal with the event-triggering control of positive systems with input saturation. More results on event-triggered control can refer to [27–29] and references therein.

The saturation of control systems is a universal phenomenon due to limited capacity of actuator elements. Actuator saturation may lead to undesirable oscillatory behavior or, even worse, instability of the systems. Some valuable results on the saturation problems of general systems[30, 31] and positive systems[32, 33] have been reported. Hu et al.[30] provided an approach to estimate the domain of attraction of discrete-time linear systems using a linear saturated feedback. Randomly occurring sensor saturation was considered in [31] for networked systems to improve the network environment. The convex hull technique was developed to solve the actuator saturation problems of positive switched systems in [32]. The saturation control of switched nonlinear systems was addressed using a linear programming approach in [33]. On the other hand, delays are universal in the practical applications and have complex impacts on system dynamics[34–37]. A delay-dependent criterion for determining the stability of systems with delays was addressed to obtain efficient stabilizing effects[38]. A controller of linear systems with stochastic input delays was designed in [39]. In [40–42], the control synthesis of positive systems with time-delay was explored in terms of linear programming. A linear matrix inequality approach was also used to reach the delay-dependent stabilization of positive switched systems[43]. Randomly occurring saturation and time-delay are more general and practical than the determined cases in the literature mentioned above. Up to now, there are few results on the event-triggered control of positive switched systems with randomly occurring actuator saturation and time-delay. It is necessary to point out that the research approaches of positive switched systems are different from general systems and the control issues of positive switched systems are more complex. These issues motivate us to carry out the work.

This paper investigates the event-triggered control of positive switched systems with randomly occurring actuator saturation and time-delay based on linear programming approach. First, an event-triggering condition is proposed for the systems in terms of 1-norm. Then, the controller of the systems is designed by virtue of a matrix decomposition approach. A general control framework is introduced, which can be developed for interval and polytopic uncertain systems. All the presented conditions are solved by linear programming approach. The rest of this paper is organized as follows. In Section 2, the problem formulation is given. In Section 3, main results are presented. Section 4 provides two examples to verify the effectiveness of design. Finally, Section 5 concludes the paper.

Notations. Let $\mathbf{R}$, $\mathbf{R}^n$ (or $\mathbf{R}^n_+$), and $\mathbf{R}^{n\times m}$ be the sets of real numbers, n-dimensional vectors (or nonnegative), and $n\times m$ matrices, respectively. $\mathbf{N}$ and $\mathbf{N}^+$ denote the sets of nonnegative and positive integers. For a vector $x=(x_1,\cdots,x_n)^{\mathrm{T}}$, $x\succeq 0$ ($\succ 0$) implies that $x_i\geq 0$ ($x_i>0$) $\forall i=1,\cdots,n$. Given a matrix $A=[a_{ij}]\in\mathbf{R}^{n\times n}$, $A\succeq 0$ ($\succ 0$) implies that $a_{ij}\geq 0$ ($a_{ij}>0$), $\forall i,j=1,\cdots,n$. Similarly, $A\succeq B$ ($A\preceq B$) means that $a_{ij}\geq b_{ij}$ ($a_{ij}\leq b_{ij}$), $\forall i,j=1,\cdots,n$. $co\{\cdot\}$ represents the convex hull. Define $\mathbf{1}_m=(\underbrace{1,\cdots,1}_{m})^{\mathrm{T}}$, $\mathbf{1}_m^{(\iota)}=(\underbrace{0,\cdots,0,1}_{\iota},\underbrace{0,\cdots,0}_{m-\iota})^{\mathrm{T}}$, and $\mathbf{1}_{n\times n}$ is an $n\times n$ matrix with all elements 1. The symbol $\|x\|_1=\sum_{i=1}^{n}|x_i|$ stands for the 1-norm of vector $x=(x_1,\cdots,x_n)^{\mathrm{T}}$. The symbol $\mathbb{E}$ refers to mathematics expectation.

2 Preliminaries

Consider the following switched systems with randomly occurring time-delay and input saturation:

$$x(k+1)=A_{\sigma(k)}x(k)+B_{\sigma(k)}\bar{u}(k) \tag{1}$$

and

$$\begin{aligned} x(k+1)=&A_{\sigma(k)}x(k)+B_{\sigma(k)}\bar{u}(k)+\\ &\alpha_{\sigma(k)}(k)A_{d\sigma(k)}x(k-h)\\ x(k)=&\varphi(k),\ k=-h,\cdots,0 \end{aligned} \tag{2}$$

where $x(k)\in\mathbf{R}^n$ and $u(k)\in\mathbf{R}^m$ are the system state, control input, respectively, and $\bar{u}(k)=\beta_{\sigma(k)}(k)\mathrm{sat}(u(k))+(1-\beta_{\sigma(k)}(k))\gamma_{\sigma(k)}(k)u(k)$. The function $\mathrm{sat}(\cdot):\mathbf{R}^m\to\mathbf{R}^m$ is the vector valued standard saturation function, i.e., $\mathrm{sat}(u)=(\mathrm{sat}(u_1),\mathrm{sat}(u_2),\cdots,\mathrm{sat}(u_m))^{\mathrm{T}}$, where $\mathrm{sat}(u_i)=\mathrm{sgn}(u_i)\min\{1,|u_i|\}$, $i=1,2,\cdots,m$. The function $\sigma(k)$ represents the switching law, which takes values at a finite set $S=\{1,2,\cdots,J\}, J\in\mathbf{N}^+$. $h>0$ denotes the constant delay and $\varphi(k)$ is the initial condition. The i-th subsystem is invoked when $\sigma(k)=i\in S$. The system matrices satisfy $A_i\in\mathbf{R}^{n\times n}$, $A_{di}\in\mathbf{R}^{n\times n}$ and $B_i\in\mathbf{R}^{n\times m}$. Throughout this paper, assumed that $A_i\succeq 0$, $B_i\succeq 0$ and $A_{di}\succeq 0$. The randomly occurring actuator saturation and time-delay are governed by stochastic variables $\alpha_i(k)$, $\beta_i(k)$ and $\gamma_i(k)$ with

$$\mathrm{Prob}\{\alpha_i(k)=1\}=\bar{\alpha}_i,\mathrm{Prob}\{\alpha_i(k)=0\}=1-\bar{\alpha}_i \tag{3a}$$

$$\mathrm{Prob}\{\beta_i(k)=1\}=\bar{\beta}_i,\mathrm{Prob}\{\beta_i(k)=0\}=1-\bar{\beta}_i \tag{3b}$$

$$\mathrm{Prob}\{\gamma_i(k)=1\}=\bar{\gamma}_i,\mathrm{Prob}\{\gamma_i(k)=0\}=1-\bar{\gamma}_i \tag{3c}$$

where $0\leq\bar{\alpha}_i\leq 1$, $0\leq\bar{\beta}_i\leq 1$ and $0\leq\bar{\gamma}_i\leq 1$.

Remark 1. In (3), the Bernoulli distribution is used to describe the randomly occurring actuator saturation and time-delay. Indeed, there are other approaches to describe the randomly occurring phenomenon such as binomial distribution, Markov chain, etc. When the stochastic variables obey a Markov chain, the systems (1) and (2) become Markov jump systems. Thus, existing results on Markov jump systems can be used to investigate the corresponding issues. The binomial distribution is more general than the Bernoulli distribution for modeling the randomly occurring phenomenon. In further work, it is interesting to investigate the event-triggered control of positive switched systems with the binomial distribution based randomly occurring phenomenon.

Definition 1.[1, 2] A system is positive if all its states and outputs are nonnegative for any nonnegative initial conditions and any nonnegative control inputs.

Definition 2. System (2) is said to be positive if $x(k) \succeq 0$ for $\varphi(k) \succeq 0$ and $u(k) \succeq 0$.

Lemma 1.[1, 2] A system $x(k+1) = Ax(k)$ is positive if and only if $A \succeq 0$. A time-delay system $x(k+1) = Ax(k) + A_d x(k-h)$ is positive if and only if $A \succeq 0$ and $A_d \succeq 0$.

By Lemma 1 and the assumptions for systems (1) and (2), we can obtain that systems (1) and (2) are positive.

Lemma 2.[1, 2] Let $A \succeq 0$, then the following statements are equivalent:

1) The matrix A is Schur.

2) There is a vector $v \succ 0$ in $\mathbf{R}^n$ such that $(A - I)v \prec 0$.

Definition 3.[35] System (2) is exponentially stable under switching signal $\sigma(k)$ if there exist constants $a > 0$ and $b > 0$ such that the solution of the system satisfies

$$\|x(k)\|_1 \leq a\|x(0)\|_c \mathrm{e}^{-bk}, \forall k \in \mathbf{N}^+$$

where $\|x(0)\|_c = \sup\limits_{-h \leq \theta \leq 0} \|x(\theta)\|_1$.

In [35], the norm in Definition 3 was chosen as the Euclidean norm. Noting the fact that the Euclidean norm is equivalent to the 1-norm, we modify the definition in [35] as Definition 3.

Definition 4.[44] Given a switching signal $\sigma(k)$, let $N_\sigma(k_0, k_1)$ denote the number of the switching in the interval (k_0, k_1). If the condition

$$N_\sigma(k_0, k_1) \leq N_0 + \frac{k_1 - k_0}{\tau}$$

holds, then τ is said to be average dwell time (ADT) of the switching signal $\sigma(k)$.

Lemma 3.[34] Given $u = (u_1, u_2, \cdots, u_m)^{\mathrm{T}}$ and $v = (v_1, v_2, \cdots, v_m)^{\mathrm{T}}$, let $|v_j| \leq 1$ for all $j = 1, 2, \cdots, m$. Then,

$$\mathrm{sat}(u) \in co\{E_1 u + E_1^- v, E_2 u + E_2^- v, \cdots, E_{2^m} u + E_{2^m}^- v\}$$

where $E_\ell, \ell = 1, 2, \cdots, 2^m$ is an $m \times m$ diagonal matrix with elements being either 1 or 0 and $E_\ell^- = I - E_\ell$.

From Lemma 3, we have $\mathrm{sat}(u) = \sum\limits_{\ell=1}^{2^m} \hbar_\ell (E_\ell u + E_\ell^- v)$, where $0 \leq \hbar_\ell \leq 1$ and $\sum\limits_{\ell=1}^{2^m} \hbar_\ell = 1$. A cone domain $\Im(v_i, 1)$ is defined as

$$\Im(v_i, 1) = \{x(k) \in \mathbf{R}_+^n : x^{\mathrm{T}}(k) v_i \leq 1\}$$

where $v_i \succ 0$. Let $H_{i\jmath}$ be the $\jmath$-th row of matrix $H_i \in \mathbf{R}^{m \times n}$ and define the symmetric polyhedron:

$$L(H_i) := \{x(k) \in \mathbf{R}_+^n : |H_{i\jmath} x(k)| \leq 1\}$$

where $i \in S$ and $\jmath = 1, 2, \cdots, m$.

3 Main results

In this section, we will establish an event-triggered control approach to the systems (1) and (2). Fig. 1 describes the event-triggered control framework of positive switched systems.

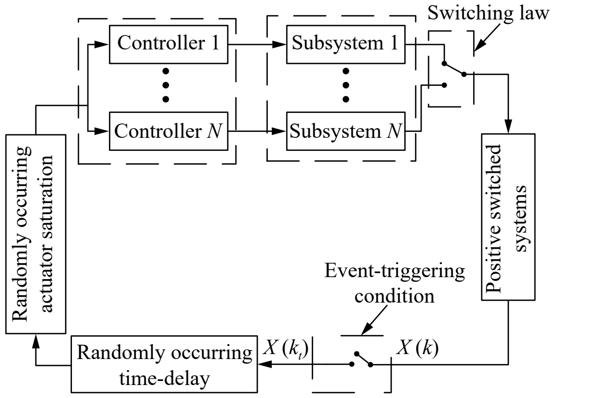

Fig. 1 Event-triggered control framework of positive switched systems

First, an event-triggering condition is established based on a 1-norm inequality to replace time-triggering control scheme. The event-triggered control law is given as

$$u(k) = F_i \hat{x}(k) \tag{4}$$

where $F_i \in \mathbf{R}^{m \times n}$ is the event-triggered controller gain matrix, $\hat{x}(k) = x(k_t)$ for any $k \in [k_t, k_{t+1}), t \in \mathbf{N}$, and k_t is the event-triggering time instant. The event-triggered condition is generated by

$$\|e(k)\|_1 > \delta \|x(k)\|_1 \tag{5}$$

where $1 > \delta > 0$, $e(k)$ is the sample error, and $e(k) = \hat{x}(k) - x(k)$.

Theorem 1. If there exist constants $1 > \mu > 0$, $\lambda > 1, \eta > 0, \zeta > 0$ and $\mathbf{R}^n$ vectors $v_i \succ 0$, $\rho_{ii}^+ \succ 0$, $\rho_{ii}^- \prec 0$, $\varrho_{ii}^+ \succ 0$, $\varrho_{ii}^- \prec 0$, $\rho_i^+ \succ 0$, $\bar{\rho}_i^- \prec 0$, $\underline{\rho}_i^- \prec 0$, $\varrho_i^+ \succ 0$, $\bar{\varrho}_i^- \prec 0$, $\underline{\varrho}_i^- \prec 0$ such that

$$\begin{aligned}
&\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i A_i + B_i E_{i\ell} \sum_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \rho_{i\imath}^{+\mathrm{T}} \Phi_1 + \\
&\quad B_i E_{i\ell} \sum_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \rho_{i\imath}^{-\mathrm{T}} \Phi_2 + B_i E_{i\ell}^- \sum_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \varrho_{i\imath}^{+\mathrm{T}} \Phi_1 + \\
&\quad B_i E_{i\ell}^- \sum_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \varrho_{i\imath}^{-\mathrm{T}} \Phi_2 \succeq 0
\end{aligned} \tag{6a}$$

$$\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i A_i + B_i \sum_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \rho_{i\imath}^{-\mathrm{T}} \Phi_2 \succeq 0 \tag{6b}$$

$$\begin{aligned}&A_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\varrho_i^{+}+\bar{\varrho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+\\&\quad\psi-\mu v_i\prec 0, E_{i\ell}=0\end{aligned}\tag{6c}$$

$$\begin{aligned}&A_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\rho_i^{+}+\bar{\rho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})+\\&\quad\psi-\mu v_i\prec 0, E_{i\ell}=I\end{aligned}\tag{6d}$$

$$\begin{aligned}&A_i^{\mathrm{T}}v_i+\bar{\beta}_i\Phi_2(\rho_i^{+}+\varrho_i^{+})+\bar{\beta}_i(\eta_1\bar{\rho}_i^{-}+\eta_1\bar{\varrho}_i^{-}-\\&\quad\eta_2\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-}-\eta_2\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+\\&\quad\psi-\mu v_i\prec 0, E_{i\ell}\neq I, E_{i\ell}\neq 0\end{aligned}\tag{6e}$$

$$\rho_{i\imath}^{+}\preceq\rho_i^{+},\ \underline{\rho}_i^{-}\preceq\rho_{i\imath}^{-}\preceq\bar{\rho}_i^{-},\ \varrho_{i\imath}^{+}\preceq\varrho_i^{+},\ \underline{\varrho}_i^{-}\preceq\varrho_{i\imath}^{-}\preceq\bar{\varrho}_i^{-}\tag{6f}$$

$$\begin{aligned}&\eta_1\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i\leq\mathbf{1}_m^{\mathrm{T}}E_{i\ell}^{\mathrm{T}}B_i^{\mathrm{T}}v_i\leq\eta_2\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i,\\&\quad E_{i\ell}\neq I, E_{i\ell}\neq 0\end{aligned}\tag{6g}$$

$$-\zeta v_i\leq\varrho_{i\imath}^{+}+\varrho_{i\imath}^{-}\leq\zeta v_i\tag{6h}$$

$$\zeta\leq\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i\tag{6i}$$

$$v_i\preceq\lambda v_j\tag{6j}$$

hold $\forall(i,j)\in S, i\neq j$, and $\imath=1,\cdots,m$, then under the event-triggered control law (4) with

$$F_i=F_i^{+}+F_i^{-}, H_i=H_i^{+}+H_i^{-}\tag{7}$$

and

$$F_i^{+}=\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i},\quad F_i^{-}=\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\tag{8a}$$

$$H_i^{+}=\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i},\quad H_i^{-}=\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\tag{8b}$$

the resulting closed-loop system (1) is positive and stable with ADT switching satisfying

$$\tau\geq-\frac{\ln\lambda}{\ln\mu}\tag{9}$$

where $E_{i\ell},\ell=1,2,\cdots,2^m$ are $m\times m$ diagonal matrices with elements being either 1 or 0 and $E_{i\ell}^{-}=I-E_{i\ell}$, $\Phi_1=I-\delta\mathbf{1}_{n\times n}$, $\Phi_2=I+\delta\mathbf{1}_{n\times n}$, and $\psi=(1-\bar{\beta}_i)\bar{\gamma}_i(\Phi_2\rho_i^{+}+\bar{\rho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})$. Moreover, the system states starting from $x(k_0)\in\Im(v_i,1)$ will remain inside $\bigcup_i^J\Im(v_i,1)$ for $N_0=0$.

Proof. By Lemma 3 and the event-triggering control law (4),

$$\begin{aligned}\mathrm{sat}(F_i\hat{x}(k))=&\sum_{\ell=1}^{2^m}\hbar_{i\ell}(E_{i\ell}F_ix(k)+E_{i\ell}F_ie(k)+\\&E_{i\ell}^{-}H_ix(k)+E_{i\ell}^{-}H_ie(k)).\end{aligned}\tag{10}$$

The closed-loop system (1) becomes as

$$\begin{aligned}x(k+1)=&A_ix(k)+\beta_i(k)B_i\sum_{\ell=1}^{2^m}\hbar_{i\ell}(E_{i\ell}F_ix(k)+\\&E_{i\ell}F_ie(k)+E_{i\ell}^{-}H_ix(k)+E_{i\ell}^{-}H_ie(k))+\\&(1-\beta_i(k))\gamma_i(k)B_iF_i\hat{x}(k)\end{aligned}\tag{11}$$

$$\begin{aligned}x(k+1)=&A_ix(k)+\beta_i(k)B_i\sum_{\ell=1}^{2^m}\hbar_{i\ell}\Big(E_{i\ell}F_i^{+}x(k)+\\&E_{i\ell}F_i^{-}x(k)+E_{i\ell}F_i^{+}e(k)+\\&E_{i\ell}F_i^{-}e(k)+E_{i\ell}^{-}H_i^{+}x(k)+\\&E_{i\ell}^{-}H_i^{-}x(k)+E_{i\ell}^{-}H_i^{+}e(k)+E_{i\ell}^{-}H_i^{-}e(k)\Big)+\\&(1-\beta_i(k))\gamma_i(k)\Big(B_iF_i^{+}x(k)+\\&B_iF_i^{-}x(k)+B_iF_i^{+}e(k)+B_iF_i^{-}e(k)\Big).\end{aligned}\tag{12}$$

We can obtain (12) from (7). Using the event-triggering condition (5), it follows that

$$\|e(k_0)\|_1\leq\delta\|x(k_0)\|_1=\delta\mathbf{1}_n^{\mathrm{T}}x(k_0)\tag{13}$$

for any initial state $x(k_0)\succeq 0$. Thus,

$$-\delta\mathbf{1}_{n\times n}x(k_0)\preceq e(k_0)\preceq\delta\mathbf{1}_{n\times n}x(k_0)\tag{14}$$

Then, we have

$$\begin{aligned}x(k_0\ +1)\succeq\ &A_ix(k_0)+\\&\beta_i(k_0)B_i\sum_{\ell=1}^{2^m}\hbar_{i\ell}\Big(E_{i\ell}F_i^{+}x(k_0)+E_{i\ell}F_i^{-}x(k_0)+\\&E_{i\ell}F_i^{+}e(k_0)+E_{i\ell}F_i^{-}e(k_0)+E_{i\ell}^{-}H_i^{+}x(k_0)+\\&E_{i\ell}^{-}H_i^{-}x(k_0)+E_{i\ell}^{-}H_i^{+}e(k_0)+E_{i\ell}^{-}H_i^{-}e(k_0)\Big)+\\&(1-\beta_i(k_0))\gamma_i(k)\Big(B_iF_i^{+}x(k_0)+\\&B_iF_i^{-}x(k_0)+B_iF_i^{+}e(k_0)+B_iF_i^{-}e(k_0)\Big)\succeq\\&\beta_i(k_0)\sum_{\ell=1}^{2^m}\hbar_{i\ell}\Big(A_i+B_iE_{i\ell}F_i^{+}\Phi_1+B_iE_{i\ell}F_i^{-}\Phi_2+\\&B_iE_{i\ell}^{-}H_i^{+}\Phi_1+B_iE_{i\ell}^{-}H_i^{-}\Phi_2\Big)x(k_0)+\\&(1-\beta_i(k_0))\Big(A_i+B_iF_i^{-}\Phi_2\Big)x(k_0)\end{aligned}\tag{15}$$

Using (6a), (6b) and (6h) gives

$$\begin{aligned}A_i\ &+B_iE_{i\ell}\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\Phi_1+B_iE_{i\ell}\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\Phi_2+\\&B_iE_{i\ell}^{-}\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\Phi_1+B_iE_{i\ell}^{-}\frac{\sum\limits_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_i}\Phi_2\succeq 0\end{aligned}$$

and

$$A_i + B_i \frac{\sum\limits_{\imath=1}^{m} \mathbf{1}_m^{(\imath)} \rho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \Phi_2 \succeq 0.$$

Using (8), it follows that $A_i + B_i E_{i\ell} F_i^+ \Phi_1 + B_i E_{i\ell} F_i^- \Phi_2 + B_i E_{i\ell}^- H_i^+ \Phi_1 + B_i E_{i\ell}^- H_i^- \Phi_2 \succeq 0$ and $A_i + B_i F_i^- \Phi_2 \succeq 0$. Therefore, $x(k_0+1) \succeq 0$ for any initial state $x(k_0) \succeq 0$. By recursive deduction, it is not hard to obtain $x(k+1) \succeq 0$ for $\forall k \in \mathbf{N}$. Therefore, the closed-loop system (1) is positive.

Choose a linear Lyapunov function $V_i(k) = x^{\mathrm{T}}(k) v_i$, where $v_i \succ 0$ in $\mathbf{R}^n$. By (12)–(14), we obtain

$$\begin{aligned}
&\mathbb{E}\{\triangle V(k)\} = \mathbb{E}\{x^{\mathrm{T}}(k+1) v_i - x^{\mathrm{T}}(k) v_i\} \leq \\
&\mathbb{E}\Big\{x^{\mathrm{T}}(k) A_i^{\mathrm{T}} v_i + x^{\mathrm{T}}(k)\Big(\bar{\beta}_i \sum_{\ell=1}^{2^m} \hbar_{i\ell}(E_{i\ell} F_i^+ + \\
&\quad E_{i\ell} F_i^- + E_{i\ell}^- H_i^+ + E_{i\ell}^- H_i^-) + \\
&\quad (1-\bar{\beta}_i)\bar{\gamma}_i (F_i^+ + F_i^-)\Big)^{\mathrm{T}} B_i^{\mathrm{T}} v_i - x^{\mathrm{T}}(k) v_i + \\
&\quad e^{\mathrm{T}}(k)\Big((1-\bar{\beta}_i)\bar{\gamma}_i (F_i^+ + F_i^-) + \\
&\quad \bar{\beta}_i \sum_{\ell=1}^{2^m} \hbar_{i\ell}(E_{i\ell} F_i^+ + E_{i\ell} F_i^- + \\
&\quad H_i^+ E_{i\ell} + E_{i\ell}^- H_i^-)\Big)^{\mathrm{T}} B_i^{\mathrm{T}} v_i\Big\}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\mathbb{E}\{\triangle V(k)\} \leq x^{\mathrm{T}}(k) A_i^{\mathrm{T}} v_i + \\
&\Big(\bar{\beta}_i \sum_{\ell=1}^{2^m} \hbar_{i\ell}(E_{i\ell} F_i^+ \Phi_2 x(k) + E_{i\ell} F_i^- x(k) - \\
&\delta E_{i\ell} F_i^- \mathbf{1}_{n\times n} e(k) + E_{i\ell}^- H_i^+ \Phi_2 x(k) + \\
&E_{i\ell}^- H_i^- x(k) - \delta E_{i\ell}^- H_i^- \mathbf{1}_{n\times n} e(k)) + \\
&(1-\bar{\beta}_i)\bar{\gamma}_i (F_i^+ x(k) \Phi_2 + F_i^- x(k) - \\
&\delta F_i^- \mathbf{1}_{n\times n} e(k)) - I\Big)^{\mathrm{T}} B_i^{\mathrm{T}} v_i. \qquad (16)
\end{aligned}$$

By (6f) and (8a),

$$\begin{aligned}
&(B_i F_i^+)^{\mathrm{T}} v_i \preceq \frac{\rho_i^+ \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \rho_i^+ \\
&(B_i F_i^-)^{\mathrm{T}} v_i \preceq \frac{\bar{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \bar{\rho}_i^- \\
&(B_i F_i^-)^{\mathrm{T}} v_i \succeq \frac{\underline{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \underline{\rho}_i^-. \qquad (17)
\end{aligned}$$

Case I. $E_{i\ell} = 0$. It is easy to get $(B_i E_{i\ell} F_i^+)^{\mathrm{T}} v_i = 0$, $(B_i E_{i\ell} F_i^-)^{\mathrm{T}} v_i = 0$ and $E_{i\ell}^- = I$. Using (6f) and (8b) yields that

$$\begin{aligned}
&(B_i E_{i\ell}^- H_i^+)^{\mathrm{T}} v_i \preceq \frac{\varrho_i^+ \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \varrho_i^+ \\
&(B_i E_{i\ell}^- H_i^-)^{\mathrm{T}} v_i \preceq \frac{\bar{\varrho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \bar{\varrho}_i^- \\
&(B_i E_{i\ell}^- H_i^-)^{\mathrm{T}} v_i \succeq \frac{\underline{\varrho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \underline{\varrho}_i^-. \qquad (18)
\end{aligned}$$

By (17) and (18), (16) can be transformed into

$$\begin{aligned}
&\mathbb{E}\{\triangle V(k)\} \leq x^{\mathrm{T}}(k)\big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i(\varrho_i^+ + \delta \mathbf{1}_{n\times n} \varrho_i^+ + \\
&\quad \bar{\varrho}_i^- - \delta \mathbf{1}_{n\times n} \underline{\varrho}_i^-) + (1-\bar{\beta}_i)\bar{\gamma}_i(\rho_i^+ + \\
&\quad \delta \mathbf{1}_{n\times n} \rho_i^+ + \bar{\rho}_i^- - \delta \mathbf{1}_{n\times n} \underline{\rho}_i^-) - v_i\big).
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\mathbb{E}\{\triangle V(k)\} \leq x^{\mathrm{T}}(k)\big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i(\Phi_2 \varrho_i^+ + \bar{\varrho}_i^- - \\
&\quad \delta \mathbf{1}_{n\times n} \underline{\varrho}_i^-) + \psi - v_i\big). \qquad (19)
\end{aligned}$$

Case II. $E_{i\ell} = I$. It is clear that $E_{i\ell}^- = 0$. Then, $(B_i E_{i\ell}^- H_i^+)^{\mathrm{T}} v_i = 0$ and $(B_i E_{i\ell}^- H_i^-)^{\mathrm{T}} v_i = 0$. Using (6f) and (8a) gives

$$\begin{aligned}
&(B_i E_{i\ell} F_i^+)^{\mathrm{T}} v_i \preceq \frac{\rho_i^+ \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \rho_i^+ \\
&(B_i E_{i\ell} F_i^-)^{\mathrm{T}} v_i \preceq \frac{\bar{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \bar{\rho}_i^- \\
&(B_i E_{i\ell} F_i^-)^{\mathrm{T}} v_i \succeq \frac{\underline{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = \underline{\rho}_i^-. \qquad (20)
\end{aligned}$$

Combining (16), (17) and (20) gives

$$\begin{aligned}
&\mathbb{E}\{\triangle V(k)\} \leq x^{\mathrm{T}}(k)(A_i^{\mathrm{T}} v_i + \bar{\beta}_i(\Phi_2 \rho_i^+ + \bar{\rho}_i^- - \\
&\quad \delta \mathbf{1}_{n\times n} \underline{\rho}_i^-) + \psi - v_i). \qquad (21)
\end{aligned}$$

Case III. $E_{i\ell} \neq 0$ and $E_{i\ell} \neq I$. By (6f), (6g) and (8), it follows that

$$\begin{aligned}
&(B_i E_{i\ell} F_i^+)^{\mathrm{T}} v_i \preceq \frac{\rho_i^+ \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \preceq \rho_i^+ \\
&(B_i E_{i\ell} F_i^-)^{\mathrm{T}} v_i \preceq \frac{\bar{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \preceq \eta_1 \bar{\rho}_i^- \\
&(B_i E_{i\ell} F_i^-)^{\mathrm{T}} v_i \succeq \frac{\underline{\rho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \succeq \eta_2 \underline{\rho}_i^- \\
&(B_i E_{i\ell}^- H_i^+)^{\mathrm{T}} v_i \preceq \frac{\varrho_i^+ \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \preceq \varrho_i^+ \\
&(B_i E_{i\ell}^- H_i^-)^{\mathrm{T}} v_i \preceq \frac{\bar{\varrho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \preceq \eta_1 \bar{\varrho}_i^- \\
&(B_i E_{i\ell}^- H_i^-)^{\mathrm{T}} v_i \succeq \frac{\underline{\varrho}_i^- \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{-\mathrm{T}} B_i^{\mathrm{T}} v_i}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} \succeq \eta_2 \underline{\varrho}_i^-. \qquad (22)
\end{aligned}$$

Substituting (22) into (16) gives

$$\begin{aligned}\mathbb{E}\{\triangle V(k)\} \le\; & x^{\mathrm{T}}(k)\Big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i \Phi_2(\rho_i^+ + \varrho_i^+) + \\ & \bar{\beta}_i(\eta_1 \bar{\rho}_i^- + \eta_1 \bar{\varrho}_i^- - \eta_2 \delta \mathbf{1}_{n\times n} \underline{\rho}_i^- - \\ & \eta_2 \delta \mathbf{1}_{n\times n} \underline{\varrho}_i^-) + \psi - v_i\Big). \end{aligned} \tag{23}$$

Applying (6c)–(6e) to (19), (21) and (23), it follows that

$$\mathbb{E}\{\triangle V(k)\} \le -(1-\mu) V_i(k). \tag{24}$$

Then, $V_{\sigma(k_m)}(k) \le \mu^{(k-k_m)} V_{\sigma(k_m)}(k_m)$, $k \in (k_m, k_{m+1})$. Suppose $0 = k_0 < k_1 < k_2 < \cdots < k_m = k_{N_\sigma(k_0,k)} < k$ is the switching time sequences of $\sigma(k)$ in the interval $[0, k)$. By (5) and (6j), we have

$$x^{\mathrm{T}}(k) v_{\sigma(k_m)} \le \mu^{k-k_0} \lambda^{N_\sigma(k_0,k)} x^{\mathrm{T}}(k_0) v_{\sigma(k_0)}. \tag{25}$$

Then $\|x(k)\|_1 \le \Gamma \Upsilon^{k-k_0} \|x(k_0)\|_1, \forall k \ge k_0$, where $\Gamma = \dfrac{\chi_2}{\chi_1} \mathrm{e}^{N_0 \ln \lambda} > 0$ and $\Upsilon = \mathrm{e}^{(\ln \mu + \frac{\ln \lambda}{\tau})}$, χ_1 and χ_2 are the minimal and maximal elements of $v_i, \forall i \in S$, respectively. We obtain from (9) that $0 < \Upsilon < 1$. Therefore, the closed-loop system (1) is stable.

For the initial state $x(k_0)$ in $\Im(v_i, 1)$, we can obtain from (25) that $x(k)\, v_i \le 1$, i.e., $x(k) \in \bigcup_i^J \Im(v_i, 1)$, $\forall k \in \mathbf{N}^+$. By (6h) and (6i), we have $-v_i \preceq \frac{\varrho_{ii}^+ + \varrho_{ii}^-}{\mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i} = H_{ii}^{\mathrm{T}} \preceq v_i$. Thus, we have $-1 \le -x^{\mathrm{T}}(k) v_i \le H_{ii} x(k) \le x^{\mathrm{T}}(k) v_i \le 1$. This implies that $\Im(v_i, 1) \subseteq L(H_i)$. □

Remark 2. The conditions (6a) and (6b) are to guarantee the positivity of the system (2). The conditions (6c)–(6e) are to reach the stability of the system. The condition (6h) is to construct a cone domain of attraction. The condition (6j) is usually used in switched systems to guarantee a finite increment of Lyapunov functions of different subsystems. The conditions (6f), (6g), and (6i) are to transform the positivity and stability conditions into linear programming. Next, an approach to estimate the maximal attraction domain of the system (1) is presented. Suppose that a cone set $\Im(h_i, 1)$ satisfies $\phi\Im(h_i, 1) \subseteq (v_i, 1)$, where $h_i \succ 0$ and $\phi > 0$. Since $x(k) \succeq 0$, we obtain $v_i \preceq \dfrac{1}{\phi} h_i$. Set $\phi^* = \dfrac{1}{\phi}$. Then, the maximal attraction domain estimation problem can be formulated by the optimization problem:

$$\min \phi^* \ \text{ s.t. } v_i \preceq \phi^* h_i \text{ and } (6). \tag{26}$$

We provide a suggestive algorithm to compute the condition (6).

Algorithm 1. Algorithm of condition (6)

Step 1. Set initial search intervals $\mu \in [\mu_{\min}, \mu_{\max}]$, $\kappa_1 \in [\eta_{1\min}, \eta_{1\max}]$, $\eta_2 \in [\eta_{2\min}, \eta_{2\max}]$ and $\alpha \in [\delta_{\min}, \delta_{\max}]$, where $\mu_{\min}$, $\mu_{\max}$, $\eta_{1\min}$, $\eta_{1\max}$, $\eta_{2\min}$, $\eta_{2\max}$, $\delta_{\min}$ and $\delta_{\max}$ are given positive constants. Give $\mu_l = \mu_{\min} + \Im_l(\mu_{\max} - \mu_{\min})$, $\eta_{1i} = \eta_{1\min} + \Im_i(\eta_{1\max} - \eta_{1\min})$, $\eta_{2j} = \eta_{2\min} + \Im_j(\eta_{2\max} - \eta_{2\min})$ and $\delta_\ell = \delta_{\min} + \Im_\ell(\delta_{\max} - \delta_{\min})$ for $l, i, j, \ell = 0, 1, 2, \cdots$, where $\Im_l$ is a uniformly distributed random number and $\Im_l \in [0, 1]$. Compute (6a)–(6f).

If the conditions (6a)–(6g) are unfeasible, compute (6a)–(6g) by fixing the value of μ_l and choosing $\eta_{1i+1} = \eta_{1\min} + \Im_{i+1}(\eta_{1\max} - \eta_{1\min})$, $\eta_{2j+1} = \eta_{2\min} + \Im_{j+1}(\eta_{2\max} - \eta_{2\min})$ and $\delta_{\ell+1} = \delta_{\min} + \Im_{\ell+1}(\delta_{\max} - \delta_{\min})$;

Else if the conditions (6a)–(6g) are unfeasible, implement (6a)–(6g) by fixing the value of η_{1l} and setting $\delta_{i+1} = \delta_{\min} + \Im_{j+1}(\delta_{\max} - \delta_{\min})$, $\eta_{2j+1} = \eta_{2\min} + \Im_{j+1}(\eta_{2\max} - \eta_{2\min})$ and $\mu_{\ell+1} = \mu_{\min} + \Im_{\ell+1}(\mu_{\max} - \mu_{\min})$;

Else if the conditions (6a)–(6g) are unfeasible, implement (6a)–(6g) by fixing the value of η_{2l} and setting $\mu_{i+1} = \mu_{\min} + \Im_{i+1}(\mu_{\max} - \mu_{\min})$, $\eta_{2j+1} = \eta_{1\min} + \Im_{j+1}(\eta_{1\max} - \eta_{1\min})$ and $\delta_{\ell+1} = \delta_{\min} + \Im_{\ell+1}(\delta_{\max} - \delta_{\min})$;

Else if the conditions (6a)–(6g) are unfeasible, implement (6a)–(6g) by fixing the value of δ_l and setting $\mu_{i+1} = \mu_{\min} + \Im_{i+1}(\mu_{\max} - \mu_{\min})$, $\eta_{1\ell+1} = \eta_{1\min} + \Im_{\ell+1}(\eta_{1\max} - \eta_{1\min})$ and $\eta_{2j+1} = \eta_{2\min} + \Im_{j+1}(\eta_{2\max} - \eta_{2\min})$;

Else if the conditions (6a)–(6g) are unfeasible, implement (6a)–(6g) by setting $\mu_{i+1} = \mu_{\min} + \Im_{i+1}(\mu_{\max} - \mu_{\min})$, $\eta_{1i+1} = \eta_{1\min} + \Im_{i+1}(\eta_{1\max} - \eta_{1\min})$, $\eta_{2i+1} = \eta_{2\min} + \Im_{i+1}(\eta_{2\max} - \eta_{2\min})$ and $\delta_{i+1} = \delta_{\min} + \Im_{i+1}(\delta_{\max} - \delta_{\min})$.

End until (6a)–(6g) are feasible.

Step 2. Compute the decision variables by (6a)–(6g) and substitute the obtained solutions into (6h)–(6j). Let ζ and λ be unknown. Solve the conditions (6h)–(6j).

Step 3. Denote the obtained parameters in Steps 1 and 2 as μ_0, λ_0, δ_0, η_1, η_2 and ζ. Fix the values of η_1, η_2, ζ, δ_0 and λ_0. Let $\mu \in [\mu_0, \mu_0 + \xi]$, where ξ is a given scalar. By the dichotomy approach, find a value of μ in the interval. Implement finite times iterative computation until the conditions (6h)–(6j) are unfeasible. Otherwise, let $\mu \in [\mu_0 + \xi, \mu_0 + l\xi]$ and repeat the previous step until a feasible value of μ is obtained as large as possible.

Step 4. Fix the parameters μ, ζ, δ, η_1 and η_2 obtained in Step 3. A value of λ as small as possible can be found in the interval $[1, \lambda_0]$ using a similar method in Step 3.

Step 5. Fix the parameters μ, ζ, λ, η_1 and η_2 obtained in Step 3. A value of δ as small as possible can be found in the interval $[0, \delta_0]$ using a similar method in Step 3.

Step 6. Substitute the parameters μ, ζ, η_1, η_2, δ and λ obtained in Steps 1–5 into (6) and compute the variables.

In Theorem 1, we consider the event-triggered control of the system (1). In the following, we will extend the proposed design to the system (1) with interval uncertainty:

$$\underline{A}_i \preceq A_i \preceq \overline{A}_i,\ \underline{B}_i \preceq B_i \preceq \overline{B}_i \tag{27}$$

where $\underline{A}_i \succeq 0$ and $\underline{B}_i \succeq 0$.

Corollary 1. If there exist constants $1 > \mu > 0$, $\lambda > 1$, $\eta > 0$, $\zeta > 0$ and $\mathbf{R}^n$ vectors $v_i \succ 0$, $\rho_{ii}^+ \succ 0$, $\rho_{ii}^- \prec 0$, $\varrho_{ii}^+ \succ 0$, $\varrho_{ii}^- \prec 0$, $\rho_i^+ \succ 0$, $\bar{\rho}_i^- \prec 0$, $\underline{\rho}_i^- \prec 0$, $\varrho_i^+ \succ 0$, $\bar{\varrho}_i^- \prec 0$, $\underline{\varrho}_i^- \prec 0$ such that

$$\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i\underline{A}_i+\underline{B}_iE_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{+\mathrm{T}}\Phi_1+\underline{B}_iE_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}\Phi_2+$$
$$\underline{B}_iE_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{+\mathrm{T}}\Phi_1+\underline{B}_iE_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{-\mathrm{T}}\Phi_2\succeq 0$$

$$\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i\underline{A}_i+\underline{B}_i\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}\Phi_2\succeq 0$$

$$\overline{A}_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\varrho_i^{+}+\bar{\varrho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+\psi-\mu v_i\prec 0, E_{i\ell}=0$$

$$\overline{A}_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\rho_i^{+}+\bar{\rho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})+\psi-\mu v_i\prec 0, E_{i\ell}=I$$

$$\overline{A}_i^{\mathrm{T}}v_i\bar{\beta}_i\Phi_2(\rho_i^{+}+\varrho_i^{+})+\bar{\beta}_i(\eta_1\bar{\rho}_i^{-}+\eta_1\bar{\varrho}_i^{-}-\eta_2\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-}$$
$$-\eta_2\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+\psi-\mu v_i\prec 0, E_{i\ell}\neq I, E_{i\ell}\neq 0$$

$$\rho_{i\imath}^{+}\preceq\rho_i^{+},\underline{\rho}_i^{-}\preceq\rho_{i\imath}^{-}\preceq\bar{\rho}_i^{-},\varrho_{i\imath}^{+}\preceq\varrho_i^{+},\underline{\varrho}_i^{-}\preceq\varrho_{i\imath}^{-}\preceq\bar{\varrho}_i^{-}$$

$$\eta_1\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i\leq\mathbf{1}_m^{\mathrm{T}}E_{i\ell}^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i\leq\eta_2\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i, E_{i\ell}\neq 0I$$

$$-\zeta v_i\leq\varrho_{i\imath}^{+}+\varrho_{i\imath}^{-}\leq\zeta v_i$$

$$\zeta\leq\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i$$

$$v_i\preceq\lambda v_j$$

hold $\forall(i,j)\in S, i\neq j$, and $\imath=1,\cdots,m$, then under the event-triggered control law (4) with (7) and

$$F_i^{+}=\frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i}, F_i^{-}=\frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i}$$
$$H_i^{+}=\frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i}, H_i^{-}=\frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\underline{B}_i^{\mathrm{T}}v_i}$$

the resulting closed-loop system (1) is positive and stable with ADT switching satisfying (9), where $E_{i\ell},\ell=1,2,\cdots,2^m$ are $m\times m$ diagonal matrices with elements being either 1 or 0 and $E_{i\ell}^{-}=I-E_{i\ell}$, $\Phi_1=I-\delta\mathbf{1}_{n\times n}$, $\Phi_2=I+\delta\mathbf{1}_{n\times n}$ and $\psi=(1-\bar{\beta}_i)\bar{\gamma}_i(\Phi_2\rho_i^{+}+\bar{\rho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})$. Moreover, the system states starting from $x(k_0)\in\Im(v_i,1)$ will remain inside $\bigcup_i^J\Im(v_i,1)$ for $N_0=0$. The proof of Corollary 1 can be easily obtained by using a similar method in Theorem 1.

Remark 3. Most of control synthesis of positive switched systems are concerned with time-triggered sample control[10–12], in which the state of the controller is updated at each time instant. This may lead to resource waste and increase the design cost. In some control systems such as communication networks and power systems, it is impossible to obtain the real-time information of states owing to limited capacity of sampling elements. In Theorem 1, we design an event-triggered controller for the systems using a linear approach. The controller only needs to obtain the state information when the event-triggering condition is satisfied. In practical, such a design is not only practical but also with lower cost.

Remark 4. Wang et al.[32, 33] considered the saturation issues of positive switched systems, where it was assumed that actuator saturation occurs in a determined way. Indeed, the occurrence of actuator saturation is dependent on the input information of the controller. Thus, the saturation is likely to arise in a random way. Up to now, few results are devoted to the randomly occurring saturation. Theorem 1 investigates the randomly occurring actuator saturation problems of positive switched systems as the first attempt.

Remark 5. For general systems, a Lyapunov function is usually constructed in a quadratic form since such a form can guarantee the positive definite property of the Lyapunov function. The state of positive systems is nonnegative. Therefore, a linear function: $V(k)=x^{\mathrm{T}}(k)v$ can be chosen as the Lyapunov function of positive systems, where $v\succ 0$ with compatible dimension. Under the linear Lyapunov function, linear programming is naturally employed as computation tool. A linear approach including linear Lyapunov functions and linear programming has been widely used in the literature[1, 2, 10–17] to deal with the issues of positive systems. It has also been verified in this literature that the linear approach is more suitable for positive systems than other approaches.

Theorem 2. If there exist constants $0<\mu<1$, $\lambda>1,\eta>0,\zeta>0$ and $\mathbf{R}^n$ vectors $v_i\succ 0$, $\nu_i\succ 0$, $\rho_{i\imath}^{+}\succ 0$, $\rho_{i\imath}^{-}\prec 0$, $\varrho_{i\imath}^{+}\succ 0$, $\varrho_{i\imath}^{-}\prec 0$, $\rho_i^{+}\succ 0$, $\bar{\rho}_i^{-}\prec 0$, $\underline{\rho}_i^{-}\prec 0$, $\varrho_i^{+}\succ 0$, $\bar{\varrho}_i^{-}\prec 0$, $\underline{\varrho}_i^{-}\prec 0$ such that

$$\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_iA_i+B_iE_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{+\mathrm{T}}\Phi_1+$$
$$B_iE_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}\Phi_2+B_iE_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{+\mathrm{T}}\Phi_1+$$
$$B_iE_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{i\imath}^{-\mathrm{T}}\Phi_2\succeq 0 \tag{28a}$$

$$\mathbf{1}_m^{\mathrm{T}}B_i^{\mathrm{T}}v_iA_i+B_i\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{i\imath}^{-\mathrm{T}}\Phi_2\succeq 0 \tag{28b}$$

$$A_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\varrho_i^{+}+\bar{\varrho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+$$
$$\psi-\mu v_i+\nu_i\prec 0, E_{i\ell}=0 \tag{28c}$$

$$A_i^{\mathrm{T}}v_i+\bar{\beta}_i(\Phi_2\rho_i^{+}+\bar{\rho}_i^{-}-\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})+$$
$$\psi-\mu v_i+\nu_i\prec 0, E_{i\ell}=I \tag{28d}$$

$$A_i^{\mathrm{T}}v_i+\bar{\beta}_i\Phi_2(\rho_i^{+}+\varrho_i^{+})+\bar{\beta}_i(\eta_1\bar{\rho}_i^{-}+$$
$$\eta_1\bar{\varrho}_i^{-}-\eta_2\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-}-\eta_2\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-})+$$
$$\psi-\mu v_i+\nu_i\prec 0, E_{i\ell}\neq 0, I \tag{28e}$$

$$\bar{\alpha}_i A_{di}^{\mathrm{T}} v_i - \mu^h \nu_i \prec 0 \tag{28f}$$

$$\rho_{ii}^+ \preceq \rho_i^+, \underline{\rho}_i^- \preceq \rho_{ii}^- \preceq \bar{\rho}_i^-, \varrho_{ii}^+ \preceq \varrho_i^+, \underline{\varrho}_i^- \preceq \varrho_{ii}^- \preceq \bar{\varrho}_i^- \tag{28g}$$

$$\eta_1 \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i \le \mathbf{1}_m^{\mathrm{T}} E_{i\ell}^{\mathrm{T}} B_i^{\mathrm{T}} v_i \le \eta_2 \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i, E_{i\ell} \neq 0, I \tag{28h}$$

$$-\zeta v_i \le \varrho_{ii}^+ + \varrho_{ii}^- \le \zeta v_i \tag{28i}$$

$$\zeta \le \mathbf{1}_m^{\mathrm{T}} B_i^{\mathrm{T}} v_i \tag{28j}$$

$$v_i \preceq \lambda v_j, \nu_i \preceq \lambda \nu_j \tag{28k}$$

hold $\forall (i,j) \in S, i \neq j$, and $\imath = 1, 2, \cdots, m$, then under the event-triggered control law (4) with (7) and (8), the resulting closed-loop system (2) is positive and stable with ADT switching satisfying (9), where $E_{i\ell}, \ell = 1, 2, \cdots, 2^m$ are $m \times m$ diagonal matrices with elements being either 1 or 0 and $E_{i\ell}^- = I - E_{i\ell}$, $\Phi_1 = I - \delta \mathbf{1}_{n\times n}$, $\Phi_2 = I + \delta \mathbf{1}_{n\times n}$ and $\psi = (1-\bar{\beta}_i)\bar{\gamma}_i(\Phi_2 \rho_i^+ + \bar{\rho}_i^- - \delta \mathbf{1}_{n\times n} \underline{\rho}_i^-)$. Moreover, the system states starting from $x(k_0) \in \Im(v_i, 1)$ will remain inside $\bigcup_i^J \Im(v_i, 1)$ for $N_0 = 0$.

Proof. First, the system (2) can be transformed into (29). Using (13) and (14) gives (30). Combining (28a), (28b) and (8) gives $A_i + B_i E_{i\ell} F_i^+ \Phi_1 + B_i E_{i\ell} F_i^- \Phi_2 + B_i E_{i\ell}^- H_i^+ \Phi_1 + B_i E_{i\ell}^- H_i^- \Phi_2 \succeq 0$ and $A_i + B_i F_i^- \Phi_2 \succeq 0$. This together with $A_{di} \succ 0$ follows that $x(k_0+1) \succeq 0$. By recursive deduction, we have $x(k+1) \succeq 0$ for any non-negative initial state.

$$\begin{aligned} x(k+1) = {} & A_i x(k) + \alpha_i(k) A_{di} x(k-h) + \\ & \beta_i(k) B_i \sum_{\ell=1}^{2^m} \hbar_{i\ell} \Big(E_{i\ell} F_i x(k) + E_{i\ell} F_i e(k) + \\ & E_{i\ell}^- H_i x(k) + E_{i\ell}^- H_i e(k) \Big) + \\ & (1-\beta_i(k)) \gamma_i(k) B_i F_i \hat{x}(k). \end{aligned} \tag{29}$$

$$\begin{aligned} x(k_0+1) \succeq {} & \beta_i(k_0) \sum_{\ell=1}^{2^m} \Big(A_i + B_i E_{i\ell} F_i^+ \Phi_1 + B_i E_{i\ell} F_i^- \Phi_2 + \\ & B_i E_{i\ell}^- H_i^+ \Phi_1 + B_i E_{i\ell}^- H_i^- \Phi_2 \Big) x(k_0) + \\ & (1-\beta_i(k_0))(A_i + B_i F_i^- \Phi_2) x(k_0) + \\ & \alpha_i(k_0) A_{di} x(k_0 - h). \end{aligned} \tag{30}$$

Choose a co-positive Lyapunov functional:

$$V_i(k) = x^{\mathrm{T}}(k) v_i + \sum_{s=k-h}^{k-1} \mu^{s-k+1} x^{\mathrm{T}}(s) \nu_i \tag{31}$$

where $v_i \succ 0$ and $\nu_i \succ 0$ in $\mathbf{R}^n$. Then, we get

$$\begin{aligned} x(k+1) = {} & A_i x(k) + \alpha_i(k) A_{di} x(k-h) + \\ & \beta_i(k) B_i \sum_{\ell=1}^{2^m} \hbar_{i\ell} \Big(E_{i\ell} F_i x(k) + E_{i\ell} F_i e(k) + \\ & E_{i\ell}^- H_i x(k) + E_{i\ell}^- H_i e(k) \Big) + \\ & (1-\beta_i(k)) \gamma_i(k) B_i F_i \hat{x}(k). \end{aligned} \tag{32}$$

By (14), we have

$$\begin{aligned} & \mathbb{E}\{\Delta V(k)\} \le \\ & x^{\mathrm{T}}(k) \Big(\bar{\beta}_i \sum_{\ell=1}^{2^m} \hbar_{i\ell} (E_{i\ell} F_i^+ \Phi_2 + \\ & E_{i\ell} F_i^- \Phi_1 + E_{i\ell}^- H_i^+ \Phi_2 + E_{i\ell}^- H_i^- \Phi_1) + A_i^{\mathrm{T}} v_i + \nu_i + \\ & (1-\bar{\beta}_i) \bar{\gamma}_i (F_i^+ \Phi_2 + F_i^- \Phi_1) \Big)^{\mathrm{T}} B_i^{\mathrm{T}} v_i - \\ & \mu x^{\mathrm{T}}(k) v_i + \bar{\alpha}_i x^{\mathrm{T}}(k-h) A_{di}^{\mathrm{T}} v_i - \mu^h x^{\mathrm{T}}(k-h) \nu_i. \end{aligned} \tag{33}$$

Using (17) and (18) gives

$$\begin{aligned} & \mathbb{E}\{V_i(k+1) - \mu V_i(k)\} \le x^{\mathrm{T}}(k) \Big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i (\Phi_2 \varrho_i^+ + \\ & \bar{\varrho}_i^- - \delta \mathbf{1}_{n\times n} \underline{\varrho}_i^-) + \psi - \mu v_i + \nu_i \Big) + \\ & x^{\mathrm{T}}(k-h)(\bar{\alpha}_i A_{di}^{\mathrm{T}} v_i - \mu^h \nu_i) \end{aligned} \tag{34}$$

for Case I. Connecting (17) and (20) gives

$$\begin{aligned} & \mathbb{E}\{V_i(k+1) - \mu V_i(k)\} \le x^{\mathrm{T}}(k) \Big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i (\Phi_2 \rho_i^+ + \\ & \bar{\rho}_i^- - \delta \mathbf{1}_{n\times n} \underline{\rho}_i^-) + \psi - \mu v_i + \nu_i \Big) + \\ & x^{\mathrm{T}}(k-h)(\bar{\alpha}_i A_{di}^{\mathrm{T}} v_i - \mu^h \nu_i) \end{aligned} \tag{35}$$

for Case II. By (17) and (22), we can obtain

$$\begin{aligned} & \mathbb{E}\{V_i(k+1) - \mu V_i(k)\} \le x^{\mathrm{T}}(k) \Big(A_i^{\mathrm{T}} v_i + \bar{\beta}_i \Phi_2 (\rho_i^+ + \\ & \varrho_i^+) + \bar{\beta}_i (\eta_1 \bar{\rho}_i^- + \eta_1 \bar{\varrho}_i^- - \eta_2 \delta \mathbf{1}_{n\times n} \underline{\rho}_i^- - \\ & \eta_2 \delta \mathbf{1}_{n\times n} \underline{\varrho}_i^-) + \psi - \mu v_i + \nu_i \Big) + \\ & x^{\mathrm{T}}(k-h)(\bar{\alpha}_i A_{di}^{\mathrm{T}} v_i - \mu^h \nu_i) \end{aligned} \tag{36}$$

for Case III. Combining (28c)–(28f), (34), (35) and (36), it follows that $\mathbb{E}\{\triangle V(k)\} \le 0$. Thus, we obtain $V_{\sigma(k_m)}(k+1) < \mu V_{\sigma(k_m)}(k)$ for $k \in [k_m, k_{m+1})$, $m \in \mathbf{N}$. Therefore, we have

$$V_{\sigma(k_m)}(k) < \mu V_{\sigma(k_m)}(k-1) < \cdots < \mu^{k-k_m} V_{\sigma(k_m)}(k_m). \tag{37}$$

Suppose that $0 = k_0 < k_1 < k_2 < \cdots < k_m = k_{N_\sigma(k_0,k)} < k$ is a switching time sequence of $\sigma(k)$ in the interval $[0, k)$. Using (28j) gives

$$V_{\sigma(k_m)}(k) < \mu^{k-k_0} \lambda^{N_\sigma(k_0,k)} V_{\sigma(k_0)}(k_0). \tag{38}$$

By Definition 4, it follows that

$$V_{\sigma(k_m)}(k) < \mathrm{e}^{(\ln\mu+\frac{\ln\lambda}{\tau})(k-k_0)}V_{\sigma(k_0)}(k_0). \tag{39}$$

Thus, $\|x(k)\|_1 \le \Gamma\Upsilon^{(k-k_0)} \sup\limits_{-h\le\theta\le 0}\|x(\theta)\|, \forall k\ge k_0$, where $\Gamma = \dfrac{\chi_2+\chi_3}{\chi_1}$, $\Upsilon = \mathrm{e}^{\ln\mu+\frac{\ln\lambda}{\tau}}$, χ_1 is the minimal element of $v_i(k)$, χ_2 and χ_3 are the maximal elements of $v_i(k_0)$ and $\nu_i(k_0), \forall i\in S$, respectively. □

Next, we will extend the proposed approach in Theorem 2 for the system (2) with polytopic uncertainty:

$$[A_i\ B_i] = \sum_{p=1}^{J}\varsigma_p[A_i^{(p)}\ B_i^{(p)}] \tag{40}$$

where $A_i^{(p)}\succeq 0$, $B_i^{(p)}\succeq 0$ and $\sum_{p=1}^{J}\varsigma_p = 1$, $0\le\varsigma_p\le 1$.

Corollary 2. If there exist constants $0<\nu<1, \lambda>1, \eta>0, \zeta>0$ and $\mathbf{R}^n$ vectors $v_i\succ 0$, $\nu_i\succ 0$, $\rho_{ii}^{+}\succ 0$, $\rho_{ii}^{-}\prec 0$, $\varrho_{ii}^{+}\succ 0$, $\varrho_{ii}^{-}\prec 0$, $\rho_i^{+}\succ 0$, $\bar{\rho}_i^{-}\prec 0$, $\underline{\rho}_i^{-}\prec 0$, $\varrho_i^{+}\succ 0$, $\bar{\varrho}_i^{-}\prec 0$, $\underline{\varrho}_i^{-}\prec 0$ such that

$$\begin{aligned}
&\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_iA_i^{(p)} + B_i^{(p)}E_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{ii}^{+\mathrm{T}}\Phi_1 + \\
&\quad B_i^{(p)}E_{i\ell}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{ii}^{-\mathrm{T}}\Phi_2 + B_i^{(p)}E_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{ii}^{+\mathrm{T}}\Phi_1 + \\
&\quad B_i^{(p)}E_{i\ell}^{-}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{ii}^{-\mathrm{T}}\Phi_2 \succeq 0\\
&\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_iA_i^{(p)} + B_i^{(p)}\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{ii}^{-\mathrm{T}}\Phi_2 \succeq 0\\
&A_i^{(p)\mathrm{T}}v_i + \bar{\beta}_i(\Phi_2\varrho_i^{+} + \bar{\varrho}_i^{-} - \delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-}) + \\
&\quad \psi - \mu v_i + \nu_i \prec 0, E_{i\ell} = I\\
&A_i^{(p)\mathrm{T}}v_i + \bar{\beta}_i(\Phi_2\rho_i^{+} + \bar{\rho}_i^{-} - \delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-}) + \\
&\quad \psi - \mu v_i + \nu_i \prec 0, E_{i\ell} = 0\\
&A_i^{(p)\mathrm{T}}v_i + \bar{\beta}_i\Phi_2(\rho_i^{+}+\varrho_i^{+}) + \bar{\beta}_i(\eta_1\bar{\varrho}_i^{-} - \eta_2\delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-} - \\
&\quad \eta_2\delta\mathbf{1}_{n\times n}\underline{\varrho}_i^{-}) + \psi - e^{-a}v_i + \nu_i \prec 0, E_{i\ell}\ne 0, I\\
&\bar{\alpha}_iA_{di}^{(p)\mathrm{T}}v_i - \mu^h\nu_i \prec 0\\
&\rho_{ii}^{+}\preceq\rho_i^{+}, \underline{\rho}_i^{-}\preceq\rho_{ii}^{-}\preceq\bar{\rho}_i^{-}, \varrho_{ii}^{+}\preceq\varrho_i^{+}, \underline{\varrho}_i^{-}\preceq\varrho_{ii}^{-}\preceq\bar{\varrho}_i^{-}\\
&\eta_1\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i \le \mathbf{1}_m^{\mathrm{T}}E_{i\ell}^{\mathrm{T}}B_i^{\mathrm{T}}v_i \le \eta_2\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i, E_{i\ell}\ne 0, I\\
&-\zeta v_i \le \varrho_{ii}^{+} + \varrho_{ii}^{-} \le \zeta v_i\\
&\zeta \le \mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i\\
&v_i \preceq \lambda v_j, \nu_i\preceq\lambda\nu_j
\end{aligned}$$

hold $\forall(i,j)\in S, i\ne j$, and $\imath = 1,\cdots,m$, then under the event-triggered control law (4) with (7) and

$$\begin{aligned}
&F_i^{+} = \frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{ii}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i}, F_i^{-} = \frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\rho_{ii}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i}\\
&H_i^{+} = \frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{ii}^{+\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i}, H_i^{-} = \frac{\sum_{\imath=1}^{m}\mathbf{1}_m^{(\imath)}\varrho_{ii}^{-\mathrm{T}}}{\mathbf{1}_m^{\mathrm{T}}\widehat{B}_i^{\mathrm{T}}v_i}
\end{aligned} \tag{41}$$

the resulting closed-loop system (2) is positive and stable with ADT switching satisfying (9), where $E_{i\ell}, \ell = 1,2,\cdots,2^m$ are $m\times m$ diagonal matrices with elements being either 1 or 0 and $E_{i\ell}^{-} = I - E_{i\ell}$, $\Phi_1 = I - \delta\mathbf{1}_{n\times n}$, $\Phi_2 = I + \delta\mathbf{1}_{n\times n}$ and $\psi = (1-\bar{\beta}_i)\bar{\gamma}_i(\Phi_2\rho_i^{+} + \bar{\rho}_i^{-} - \delta\mathbf{1}_{n\times n}\underline{\rho}_i^{-})$. Moreover, the system states starting from $x(k_0)\in\Im(v_i,1)$ will remain inside $\bigcup_i^J\Im(v_i,1)$, where $\widehat{B}_i = [\hat{b}_{ij}]$ with $\hat{b}_{ij} = \min\limits_{p=1,\cdots,L}\{b_{ij}^{(p)}\}$ and $b_{ij}^{(p)}$ is the i-th row j-th column element of the matrix $B^{(p)}$.

Remark 6. In [25], the event-triggered control for positive systems with input saturation was considered by means of linear matrix inequalities. The advantages of the event-triggered control approach in this paper are that: 1) A linear event-triggering framework is constructed for positive systems; 2) A design approach to the attraction domain gain H_i is proposed; 3) A linear programming computation method is employed. The proposed event-triggered control approach is simpler and the corresponding computation burden is lower.

Remark 7. In [40–43], the control issues of positive systems with time delay were considered. These works were concerned with the time-triggered control strategy. As stated in the introduction, the event-triggered control is more practical and effective in practice, especially for communication and power systems, than the time-triggered control. In [26], the event-triggered control of positive switched systems with input saturation was proposed using a linear programming. It should be pointed out that the time delay in [40–43] and the saturation in [26] are described in a certain form. This paper further considers the randomly occurring saturation and time delay issue for positive switched systems. Based on these points, the control design in this paper is more general and practicable than the results in the literature.

Remark 8. The time-delay in this paper is constant. It should be pointed out that the proposed control approach can be developed for positive switched systems with time-varying delay. In such a case, one can construct the following co-positive Lyapunov functional:

$$\begin{aligned}
V_i(k) = {} & x^{\mathrm{T}}(k)v_i + \sum_{s=k-d_1}^{k-1}\alpha^{s-k+1}x^{\mathrm{T}}(s)v_{1i} + \\
& \sum_{s=k-d_2}^{k-1}\alpha^{s-k+1}x^{\mathrm{T}}(s)v_{2i} + \\
& \sum_{s=-d_2}^{-d_1}\sum_{m=k+s}^{k-1}\alpha^{m-k+1}x^{\mathrm{T}}(m)v_{3i}
\end{aligned}$$

where $0<\alpha<1$ and $v_i\succ 0, v_{1i}\succ 0, v_{2i}\succ 0, v_{3i}\succ 0$. Then, the corresponding control problems can be solved using some similar approaches in theorems and corollaries presented in the paper. This paper refers to several issues: switching mechanism, event-triggering mechanism, randomly occurring time-delay, and randomly occurring actuator saturation. We provide a simple discussion on these issues from two aspects: the running of the systems and the feasibility of conditions. The switching law orchestrates the running rule of subsystems of positive switched systems. Each subsystem has its own saturation and time-delay behavior. The event-triggering condition is activated during the running processes of some subsystems. This means that the saturation and time-delay are the essential features of subsystems and the event-triggering mechanism is for each subsystem. We give two statements on the effect of the event-triggering mechanism and randomly occurring behavior on the feasibility of Theorem 2. Under the event-triggering condition (5), an estimate to $e(k)$ is given in (14). Then, the lower bound of the closed-loop system (2) is obtained in (30). The conditions (28a) and (28b) in Theorem 2 can guarantee the positivity of the lower bound. Thus, the closed-loop system (2) is positive. It is clear that an inaccurate estimate (14) may lead to the infeasibility of Theorem 2. Noting (29), a worse case is that $\beta_i(k) = \gamma_i(k) = 0$. In such a case, the closed-loop system (2) is reduced to $x(k+1) = A_i x(k) + \alpha_i(k) A_{di} x(k-h)$, which may be unstable. This leads to the invalidity of Theorem 2.

4 Examples

In [4], a communication network is constructed based on positive switched systems (see Fig. 2). A data communication network is generally divided into two modes: the busy time model and the idle time model. The busy-time model and the idle-time model mean that there are a large number of packets and a small number of packets in the network, respectively. When the network is busy, there is a lot of data transmission in the network channel. The network may become congested, which will lead to network collapse. The event-triggered control strategy[28, 45] can ensure the normal operation of the network. In detail, the control center can reduce the network speed to avoid the crash of the network in the busy time and increase the number of packages in the idle time. If the input of the actuator exceeds a certain limit, saturation will occur. Meanwhile, the network congestion can cause delays in packet transmission. It should also be pointed out that the saturation and time-delay of communication networks are likely to arise in a random form. Considering these points, we use the systems (1) and (2) for modeling data communication networks.

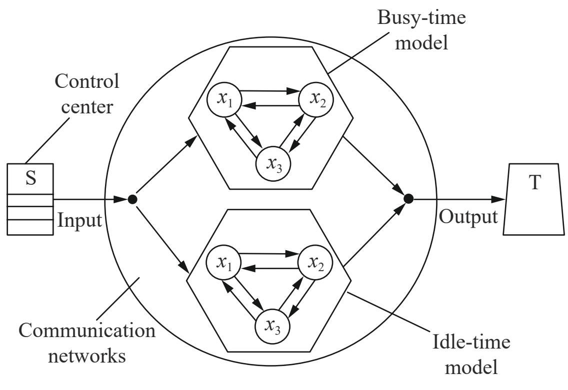

Fig. 2 A data communication network

Example 1. Consider the system (1) with two subsystems:

$$A_1 = \begin{pmatrix} 0.38 & 0.31 & 0.34 \\ 0.33 & 0.33 & 0.34 \\ 0.32 & 0.35 & 0.32 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \\ 0.001 & 0.002 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.37 & 0.32 & 0.34 \\ 0.31 & 0.33 & 0.33 \\ 0.35 & 0.33 & 0.36 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.003 & 0.002 \\ 0.002 & 0.001 \\ 0.001 & 0.002 \end{pmatrix}.$$

Choose $\mu = 0.98, \lambda = 1.02, \bar{\beta}_1 = 0.5, \bar{\beta}_2 = 0.49, \bar{\gamma}_1 = 0.51, \bar{\gamma}_2 = 0.46, \eta_1 = 0.2, \eta_2 = 1.7, \zeta = 5$ and $\delta = 0.06$. By Theorem 1, we can obtain

$$v_1 = \begin{pmatrix} 513.3620 \\ 491.9083 \\ 497.4269 \end{pmatrix}, v_2 = \begin{pmatrix} 503.3059 \\ 482.2728 \\ 508.0821 \end{pmatrix}, \rho_{11}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix},$$

$$\rho_{12}^+ = \begin{pmatrix} 0.020 \\ 0.010 \\ 0.030 \end{pmatrix}, \rho_{21}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix}, \rho_{22}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix},$$

$$\rho_{11}^- = \begin{pmatrix} -86.8357 \\ -86.8557 \\ -86.8457 \end{pmatrix}, \rho_{12}^- = \begin{pmatrix} -86.8357 \\ -86.8557 \\ -86.8457 \end{pmatrix},$$

$$\rho_{21}^- = \begin{pmatrix} -131.1538 \\ -115.9495 \\ -115.9595 \end{pmatrix}, \rho_{22}^- = \begin{pmatrix} -131.1538 \\ -115.9495 \\ -115.9595 \end{pmatrix},$$

$$\rho_1^+ = \begin{pmatrix} 0.030 \\ 0.030 \\ 0.040 \end{pmatrix}, \bar{\rho}_1^- = \begin{pmatrix} -86.8257 \\ -86.8457 \\ -86.8357 \end{pmatrix}, \rho_2^+ = \begin{pmatrix} 0.020 \\ 0.030 \\ 0.020 \end{pmatrix},$$

$$\bar{\rho}_2^- = \begin{pmatrix} -131.1438 \\ -115.9395 \\ -115.9495 \end{pmatrix}, \varrho_{11}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix}, \varrho_{12}^+ = \begin{pmatrix} 0.020 \\ 0.010 \\ 0.010 \end{pmatrix},$$

$$\varrho_{21}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix}, \varrho_{22}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix}, \varrho_{11}^- = \begin{pmatrix} -0.020 \\ -0.030 \\ -0.020 \end{pmatrix},$$

$$\varrho_{12}^- = \begin{pmatrix} -0.0200 \\ -0.0200 \\ -0.0200 \end{pmatrix}, \varrho_{21}^- = \begin{pmatrix} -0.020 \\ -0.030 \\ -0.020 \end{pmatrix}, \varrho_{22}^- = \begin{pmatrix} -0.020 \\ -0.020 \\ -0.020 \end{pmatrix},$$

$$\varrho_1^+ = \begin{pmatrix} 0.030 \\ 0.020 \\ 0.020 \end{pmatrix}, \bar{\varrho}_1^- = \begin{pmatrix} -0.010 \\ -0.010 \\ -0.010 \end{pmatrix}, \varrho_2^+ = \begin{pmatrix} 0.020 \\ 0.030 \\ 0.020 \end{pmatrix},$$

$$\bar{\varrho}_2^- = \begin{pmatrix} -0.010 \\ -0.010 \\ -0.010 \end{pmatrix}, \underline{\rho}_1^- = \begin{pmatrix} -86.8457 \\ -86.8657 \\ -86.8557 \end{pmatrix}, \underline{\rho}_2^- = \begin{pmatrix} -131.1638 \\ -115.9595 \\ -115.9695 \end{pmatrix},$$

$$\underline{\varrho}_1^- = \begin{pmatrix} -0.030 \\ -0.040 \\ -0.030 \end{pmatrix}, \underline{\varrho}_2^- = \begin{pmatrix} -0.030 \\ -0.040 \\ -0.030 \end{pmatrix}$$

and $\tau \geq 0.9802$. Thus, the event-triggered controller gain matrices and the domain of attraction gain matrices are:

$$F_1 = \begin{pmatrix} -17.3651 & -17.3671 & -17.3671 \\ -17.3631 & -17.3691 & -17.3631 \end{pmatrix}$$
$$F_2 = \begin{pmatrix} -23.8982 & -21.1276 & -21.1294 \\ -23.8982 & -21.1257 & -21.1294 \end{pmatrix}$$
$$H_1 = \begin{pmatrix} -0.0020 & -0.0040 & -0.0020 \\ 0.0000 & -0.0020 & -0.0020 \end{pmatrix}$$
$$H_2 = \begin{pmatrix} -0.0018 & -0.0018 & -0.0018 \\ -0.0018 & -0.0018 & -0.0018 \end{pmatrix}.$$

Given initial state $x(t_0) = (3\ 5\ 4)^{\mathrm{T}}$, the simulation results of the states $x(k)$ under the ADT switching are shown in Fig. 3. Fig. 4 shows release instants and release interval, where the solid and dotted lines represent the event-triggering time instants of the first and second subsystems, respectively. Fig. 5 is the domain of attraction, where it is shown that the states will be kept in the domain for different initial conditions.

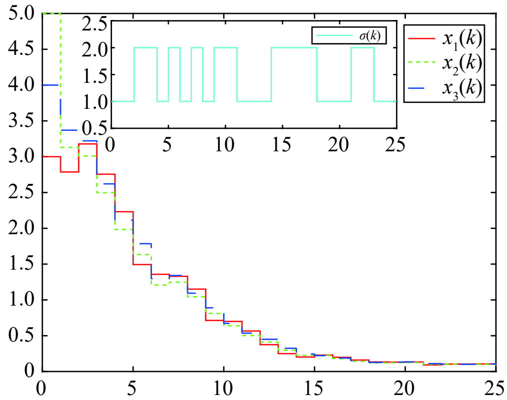

Fig. 3 Simulations of the state $x(k)$ under ADT switching

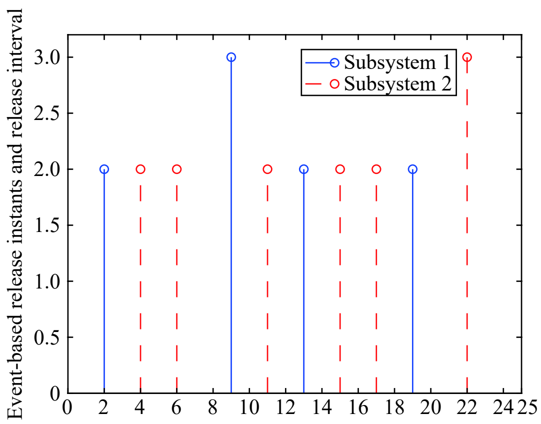

Fig. 4 Release instants and release interval

Remark 9. In Figs. 3 and 4, the sampling step is 1 s. Under the time-triggered control, the control law will be updated 25 times in 25 s. By means of the event-triggered control strategy in Theorem 1, the control law is updated only 10 times. Compared with the time-triggered control strategy, the event-triggered control strategy significantly reduces the number of state updates.

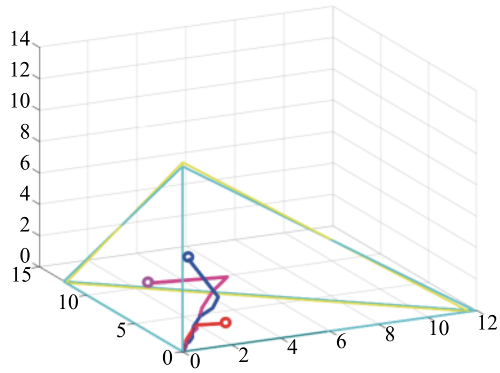

Fig. 5 Domain of attraction

Example 2. Consider the system (2) with two subsystems:

$$A_1 = \begin{pmatrix} 0.39 & 0.32 & 0.33 \\ 0.33 & 0.34 & 0.34 \\ 0.32 & 0.36 & 0.32 \end{pmatrix}, B_1 = \begin{pmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \\ 0.001 & 0.002 \end{pmatrix},$$
$$A_{d1} = \begin{pmatrix} 0.010 & 0.020 & 0.004 \\ 0.003 & 0.001 & 0.030 \\ 0.002 & 0.020 & 0.010 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} 0.38 & 0.32 & 0.35 \\ 0.32 & 0.34 & 0.33 \\ 0.34 & 0.34 & 0.36 \end{pmatrix}, B_2 = \begin{pmatrix} 0.003 & 0.002 \\ 0.002 & 0.001 \\ 0.001 & 0.002 \end{pmatrix},$$
$$A_{d2} = \begin{pmatrix} 0.02 & 0.001 & 0.010 \\ 0.001 & 0.003 & 0.001 \\ 0.001 & 0.002 & 0.002 \end{pmatrix}.$$

Choose $\mu = \mathrm{e}^{-0.02}$, $\lambda = 1.02$, $\bar{\alpha}_1 = 0.32$, $\bar{\alpha}_2 = 0.28$, $\bar{\beta}_1 = 0.56$, $\bar{\beta}_2 = 0.6$, $\bar{\gamma}_1 = 0.39$, $\bar{\gamma}_2 = 0.41$, $\eta_1 = 0.33$, $\eta_2 = 2.1$, $\zeta = 4$, $h = 2$ and $\delta = 0.05$. By Theorem 2, we can obtain

$$v_1 = \begin{pmatrix} 406.5478 \\ 402.9300 \\ 389.5456 \end{pmatrix}, v_2 = \begin{pmatrix} 398.5861 \\ 395.0392 \\ 406.8460 \end{pmatrix}, \nu_1 = \begin{pmatrix} 1.9471 \\ 5.2339 \\ 5.6451 \end{pmatrix},$$
$$\nu_2 = \begin{pmatrix} 2.4666 \\ 5.1411 \\ 5.5442 \end{pmatrix}, \rho_{11}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix}, \rho_{12}^+ = \begin{pmatrix} 0.020 \\ 0.010 \\ 0.030 \end{pmatrix},$$
$$\rho_{21}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix}, \rho_{22}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix}, \rho_{11}^- = \begin{pmatrix} -127.5204 \\ -127.5404 \\ -127.5304 \end{pmatrix},$$
$$\rho_{12}^- = \begin{pmatrix} -127.5204 \\ -123.0062 \\ -127.5304 \end{pmatrix}, \rho_{21}^- = \begin{pmatrix} -168.3254 \\ -145.0685 \\ -157.4638 \end{pmatrix},$$
$$\rho_{22}^- = \begin{pmatrix} -168.3254 \\ -145.0685 \\ -157.4638 \end{pmatrix}, \bar{\rho}_1^+ = \begin{pmatrix} 0.030 \\ 0.030 \\ 0.040 \end{pmatrix}, \bar{\rho}_1^- = \begin{pmatrix} -127.5104 \\ -122.9962 \\ -127.5204 \end{pmatrix},$$
$$\bar{\rho}_2^+ = \begin{pmatrix} 0.020 \\ 0.030 \\ 0.020 \end{pmatrix}, \bar{\rho}_2^- = \begin{pmatrix} -168.3154 \\ -145.0585 \\ -157.4538 \end{pmatrix}, \varrho_{11}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix},$$
$$\varrho_{12}^+ = \begin{pmatrix} 0.020 \\ 0.010 \\ 0.010 \end{pmatrix}, \varrho_{21}^+ = \begin{pmatrix} 0.010 \\ 0.020 \\ 0.010 \end{pmatrix}, \varrho_{22}^+ = \begin{pmatrix} 0.010 \\ 0.010 \\ 0.010 \end{pmatrix},$$

$$\varrho_{11}^- = \begin{pmatrix} -2.243\,4 \\ -2.253\,4 \\ -2.233\,4 \end{pmatrix}, \varrho_{12}^- = \begin{pmatrix} -2.243\,4 \\ -0.020\,0 \\ -2.233\,4 \end{pmatrix},$$

$$\varrho_{21}^- = \begin{pmatrix} -7.761\,1 \\ -2.601\,5 \\ -2.581\,5 \end{pmatrix}, \varrho_{22}^- = \begin{pmatrix} -7.761\,1 \\ -2.591\,5 \\ -2.581\,5 \end{pmatrix}, \varrho_1^+ = \begin{pmatrix} 0.030 \\ 0.020 \\ 0.020 \end{pmatrix},$$

$$\underline{\varrho}_1^- = \begin{pmatrix} -2.233\,4 \\ -0.010\,0 \\ -2.223\,4 \end{pmatrix}, \varrho_2^+ = \begin{pmatrix} 0.020 \\ 0.030 \\ 0.020 \end{pmatrix}, \bar{\varrho}_2^- = \begin{pmatrix} -7.751\,1 \\ -0.010\,0 \\ -2.571\,5 \end{pmatrix},$$

$$\underline{\rho}_1^- = \begin{pmatrix} -127.530\,4 \\ -127.550\,4 \\ -127.540\,4 \end{pmatrix}, \underline{\rho}_2^- = \begin{pmatrix} -168.335\,4 \\ -145.078\,5 \\ -157.473\,8 \end{pmatrix},$$

$$\underline{\varrho}_1^- = \begin{pmatrix} -2.253\,4 \\ -2.263\,4 \\ -2.243\,4 \end{pmatrix}, \underline{\varrho}_2^- = \begin{pmatrix} -7.771\,1 \\ -2.611\,5 \\ -2.591\,5 \end{pmatrix}$$

and $\tau \geq 0.9901$. Thus, the event-triggered controller gain matrices and the domain of attraction gain matrices are

$$F_1 = \begin{pmatrix} -31.877\,6 & -31.880\,1 & -31.880\,1 \\ -31.875\,1 & -30.749\,1 & -31.875\,1 \end{pmatrix}$$
$$F_2 = \begin{pmatrix} -42.078\,8 & -36.264\,6 & -39.363\,5 \\ -42.078\,8 & -36.262\,1 & -39.363\,5 \end{pmatrix}$$
$$H_1 = \begin{pmatrix} -0.558\,4 & -0.560\,9 & -0.555\,9 \\ -0.555\,9 & -0.002\,5 & -0.555\,9 \end{pmatrix}$$
$$H_2 = \begin{pmatrix} -1.937\,8 & -0.645\,4 & -0.642\,9 \\ -1.937\,8 & -0.645\,4 & -0.642\,9 \end{pmatrix}.$$

Given initial state $x(k_0) = (4\ 6\ 2)^{\mathrm{T}}$, the simulation results of the states $x(k)$ under the ADT switching are shown in Fig. 6. Fig. 7 shows release instants and release interval, where the solid and dotted lines represent the event-triggering time instants of the first and second subsystems, respectively. Fig. 8 is the domain of attraction, where it is shown that the states will be kept in the domain for different initial conditions.

5 Conclusions

This paper investigates the event-triggered control of positive switched systems with randomly occurring actuator saturation and time-delay using linear programming. The randomly occurring actuator saturation and time-delay are governed by random variables obeying the Bernoulli distribution. A general framework is established for event-triggered control of the systems. The obtained approach is extended for interval and polytopic uncertainty systems. A linear programming based optimization algorithm is presented to estimate the maximal domain of attraction.

In the future, it would be interesting to develop the obtained approaches for positive switched systems with randomly occurring saturation and time-delay obeying the binomial distribution. In this paper, the stochastic variables with respect to saturation and time-delay are independent on each other. It is interesting but challenging to consider the interdependent case of stochastic variables.

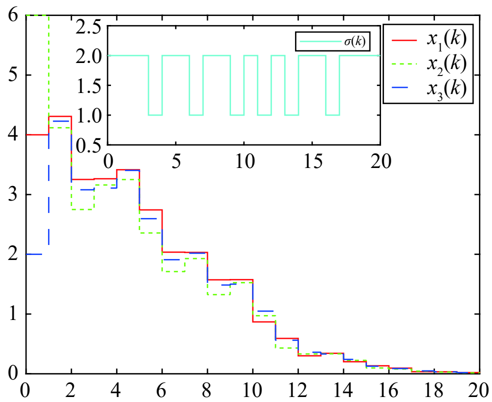

Fig. 6 Simulations of the state $x(k)$ under ADT switching

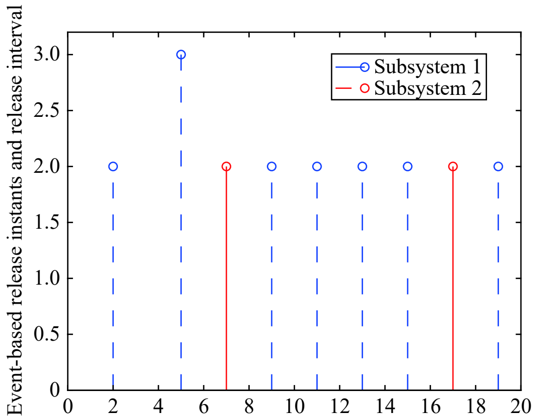

Fig. 7 Release instants and release interval

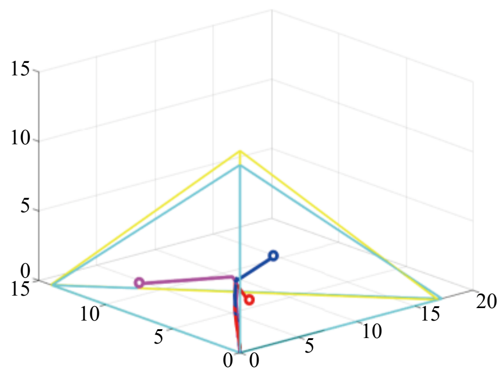

Fig. 8 Domain of attraction

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References

[1] L. Farina, S. Rinaldi. *Positive Linear Systems: Theory and Applications*, New York: John Wiley & Sons, 2000. DOI: 10.1002/9781118033029.

[2] J. Lam, Y. Chen, X. W. Liu, X. D. Zhao, J. F. Zhang. *Positive Systems*, Cham Switzerland: Springer, 2019. DOI: 10.1007/978-3-030-04327-8.

[3] S. Y. Xiao, X. H. Ge, Q. L. Han, Y. J. Zhang. Distributed resilient estimator design for positive systems under topological attacks. *IEEE Transactions on Cybernetics*, published online. DOI: 10.1109/TCYB.2020.2981646.

[4] R. Shorten, F. Wirth, D. Leith. A positive systems model of TCP-like congestion control: Asymptotic results. *IEEE/ACM Transactions on Networking*, vol. 14, no. 3, pp. 616–629, 2006. DOI: 10.1109/TNET.2006.876178.

[5] J. F. Zhang, L. W. Zhang, T. Raïssi. A linear framework on the distributed model predictive control of positive systems. *Systems & Control Letters*, vol. 138, Article number 104665, 2020. DOI: 10.1016/j.sysconle.2020.104665.

[6] C. C. Sun. Stabilization for a class of discrete-time switched large-scale systems with parameter uncertainties. *International Journal of Automation and Computing*, vol. 16, no. 4, pp. 543–552, 2019. DOI: 10.1007/s11633-016-0966-6.

[7] H. L. Ren, G. D. Zong, H. R. Karimi. Asynchronous finite-time filtering of networked switched systems and its application: An event-driven method. *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 1, pp. 391–402, 2019. DOI: 10.1109/TCSI.2018.2857771.

[8] Q. Zhou, S. Y. Zhao, H. Y. Li, R. Q. Lu, C. W. Wu. Adaptive neural network tracking control for robotic manipulators with dead zone. *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 12, pp. 3611–3620, 2019. DOI: 10.1109/TNNLS.2018.2869375.

[9] B. D. O. Anderson, M. B. Ye. Recent advances in the modelling and analysis of opinion dynamics on influence networks. *International Journal of Automation and Computing*, vol. 16, no. 2, pp. 129–149, 2019. DOI: 10.1007/s11633-019-1169-8.

[10] X. D. Zhao, L. X. Zhang, P. Shi, M. Liu. Stability of switched positive linear systems with average dwell time switching. *Automatica*, vol. 48, no. 6, pp. 1132–1137, 2012. DOI: 10.1016/j.automatica.2012.03.008.

[11] X. W. Liu, C. Y. Dang. Stability analysis of positive switched linear systems with delays. *IEEE Transactions on Automatic Control*, vol. 56, no. 7, pp. 1684–1690, 2011. DOI: 10.1109/TAC.2011.2122710.

[12] E. Fornasini, M. E. Valcher. Linear copositive Lyapunov functions for continuous-time positive switched systems. *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1933–1937, 2010. DOI: 10.1109/TAC.2010.2049918.

[13] F. Blanchini, P. Colaneri, M. E. Valcher. Co-positive Lyapunov functions for the stabilization of positive switched systems. *IEEE Transactions on Automatic Control*, vol. 57, no. 12, pp. 3038–3050, 2012. DOI: 10.1109/TAC.2012.2199169.

[14] O. Mason, R. Shorten. On linear copositive Lyapunov functions and the stability of switched positive linear systems. *IEEE Transactions on Automatic Control*, vol. 52, no. 7, pp. 1346–1349, 2007. DOI: 10.1109/TAC.2007.900857.

[15] A. Zappavigna, P. Colaneri, J. C. Geromel, R. Shorten. Dwell time analysis for continuous-time switched linear positive systems. In *Proceedings of American Control Conference*, IEEE, Baltimore, USA, pp. 6256-6261, 2010. DOI: 10.1109/ACC.2010.5531524.

[16] P. Colaneri, R. H. Middleton, Z. Y. Chen, D. Caporale, F. Blanchini. Convexity of the cost functional in an optimal control problem for a class of positive switched systems. *Automatica*, vol. 50, no. 4, pp. 1227–1234, 2014. DOI: 10.1016/j.automatica.2014.02.025.

[17] E. Fornasini, M. E. Valcher. Stability and stabilizability of special classes of discrete-time positive switched systems. In *Proceedings of American Control Conference*, IEEE, San Francisco, USA, pp. 2619–2624, 2011. DOI: 10.1109/ACC.2011.5990765.

[18] R. Dorf, M. Farren, C. Phillips. Adaptive sampling frequency for sampled-data control systems. *IRE Transactions on Automatic Control*, vol. 7, no. 1, pp. 38–47, 1962. DOI: 10.1109/TAC.1962.1105415.

[19] J. Lunze, D. Lehmann. A state-feedback approach to event-based control. *Automatica*, vol. 46, no. 1, pp. 211–215, 2010. DOI: 10.1016/j.automatica.2009.10.035.

[20] R. Postoyan, P. Tabuada, D. Nešić, A. Anta. A framework for the event-triggered stabilization of nonlinear systems. *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 982–996, 2015. DOI: 10.1109/TAC.2014.2363603.

[21] L. T. Xing, C. Y. Wen, Z. T. Liu, H. Y. Su, J. P. Cai. Event-triggered adaptive control for a class of uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071–2076, 2017. DOI: 10.1109/TAC.2016.2594204.

[22] W. P. M. H. Heemels, M. C. F. Donkers, A. R. Teel. Periodic event-triggered control for linear systems. *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 847–861, 2013. DOI: 10.1109/TAC.2012.2220443.

[23] T. F. Li, J. Fu. Event-triggered control of switched linear systems. *Journal of the Franklin Institute*, vol. 354, no. 15, pp. 6451–6462, 2017. DOI: 10.1016/j.jfranklin.2017.05.018.

[24] Y. W. Qi, Z. Cao, X. L. Li. Decentralized event-triggered H_∞ control for switched systems with network communication delay. *Journal of the Franklin Institute*, vol. 356, no. 3, pp. 1424–1445, 2019. DOI: 10.1016/j.jfranklin.2018.12.008.

[25] Y. Y. Yin, Z. L. Lin, Y. Q. Liu, K. L. Teo. Event-triggered constrained control of positive systems with input saturation. *International Journal of Robust and Nonlinear Control*, vol. 28, no. 11, pp. 3532–3542, 2018. DOI: 10.1002/rnc.4097.

[26] L. Y. Liu, J. F. Zhang, Y. Shao, X. J. Deng. Event-triggered control of positive switched systems based on linear programming. *IET Control Theory & Applications*, vol. 14, no. 1, pp. 145–155, 2020. DOI: 10.1049/iet-cta.2019.0606.

[27] H. J. Wang, P. Shi, C. C. Lim, Q. Q. Xue. Event-triggered control for networked Markovian jump systems. *International Journal of Robust and Nonlinear Control*, vol. 25, no. 17, pp. 3422–3438, 2015. DOI: 10.1002/rnc.3273.

[28] A. Selivanov, E. Fridman. Event-triggered H_∞ control: A switching approach. *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 3221–3226, 2016. DOI: 10.1109/TAC.2015.2508286.

[29] H. Ma, H. Y. Li, R. Q. Lu, T. W. Huang. Adaptive event-triggered control for a class of nonlinear systems with periodic disturbances. *Science China Information Sciences*, vol. 63, no. 5, Article number 150212, 2020. DOI: 10.1007/s11432-019-2680-1.

[30] T. S. Hu, Z. L. Lin, B. M. Chen. Analysis and design for discrete-time linear systems subject to actuator saturation. *Systems & Control Letters*, vol. 45, no. 2, pp. 97–112, 2002. DOI: 10.1016/S0167-6911(01)00168-2.

[31] Z. D. Wang, B. Shen, X. S. Liu. H_∞ filtering with randomly occurring sensor saturations and missing measurements. *Automatica*, vol. 48, no. 3, pp. 556–562, 2012. DOI: 10.1016/j.automatica.2012.01.008.

[32] J. Wang, J. Zhao. Stabilisation of switched positive systems with actuator saturation. *IET Control Theory & Applications*, vol. 10, no. 6, pp. 717–723, 2016. DOI: 10.1049/iet-cta.2015.0064.

[33] J. F. Zhang, T. Raïssi. Saturation control of switched nonlinear systems. *Nonlinear Analysis*: *Hybrid Systems*, vol. 32, pp. 320–336, 2019. DOI: 10.1016/j.nahs.2019.01.005.

[34] G. W. Zhang, P. Yang, J. Wang, J. J. Sun, Y. Zhang. Integrated observer-based fixed-time control with backstepping method for exoskeleton robot. *International Journal of Automation and Computing*, vol. 17, no. 1, pp. 71–82, 2020. DOI: 10.1007/s11633-019-1201-z.

[35] X. M. Sun, W. Wang, G. P. Liu, J. Zhao. Stability analysis for linear switched systems with time-varying delay. *IEEE Transactions on Systems*, *Man*, *and Cybernetics – Part B*: *Cybernetics*, vol. 38, no. 2, pp. 528–533, 2008. DOI: 10.1109/TSMCB.2007.912078.

[36] Z. G. Feng, H. Y. Zhang, H. P. Du, Z. Y. Jiang. Admissibilisation of singular interval type-2 Takagi-sugeno fuzzy systems with time delay. *IET Control Theory & Applications*, vol. 14, no. 8, pp. 1022–1032, 2020. DOI: 10.1049/iet-cta.2019.0791.

[37] D. W. Zhang, Q. L. Han, X. M. Zhang. Network-based modeling and proportional-integral control for direct-drive-wheel systems in wireless network environments. *IEEE Transactions on Cybernetics*, vol. 50, no. 6, pp. 2462–2474, 2020. DOI: 10.1109/TCYB.2019.2924450.

[38] E. Fridman, U. Shaked. An improved stabilization method for linear time-delay systems. *IEEE Transactions on Automatic Control*, vol. 47, no. 11, pp. 1931–1937, 2002. DOI: 10.1109/TAC.2002.804462.

[39] D. Yue, E. G. Tian, Z. D. Wang, J. Lam. Stabilization of systems with probabilistic interval input delays and its applications to networked control systems. *IEEE Transactions on Systems*, *Man*, *and Cybernetics – Part A*: *Systems and Humans*, vol. 39, no. 4, pp. 939–945, 2009. DOI: 10.1109/TSMCA.2009.2019875.

[40] M. Xiang, Z. R. Xiang. Stability, L_1-gain and control synthesis for positive switched systems with time-varying delay. *Nonlinear Analysis*: *Hybrid Systems*, vol. 9, pp. 9–17, 2013. DOI: 10.1016/j.nahs.2013.01.001.

[41] T. T. Liu, B. W. Wu, L. L. Liu, Y. E. Wang. Asynchronously finite-time control of discrete impulsive switched positive time-delay systems. *Journal of the Franklin Institute*, vol. 352, no. 10, pp. 4503–4514, 2015. DOI: 10.1016/j.jfranklin.2015.06.015.

[42] J. F. Zhang, X. D. Zhao, F. B. Zhu, Z. Z. Han. L_1/ℓ_1-gain analysis and synthesis of Markovian jump positive systems with time delay. *ISA Transactions*, vol. 63, pp. 93–102, 2016. DOI: 10.1016/j.isatra.2016.03.015.

[43] W. Elloumi, A. Benzaouia, M. Chaabane. Delay-dependent stabilization conditions of controlled positive continuous-time systems. *International Journal of Automation and Computing*, vol. 11, no. 6, pp. 653–660, 2014. DOI: 10.1007/s11633-014-0816-3.

[44] J. P. Hespanha, A. S. Morse. Stability of switched systems with average dwell-time. In *Proceedings of the 38th IEEE Conference on Decision and Control*, IEEE, Phoenix, USA, pp. 2655–2660, 1999. DOI: 10.1109/CDC.1999.831330.

[45] X. M. Zhang, Q. L. Han, X. H. Ge, D. R. Ding, L. Ding, D. Yue, C. Peng. Networked control systems: A survey of trends and techniques. *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 1–17, 2020. DOI: 10.1109/JAS.2019.1911651.

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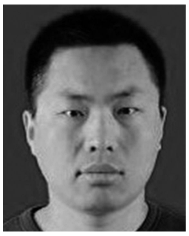

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