Original Paper

MEASUREMENT AND CONTROL

Measurement and Control
2020, Vol. 53(9-10) 2129–2136
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DOI: 10.1177/0020294020970221
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Event-triggered guaranteed cost control of time-varying delayed fuzzy systems with limited communication

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Abstract

Modern network applications place higher demands on its controller, especially for those with time-varying delays and limited communication capacity. For such cases, the fuzzy system has already become an advanced and powerful tool to deal with the control problem in consideration of a guaranteed cost performance. In this paper, we introduce the event-triggered mechanism with quantization effect to the controller, which proves to be more effective in terms of the information transmission. We adopt the classical Lyapunov approach to find the sufficient conditions for the controller and we illustrate the effectiveness of the controller with a numerical simulation.

Keywords

Event-triggered control, fuzzy systems, guaranteed cost controller, time-varying delay, limited communication capacity

Date received: 22 June 2020; accepted: 12 October 2020

Introduction

Fuzzy systems have been proven to be an advanced and powerful tool in modelling and controlling of nonlinear and complex systems in a series of robotic applications,[1–3] such as robot manipulator,[4] unmanned autonomous mobile[5] and rehabilitation robot.[6] In particular, much effort has been made towards the classical type fuzzy control systems,[7] which turns out to be a compelling tool for representing nonlinear dynamics.[8,9] As a result, fruitful analytical and synthetical methodologies have been proposed. General examples can be found as filtering problems, stability problems, synchronization problems and so on.[10–12] Moreover, the control strategies based on network have been discussed targeting at fuzzy systems relying on the substantial progress of network research. These works are mainly based on sample-data control framework. Meanwhile, some network constraints have also been taken into account, such as data packet dropout, bandwidth limitation and network communication delay.[13–15] More precisely, in Ma et al.[14] the discrete-time information exchange is proposed instead of continuous-time approaches to decrease the information exchanges under networked communication environment. Moreover, in Guan et al.[15] and Zhang and Han,[16] the limited communication capacity is discussed with the corresponding effective control methods. These constraints are always inevitable and may lead to system performance degradation or even system divergence, which make it critical and sensible to specify the corresponding effect during the design procedure.

On another active research area, the so-called event-triggered strategy has attracted rapidly growing attention for control systems. In comparison to the traditional time-triggered methods according to the fixed-time instances, the event-triggered strategy is based on a prescribed triggering function monitoring the event thresholds.[16–18] For example, based on the sign function and backstepping design, a novel event-triggered strategy is studied for a class of uncertain non-linear systems with global finite-time controller.[19] Another event-based adaptive control approach is developed to deal with a class of uncertain nonlinear systems with unknown control direction and actuator failures.[20] In Pan et al.[21] an adaptive robust control approach with event-triggering mechanism is designed to handle the

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communication burden, physical limitation and measurement errors in uncertain constrained nonlinear systems. The effective finite-time control is studied in Pan and Sun[22] for vehicle active suspension systems with desired control performance. As such, this effective mechanism would bring considerable benefits, which can be obtained by decreasing the network burden and increase the signal transmission efficiency. Especially, additional advantages can be obtained in energy saving in some wireless network scenarios.[23,24] It is worth mentioning that although successful performance on event-triggered control systems have been addressed, the concerns on limited communication capacity issue with the event-triggered approaches are few. As is well known, there is no perfect capacity in the digital communication channels. This would lead certain conservatism when modeling these controller or sensor information transmissions. As a result, it is reasonable and necessary to investigate a more practical communication strategy considering the communication capacity limitations. However, to our best knowledge, the event-triggered control problem for delayed T–S fuzzy systems with (a) guaranteed cost performance and (b) limited communication capacity is still remaining unresolved.

Provoked by the aforementioned discussions, we aim at solving the guaranteed cost control problem of time-varying delayed T–S fuzzy systems based on event-triggered strategy with limited communication capacity. Compared with the most of the existing literature, our novelties include three points: (1) To deal with the limited communication capacity issue, an event-triggered strategy with transmission quantization is investigated for a time-varying delayed T–S fuzzy system while the desired guaranteed cost performance can be satisfied. (2) By adopting a Lyapunov–Krasovskii function, the delay-dependent control criteria are derived, and the corresponding fuzzy controller is designed with the aid of linear matrix inequality (LMI). (3) The established theoretical results are further illustrated with a numerical simulation case study.

The organization of the paper is as follows: In section ‘ Preliminaries and problem formulation’, some preliminaries are presented, and the control problem is established. Section ‘ Main results’ presents the theoretical conditions for the proposed control scheme. In section ‘ Illustrative case study’, the proposed control scheme is discussed in a simulated case study. In section ‘Conclusion’, the conclusion with some future perspectives is reported.

Notation: The following standard notations are utilized throughout the paper: (1) $\mathbb{R}^n$: n dimensional Euclidean space. (2)$A \succ 0$: A is positive definite and vice versa. (3) $\mathbb{R}^{m\times n}$: the set of $m\times n$ real matrices. (4) $*$: the ellipsis terms in symmetry matrices.

Preliminaries and problem formulation

System model

Consider the following time-varying delayed T–S fuzzy system based on IF–THEN rules:

System Rule i:

IF

ϑ_1 is $\mathcal{M}_{i1}$, ϑ_2 is $\mathcal{M}_{i2}$, ... and ϑ_p is $\mathcal{M}_{ip}$,

THEN

$$\begin{cases} \dot{x}(t)| = A_i x(t) + A_{di} x(t-d(t)) + B_i u(t), \\ x(t)| = \delta(t), \end{cases} \quad t \in [-\bar{d}, 0]$$

where ϑ_j are premise variables, $j = 1, 2, \ldots, p$, $\mathcal{M}_{ij}$ are the fuzzy sets, $i = 1, 2, \ldots, r$ and r represents the number of IF-THEN rules, A_i, A_{di} and B_i are constant system matrices, $x(t)$ and $u(t)$ are system state and control input, respectively. In addition,$\bar{d}$ is a known constant, and $d(t)$ is the time-varying delay satisfying $0 \leqslant d(t) \leqslant \bar{d}$.

As a result, the fuzzy system could be given by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\vartheta(t))[A_i x(t) + A_{di} x(t-d(t)) + B_i u(t)],$$

where $\vartheta(t) = [\vartheta_1, \vartheta_2, \ldots, \vartheta_p]$ and

$$\mu_i(\vartheta(t)) = \prod_{j=1}^{p} \mathcal{M}_{ij}(\vartheta_j(t)),$$
$$\sum_{i=1}^{r} h_i(\vartheta(t))| = 1,$$
$$h_i(\vartheta(t)) = \frac{\mu_i(\vartheta(t))}{\sum_{i=1}^{r} \mu_i(\vartheta(t))},$$

with $\mathcal{M}_{ij}(\vartheta_j(t))$ being the grade of membership of $\vartheta_j(t)$.

Fuzzy event-triggered controller

Under the networked communications, the sampler of the system is supposed to be time-driven by sampling sequence: $0 = t_0 < t_1 < \cdots < t_k < \cdots$, and $t_{k+1} - t_k = \tau$ as $t \to \infty$, while the actuator and the controller are event-driven with zero-order hold (ZOH).

Considering the limited network environment, the event-triggering strategy is proposed for networked controller design. The control input only updates when the following event-triggering function (equation (1)) can be satisfied.

$$t_{k+1} h = \begin{matrix} t_k h + \min\limits_{l \geqslant 1}\{lh | e_k^\top(t) W_1 e_k(t) \\ > \varepsilon x^\top(t_k h + lh) W_2 x(t_k h + lh)\} \end{matrix} \quad (1)$$

where $t_k h$ is the latest triggering instant, $0 < \varepsilon < 1$ is the threshold, $W_1 > 0$ and $W_2 > 0$ are weighting matrices and $e_k(t) = x(t_k h + lh) - x(t_k h)$.

Furthermore, the following quantizer (equation (2)) is considered for the limited communication work bandwidth:

$$\Gamma = \left\{ w_k = \mu^k w_0, k = 0, \pm 1, \pm 2, \ldots \right\} \cup^{\{0\}}, w_0 > 0, \tag{2}$$

where w_k is the quantization density, $\mu \in [0, 1]$ and $q(\cdot) : \mathbb{R} \to \Gamma$ is defined as follows:

$$q(x(t_k h)) = \begin{cases} w_k, & \text{if} \frac{1}{1+\kappa} w_k < x(t_k h) \leqslant \frac{1}{1-\kappa} w_k, \\ 0, & \text{if} x(t_k h) = 0, \\ -q(-x(t_k h)), & \text{if} x(t_k h) < 0, \end{cases} \tag{3}$$

where $\kappa = \frac{1-\mu}{1+\mu}$ denotes sector bound.[25] The quantization density for the quantizer (equation (3)) is defined as $\frac{-2}{\ln \mu}$. Then, it follows that

$$q(x(t_k h)) = (I + \Delta) x(t_k h), \Delta \in [-\kappa, \kappa]. \tag{4}$$

Remark 1. For the networked control scheme in most systems, there always exists a certain limited communication capacity. An effective method to deal with the limited communication capacity issue is the adoption of the quantizer, which can considerably reduce the communication consumption with desired accuracy of information. In particular, the logarithmic quantizer can well achieve the signal quantization and is widely studied in voluminous literature.

As a result, the corresponding fuzzy controller can be defined as below:

Controller Rule i:

IF

ϑ_1 is $\mathcal{M}_{i1}$, and ϑ_2 is $\mathcal{M}_{i2}$, and … and ϑ_p is $\mathcal{M}_{ip}$,

THEN

$$u(t) = K_i q(x(t_k h)), t \in [t_k h, t_{k+1} h],$$

where K_i is the local gain matrix to be determined.

Similarly, it can be obtained that

$$u(t) = \sum_{j=1}^{r} h_j(\vartheta(t_k)) K_j q(x(t_k h)), \ t \in [t_k h, t_{k+1} h],$$

and we can rewrite the overall closed-loop system by parallel distributed compensation as follows[26]:

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\vartheta(t)) h_j(\vartheta(t_k)) [A_i x(t) \\ & + A_{di} x(t - d(t)) + B_i K_j q(x(t_k h))], \\ & t \in [t_k h, t_{k+1} h]. \end{aligned}$$

Control objective

With the aforementioned discussions, the cost performance index is introduced for the T–S fuzzy system with time-varying delays, such that the desired control performance can be achieved with the proposed control scheme. In this paper, the following guaranteed cost performance is given.

$$J = \int_0^{\infty} x^\top(t) M_1 x(t) + u^\top(t) M_2 u(t) \mathrm{d}t,$$

where $M_1 > 0$, $M_2 > 0$.

Consequently, the goal is to ensure that the system can be asymptotically stable while the cost performance $J \leqslant J^*$ holds, where J^* is the guaranteed cost.

Before proceeding, the following lemmas are useful for later results.

Lemma 1.[27] Given an arbitrary matrix $\mathcal{X} > 0$ and a scalar $\bar{\tau} > 0$, $\tau(t)$ satisfying $0 \leqslant \tau(t) \leqslant \bar{\tau}$, if the vector function $\dot{x}(t) : [-\bar{\tau}, 0] \to \mathbb{R}^n$ such that the involved integrations are well defined, then

$$-\bar{\tau} \int_{t-\bar{\tau}}^{t} \dot{x}^\top(s) \mathcal{X} \dot{x}(s) \mathrm{d}s \leqslant \eta^\top(t) \mathcal{Y} \eta(t),$$

where

$$\eta(t) = [x^\top(t), x^\top(t - \tau(t)), x^\top(t - \bar{\tau})]^\top,$$
$$\mathcal{Y} = \begin{bmatrix} -\mathcal{X} & \mathcal{X} & 0 \\ * & -2\mathcal{X} & \mathcal{X} \\ * & * & -\mathcal{X} \end{bmatrix}.$$

Lemma 2.[28] Let $\mathcal{X}^\top = \mathcal{L}$, $\mathcal{H}$ and $\mathcal{E}$ correspondingly be the real matrices with appropriate dimensions. $\mathcal{F}(t)$ satisfies that $\mathcal{F}^\top(t) \mathcal{F}(t) \leqslant I$. Then it holds that $\mathcal{X} + \mathcal{H}\mathcal{F}\mathcal{E} + \mathcal{E}^\top \mathcal{F}^\top \mathcal{H}^\top \prec 0$ if and only if there exists a scalar $\varepsilon > 0$ for $\mathcal{L} + \varepsilon^{-1} \mathcal{H}\mathcal{H}^\top + \varepsilon \mathcal{E}^\top \mathcal{E} \prec 0$, or equivalently

$$\begin{bmatrix} \mathcal{X} & \mathcal{H} & \varepsilon \mathcal{E}^\top \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} \prec 0.$$

Main results

Following the preliminaries, we present the derivation of the sufficient conditions of the proposed controller. We introduce the design procedure of the controller hereafter.

Theorem 1. For parameters $\bar{d}, h$, the closed-loop fuzzy system could realize the guaranteed cost performance with designed controller gains, if for $i = 1, \ldots, p$, and $i < j \leqslant p$, matrices $P > 0$, $Q_k > 0$, $R_k > 0, W_k > 0$ $(k = 1, 2)$ exist, such that it holds that $\Theta_{ij} < 0$ where

$$\Theta_{ij} = \begin{bmatrix} \Theta_{ij1} & \Theta_{ij2} \\ * & \Theta_{ij3} \end{bmatrix},$$

$$\Theta_{ij1} = \begin{bmatrix} 2PA_i + Q_1 + Q_2 - R_1 - R_2 - M_1 + M_2 & PA_{di} + R_2 & 0 \\ * & -2R_2 & R_2 \\ * & * & -Q_1 - R_2 \end{bmatrix},$$

$$\Theta_{ij2} = \begin{bmatrix} PB_iK_j + R_1 & 0 & PB_iK_j - M_2 & hA_i^\top & \bar{d}A_i^\top & PB_iK_j & 0 \\ 0 & 0 & 0 & hA_{di}^\top & \bar{d}A_{di}^\top & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{ij3} = \begin{bmatrix} -2R_1 + \varepsilon W_2 & R_1 & 0 & hK_j^\top B_i^\top & \bar{d}K_j^\top B_i^\top & 0 & \epsilon I \\ * & -Q_2 - R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -W_1 + M_2 & -hK_j^\top B_i^\top & -\bar{d}K_j^\top B_i^\top & 0 & -\epsilon I \\ * & * & * & -R_1^{-1} & 0 & hB_iK_j & 0 \\ * & * & * & * & -R_2^{-1} & \bar{d}B_iK_j & 0 \\ * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & -\epsilon I \end{bmatrix}.$$

Proof. Firstly, applying the virtual delay approach, one has

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k)) [A_i x(t) + A_{di}x(t - d(t)) + B_iK_j(I + \Delta) (x(t - \tau(t)) - e_k(t))],$$

where $\tau(t) = t - t_k h - lh$, $0 \leqslant \tau(t) \leqslant h$, $\dot{\tau}(t) = 1$.

Secondly, the Lyapunov–Krasovskii function is constructed:

$$V(t) = \sum_{i=1}^{3} V_i(t),$$

Where

$$V_1(t) = x^\top(t)Px(t),$$

$$V_2(t) = \int_{t-h}^{t} x^\top(s)Q_1x(s)\mathrm{d}s + \int_{t-\bar{d}}^{t} x^\top(s)Q_2x(s)\mathrm{d}s,$$

$$V_3(t) = h\int_{-h}^{0}\int_{t+\delta}^{t} \dot{x}^\top(s)R_1\dot{x}(s)\mathrm{d}s\mathrm{d}\delta + \bar{d}\int_{-\bar{d}}^{0}\int_{t+\delta}^{t} \dot{x}^\top(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\delta.$$

Then, it can be obtained that

$$\begin{aligned} \dot{V}_1(t) &= 2x^\top(t)P\dot{x}(t) \\ &= 2x^\top(t)P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k))[A_i x(t) \\ &\quad + A_{di}x(t - d(t)) + B_iK_j(I + \Delta)(x(t - \tau(t)) - e_k(t))] \\ &= 2x^\top(t)P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k))A_i x(t) \\ &\quad + 2x^\top(t)P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k))A_{di}x(t - d(t)) \\ &\quad + 2x^\top(t)P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k))B_iK_j(I+\Delta)x(t-\tau(t)) \\ &\quad - 2x^\top(t)P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t))h_j(\vartheta(t_k))B_iK_j(I + \Delta)e_k(t), \end{aligned}$$

and

$$\dot{V}_2(t) = x^\top(t)Q_1x(t) - x^\top(t - h)Q_1x(t - h) + x^\top(t)Q_2x(t) - x^\top\left(t - \bar{d}\right)Q_2x\left(t - \bar{d}\right),$$

and

$$\dot{V}_3(t) = h^2\dot{x}^\top(t)R_1\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^\top(\varphi)R_1\dot{x}(\varphi)d\varphi + \bar{d}^2\dot{x}^\top(t)R_2\dot{x}(t) - \bar{d}\int_{t-d}^{t} \dot{x}^\top(\varphi)R_2\dot{x}(\varphi)d\varphi.$$

To determine the sign of each part of $\dot{V}_3(t)$, based on Lemma 1, it can be derived that

$$-h\int_{t-h}^{t} \dot{x}^\top(s)R_1\dot{x}(s)\mathrm{d}s \leqslant -\begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-h) \end{bmatrix}^\top \begin{bmatrix} R_1 & -R_1 & 0 \\ * & 2R_1 & -R_1 \\ * & * & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-h) \end{bmatrix},$$

and similarly, that

$$-\bar{d}\int_{t-\bar{d}}^{t} \dot{x}^\top(s)R_1\dot{x}(s)\mathrm{d}s \leqslant -\begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-h) \end{bmatrix}^\top \begin{bmatrix} R_1 & -R_1 & 0 \\ * & 2R_1 & -R_1 \\ * & * & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-h) \end{bmatrix}.$$

In addition, it can be derived that

$$\dot{x}^{\top}(t)(R_1 + R_2)\dot{x}(t) = \eta^{\top}(t)\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\vartheta(t))h_j(\vartheta(t_k)) \begin{bmatrix} A_i \\ A_{di} \\ 0 \\ B_iK_j(I+\Delta) \\ 0 \\ -B_iK_j(I+\Delta) \end{bmatrix}(R_1 + R_2) \times \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\vartheta(t))h_j(\vartheta(t_k)) \begin{bmatrix} A_i \\ A_{di} \\ 0 \\ B_iK_j(I+\Delta) \\ 0 \\ -B_iK_j(I+\Delta) \end{bmatrix}^{\top}\eta(t),$$

where

$$\eta(t) = [x^{\top}(t), x^{\top}(t-d(t)), x^{\top}(t-\bar{d}), x^{\top}(t-\tau(t)), x^{\top}(t-h), e_k^{\top}(t)]^{\top}.$$

With the event-triggering function, it follows that

$$\varepsilon x^{\top}(t-\tau(t))W_2x(t-\tau(t)) - e_k(t)^{\top}W_1e_k(t) > 0.$$

Basing on the above results, one has

$$\begin{aligned} &\dot{V}(t) + x^{\top}(t)M_1x(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\vartheta(t))h_j(\vartheta(t_k))(x(t-\tau(t)) - e_k(t))^{\top}(I+\Delta)^{\top}K_j^{\top}\times \\ &M_2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\vartheta(t))h_j(\vartheta(t_k))K_j(I+\Delta)(x(t-\tau(t)) - e_k(t)) \\ &+ \varepsilon x^{\top}(t-\tau(t))W_2x(t-\tau(t)) - e_k(t)^{\top}W_1e_k(t) \\ &\leqslant \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\vartheta(t))h_j(\vartheta(t_k)) \\ &\left(\eta^{\top}(t)\tilde{\Theta}_{ij}\eta^{\top}(t) + h^2\dot{x}^{\top}(t)R_1\dot{x}(t) + \bar{d}^2\dot{x}^{\top}(t)R_2\dot{x}(t)\right), \end{aligned}$$

Where

$$\tilde{\Theta}_{ij} = \begin{bmatrix} \tilde{\Theta}_{ij1} & \tilde{\Theta}_{ij2} \\ * & \tilde{\Theta}_{ij3} \end{bmatrix},$$

$$\tilde{\Theta}_{ij1} = \begin{bmatrix} 2PA_i + Q_1 + Q_2 - R_1 - R_2 - M_1 + M_2 & PA_{di} + R_2 \\ * & -2R_2 \end{bmatrix},$$

$$t\,\tilde{\Theta}_{ij2} = \begin{bmatrix} 0 & PB_iK_j(I+\Delta) + R_1 & 0 & -PB_iK_j(I+\Delta) - M_2 \\ R_2 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Theta}_{ij3} = \begin{bmatrix} -Q_1 - R_2 & 0 & 0 & 0 \\ * & -2R_1 + \varepsilon W_2 & R_1 & 0 \\ * & * & -Q_2 - R_1 & 0 \\ * & * & * & -W_1 + M_2 \end{bmatrix}.$$

Together with Lemma 2 and the Schur complement, it holds that

$$\dot{V}(t) + x^{\top}(t)M_1x(t) + u^{\top}(t)M_2u(t) < 0,$$

when $\Theta_{ij} < 0$ is satisfied. Finally, it can be deduced that

$$J = \lim_{t\to\infty}\sum_{k=0}^{T}\int_{t_kh}^{t_{k+1}h} x^{\top}(t)M_1x(t) + u^{\top}(t)M_2u(t)dt,$$

such that $V(\infty) - V(0) \leqslant -J$, which implies that

$$\begin{aligned} J \leqslant J^* &= V(0) \\ &= \delta^{\top}(0)P\delta(0) + \int_{-h}^{0}\delta^{\top}(s)Q_1\delta(s)\mathrm{d}st^{0}_{-h} \\ &+ \int_{-\bar{d}}^{0}\delta^{\top}(s)Q_2\delta(s)\mathrm{d}s \\ &+ h\int_{-h}^{0}\int_{\delta}^{0}\dot{\delta}^{\top}(s)R_1\dot{\delta}(s)\mathrm{d}s\mathrm{d}\delta + \bar{d}\int_{-\bar{d}}^{0}\int_{\delta}^{0}\dot{\delta}^{\top}(s)R_2\dot{\delta}(s)\mathrm{d}s\mathrm{d}\delta \end{aligned}$$

as $t \to \infty$ and thus completes the proof.

Theorem 2. For parameters $\bar{d}, h$, the closed-loop fuzzy system could realize the guaranteed cost performance if for $i = 1, \ldots, p$, and $i < j \leqslant p$, matrices $\tilde{P} > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$, $\tilde{W}_k > 0$ $(k = 1, 2)$ and $\tilde{K}_j$ exist, such that it holds that $\bar{\Theta}_{ij} < 0$ where

$$\bar{\Theta}_{ij} = \begin{bmatrix} \bar{\Theta}_{ij1} & \bar{\Theta}_{ij2} \\ * & \bar{\Theta}_{ij3} \end{bmatrix},$$

$$\bar{\Theta}_{ij1} = \begin{bmatrix} 2A_i\tilde{P} + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2 - \tilde{M}_1 + \tilde{M}_2 & A_{di}\tilde{P} + \tilde{R}_2 & 0 & B_i\tilde{K}_j + \tilde{R}_1 \\ * & -2\tilde{R}_2 & \tilde{R}_2 & 0 \\ * & * & -\tilde{Q}_1 - \tilde{R}_2 & 0 \\ * & * & * & -2\tilde{R}_1 + \varepsilon\tilde{W}_2 \end{bmatrix},$$

$$\bar{\Theta}_{ij2} = \begin{bmatrix} 0 & B_i\tilde{K}_j - \tilde{M}_2 & h\tilde{P}A_i^{\top} & \bar{d}\tilde{P}A_i^{\top} & B_i\tilde{K}_j & 0 \\ 0 & 0 & h\tilde{P}A_{di}^{\top} & \bar{d}\tilde{P}A_{di}^{\top} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{R}_1 & 0 & h\tilde{K}_j^{\top}B_i^{\top} & \bar{d}\tilde{K}_j^{\top}B_i^{\top} & 0 & \epsilon\tilde{P} \end{bmatrix},$$

$$\bar{\Theta}_{ij3} = \begin{bmatrix} -\tilde{Q}_2 - \tilde{R}_1 & 0 & 0 & 0 & 0 & 0 \\ * & -\tilde{W}_1 + \tilde{M}_2 & -h\tilde{K}_j^{\top}B_i^{\top} & -\bar{d}\tilde{K}_j^{\top}B_i^{\top} & 0 & -\epsilon\tilde{P} \\ * & * & \tilde{R}_1 - 2\tilde{P} & 0 & hB_i\tilde{K}_j & 0 \\ * & * & * & \tilde{R}_2 - 2\tilde{P} & \bar{d}B_i\tilde{K}_j & 0 \\ * & * & * & * & \epsilon I - 2\epsilon\tilde{P} & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix},$$

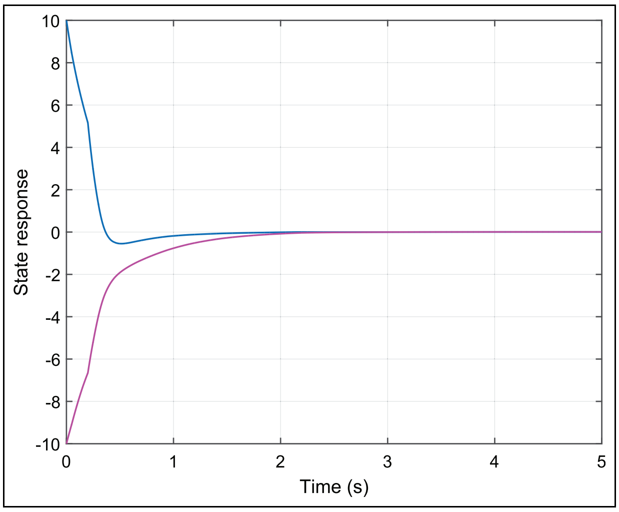

Figure 1. The simulated state response of the closed-loop system.

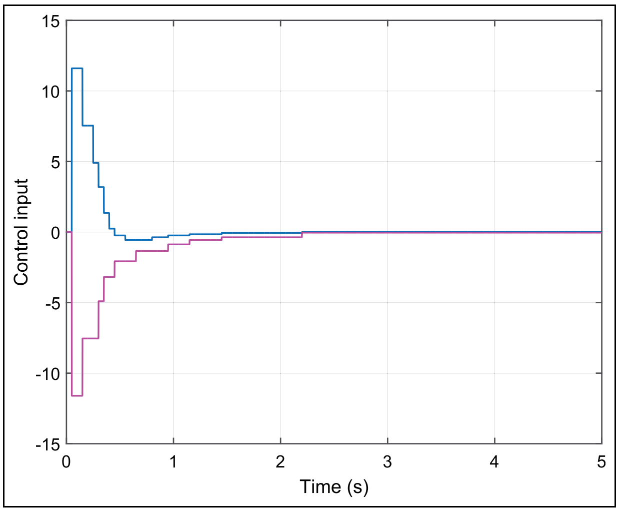

Figure 2. The control input in simulation.

and the expected controller gains could be computed with:

$$K_j = \tilde{K}_j \tilde{P}^{-1}.$$

Proof. Let $\tilde{P} = P^{-1}$, $\tilde{Q}_k = P^{-1}Q_kP^{-1}$, $\tilde{R}_k = P^{-1}R_kP^{-1}$, $\tilde{W}_k = P^{-1}W_kP^{-1}$ (k = 1,2), and perform matrix congruent transformation. Then, the theorem could be derived directly from Theorem 1.

Illustrative case study

We present a numerical simulation in this section, which could be seen as an abstraction of the controlled complex non-linear system, to verify the effectiveness of the controller.

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\vartheta(t))[A_i x(t) + A_{di}x(t-d(t)) + B_i u(t)],$$

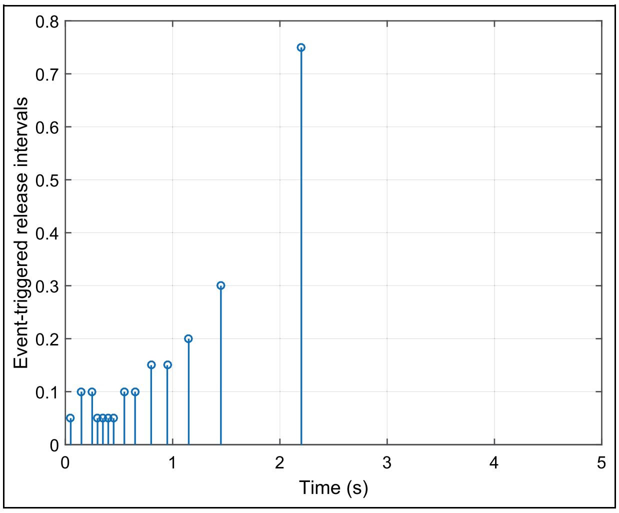

Figure 3. The event triggering signals in simulation.

where $h_1(\vartheta(t)) = \frac{1}{1+\exp(-2x_1(t))}$, $h_2(\vartheta(t)) = 1 - h_1(\vartheta(t))$, and

$$A_1 = \begin{bmatrix} -1.2 & 1 \\ 1 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2 & 0.8 \\ 0.8 & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}.$$

In the simulation, it is assumed that $d(t) = 0.2\sin(t)$, $h = 0.05$ and $M_1 = 0.5I$, $M_2 = 1$. The parameters of the quantizer is set as $\mu = 0.65$ and $w_0 = 100$. As a result, the desired fuzzy controller gains can be calculated based on the Theorem 2 as follows:

$$K_1 = [-0.1481 \quad -0.2400],$$
$$K_2 = [-0.1570 \quad -0.2240].$$

By setting the above initial values and parameters as $[10, 10]^\top$, the resulting closed-loop system state response and the event-triggered signals can be seen from Figures 1 to 3, respectively. We could draw the conclusion from the results, that the proposed controller can stabilize the system with guaranteed cost. The guaranteed cost performance is shown in Figure 4, from which one can see that the guaranteed cost can be satisfied. Moreover, Figure 5 depicts the control input comparison results of our proposed event-triggered scheme and the common time-triggered scheme. It can be seen that the control input trajectory of the event-triggered approach is almost same with the time-triggered one by desired control performance. However, the event-triggered strategy can considerably decrease the signal transmissions with distinguishing advantages.

Conclusion

In this work, we implemented a fuzzy controller for complex, non-linear systems in consideration of the time-vary delay and the limited communication capacity. We adopted the event-triggered mechanism in the design procedure to deal with the limited

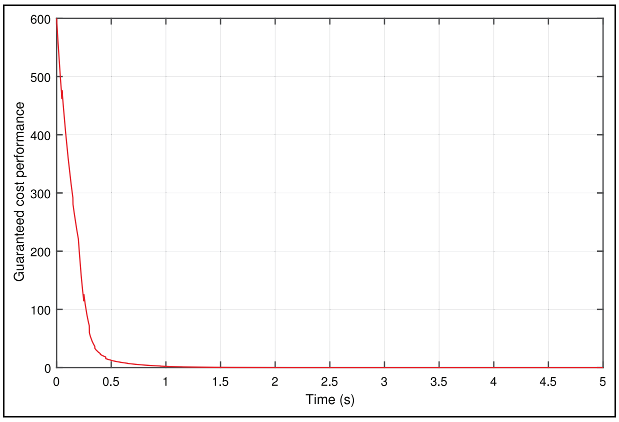

Figure 4. The guaranteed cost performance.

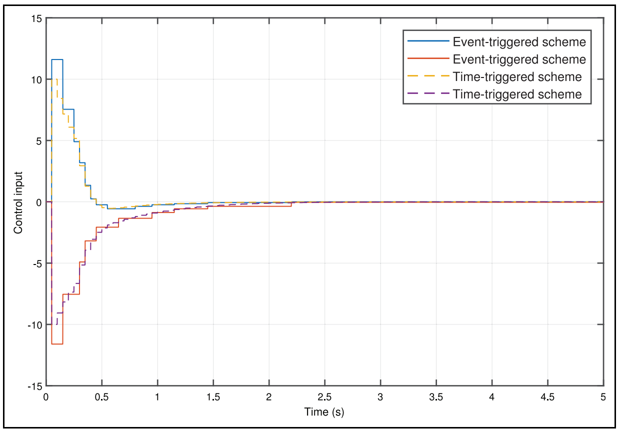

Figure 5. Comparison of the event-triggered and the time-triggered schemes.

communication situation. By constructing the Lyapunov–Krasovskii function, the fuzzy controller is designed with parallel distributed compensation strategy for ensuring both asymptotic stability and guaranteed cost performance. Furthermore, a numerical simulation case study is performed for showing the correctness of the proposed approach. In our future researches, we would extend our current results to the cases with Type II fuzzy systems, which are more complex but with fuzzier modeling ability.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is partly supported by the National Key Research and Development Program of China (2017 YFB1300200, 2017YFB1300203, 2020AAA0105900), the National Natural Science Foundation of China (Grant no. 61702516, 51705515, 61933001, 62003059), the Joint Research Fund between the National Natural Science Foundation of China (NSFC) and Shen Zhen (Grant no.U1713201) and the Research Fund from Science and Technology on Underwater Vehicle Technology Laboratory (No.6142215190103) and the China Postdoctoral Science Foundation (Grant No. 2020M673136).

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