A Variable-Parameter-Model-Based Feedforward Compensation Method for Tracking Control

Dailin Zhang, Zining Wang, and Masayoshi Tomizuka, *Fellow, IEEE*

***Abstract*—Base on the accurate inverse of a system, the feedforward compensation method can compensate the tracking error of a linear system dramatically. However, many control systems have complex dynamics and their accurate inverses are difficult to obtain. In the paper, a variable parameter model is proposed to describe a system and a multi-step adaptive seeking approach is used to obtain its parameters in real time. Based on the proposed model, a variable-parameter-model-based feedforward compensation method is proposed, and a disturbance observer is used to overcome the influence of the model uncertainty. Theoretical analysis and simulation results show that the variable-parameter-model-based feedforward compensation method can obtain better performance than the traditional feedforward compensation.**

***Index Terms*—Disturbance observer, feedforward compensation, iterative learning control, parameter identification, system model.**

I. Introduction

GENERALLY, feedforward compensation is used to reduce the tracking error of a control system, and then the control accuracy can be improved [1]–[4]. The inverse of a system is commonly obtained from the system model, and the compensation values of the feedforward compensator are calculated according to the inverse and the planned trajectory.

Many methods were proposed to identify system models and achieve their inverses. A simple method is to use the parameter identification to obtain a system model, and then its inverse can be obtained by inversing the system model. Least squares (LS) and recursive least squares (RLS) [5], [6] are effective to identify the transfer function of a linear system. In [1], the system model of a permanent magnet linear motor (PMLM) was obtained by the RLS identification method and an adaptive feedforward component based on the inverse dominant linear model was used to reduce the tracking error. In [7], an inverse Preisach model was used for feedforward compensation of hysteresis compliance and the model was identified from drive experiments. In [8], measured data in every task was used for system identification and the feedforward controller could be updated after each task. In [9], a multi-model adaptive preview control using a set of augmented systems was proposed to enhance the feedforward performance.

The accurate model is difficult to obtain in a real control system with complex dynamics, so the effectiveness of feedforward compensation may be limited. Some methods are able to avoid the difficulties to build accurate models. For example, ILC (iterative learning control) adjusts its control signal to a control system in every iteration by using feedback information from previous iterations, which can improve the control accuracy without knowing the accurate system model [10]–[13]. In effect, ILC can find the perfect inverse of a system for a repetitive trajectory, which makes ILC be able to compensate for the disturbances optimally [14]–[16]. ILC, however, performs badly in the systems with uncertain factors. For example, the change of reference trajectory will result in varying disturbances, and changing the moment of inertia will result in the change of system model. Both uncertainties above will result in performance deterioration.

The paper proposes to use a variable parameter model to describe a system with uncertain factors and achieve the inverse of the system in real time. The basic idea is to calculate the parameters of the variable parameter model by solving the strict equality between variable parameter model output and actual output.

The contribution of the paper includes: Firstly, a variable parameter model with constraints is proposed and a multi-step adaptive seeking approach is used to obtain the parameters of the proposed model in real time. The multi-step adaptive seeking approach can obtain the optimal parameter in every control period by adaptive control approaches [17], [18]. Secondly, a variable-parameter-model-based feedforward compensation method is proposed. The proposed variable-parameter-model-based feedforward compensation method can reduce the tracking error effectively. Thirdly, a disturbance observer is used to compensate for the model uncertainty, which helps to reduce the influence of disturbances.

The remaining of the paper is organized as follows. Section II gives the problem formulation, Sections III and IV propose the variable parameter model and variable-parameter-model-based feedforward compensation method, respectively. Section V presents the experimental results. Section VI gives the conclusion and future works.

Manuscript received January 11, 2020; revised February 20, 2020; accepted March 9, 2020. This work was supported in part by the National Natural Science Foundation of China (51775215, 51535004). Recommended by Associate Editor Chengdong Li. *(Corresponding author: Dailin Zhang and Zining Wang.)*

Citation: D. L. Zhang, Z. N. Wang, and M. Tomizuka, "A variable-parameter-model-based feedforward compensation method for tracking control," *IEEE/CAA J. Autom. Sinica*, vol. 7, no. 3, pp. 693–701, May 2020.

D. L. Zhang is with the School of Mechanical Science and Engineering, State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: mnizhang@mail.hust.edu.cn).

Z. N. Wang and M. Tomizuka are with the Department of Mechanical Engineering, University of California at Berkeley, Berkeley CA 94720 USA (e-mail: wangzining@berkeley.edu; tomizuka@berkeley.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JAS.2020.1003135

II. Problem Formulation

A single input single output system is depicted in Fig. 1. The plant is described as a fixed transfer function $P(z)$, and the disturbances at time k are $d(k)$. The control configuration consists of a feedback controller $C(z)$ and a feedforward controller $F(z)$. With the input $r(k)$, the output is $y(k)$, and then the tracking error $e(k)$ can be calculated. The closed-loop system output can be calculated by

$$y(k) = (u_{ff}(k)+d(k))\frac{P(z)}{1+P(z)C(z)} + r(k)\frac{P(z)C(z)}{1+P(z)C(z)} \tag{1}$$

where $u_{ff}(k)$ is the feedforward compensation value at time k.

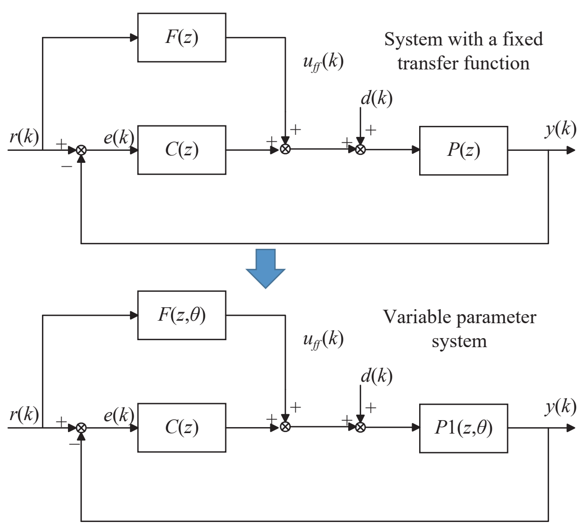


Fig. 1. A system with a fixed transfer function is changed to a variable parameter system.

The feedforward controller can be described by

$$u_{ff}(k) = r(k)F(z). \tag{2}$$

Substituting (2) into (1), we have

$$y(k) = d(k)\frac{P(z)}{1+P(z)C(z)} + r(k)\frac{P(z)(C(z)+F(z))}{1+P(z)C(z)}. \tag{3}$$

According to Stearns *et al*. [11], the ideal feedforward controller is

$$F(z) = P^{-1}(z). \tag{4}$$

If the disturbance is zero and will be considered in the following uncertainty portion, the obtained inverse of the system has three possible solutions.

Solution 1: Substituting (4) into (3), we have

$$y(k) = r(k). \tag{5}$$

From (5), it can be concluded that the tracking error $e(k) = r(k) - y(k)$ will be reduced to zero with the feedforward controller.

Solution 2: If the ideal feedforward controller is not available because the number of zeros is bigger than that of poles, the feedforward controller can be described by

$$u_{ff}(k) = r(k+d)(z^{-d}P^{-1}(z)). \tag{6}$$

The feedforward controller becomes

$$F(z) = z^{-d}P^{-1}(z). \tag{7}$$

And the reference input $r(k)$ becomes $r(k+d)$ by previewing $r(k)$ d time steps. Obviously, $F(z)$ is realizable and the feedforward compensation effect remains unchanged.

Solution 3: If $P(z)$ has an unstable zero, zero phase error tracking control (ZPETC) can be used to reduce the tracking error [19]. In this method, the feedforward tracking control is designed as

$$F(z) = G_{\text{ZPETC}}(z) = \frac{z^l B_c^u(z) A_c(z^{-1})}{B_c^a(z^{-1})[B_c^u(1)]^2} \tag{8}$$

where $G_{\text{ZPETC}}(z)$ is the zero phase error tracking controller, $A_c(z^{-1})$ is the denominator of $P(z)$. $B_c^a(z^{-1})$ and $B_c^u(z)$ are two parts of the numerator, which contain cancelable and un-cancelable zeros, respectively. $B_c^u(1)$ is a scale.

Equation (8) does not produce the perfect inverse, but can help reduce the tracking error effectively. Because the variable parameter model is the main point, the paper simply focuses on (5) and (7).

When the plant has realtime changing parameters and the system becomes a variable parameter one, $F(z)$ should follow the corresponding changes in order to ensure the last term in (3) be equal to one.

The variable parameter system is shown in Fig.1. The variable parameter plant is $P1(z,\theta)$ instead of $P(z)$ and the corresponding feedforward controller is $F(z,\theta)$, where θ is a variable parameter. For every value of θ, the ideal feedforward controller can be obtained according to (4), (7), and (8)

$$F(z,\theta) = P1^{-1}(z,\theta) \text{ or } z^{-d}P1^{-1}(z,\theta) \text{ or } G_{\text{ZPETC}}(z,\theta) \tag{9}$$

At every control period, the parameter θ should be identified and the corresponding variable parameter function is obtained. the identified function of $P1(z,\theta)$ is supposed to be $P_v(z,\theta)$, which is a variable parameter model shown in Fig. 2. The variable parameter model means the identified transfer function $P_v(z,\theta)$ will be changing with the parameter θ. $G_{\text{ZPETC}}(z,\theta)$ is the zero phase error tracking controller with variable parameter θ. And the uncertainty in the identification process will be resolved by the model uncertainty compensation based on disturbance observer in Section IV. For example, the disturbance $d(k)$ will be considered as an uncertainty portion in the paper.

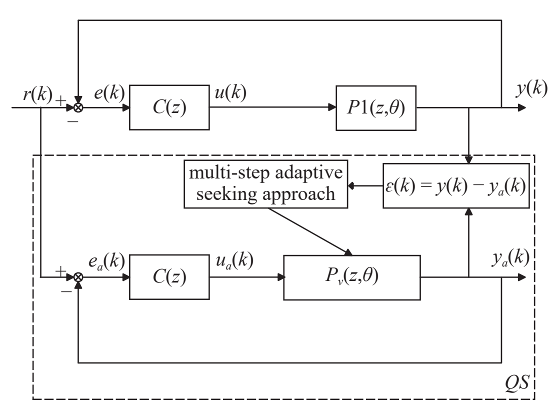


Fig. 2. variable parameter model.

Remark 1: If a variable parameter model $P_v(z,\theta)$, where $\theta = [\theta_1, \theta_2, \ldots, \theta_n]$ is a variable parameter and its the initial value is $\theta(0) = [1\,1\ldots 1]$, has the same input and output as $P1(z,\theta)$ at every discrete time k, $P_v(z,\theta)$ is the perfect description of $P1(z,\theta)$.

Therefore, if we can find a variable parameter model $P_v(z,\theta)$ meeting the requirements in Remark 1 in every sampling instant, the feedforward controller $F(z,\theta)$ will result in a very perfect tracking performance according to (4)–(9). In the next section, the variable parameter model will be proposed.

III. The Variable Parameter Model With Constraints

A variable parameter model $P_v(z,\theta)$ can have the same output as $P1(z,\theta)$ by tuning the variable parameter θ, but realtime performance to obtain the optimal parameters of $P_v(z,\theta)$ is very difficult to be guaranteed because $P_v(z,\theta)$ is a complex variable parameter system. This paper proposes a variable parameter model with constraints and a multi-step adaptive seeking approach to obtain the optimal parameters of $P_v(z,\theta)$ in real time.

The constraints of the variable parameter model are set to limit the variation range of parameters, which ensure the optimal parameters be achieved within a limited seeking range [20].

In this section, a structure combining a variable gain and an identified transfer function is used to simplify the variable parameter model. And the multi-step adaptive seeking approach computes the optimal parameters within the constrained range in real time.

A. Variable Parameter Model With Constraints

As shown in Fig. 2, the object under study is $P1(z,\theta)$. No disturbance is applied and the influence of disturbance will be discussed in the next section. Given a reference trajectory, we can get the output y. At the same time, a parallel identification system QS is set up with the variable parameter model $P_v(z,\theta)$. Except $P_v(z,\theta)$, the identification system QS has the same structure as the given system and the output is assumed to be $y_a(k)$. Then the identification error is

$$\varepsilon(k) = y(k) - y_a(k). \tag{10}$$

At every period, the parameters of $P_v(z,\theta)$ need to be sought because $P_v(z,\theta)$ is a variable parameter model.

The proposed multi-step adaptive seeking approach can iteratively run QS and tune its model parameters. The goal of the multi-step adaptive seeking approach is to obtain the optimal parameters of $P_v(z,\theta)$ to let y_a be equal to y, that is, to let every $\varepsilon(k)$ be equal to zero. Constraints of a variable parameter model will be used to accelerate the seeking process by restricting the seeking range of parameters.

Assumption 1: It is assumed that every variable parameter of the variable parameter model has an upper and lower bound, which is to reduce the seeking range and guarantee the system stability. In other words, it can be described by

$$\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$$

where $i = 1, 2, \ldots, n$, and $\underline{\theta}_i$ and $\bar{\theta}_i$ are upper and lower bounds of the ith parameter θ_i, respectively.

With Assumption 1, θ_i is within the range. If the actual parameter θ_i violates the constraints due to big parameter change or disturbances, θ_i is equal to $\bar{\theta}_i$ at the upper bound or $\underline{\theta}_i$ at the lower bound. In this case, the model uncertainty needs to be compensated by the disturbance observer in Section IV.

It is supposed that there are multiple steps of seeking process to achieve the optimal value θ. The optimal value of θ at every time k can be obtained by the seeking approach

$$\theta^*(k) = \arg\min_j J(\theta_{x,j}(k)) \tag{11}$$

where $\theta^*(k)$ is the optimal value at time k. At the jth step of seeking process, $\theta_{x,j}(k) = [\theta_{1,j}(k)\ \theta_{2,j}(k)\ \ldots\ \theta_{n,j}(k)]$, where $\theta_{n,j}(k)$ is the value of the nth parameter θ_n, is the value of the parameter θ at time k. $J(\theta_{x,j}(k))$ is the cost function of $\theta_{x,j}(k)$

$$J(\theta_{x,j}(k)) = \frac{1}{2}(\varepsilon_j(k))^2 \tag{12}$$

where $\varepsilon_j(k)$ is the identification error at the jth step of seeking process, respectively.

Considering the real-time performance and complexity, too many variable parameters are not practical. In the proposed variable parameter model, the identified transfer function $P(z)$ is used as a fixed plant and a variable gain is used to tune the output, so only one variable parameter is used in the variable parameter model.

The variable parameter model is described as

$$P_v(z,\theta) = P(z) f(\theta) \tag{13}$$

where $f(\theta)$ is a variable proportional gain, and θ includes only one variable parameter θ_1, that is, $f(\theta) = \theta_1$.

B. Multi-step Variable Parameter Seeking Approach

A multi-step variable parameter seeking approach is proposed to seek the optimal parameter $f(\theta)$ in (13). The goal is to adjust the variable proportional gain $f(\theta)$ by a multi-step adaptive seeking approach to let $y(k)$ equal $y_a(k)$ at every moment k. Here "multi-step" means the optimal parameter $\theta(k)$ is achieved through multiple calculations within one control period.

From Fig. 2, it can be seen that the given system and the parallel identification system QS have the same controller $C(k)$. With the same reference input, there exists

$$u(k) = u_a(k). \tag{14}$$

The outputs of the two systems are

$$y(k) = u(k) P1(z,\theta) \tag{15}$$

$$y_a(k) = u_a(k) P_v(z,\theta). \tag{16}$$

Substituting (14) into (16), it can be concluded that $P_v(z,\theta)$ will be equal to $P1(z)$ only if $y(k)$ equals $y_a(k)$ at every time k.

$f(\theta)$ can be described by

$$f(\theta(k)) = \theta(k) = \theta_1(k), \quad k = 1, 2, \ldots, n. \tag{17}$$

At time k, the actual output is $y(k)$, and the seeking approach is to run the variable parameter system QS in parallel to adaptively find the optimal value of $\theta(k)$. At the jth step, $\theta(k)$ is supposed to be equal to $\theta_{1,j}(k)$, $y_a(k)$ can be obtained at the step, then $\varepsilon_j(k)$ is obtained. The optimal value of $\theta(k)$ can be obtained by

$$\theta^*(k) = \arg\min_j J(\theta_{1,j}(k)) \tag{18}$$

where $\theta^*(k)$ is the optimal value, $\varepsilon_j(k)$ is the jth identification error.

It must be noticed that $\theta^*(k)$ is obtained in one control period by multiple steps of seeking process. The traditional adaptive method runs only one step and the seeking result is $\theta_{1,j}(k)$, but the variable parameter seeking approach obtains the optimal parameter $\theta^*(k)$ by (18). So the variable parameter seeking approach works better than the traditional adaptive method. Considering the realtime performance, limited steps will be used in the multi-step adaptive seeking approach. The detailed procedure for the multi-step variable parameter seeking approach is shown in Algorithm 1.

Algorithm 1 Multi-step variable parameter seeking approach

Initialize: θ and its change range of θ
For $k = 1 : n$
  Run the system to sample $r(k)$ and $y(k)$
  $j = 1$
  Repeat
    Use the update law in (16) to adjust $\theta_{1,j}(k)$
    Calculate $\varepsilon_j(k)$ and $\xi(k)$ at every step j
    Choose the minimum $J(\theta_{1,j}(k))$
    $j = j + 1$
  Until $\varepsilon_j(k) < \delta$ or $j = N_{\max}$
  Set $\theta^*(k)$ according to (15)
End for
Obtain $f(\theta)$ by collecting all $\theta^*(k)$

The jth parameter update law is

$$\dot{\xi}_j(k) = \kappa_\theta(k) \times \varepsilon_j(k)\dot{\varepsilon}_j(k) \tag{19}$$

where $\xi(k)$ is the increment of $\theta_{1,j}(k)$ at the jth step of seeking process and $\xi(k)$ is equal to $\dot{\xi}_1(k) + \dot{\xi}_2(k) + \cdots + \dot{\xi}_j(k) + \cdots$. Here $|\xi(k)|$ is set to be smaller than a positive constant φ to avoid an excessive parameter change within one control period, otherwise, the seeking process ends. $\kappa_\theta(k)$ is a constant and $\dot{\varepsilon}_j(k)$ is the differential of $\varepsilon_j(k)$. In fact, one can avoid the difficulty in choosing a best $\kappa_\theta(k)$ if $\varepsilon_j(k)\dot{\varepsilon}_j(k)$ is set for a product of a constant and a sign function of $\varepsilon_j(k)\dot{\varepsilon}_j(k)$.

$\theta_{1,j}(k)$ and $\varepsilon_j(k)$ are recorded at every step and $\theta^*(k)$ is obtained by selecting $\theta_{1,j}(k)$ when $\varepsilon_j(k)$ is smaller enough.

In order to reduce the number of steps for obtaining the optimal value $\theta^*(k)$, the last parameter value $\theta(k-1)$ can be used as the present initial value, because the system does not change dramatically during a very small period. m values within $[\underline{\theta}_i, \bar{\theta}_i]$ are tested to seek an optimal value $\theta(k-1) + \theta_{\text{opt}}(k)$. Then the initial value can be set as $\theta(k-1) + \theta_{\text{opt}}(k)$ and the parameter $\theta(k)$ can be obtained by

$$\theta(k) = \theta(k-1) + \theta_{\text{opt}}(k) + \xi(k),\ k > 2 \tag{20}$$

In order to avoid local minima, $\theta_{\text{opt}}(k)$ can be replaced by random values every h steps in Algorithm 1 under the conditions that (18) is met and $\theta(k-1) + \theta_{\text{opt}}(k)$ is within $[\underline{\theta}_i, \bar{\theta}_i]$.

After n steps of running, the optimal value $\theta^*(k)$ can be obtained by (20) if $|\varepsilon_j(k)| < \delta$ where δ is a very small positive constant or when j reaches maximum $N_{\max}$. Then the equivalent variable parameter model in Fig. 2 can be obtained.

It can be seen that the variable parameter model in (13) and its parameter seeking approach are simple and practical.

IV. Variable-Parameter-Model-Based Feedforward Compensation Method

With the built variable parameter model, the feedforward compensation in Fig. 1 becomes variable-parameter-model-based feedforward compensation.

A. Variable-Parameter-Model-Based Feedforward Compensation Method

The proposed variable-parameter-model-based feedforward compensation method is shown in Fig. 3. Compared with Fig. 1, $P(z)$ and $F(z)$ are replaced by $P_v(z,\theta)$ and $F(z,\theta)$.

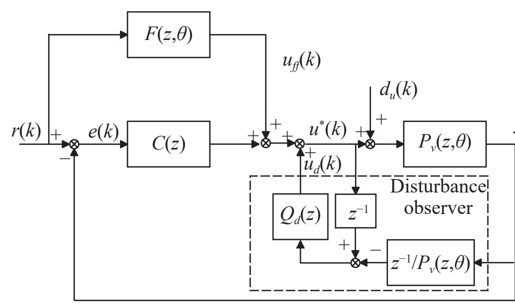


Fig. 3. The variable-parameter-model-based feedforward compensation.

With the variable parameter model in (13), the feedforward controller can be obtained by

$$F(z,\theta) = F(z) f^{-1}(\theta). \tag{21}$$

The basic variable-parameter-model-based feedforward compensation method uses $F(z,\theta)$ in (21) to compensate the tracking error.

Because $f(\theta)$ consists of only one parameter θ_1 where $\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$, the corresponding feedforward compensation value can be obtained easily by the inverse of the parameter. To avoid the influence of high-frequency disturbances, a low-pass filter $Q(z)$ is used in (13) and (21), then $f(\theta)$ can be rewritten as

$$f(\theta) = f(\theta_1)Q(z). \tag{22}$$

With the proposed variable parameter system shown in Fig. 1, the fixed plant $P(z)$ becomes $P1(z,\theta)$. Under the conditions in Remark 1, $P1(z,\theta)$ can be identified as $P_v(z,\theta)$ by the method in Section III. If the disturbance $d(k)$ is zero and will be considered in the next section, (1) becomes

$$y(k) = u_{ff}(k)\frac{P_v(z,\theta)}{1 + P_v(z,\theta)C(z)} + r(k)\frac{P_v(z,\theta)C(z)}{1 + P_v(z,\theta)C(z)}. \tag{23}$$

From Fig. 1, we have $u_{ff}(k) = r(k) \cdot F(z,\theta)$. Then substituting it into (23), we have

$$\begin{aligned} y(k) &= \frac{r(k)F(z,\theta)P_v(z,\theta)}{1+P_v(z,\theta)C(z)} + r(k)\frac{P_v(z,\theta)C(z)}{1+P_v(z,\theta)C(z)} \\ &= \frac{r(k)F(z)f^{-1}(\theta)P(z)f(\theta)}{1+P_v(z,\theta)C(z)} + r(k)\frac{P_v(z,\theta)C(z)}{1+P_v(z,\theta)C(z)} \\ &= \frac{r(k)F(z)P(z)}{1+P_v(z,\theta)C(z)} + r(k)\frac{P_v(z,\theta)C(z)}{1+P_v(z,\theta)C(z)}. \end{aligned} \tag{24}$$

If $F(z)$ has Solution 1, $F(z)P(z)$ will be one and $y(k)$ is equal to $r(k)$.

If $F(z)$ has Solution 2, substituting (6) into (23) and let $r(k+d) = r(k)z^d$, we have the same result.

If $F(z)$ has Solution 3, $u_{ff}(k)P_v(z,\theta)$ will be equal to one and $y(k)$ is equal to $r(k)$.

Therefore, the tracking error is theoretically equal to zero.

However, the variable parameter model has difficulties in obtaining the accurate model under the following conditions: 1) large disturbances cause too big parameter deviation; 2) parameter value exceeds the upper or lower bounds.

B. Model Uncertainty Compensation Based on Disturbance Observer

To filter disturbances and improve the control accuracy, a disturbance observer [21], [22] can be applied. Based on the variable parameter model, the designed disturbance observer is shown in Fig. 3. It can be seen that the variable parameter model $P_v(z,\theta)$ has replaced $P1(z,\theta)$ in the disturbance observer. If $d(k)$ in (1) is not zero, $d(k)$ is supposed to become $d_u(k)$ with $P1(z,\theta)$ replaced by $P_v(z,\theta)$. $d_u(k)$ will be compensated by the disturbance observer.

In Fig. 3, $Q_d(z)$ is a low-pass filter. Before running the disturbance observer, the variable parameter model $P_v(z,\theta)$ should be sought and the feedforward controller $F(z)$ can be obtained by (21).

It is supposed that the model uncertainty is ΔP, which is calculated by

$$\Delta P = P1(z,\theta) - P_v(z,\theta). \tag{25}$$

From Fig.3, the output of the disturbance observer can be obtained by

$$\begin{aligned} u_d(k) &= (u^*(k)z^{-1} - y(k)\frac{z^{-1}}{P_v(z,\theta)})Q_d(z) \\ &= (u^*(k-1) - y(k)\frac{1}{z\cdot P_v(z,\theta)}F(z,\theta))Q_d(z) \end{aligned} \tag{26}$$

where $u^*(k)$ is the sum of the outputs of the feedback controller $C(z)$ and feedforward controller $F(z)$.

From the schematic diagram of the variable-parameter-model-based feedforward compensation shown in Fig. 3, $u^*(k)$ can be calculated by

$$u^*(k) = e(k)C(z) + r(k)F(z,\theta) + u_d(k). \tag{27}$$

Remark 2: With a disturbance observer helping to observe the model uncertainty, $e(k)$ is close to $e_a(k)$ if the disturbance is fully compensated, then the variable parameter model is able to acquire the optimal estimation of the system $P1(z,\theta)$.

Proof: Suppose the transfer function from the disturbance $d_u(k)$ to the tracking error is $H'_{DE}(z)$ in the proposed variable-parameter-model-based feedforward compensation in Fig. 3. It is noticed that the transfer function is $H_{DE}(z)$ in the traditional feedforward compensation method. According to Yu and Tomizuka [21], the following relationship exists if the feedforward controller replaces the ILC in the paper

$$H'_{DE}(z) \approx H_{DE}(z)(1 - z^{-1}Q_d(z)). \tag{28}$$

Because $Q_d(z)$ is a low-pass filter, $Q_d(z)$ is equal to one and $H'_{DE}(z)$ can be considered as zero when the frequency is low. According to (26) the model uncertainty at low frequency can be estimated by the disturbance observer and then compensated. So the tracking error can be further reduced compared with the traditional feedforward compensation method.

From Fig. 2, it can be concluded that $e(k)$ is close to $e_a(k)$ if the disturbance observer is applied, so it exists $u(k) \approx u_a(k)$ and $y(k) \approx y_a(k)$, which implies that the variable parameter model is able to acquire the optimal estimation of $P1(z,\theta)$. For a traditional disturbance observer used in the paper, a more accurate model will help achieve more accurate estimation of uncertainty. So the variable parameter model works better with the realtime update of the model parameter.

In summary, there are two cases for the variable parameter model. On the one hand, the variable parameter model is theoretically equal to $P1(z,\theta)$ under the conditions that the disturbance is small and θ can be sought within its bounds. On the other hand, if the above conditions cannot be met, the model uncertainty resulted from the variable parameter model can be obtained by the disturbance observer. Therefore, the feedforward controller can reduce the tracking error to the greatest degree in the two cases.

C. Advantage Analysis Compared With the Fixed Parameters Feedforward Compensation Method

Considering the model uncertainty ΔP, $P1(z,\theta)$ in every control period can be estimated as

$$P1(z,\theta) = P_v(z,\theta) + \Delta P. \tag{29}$$

In Fig. 3, it is supposed that ΔP is caused by $d_u(k)$. The output can be obtained by

$$\begin{aligned} y(k) = r(k)&\frac{P_v(z,\theta)F(z,\theta) + P_v(z,\theta)C(z)}{1+P_v(z,\theta)C(z)} \\ &+ d_u(k)\frac{P_v(z,\theta)}{1+P_v(z,\theta)C(z)}. \end{aligned} \tag{30}$$

With the variable parameter model, if $F(z,\theta)$ has Solution 1 or Solution 2, then (30) becomes

$$y(k) = r(k) + d_u(k)\frac{P_v(z,\theta)}{1+P_v(z,\theta)C(z)}. \tag{31}$$

The tracking error can be calculated by

$$\begin{aligned} e(k) &= r(k) - y(k) \\ &= -d_u(k)\frac{P_v(z,\theta)}{1+P_v(z,\theta)C(z)}. \end{aligned} \tag{32}$$

If $F(z,\theta)$ has Solution 3, $f^{-1}(\theta)$ in $F(z,\theta)$ and $f(\theta)$ in $P_v(z,\theta)$ can be canceled, so the feedforward controller can be considered as a single ZPETC and the tracking error can be reduced. According to the theory of ZPETC, the tracking error can also be estimated by (32) if the frequency of $r(k)$ meets the requirement of ZPETC.

From (32), it can be concluded that the tracking error is determined by $d_u(k)$.

With the variable-parameter-model-based feedforward compensation, the model uncertainty can be reduced to be a very low level by using the variable parameter model, which implies

$$d_u(k) \approx 0. \tag{33}$$

When the parameters is over the setting constraints, the disturbance-observer-based compensation will help achieve the optimal estimation of $P1(z,\theta)$ according to Remark 2 and model uncertainty is compensated by $u_d(k)$ obtained by disturbance observer. According to (29), $e(k)$ can be controlled to be a very small value

$$e(k) \approx 0. \tag{34}$$

With the fixed parameters model, the model uncertainty is big if disturbances exist or a model has variable parameters. In this case, the tracking error is determined by $d_u(k)$ and can be estimated by (29).

In summary, the above theoretical analysis shows that the variable-parameter-model-based feedforward compensation is able to achieve a smaller tracking error than the fixed parameters model.

V. Illustrative Examples

Servo systems are often used in robots or numerical control machine tools. Two categories of application examples in servo systems are investigated: the one is that $P(z)$ has a varying proportional gain; the other is that a servo system is influenced by complex disturbances.

Generally, a servo system can be described by [23]

$$P(z) = \frac{b_0 z^m + b_1 z^{m-1} + \cdots + b_m}{a_0 z^n + a_1 z^{n-1} + \cdots + a_n}, \quad m \le n. \tag{35}$$

If $P(z)$ is a fixed transfer function, a_i and b_j are constants where $i = 0,1,\ldots,n$ and $j = 0,1,\ldots,m$.

If the servo system has variable parameters, a_i and b_j are varying coefficients and the transfer function will be $P_v(z,\theta)$, where θ is a vector composed of a_i and b_j. But it is time-consuming to identify the parameters and very difficult to achieve the accurate values of the vector θ. According to Section III, the variable model in (13) can be used if the conditions in Remark 1 are met. As a result, the variable parameter model is described by the product of $P(z)$ and $f(\theta)$, and the model uncertainty is considered as $d_u(k)$ which can be compensated by the disturbance observer in Fig. 3.

The plant of a servo system $P(z)$ is set for a fixed transfer function

$$P(z) = \frac{0.7979e^{-5}z + 0.7957e^{-5}}{z^2 - 1.992z + 0.992}. \tag{36}$$

The feedback controller is a proportional-derivative (PD) controller, whose transfer function is

$$C(z) = 4 + \frac{10(z-1)}{T_s \times z}. \tag{37}$$

where T_s is the sampling period and set for 0.004 s.

The reference input is a sine curve

$$r(k) = 1.4\sin(kT_s). \tag{38}$$

This section verifies the effectiveness of the variable-parameter-model-based feedforward compensation method by analyzing the performance of the variable parameter model and comparing with ILC and a traditional feedforward compensation method. And the advantages of the variable-parameter-model-based feedforward compensation are also illustrated by examples with model uncertainty.

A. Performance Analysis of the Variable Parameter Model

To analyze the performance of the variable parameter model the identification system in Fig. 2 is used to estimate the variable parameter $f(\theta)$. Because $f(\theta)$ in (13) has only one parameter θ_1, the variable parameter model becomes

$$P_v(z,\theta) = P(z)\theta_1 \tag{39}$$

where θ_1 is a variable scalar coefficient and $\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$.

In order to evaluate the performance of the proposed variable parameter model, the variable parameter θ_1 is set with different changing rules and the torque disturbance d is set as a complex function. The changing rules of the variable parameter θ_1 are given as follows:

1) a line

$$\theta_1 = 1.0 + 0.01kT_s. \tag{40}$$

2) a sine

$$\theta_1 = 1.0 + 0.1\sin(kT_s). \tag{41}$$

3) a sine over the set seeking range

$$\theta_1 = 1.0 + 0.4\sin(kT_s). \tag{42}$$

4) the sine in (41) with the following disturbance added

$$d(k) = 0.01\cos(0.2y(k-1)). \tag{43}$$

Within the set seeking range $\theta_1 \in [0.8, 1.4]$, the variable parameter θ_1 can be obtained in every time k by the seeking method in Section III. By using (40)–(43), the variable parameters are obtained and the results are shown in Figs. 4(a)–(d), respectively. In the figure, "identified" and "actual" represent the identified and actual parameters, respectively.

From the results, it can be seen that the proposed seeking method can obtain the values of the variable parameter accurately. But there is model uncertainty when the parameter θ_1 is out of the set seeking range. For example, in Fig. 4(c) θ_1 can be obtained when its value is within the seeking range, but not when its value is over the seeking range, which implies that the model uncertainty exists. In this case, the parameter over the seeking range is set as the boundary value. The disturbance in (43) is complex because it changes with the change of the output $y(k-1)$. And its result in Fig. 4(d) shows that the identified variable parameter is not fully in accordance with (41), especially at 6.2 s, 18.8 s and 31.4 s. The disturbance causes the inconsistency of the parameters, but when the identified variable parameter is used in Fig. 5(d) the excellent compensation effectiveness verifies that the identified variable parameter matches the real system model. So it can be concluded that the variable parameter is able to be identified when the disturbance is compensated by a disturbance observer.

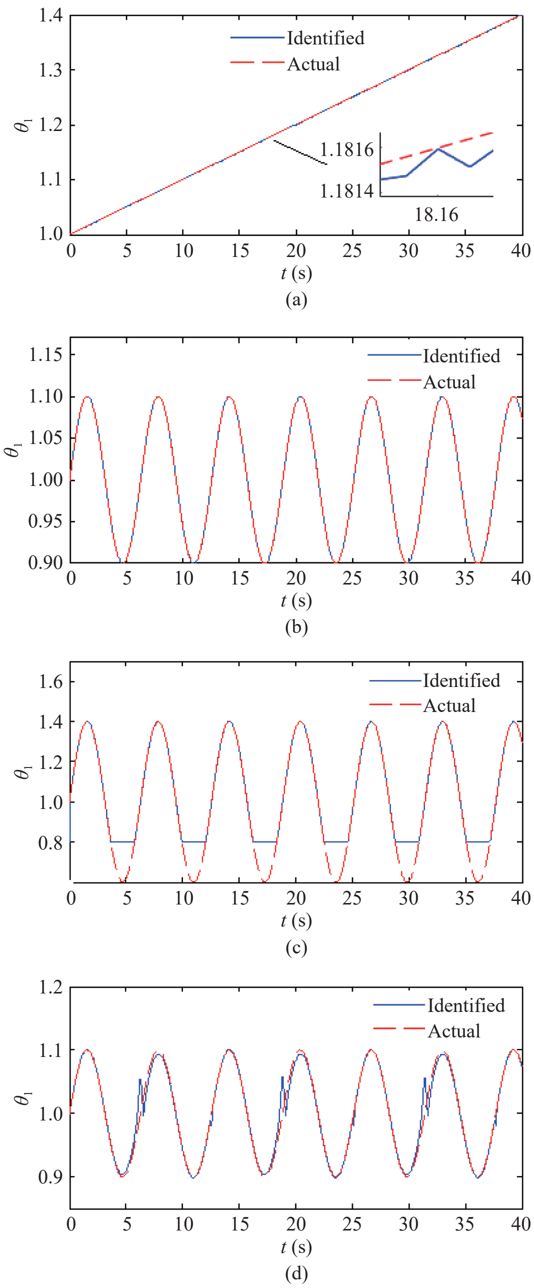


Fig. 4. Identification of θ_1 in the variable parameter model. Within the seeking range, the parameters under different changing rules including (a) a line, (b) a sine curve, (c) a sine curve over the set seeking range, and (d) a sine curve with the disturbance can be obtained.

B. Comparison With a Traditional Feedforward Compensation Method

By using the variable parameters sought in Section V-A, the variable-parameter-model-based feedforward compensation method is used to compensate the tracking error. A traditional feedforward controller shown in Fig. 1 is used to compare the compensation performance. In Fig. 3, $Q_d(z)$ and $Q(z)$ are designed as the same second ordered Butterworth low-pass filter whose transfer function is shown as follows:

$$Q_d(z) = Q(z) = \frac{0.2929z^2 + 0.5858z + 0.2929}{z^2 - 1.301e^{-16}z + 0.1716}. \qquad (44)$$

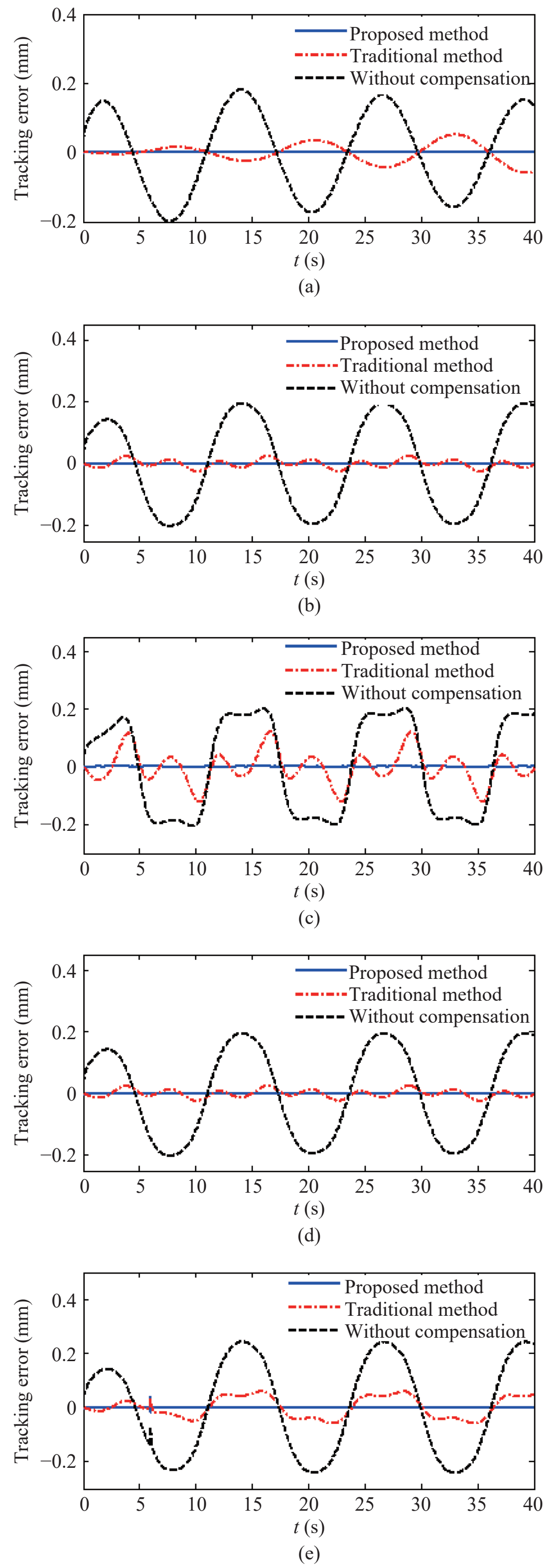


Fig. 5. The tracking errors. Different conditions, that is, (a) a line, (b) a sine curve, (c) constant 1 with the disturbance and (d) a sine over the set seeking range, (e) with a varying transfer function, are considered.

The compensation results are shown in Figs. 5(a)–(d), respectively. "proposed method", "traditional method" and "without compensation" represent the tracking errors obtained by the proposed variable-parameter-model-based feedforward compensation method, traditional feedforward method and without compensation, respectively.

From the results, it can be concluded that the variable-parameter-model-based feedforward compensation method is much better than the traditional feedforward compensation. Just like the theoretical analysis, the traditional feedforward compensation can compensate the tracking error, but the inaccurate plant model limits its effectiveness. Fig. 5 shows that the proposed variable-parameter-model-based feedforward compensation method has much lower tracking errors than the traditional feedforward compensation. Fig. 5(c) and Fig. 5(d) show that the tracking error can be compensated by the variable-parameter-model-based feedforward compensation with disturbance observer, even if the model uncertainty exists, i.e., the variable parameter is over the set seeking range in Fig. 5(c), and there are the disturbance inputs in Fig. 5(d). Fig. 5(d) uses the obtained parameters in Fig. 4(d), and the result shows the tracking error can be compensated to a very small level, which implies that the disturbance can be overcome by the proposed variable-parameter-model-based feedforward compensation method.

In addition to the variable parameter θ_1, a more complex condition is considered in the paper. Under the condition that the variable parameter θ_1 is set with the rule in (41), at the 6th second, the plant of the servo system $P(z)$ in (36) is changed as follows:

$$P'(z) = \frac{0.85e^{-5}z + 0.4e^{-5}}{z^2 - 1.992z + 0.992}. \quad (45)$$

In this case, numerator polynomial coefficients in (35) are changed except for the varying parameter θ_1.

But $P(z)$ in the variable parameter model (39) remains unchanged. Fig. 5(e) shows that the tracking error by the traditional method becomes bigger after 6 s, but that by the proposed variable-parameter-model-based feedforward compensation method remains a small value. This result further verifies that the proposed method is effective to reduce the tracking error when the parameters of a system are varying.

VI. Conclusion and Future Works

In the paper, a variable-parameter-model-based feedforward compensation method is proposed to reduce the tracking error. Based on the built variable parameter model, the nonlinear plant is constructed as a variable parameter model with constraints. A multi-step adaptive seeking approach is used to obtain the parameter of the variable parameter model, and then the inverse of the system can be calculated by the variable parameter model. Finally, the proposed variable-parameter-model-based feedforward compensation method can compensate the tracking error to the greatest degree.

By an example of a servo system, the effectiveness of the variable-parameter-model-based feedforward compensation method is verified.

1) The proposed multi-step adaptive seeking method can obtain the variable parameter accurately;

2) The variable-parameter-model-based feedforward compensation method can achieve smaller tracking errors than a traditional feedforward compensation method, and the disturbance observer can help achieve good effectiveness even when the model uncertainty exists.

In the future, more real systems are expected to use the variable-parameter-model-based feedforward compensation method to reduce the tracking error.

References

[1] S. Zhao and K. K. Tan, "Adaptive feedforward compensation of force ripples in linear motors," *Control Engineering Practice*, vol. 13, no. 9, pp. 1081–1092, Sep. 2005.

[2] P. Van Den Braembussche, J. Swevers, H. Van Brussel, and P. Vanherck, "Accurate tracking control of linear synchronous motor machine tool axes," *Mechatronics*, vol. 6, no. 5, pp. 507–521, Aug. 1996.

[3] M. Heertjes, D. Hennekens, and M. Steinbuch, "MIMO feed-forward design in wafer scanners using a gradient approximation-based algorithm," *Control Engineering Practice*, vol. 18, no. 5, pp. 495–506, May 2010.

[4] Y. Jiang, Y. Zhu, K. M. Yang, C. X. Hu, and D. D. Yu, "A data-driven iterative decoupling feedforward control strategy with application to an ultraprecision motion stage," *IEEE Trans. Industrial Electronics*, vol. 62, no. 1, pp. 620–627, Jan. 2015.

[5] Y. Gu and R. Ding, "A least squares identification algorithm for a state space model with multi-state delays," *Applied Mathematics Letters*, vol. 26, no. 7, pp. 748–753, Jul. 2013.

[6] X. Y. Ma and F. Ding, "Recursive and iterative least squares parameter estimation algorithms for observability canonical state space systems," *J. Franklin Institute*, vol. 352, no. 1, pp. 248–258, Jan. 2015.

[7] M. Ruderman and T. Bertram, "Feed-forward compensation of hysteresis compliance using inverse Preisach model," *IFAC Proceedings*, vol. 42, no. 6, pp. 243–248, 2009.

[8] F. Boeren, T. Oomen, and M. Steinbuch, "Iterative motion feedforward tuning: a data-driven approach based on instrumental variable identification," *Control Engineering Practice*, vol. 37, pp. 11–19, Apr. 2015.

[9] D. Wang, F. C. Liao, and M. Tomizuka, "Adaptive preview control for piecewise discrete-time systems using multiple models," *Applied Mathematical Modelling*, vol. 40, no. 23–24, pp. 9932–9946, Dec. 2016.

[10] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, Jul. 2006.

[11] H. Stearns, F. Benjamin, and M. Tomizuka, "Iterative identification of feedforward controllers for iterative learning control," *IFAC Proceedings Volumes*, vol. 42, no. 16, pp. 203–208, 2009.

[12] W. J. Chen and M. Tomizuka, "Dual-stage iterative learning control for mimo mismatched system with application to robots with joint elasticity," *IEEE Trans. Control Systems Technology*, vol. 22, no. 4, pp. 1350–1361, Jul. 2014.

[13] M. M. G. Ardakani, S. Z. Khong, and B. Bernhardsson, "On the convergence of iterative learning control," *Automatica*, vol. 78, pp. 266–273, Apr. 2017.

[14] J. Bolder and T. Oomen, "Rational basis functions in iterative learning control-With experimental verification on a motion system," *IEEE Trans. Control Systems Technology*, vol. 23, no. 2, pp. 722–729, Mar. 2015.

[15] D. Y. Meng and K. L. Moore, "Robust iterative learning control for nonrepetitive uncertain systems," *IEEE Trans. Autom. Control*, vol. 62, no. 2, pp. 907–913, Feb. 2017.

[16] C. X. Hu, Z. P. Hu, Y. Zhu, Z. Wang, and S. Q. He, "Model-data driven learning adaptive robust control of precision mechatronic motion systems with comparative experiments," *IEEE Access*, vol. 6, pp. 78286–78296, Dec. 2018.

[17] M. C. Jorge, S. Víctor, and M. V. Javier, "Adaptive control schemes applied to a control moment gyroscope of 2 degrees of freedom," *Mechatronics*, vol. 57, pp. 73–85, Feb. 2019.

[18] M. S. Mahmoud and M. O. Oyedeji, "Adaptive and predictive control strategies for wind turbine systems: a survey," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 2, pp. 364–378, Mar. 2019.

[19] M. Tomizuka, "Zero phase error tracking algorithm for digital control,"

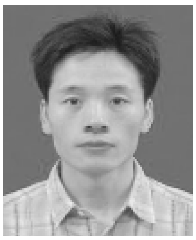
J. Dynamic Systems Measurements & *Control*, vol. 109, no. 1, pp. 65–68, Mar. 1987.

[20] M. Guay and D. Dochain, "A proportional-integral extremum-seeking controller design technique," *Automatica*, vol. 77, pp. 61–67, Mar. 2017.

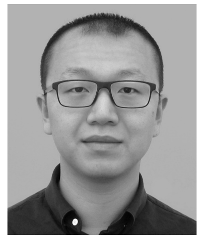
[21] S. W. Yu and M. Tomizuka, "Performance enhancement of iterative learning control system using disturbance observer," in *Proc. IEEE/ASME Int. Conf. on AIM*, pp. 987–992, 2009.

[22] C. Zhang, G. F. Ma, Y. C. Sun, and C. J. Li, "Observer-based prescribed performance attitude control for flexible spacecraft with actuator saturation," *ISA Trans.*, vol. 89, pp. 84–95, Jun. 2019.

[23] D. L. Zhang, Y. H. Niu, L. T. Sun, and M. Tomizuka, "A position-based friction error model and its application to parameter identification," *IEEE Access*, vol. 7, pp. 7759–7767, Jan. 2019.



Dailin Zhang received the bachelor and master degrees from Shandong University, in 2000 and 2002, respectively, and the Ph.D. degree from Huazhong University of Science and Technology, in 2007. Currently, he is an Associate Professor of the School of Mechanical Science and Engineering, Huazhong University of Science and Technology. From 2017 to 2019, he was also a Visiting Scholar at the Department of Mechanical Engineering, UC Berkeley, USA. His research interests include motion control and machine vision.



Zining Wang received the B.S. degree of engineering mechanics from Tsinghua University, in 2015. He is now pursuing the Ph.D. degree in mechanical engineering at University of California, Berkeley, CA, USA. His research interests include robot calibration, adaptive control, perception with deep learning, and sensor fusion.

Masayoshi Tomizuka (M'86–SM'95–F'97) received the B.S. and M.S. degrees from Keio University, Tokyo, Japan, in 1968 and 1970, respectively, and the Ph.D. degree from Massachusetts Institute of Technology, Cambridge, all in mechanical engineering. In 1974, he joined the Department of Mechanical Engineering, University of California, Berkeley, where he is currently the Cheryl and John Neerhout Jr. Distinguished Professor Chair. His research interests include optimal and adaptive control, digital control, signal processing, motion control, and control problems related to robotics, machining, manufacturing, and information storage devices and vehicles. From 1988 to 1993, he was a Technical Editor of the *American Society of Mechanical Engineers* (*ASME*), *Journal of Dynamic Systems, Measurement and Control* (*J-DSMC*). He has also been an Associate Editor of the *Journal of the International Federation of Automatic Control*, *Automatica*, and the *European Journal of Control*.