

Iterative Learning Control for Distributed Parameter Systems Based on Non-Collocated Sensors and Actuators

Jianxiang Zhang, Baotong Cui, Xisheng Dai, and Zhengxian Jiang

Abstract—In this paper, an open-loop PD-type iterative learning control (ILC) scheme is first proposed for two kinds of distributed parameter systems (DPSs) which are described by parabolic partial differential equations using non-collocated sensors and actuators. Then, a closed-loop PD-type ILC algorithm is extended to a class of distributed parameter systems with a non-collocated single sensor and m actuators when the initial states of the system exist some errors. Under some given assumptions, the convergence conditions of output errors for the systems can be obtained. Finally, one numerical example for a distributed parameter system with a single sensor and two actuators is presented to illustrate the effectiveness of the proposed ILC schemes.

Index Terms—Actuators, distributed parameter system, iterative learning control, PD-type ILC scheme, sensors.

I. INTRODUCTION

In practice, most systems can be described by a partial differential equation or a partial integral equation, referred to as distributed parameter systems. The states of distributed parameter systems are dependent on time and spatial position. Therefore, these systems are more suitable to describe system dynamics. At the same time, this has attracted many researchers to study the control and estimation of distributed parameter systems in a number of fields, most recently in [1]–[3]. Since the sensors and actuators are low-cost and low energy, the distributed parameter systems using sensors and actuators have been extensively studied by many specialists. Demetriou [4] considered a law for the guidance of a mobile collocated actuator/sensor for the enhanced control of spatially distributed

Manuscript received November 28, 2018; revised February 28, 2019, April 11, 2019; accepted May 30, 2019. This work was supported by National Natural Science Foundation of China (61807016) and Postgraduate Research and Practice Innovation Program of Jiangsu Province (KYCX18-1859). Recommended by Associate Editor Hongyi Li. (Corresponding author: Baotong Cui.)

Citation: J. X. Zhang, B. T. Cui, X. S. Dai, and Z. X. Jiang, “Iterative learning control for distributed parameter systems based on non-collocated sensors and actuators,” *IEEE/CAA J. Autom. Sinica*, vol. 7, no. 3, pp. 865–871, May 2020.

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Digital Object Identifier 10.1109/JAS.2019.1911663

uted processes. Accordingly, he suggested an algorithm to replace the full state information from a scalar multiple of the output measurement in finite horizon linear quadratic regulator control of DPSs in [5]. Meanwhile, Mu *et al.* [6] considered a scheme aimed at guiding the moving actuator/sensor pair for enhanced control and estimation of the distributed parameter systems. Jiang *et al.* [7] proposed an even-driven observer-based control for DPSs based on a mobile sensor and actuator.

Iterative learning control (ILC) is an intelligent control method which particularly suits systems working in a fixed time interval with a repetitive model. ILC aims to find proper learning control schemes of the controlled system for the actual output signal to track the given desired output signal over a finite interval time. At the same time, the constructed learning control sequences can converge to a desired control. An effective ILC algorithm can promote tracking accuracy by adjusting the system input signal according to error observations from every iteration even when the system has incomplete knowledge. Initially, ILC was proposed in 1984 by Arimoto *et al.* [8] that mainly involved a class of ILC algorithm for robots to obtain better control performance. Since then, ILC has been established as a separate field of control theory [9]–[14]. This methodology has been given consideration in various industrial applications, including industrial robots [15], health care systems [16], batch processes [17], and so on [18]. Nowadays, ILC is extensively employed in distributed parameter systems [19]–[21]. In particular, Dai *et al.* [22] proposed a closed-loop P-type iterative learning law for uncertain linear DPSs. In addition, he considered ILC for second-order hyperbolic DPSs with uncertainties [23]. A D-type ILC law for a type of distributed parameter systems with collocated sensors and actuators is considered in [24]. In many industrial processes, the sensors and actuators are always non-collocated. Hence, a type of linear parabolic distributed parameter system based on non-collocated sensors and actuators is proposed. No research papers have taken into account the problem of a PD-type ILC for this system.

The distributed parameter system based on non-collocated sensors and actuators is a complex system since it depends on time and spatial position. Furthermore, ILC can be better in controlling dynamic systems with complex modelling, uncertainty and with strong non-linear coupling effects. As such, we can obtain good control performance of a distributed parameter system by using ILC schemes. As discussed above,

there is no existing research that has been carried out using ILC for distributed parameter systems using non-collocated sensors and actuators. Thus, we first propose an open-loop PD-type ILC scheme for a distributed parameter system with non-collocated single sensor and m actuators. After that, we consider the distributed parameter system based on non-collocated m sensors as well as m actuators, which include numerous industrial processes, such as heat exchangers, industrial chemical reactors, and agricultural irrigation processes. Lastly, we present a closed-loop PD-type ILC algorithm for the distributed parameter system using a single sensor and multiple actuators when some errors exist in the initial states of the system.

In distributed parameter systems with non-collocated sensors and actuators, the sensors are capable of gathering information from the systems in real time. At the same time, the actuators can perform various tasks. When the states change, an input is imposed to control the output of the actuators. However, the actual output of the systems may not represent the desired output in the running of actuators. In this case, it is crucial to use ILC schemes to learn the output error of the systems. This facilitates the actual output in tracking the desired output. Therefore, this work improves the performance of systems. At the same time, it significantly closes the existing theoretical gap.

The remainder of this paper is as follows: In Section II, we first discuss the system and problem formulation. Next, the open-loop PD-type and closed-loop PD-type ILC schemes are presented in a distributed parameter system with a sensor and m actuators. In addition, the proposed ILC schemes are extended to a class of distributed parameter systems using non-collocated m sensors and m actuators in Section III. The effectiveness of the proposed methods are illustrated through numerical simulation in Section IV and conclusions follow in Section V.

Notations: \mathbb{R} , \mathbb{R}^n and \mathbb{R}^+ are the set of all real numbers, n -dimensional space and the set of all positive real numbers.

The definition of the L_2 -norm of the function $W(x, t) : [0, h] \times [0, T] \rightarrow \mathbb{R}$ is

$$\|W(\cdot, t)\|_{L_2} = \sqrt{\langle W(\cdot, t), W(\cdot, t) \rangle}$$

where $\langle W_1(\cdot, t), W_2(\cdot, t) \rangle = \int_0^h W_1(x, t) W_2(x, t) dx$ is the inner product.

II. THE SYSTEM AND PROBLEM FORMULATION

Consider the distributed parameter system with a non-collocated single sensor and m actuators as follows:

$$\begin{cases} \frac{\partial q_k(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial q_k(x, t)}{\partial x} \right) + \sum_{i=1}^m b(x; x_i^a) u_{(k,i)}(t) \\ y_k(t) = \int_0^h c(x) q_k(x, t) dx \end{cases} \quad (1)$$

with the Neumann boundary conditions

$$\frac{\partial q_k(x, t)}{\partial x} \Big|_{x=0} = \frac{\partial q_k(x, t)}{\partial x} \Big|_{x=h} = 0 \quad (2)$$

and the initial condition

$$q_k(x, 0) = 0 \quad (3)$$

where x and t are the spatial position and time which satisfy

$(x, t) \in [0, h] \times [0, T] \subset \mathbb{R}^+ \times \mathbb{R}^+$. $\varpi(x) \geq \varpi_0 > 0$ is a known continuous function of x (ϖ_0 is a constant). k denotes the k th iteration of the repetitive operation of the system. $q_k(x, t)$ and $y_k(t)$ denote the state and output of the system at the k th iteration. When the system operates in the k th iteration, $u_{(k,i)}(t)$ is the associated control signal of the i th actuator. $b(x; x_i^a)$ denotes the spatial distribution of the actuating device of the i th actuator and x_i^a denotes the centroid position of the i th actuator. $c(x)$ is the spatial distribution of the sensor. The sensor spatial distribution and the actuators spatial distribution satisfy

$$c(x) = \delta, \quad x \in [0, h] \quad (4)$$

and

$$b(x; x_i^a) = \begin{cases} \gamma, & x \in [x_i^a - \sigma, x_i^a + \sigma] \\ 0, & \text{others} \end{cases} \quad (5)$$

where δ and γ are constants. $\sigma > 0$ is the spatial support of the actuators.

Throughout this paper, two lemmas and one assumption are first given as follows:

Lemma 1 [22]: If $f(t)$ and $g(t)$ are two continuous nonnegative functions on $[0, T]$, and there exist nonnegative constants ρ and M satisfying

$$f(t) \leq \rho + g(t) + M \int_0^t f(s) ds$$

then

$$f(t) \leq \rho e^{Mt} + g(t) + M e^{Mt} \int_0^t e^{-Ms} g(s) ds.$$

Lemma 2 [22]: If the constant sequence $\{d_k\}_{k \geq 0}$ converges to zero, and the sequence $\{Z_k(t)\}_{k \geq 0} \subset C[0, T]$ satisfies

$$Z_{k+1}(t) = \theta Z_k(t) + M(d_k + \int_0^t Z_k(s) ds)$$

then $\{Z_k(t)\}_{k \geq 0}$ ($k \rightarrow \infty$) uniformly converges to zero, where $M > 0$ and $0 \leq \theta < 1$ are constants.

Assumption 1: For a desired output $y_d(t)$, a unique $u_{(d,i)}(t)$ exists such that

$$\begin{cases} \frac{\partial q_d(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial q_d(x, t)}{\partial x} \right) + \sum_{i=1}^m b(x; x_i^a) u_{(d,i)}(t) \\ y_d(t) = \int_0^h c(x) q_d(x, t) dx \end{cases} \quad (6)$$

where $q_d(x, 0) = 0$.

In this paper, an open-loop PD-type ILC scheme is employed as follows:

$$u_{(k+1,i)}(t) = u_{(k,i)}(t) + \Upsilon e_k(t) + \Gamma \dot{e}_k(t) \quad (7)$$

where $e_k(t)$ is the output error of k th iteration which satisfies $e_k(t) = y_k(t) - y_d(t)$. Γ and Υ are the open-loop ILC learning gains.

III. CONVERGENCE ANALYSIS

In this section, we first prove the effectiveness of the open-loop PD-type ILC for a distributed parameter system with non-collocated single sensor and m actuators. In addition, we extend the proposed scheme to the distributed parameter system with non-collocated m sensors and m actuators. The following theorem is first given.

Theorem 1: Consider the open-loop PD-type ILC scheme (7) for the repetitive distributed parameter system (1) with the desired output satisfying Assumption 1. If the learning gain exists and satisfies $(1 + 2m\delta\gamma\sigma\Gamma)^2 < 1/2$, then the output error converges to zero for all $t \in [0, T]$ as $k \rightarrow \infty$, i.e., $\lim_{k \rightarrow \infty} e_k(t) = 0$, $t \in [0, T]$.

Proof: The input error $u_{(k+1,i)}(t) - u_{(d,i)}(t)$ at the $(k+1)$ th iteration can be expressed as

$$\begin{aligned} u_{(k+1,i)}(t) - u_{(d,i)}(t) &= u_{(k,i)}(t) - u_{(d,i)}(t) + \Upsilon e_k(t) + \Gamma \dot{e}_k(t) \\ &= \bar{u}_{(k,i)}(t) + \Upsilon(y_k(t) - y_d(t)) + \Gamma(\dot{y}_k(t) - \dot{y}_d(t)) \\ &= \bar{u}_{(k,i)}(t) + \Upsilon \int_0^h c(x) \bar{q}_k(x, t) dx \\ &\quad + \Gamma \int_0^h c(x) \frac{\partial \bar{q}_k(x, t)}{\partial t} dx \end{aligned} \quad (8)$$

where $\bar{u}_{(k,i)}(t) = u_{(k,i)}(t) - u_{(d,i)}(t)$ and $\bar{q}_k(x, t) = q_k(x, t) - q_d(x, t)$.

According to the state equation of system (1), we have

$$\begin{aligned} \bar{u}_{(k+1,i)}(t) &= \bar{u}_{(k,i)}(t) + \Upsilon \int_0^h c(x) \bar{q}_k(x, t) dx \\ &\quad + \Gamma \int_0^h c(x) \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x, t)}{\partial x} \right) dx \\ &\quad + \Gamma \int_0^h c(x) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) dx. \end{aligned} \quad (9)$$

Applying integration by parts and using the boundary conditions for the third term on the right hand side of (9), we obtain

$$\begin{aligned} &\int_0^h c(x) \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x, t)}{\partial x} \right) dx \\ &= \int_0^h c(x) \frac{d(\varpi(x))}{dx} \frac{\partial \bar{q}_k(x, t)}{\partial x} dx \\ &\quad + \int_0^h c(x) \varpi(x) \frac{\partial^2 \bar{q}_k(x, t)}{\partial x^2} dx \\ &= \left[c(x) \varpi(x) \frac{\partial \bar{q}_k(x, t)}{\partial x} \right]_{x=0}^{x=h} \\ &\quad - \int_0^h c(x) \varpi(x) \frac{\partial^2 \bar{q}_k(x, t)}{\partial x^2} dx \\ &\quad - \int_0^h \varpi(x) \frac{\partial \bar{q}_k(x, t)}{\partial x} \frac{\partial c(x)}{\partial x} dx \\ &\quad + \int_0^h c(x) \varpi(x) \frac{\partial^2 \bar{q}_k(x, t)}{\partial x^2} dx = 0. \end{aligned} \quad (10)$$

Substituting (10) into (9) yields

$$\begin{aligned} \bar{u}_{(k+1,i)}(t) &= \bar{u}_{(k,i)}(t) + \Upsilon \int_0^h c(x) \bar{q}_k(x, t) dx \\ &\quad + \Gamma \int_0^h c(x) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) dx. \end{aligned} \quad (11)$$

Based on the spatial distribution of the sensor (4) and actuators (5), (11) can be further rewritten as

$$\bar{u}_{(k+1,i)}(t) = (1 + 2m\delta\gamma\sigma\Gamma) \bar{u}_{(k,i)}(t) + \Upsilon \delta \int_0^h \bar{q}_k(x, t) dx. \quad (12)$$

Squaring both sides of (12) and using the definition of the L_2 -norm, we have

$$\bar{u}_{(k+1,i)}^2(t) \leq 2(1 + 2m\delta\gamma\sigma\Gamma)^2 \bar{u}_{(k,i)}^2(t) + 2(\Upsilon\delta)^2 \|\bar{q}_k(\cdot, t)\|_{L_2}^2. \quad (13)$$

Now we investigate $\frac{\partial(\bar{q}_k(x,t))^2}{\partial t}$ by using (1) as follows:

$$\begin{aligned} \frac{\partial(\bar{q}_k(x,t))^2}{\partial t} &= 2\bar{q}_k(x,t) \frac{\partial \bar{q}_k(x,t)}{\partial t} \\ &= 2\bar{q}_k(x,t) \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x,t)}{\partial x} \right) \\ &\quad + 2\bar{q}_k(x,t) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) \\ &\leq 2\bar{q}_k(x,t) \frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x,t)}{\partial x} \right) + \bar{q}_k^2(x,t) \\ &\quad + \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t). \end{aligned} \quad (14)$$

Integrating (14) with respect to x on $[0, h]$, it satisfies

$$\begin{aligned} &\frac{d}{dt} \int_0^h \bar{q}_k^2(x,t) dx \\ &\leq 2 \int_0^h \bar{q}_k(x,t) \left(\frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x,t)}{\partial x} \right) \right) dx \\ &\quad + \int_0^h \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) dx \\ &\quad + \int_0^h \bar{q}_k^2(x,t) dx. \end{aligned} \quad (15)$$

Similar to (10), $\int_0^h \bar{q}_k(x,t) \left(\frac{\partial}{\partial x} \left(\varpi(x) \frac{\partial \bar{q}_k(x,t)}{\partial x} \right) \right) dx \leq 0$. Hence, we obtain

$$\begin{aligned} &\frac{d}{dt} \int_0^h \bar{q}_k^2(x,t) dx \\ &\leq \int_0^h \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) \sum_{i=1}^m b(x; x_i^a) \bar{u}_{(k,i)}(t) dx \\ &\quad + \int_0^h \bar{q}_k^2(x,t) dx. \end{aligned} \quad (16)$$

According to the spatial distribution of the actuators (5), and the definition of L_2 -norm, the following gives

$$\frac{d}{dt} \|\bar{q}_k(\cdot, t)\|_{L_2}^2 \leq 2m\sigma\gamma^2 \bar{u}_{(k,i)}^2(t) + \|\bar{q}_k(\cdot, t)\|_{L_2}^2. \quad (17)$$

Integrating (17) with respect to t and using the Bellman-Gronwall Lemma, we have

$$\begin{aligned} \|\bar{q}_k(\cdot, t)\|_{L_2}^2 &\leq \int_0^t (2m\sigma\gamma^2 \bar{u}_{(k,i)}^2(\tau) + \|\bar{q}_k(\cdot, \tau)\|_{L_2}^2) d\tau \\ &\leq 2m\sigma\gamma^2 \int_0^t e^{(t-\tau)} \bar{u}_{(k,i)}^2(\tau) d\tau. \end{aligned} \quad (18)$$

Substituting (18) into (13), we get

$$\begin{aligned} \bar{u}_{(k+1,i)}^2(t) &\leq 2(1 + 2m\delta\gamma\sigma\Gamma)^2 \bar{u}_{(k,i)}^2(t) \\ &\quad + 4m\sigma(\Upsilon\delta\gamma)^2 \int_0^t e^{(t-\tau)} \bar{u}_{(k,i)}^2(\tau) d\tau. \end{aligned} \quad (19)$$

Set $\|\bar{u}_{(k,i)}\|_\lambda = \sup_{t \in [0, T]} \{\|\bar{u}_{(k,i)}(t)\|^2 e^{-\lambda t}\}$ where $\lambda > 0$ and $\|\cdot\|$ is a kind of \mathbb{R}^n norm, then we can obtain

$$\|\bar{u}_{(k+1,i)}\|_\lambda \leq (2(1 + 2m\delta\gamma\sigma\Gamma)^2 + \frac{4m\sigma(\Upsilon\delta\gamma)^2}{\lambda - 1}) \|\bar{u}_{(k,i)}\|_\lambda. \quad (20)$$

Because $(1+2m\delta\gamma\sigma\Gamma)^2 < 1/2$, we can obtain $(2(1+2m\delta\gamma\sigma\Gamma)^2 + 4m\sigma(\Upsilon\delta\gamma)^2/(l-1)) < 1$ if λ is chosen large enough. Hence, $\lim_{k \rightarrow \infty} u_{(k,i)}(t) = 0$.

According to (18), we have

$$\lim_{k \rightarrow \infty} \|\tilde{q}_k(\cdot, t)\|_{L_2} = 0. \quad (21)$$

And from the output equation of system (1), we readily conclude that

$$\lim_{k \rightarrow \infty} e_k(t) = 0. \quad (22)$$

■

Remark 1: In engineering applications, there always need to be multiple sensors to finish complicated tasks. Hence, we consider the following distributed parameter system with non-collocated m sensors and m actuators which exists the same boundary conditions and initial condition as system (1) in a repeatable environment

$$\begin{cases} \frac{\partial q_k(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(\frac{\varpi(x) \partial q_k(x, t)}{\partial x} \right) + \sum_{i=1}^m b(x; x_i^a) u_{(k,i)}(t) \\ \begin{bmatrix} y_{(k,1)} \\ \vdots \\ y_{(k,m)} \end{bmatrix} = \begin{bmatrix} \int_0^h c(x; x_1^s) q_k(x, t) dx \\ \vdots \\ \int_0^h c(x; x_m^s) q_k(x, t) dx \end{bmatrix} \end{cases} \quad (23)$$

where $c(x; x_i^s)$ denotes the spatial distribution of the sensing device of the i th sensor and x_i^s is the centroid position of the i th sensor.

The spatial distribution of the sensors are assumed to be the boxcar function

$$c(x; x_i^s) = \begin{cases} \beta, & x \in [x_i^s - \varepsilon, x_i^s + \varepsilon] \\ 0, & \text{others} \end{cases} \quad (24)$$

and the spatial distribution of the actuators are also assumed to be a boxcar function

$$b(x; x_i^a) = \begin{cases} \alpha, & x \in [x_i^a - \eta, x_i^a + \eta] \\ 0, & \text{others} \end{cases} \quad (25)$$

where β and α are constants. $\varepsilon > 0$ and $\eta > 0$ are the spatial support of the sensors and actuators, respectively.

Remark 2: In some practical engineering applications, the actuator needs to perform a task so the spatial distribution of actuating device is wider than sensing device. Therefore, we assume $[x_i^s - \varepsilon, x_i^s + \varepsilon] \subset [x_i^a - \eta, x_i^a + \eta]$.

According to the system (23), the following assumption is given.

Assumption 2: For a desired output $y_{(d,i)}(t)$, a unique $u_{(d,i)}(t)$ exists such that

$$\begin{cases} \frac{\partial q_d(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(\frac{\varpi(x) \partial q_d(x, t)}{\partial x} \right) + \sum_{i=1}^m b(x; x_i^a) u_{(d,i)}(t) \\ \begin{bmatrix} y_{(d,1)} \\ \vdots \\ y_{(d,m)} \end{bmatrix} = \begin{bmatrix} \int_0^h c(x; x_1^s) q_d(x, t) dx \\ \vdots \\ \int_0^h c(x; x_m^s) q_d(x, t) dx \end{bmatrix}. \end{cases} \quad (26)$$

In this part, we consider the open-loop PD-type ILC scheme

$$u_{(k+1,i)}(t) = u_{(k,i)}(t) + \Upsilon_i e_{(k,i)}(t) + \Gamma_i \dot{e}_{(k,i)}(t) \quad (27)$$

where $e_{(k,i)}(t)$ is the output error of i th sensor during k th iteration which satisfies $e_{(k,i)}(t) = y_{(k,i)}(t) - y_{(d,i)}(t)$. Υ_i and Γ_i are i th number of open-loop PD-type learning gains.

Theorem 2: If the open-loop PD-type gain Γ_i of the ILC scheme (27) satisfies $(1 + 2\alpha\beta\varepsilon\Gamma_i)^2 < 1/2$, and the system with the desired output satisfies the Assumption 2 under the initial and boundary conditions, the output errors of (23) converge to zero when $k \rightarrow \infty$, i.e., $\lim_{k \rightarrow \infty} e_{(k,i)}(t) = 0, t \in [0, T]$.

Remark 3: In previous proof, we just consider that the initial condition is zero at every iterative learning process, however, there always exists some errors at the beginning in every iterative process. Hence, a more favorable initial condition is given as follows:

$$\|q_d(\cdot, 0) - q_k(\cdot, 0)\|_{L_2}^2 \leq l\varrho^k \quad (\varrho \in [0, 1], l > 0).$$

For the system (1), if we consider using a closed-loop PD-type ILC scheme to replace the open-loop PD-type ILC scheme (7), we can obtain the convergence conditions of tracking error. The closed-loop PD-type ILC scheme is employed

$$u_{(k+1,i)}(t) = u_{(k,i)}(t) + \Phi e_{(k+1)}(t) + \Psi \dot{e}_{(k+1)}(t) \quad (28)$$

where Φ and Ψ are the P-type learning gain and D-type learning gain, respectively.

Theorem 3: Consider the closed-loop PD-type ILC algorithm (28) for the repetitive system (1) with non-collocated sensor/actuators under the initial and boundary conditions, which satisfies Assumption 1. If the learning gain Ψ exists and satisfies $1/|1 - 2m\Psi\delta\gamma\sigma|^2 < 1/2$, then the output error converges to zero for all $t \in [0, T]$ as $k \rightarrow \infty$, i.e., $\lim_{k \rightarrow \infty} e_k(t) = 0, t \in [0, T]$.

Proof: From the desired input and actual input, the following gives

$$\begin{aligned} \tilde{u}_{(k+1,i)}(t) &= \tilde{u}_{(k,i)}(t) + \Phi e_{(k+1)}(t) + \Psi \dot{e}_{(k+1)}(t) \\ &= \tilde{u}_{(k,i)}(t) + \Phi \int_0^h c(x) \tilde{q}_{k+1}(x, t) dx \\ &\quad + \Psi \int_0^h c(x) \frac{\partial \tilde{q}_{k+1}(x, t)}{\partial t} dx \\ &= \tilde{u}_{(k,i)}(t) + \Phi \int_0^h c(x) \tilde{q}_{k+1}(x, t) dx \\ &\quad + \Psi \int_0^h c(x) \frac{\partial}{\partial x} \left(\frac{\varpi(x) \partial \tilde{q}_{k+1}(x, t)}{\partial x} \right) dx \\ &\quad + \Psi \int_0^h c(x) \sum_{i=1}^m b(x; x_i^a) \tilde{u}_{(k+1,i)}(t) dx \end{aligned} \quad (29)$$

where $\tilde{u}_{(k+1,i)}(t) = u_{(k+1,i)}(t) - u_{(d,i)}(t)$ and $\tilde{q}_{k+1}(x, t) = q_{k+1}(x, t) - q_d(x, t)$.

Similar to the proof of (10), we know $\Psi \int_0^h c(x) \frac{\partial}{\partial x} \left(\frac{\varpi(x) \partial \tilde{q}_{k+1}(x, t)}{\partial x} \right) dx = 0$. Based on the ILC scheme (28), the input of every actuator is identical at same iteration time. And from the spatial distribution of sensor and actuators, we obtain

$$\begin{aligned}
\tilde{u}_{(k+1,i)}(t) &= \tilde{u}_{(k,i)}(t) + \Phi \int_0^h c(x) \tilde{q}_{k+1}(x, t) dx \\
&\quad + \Psi \int_0^h c(x) \sum_{i=1}^m b(x; x_i^a) \tilde{u}_{(k+1,i)}(t) dx \\
&= \tilde{u}_{(k,i)}(t) + \Phi \delta \int_0^h \tilde{q}_{k+1}(x, t) dx \\
&\quad + 2m\Psi\delta\gamma\sigma \tilde{u}_{(k+1,i)}(t).
\end{aligned} \tag{30}$$

From (30), We can get

$$\begin{aligned}
\tilde{u}_{(k+1,i)}(t) &= (1 - 2m\Psi\delta\gamma\sigma)^{-1} \tilde{u}_{(k,i)}(t) \\
&\quad + \frac{\Phi\delta}{1 - 2m\Psi\delta\gamma\sigma} \int_0^h \tilde{q}_{k+1}(x, t) dx
\end{aligned} \tag{31}$$

where $(1 - 2m\Psi\delta\gamma\sigma) \neq 0$.

According to the definition of L_2 -norm, we have

$$\begin{aligned}
\tilde{u}_{(k+1,i)}^2(t) &\leq 2(1 - 2m\Psi\delta\gamma\sigma)^{-2} \tilde{u}_{(k,i)}^2(t) \\
&\quad + 2 \left| \frac{\Phi\delta}{1 - 2m\Psi\delta\gamma\sigma} \right|^2 \|\tilde{q}_{k+1}(\cdot, t)\|_{L_2}^2.
\end{aligned} \tag{32}$$

Similar to the proof of Theorem 1, we can investigate $\|\tilde{q}_{k+1}(\cdot, t)\|_{L_2}^2$ by using the spatial distribution of the actuators and the definition of L_2 -norm as follows:

$$\frac{d}{dt} \|\tilde{q}_{k+1}(\cdot, t)\|_{L_2}^2 \leq 2m\gamma^2\sigma \tilde{u}_{(k+1,i)}^2(t) + \|\tilde{q}_{k+1}(\cdot, t)\|_{L_2}^2. \tag{33}$$

Due to the initial condition $\|q_d(\cdot, 0) - q_k(\cdot, 0)\|_{L_2}^2 \leq l\varrho^k$, the following gives

$$\begin{aligned}
\|\tilde{q}_{k+1}(\cdot, t)\|_{L_2}^2 &\leq \int_0^t (2m\gamma^2\sigma \tilde{u}_{(k+1,i)}^2(\tau) + \|\tilde{q}_{k+1}(\cdot, \tau)\|_{L_2}^2) d\tau \\
&\quad + \|\tilde{q}_{k+1}(\cdot, 0)\|_{L_2}^2 \\
&\leq e^t l\varrho^{k+1} + 2m\gamma^2\sigma \int_0^t e^{(t-\tau)} \tilde{u}_{(k+1,i)}^2(\tau) d\tau.
\end{aligned} \tag{34}$$

Substituting (34) into (32), we have

$$\begin{aligned}
\tilde{u}_{(k+1,i)}^2(t) &\leq 2|1 - 2m\Psi\delta\gamma\sigma|^{-2} \tilde{u}_{(k,i)}^2(t) + \frac{2|\Phi\delta|^2 e^t l\varrho^{k+1}}{|1 - 2m\Psi\delta\gamma\sigma|^2} \\
&\quad + \frac{|4\Phi^2\delta^2 m\gamma^2\sigma|}{|1 - 2m\Psi\delta\gamma\sigma|^2} \int_0^t e^{(t-\tau)} \tilde{u}_{(k+1,i)}^2(\tau) d\tau.
\end{aligned} \tag{35}$$

Multiply both sides of (35) by e^{-t} , and let $W_{(k+1,i)}(t) = e^{-t} \tilde{u}_{(k+1,i)}^2(t)$, we can get

$$\begin{aligned}
W_{(k+1,i)}(t) &\leq 2|1 - 2m\Psi\delta\gamma\sigma|^{-2} W_{(k,i)}(t) + \frac{2|\Phi\delta|^2 l\varrho^{k+1}}{|1 - 2m\Psi\delta\gamma\sigma|^2} \\
&\quad + \frac{|4\Phi^2\delta^2 m\gamma^2\sigma|}{|1 - 2m\Psi\delta\gamma\sigma|^2} \int_0^t W_{(k+1,i)}(\tau) d\tau \\
&= o_1 W_{(k,i)}(t) + o_2 + o_3 \int_0^t W_{(k+1,i)}(\tau) d\tau
\end{aligned} \tag{36}$$

where $o_1 = 2|1 - 2m\Psi\delta\gamma\sigma|^{-2}$, $o_2 = 2|\Phi\delta|^2 l\varrho^{k+1}/|1 - 2m\Psi\delta\gamma\sigma|^2$ and $o_3 = |4\Phi^2\delta^2 m\gamma^2\sigma|/|1 - 2m\Psi\delta\gamma\sigma|^2$.

From Lemma 1, we can have that

$$W_{(k+1,i)}(t) \leq o_1 W_{(k,i)}(t) + o_2 e^{o_3 t} + o_3 e^{o_3 t} \int_0^t W_{(k,i)}(\tau) e^{-o_3 \tau} d\tau. \tag{37}$$

Multiply both sides of (37) by $e^{-o_3 t}$, and let $V_{(k+1,i)}(t) = W_{(k+1,i)}(t) e^{-o_3 t}$. We obtain

$$V_{(k+1,i)}(t) \leq o_1 V_{(k,i)}(t) + o_2 + o_3 \int_0^t V_{(k,i)}(\tau) d\tau. \tag{38}$$

From the initial condition, we know $\varrho \in [0, 1]$, hence $o_2 \rightarrow 0$, when $k \rightarrow \infty$. And when $o_1 < 1$, we can obtain $V_{(k,i)}(t) \rightarrow 0$ ($k \rightarrow \infty$) from Lemma 2. Because $V_{(k,i)}(t) = W_{(k,i)}(t) e^{-o_3 t} = e^{-o_3 t} e^{-t} \tilde{u}_{(k+1,i)}^2(t)$, we have

$$\lim_{k \rightarrow \infty} \tilde{u}_{(k+1,i)}(t) = 0. \tag{39}$$

Hence, we can obtain

$$\lim_{k \rightarrow \infty} e_{k+1}(t) = 0. \tag{40}$$

■

IV. NUMERICAL SIMULATIONS

Consider the following distributed parameter system with a sensor and two actuators in a repeatable environment

$$\begin{cases} \frac{\partial q_k(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(\frac{\varpi(x) \partial q_k(x, t)}{\partial x} \right) + \sum_{i=1}^2 b(x; x_i^a) u_{(k,i)}(t) \\ y_k(t) = \int_0^h c(x) q_k(x, t) dx \end{cases} \tag{41}$$

where $x \in [0, 1]$, $t \in [0, 0.8]$ and $\varpi(x) = 0.01 > 0$.

Assume that the sensor spatial distribution satisfies

$$c(x) = \begin{cases} 5.2, & x \in [0, 1] \\ 0, & \text{others} \end{cases} \tag{42}$$

and the actuators spatial distribution satisfy

$$b(x; x_1^a) = \begin{cases} 1, & x \in [0.3125, 0.375] \\ 0, & \text{others} \end{cases} \tag{43}$$

and

$$b(x; x_2^a) = \begin{cases} 1, & x \in [0.5, 0.5625] \\ 0, & \text{others} \end{cases} \tag{44}$$

In this example, we employ the open-loop and closed-loop PD-type ILC schemes, and assume $\Gamma = -0.7$, $\Phi = -0.3$ and $\Upsilon = \Psi = -15$. Thus, we can obtain $(1 + 2m\delta\gamma\sigma\Gamma)^2 < 1/2$ and $1/|1 - 2m\Psi\delta\gamma\sigma|^2 < 1/2$ which satisfy Theorems 1 and 3, respectively. The desired trajectory is given as $y_d = \sin(5\pi t)$.

Using the difference method for partial differential equations, the simulation results can be obtained which are shown in Figs. 1–6.

Figs. 1–3 are obtained using an open-loop PD-type ILC scheme. Fig. 1 shows the desired output and actual output of system at $k = 15, 25, 30$, respectively. Fig. 2 shows the states $q_k(x, t)$ of system at $k = 25$, and it is seen that only when $x \in [0.3125, 0.375] \cup [0.5, 0.5625]$, the states $q_k(x, t) \neq 0$. Fig. 3 shows a curve chart which describes the variation in the error of output with the number of iterations. When $k = 25$, the maximum error of the output function is 1.2×10^{-3} . The simulation results demonstrate the effectiveness of the proposed scheme.

Figs. 4–6 are obtained using a closed-loop PD-type ILC algorithm. Fig. 4 shows the desired output and actual output of system at $k = 15, 25, 30$, respectively. Fig. 5 shows the states $q_k(x, t)$ of system at $k = 25$, and it is seen that only when $x \in [0.3125, 0.375] \cup [0.5, 0.5625]$, the states $q_k(x, t) \neq 0$. Fig. 6 shows a curve chart which describes the variation in the error

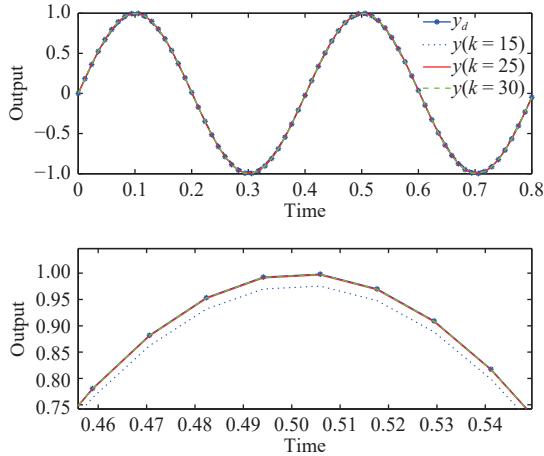


Fig. 1. The desired output $y_d(t)$ and iterations for output function $y_k(t)$ when $k = 15, 25, 30$, respectively (open-loop PD-type ILC scheme).

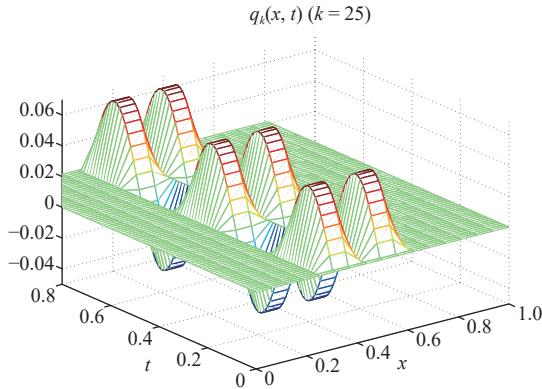


Fig. 2. The states $q_k(x, t)$ of system when $k = 25$ (open-loop PD-type ILC scheme).

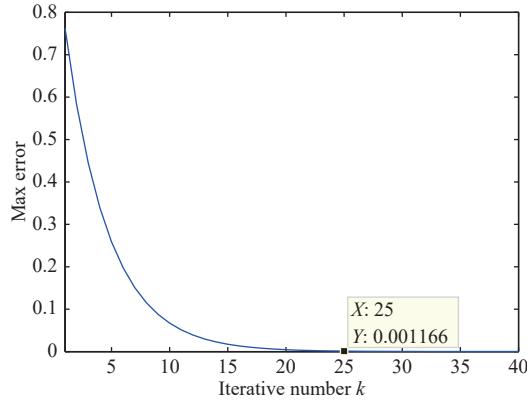


Fig. 3. The variation of maximum output error $e_k(t)$ along with iterative number (open-loop PD-type ILC scheme).

of output with the number of iterations. When $k = 25$, the maximum error of the output function is 0.64×10^{-3} . The simulation results demonstrate the effectiveness of the proposed algorithm.

V. CONCLUSION

In this paper, we extended the open-loop and closed-loop

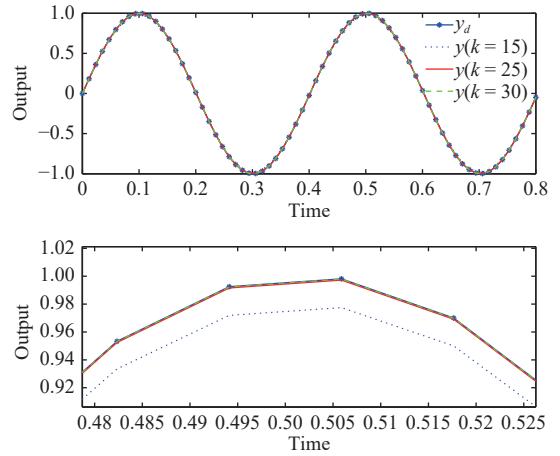


Fig. 4. The desired output $y_d(t)$ and iterations for output function $y_k(t)$ when $k = 15, 25, 30$, respectively (closed-loop PD-type ILC scheme).

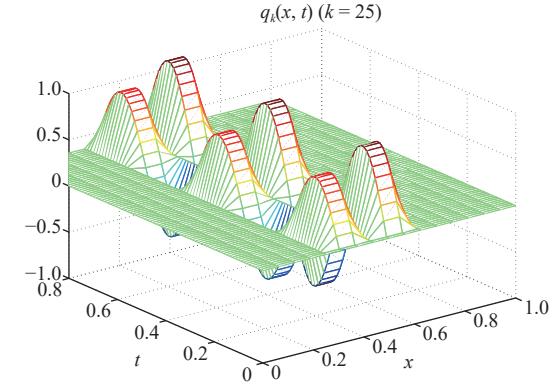


Fig. 5. The states $q_k(x, t)$ of system when $k = 25$ (closed-loop PD-type ILC scheme).

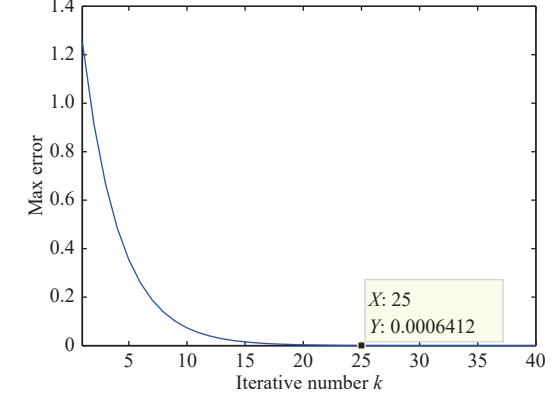


Fig. 6. The variation of maximum output error $e_k(t)$ along with iterative number (closed-loop PD-type ILC scheme).

PD-type ILC schemes for two types of the parabolic distributed parameter systems based on non-collocated sensors and actuators working in a repeatable environment. Firstly, we took into consideration the convergence condition of a type of distributed parameter system with non-collocated single sensor and m actuators by using two classes of ILC schemes. Thereafter, we discussed the convergence condition of a class

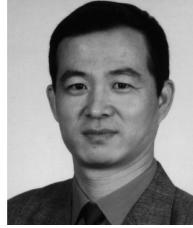
of distributed parameter systems using non-collocated m sensors and m actuators. Lastly, we presented a distributed parameter system based on one sensor and two actuators to illustrate the effectiveness of the proposed control. From Figs. 3 and 6, we know the maximum error of the output function are 1.2×10^{-3} (open-loop PD-type) and 0.64×10^{-3} (closed-loop PD-type), respectively. Hence, the closed-loop PD-type ILC scheme is more effective.

REFERENCES

- [1] R. Padhi and S. N. Balakrishnan, "Optimal dynamic inversion control design for a class of nonlinear distributed parameter systems with continuous and discrete actuators," *IET Contr. Theory Appl.*, vol. 1, no. 6, pp. 1662–1671, Dec. 2007.
- [2] H. X. Li and C. Qi, "Modeling of distributed parameter systems for applications-a synthesized review from time-space separation," *J. Process Control*, vol. 20, no. 8, pp. 891–901, Sept. 2010.
- [3] M. Wang and H. Shi, "An adaptive neural network prediction for nonlinear parabolic distributed parameter system based on block-wise moving window technique," *Neurocomputing*, vol. 133, no. 10, pp. 67–73, Jun. 2014.
- [4] M. A. Demetriou, "Guidance of a moving collocated actuator/sensor for improved control of distributed parameter systems," in *Proc. 47th IEEE Conf. Decis. Control*, pp. 215–220, Dec. 2008.
- [5] M. A. Demetriou, "Gain adaptation and sensor guidance of diffusion PDEs using on-line approximation of optimal feedback kernels," in *Proc. American Control Conf.*, pp. 2536–2541, Jul. 2016.
- [6] W. Y. Mu, B. T. Cui, W. Li, and Z. X. Jiang, "Improving control and estimation for distributed parameter systems utilizing mobile actuator-sensor network," *ISA Trans.*, vol. 53, no. 4, pp. 1087–1095, Jul. 2014.
- [7] Z. X. Jiang, B. T. Cui, W. Wu, and B. Zhuang, "Event-driven observer-based control for distributed parameter systems using mobile sensor and actuator," *Computer and Mathematics With Applications*, vol. 72, no. 12, pp. 2854–2864, Dec. 2016.
- [8] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. Robotic System*, vol. 1, no. 2, pp. 123–140, Jun. 1984.
- [9] N. Zeng and X. Ying, "Iterative learning control algorithm for nonlinear dynamical systems," *Acta Automatica Sinica*, vol. 18, no. 2, pp. 168–176, Mar. 1992.
- [10] M. X. Sun and B. J. Huang, *Iterative Learning Control*, Beijing: National Defence Industrial Press, 1999.
- [11] F. X. Piao, Q. L. Zhang, and Z. F. Wang, "Iterative learning control for a class of singular systems," *Acta Automatica Sinica*, vol. 33, no. 6, pp. 658–659, 2007.
- [12] R. H. Chi and Z. S. Hou, "Dual-stage optimal iterative learning control for nonlinear non-affine discrete-time system," *Acta Automatica Sinica*, vol. 32, no. 10, pp. 1061–1065, Oct. 2007.
- [13] D. Shen, Y. Mu, and G. Xiong, "Iterative learning control for non-linear systems with deadzone input and time delay in presence of measurement noise," *IET Contr. Theory Appl.*, vol. 5, no. 12, pp. 1418–1425, Aug. 2011.
- [14] X. H. Bu, T. H. Wang, Z. S. Hou, and R. H. Chi, "Iterative learning control for discrete-time systems with quantised measurements," *IET Contr. Theory Appl.*, vol. 9, no. 9, pp. 1455–1460, Jul. 2015.
- [15] W. Paszke, E. Rogers, K. Galkowski, and Z. Cai, "Robust finite frequency range iterative learning control design and experimental verification," *Control Eng. Practice*, vol. 21, no. 10, pp. 1310–1320, Oct. 2013.
- [16] C. T. Freeman, E. Rogers, A. Hughes, J. H. Burridge, and K. L. Meadmore, "Iterative learning control in health care: electrical stimulation and robotic-assisted upper-limb stroke rehabilitation," *IEEE Control Syst. Mag.*, vol. 32, no. 1, pp. 18–43, Feb. 2012.
- [17] J. H. Lee and K. S. Lee, "Iterative learning control applied to batch processes: an overview," *Control Eng. Practice*, vol. 15, no. 10, pp. 1306–1318, Oct. 2007.
- [18] X. E. Ruan, Z. Z. Bien, and K. H. Park, "Decentralized iterative learning control to large-scale industrial processes for nonrepetitive trajectory tracking," *IEEE Trans. Systems, Man, and Cybernetics: Systems*, vol. 38, no. 1, pp. 238–252, Feb. 2008.
- [19] H. H. Ji, Z. S. Hou, L. L. Fan, and F. L. Lewis, "Adaptive iterative learning reliable control for a class of non-linearly parameterised systems with unknown state delays and input saturation," *IET Contr. Theory Appl.*, vol. 10, no. 17, pp. 2160–2174, Jun. 2016.
- [20] D. Q. Huang, X. F. Li, J. X. Xu, C. Xu, and W. He, "Iterative learning control of inhomogeneous distributed parameter systems-frequency domain design and analysis," *System & Control Letters*, vol. 72, pp. 22–29, Oct. 2014.
- [21] T. F. Xiao and H. X. Li, "Eigenspectrum-based iterative learning control for a class of distributed parameter systems," *IEEE Trans. Automatic Control*, vol. 62, no. 2, pp. 824–836, Jan. 2016.
- [22] X. S. Dai, S. P. Tian, Y. J. Peng, and W. G. Luo, "Closed-loop P-type iterative learning control of uncertain linear distributed parameter systems," *IEEE/CAA J. Autom. Sinica*, vol. 1, no. 3, pp. 267–273, Jul. 2014.
- [23] X. S. Dai, C. Xu, and S. P. Tian, "Iterative learning control for MIMO second-order hyperbolic distributed parameter systems with uncertainties," *Adv. Differ. Equat.*, vol. 1, no. 94, Dec. 2016.
- [24] J. X. Zhang, B. T. Cui, and X. Y. Lou, "Iterative learning control for distributed parameter systems based on actuator-sensor network," in *Proc. 7th Int. Conf. Info. Science and Tech*, pp. 14–18, Apr. 2017.



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