Finite-time Control of Discrete-time Systems With Variable Quantization Density in Networked Channels

Yiming Cheng, Xu Zhang, Tianhe Liu, and Changhong Wang

***Abstract*—This paper is concerned with the problem of finite-time control for a class of discrete-time networked systems. The measurement output and control input signals are quantized before being transmitted in communication network. The quantization density of the network is assumed to be variable depending on the throughputs of network for the sake of congestion avoidance. The variation of the quantization density modes satisfies persistent dwell-time (PDT) switching which is more general than dwell-time switching in networked channels. By using a quantization-error-dependent Lyapunov function approach, sufficient conditions are given to ensure that the quantized systems are finite-time stable and finite-time bounded with a prescribed $\mathcal{H}_\infty$ performance, upon which a set of controllers depending on the mode of quantization density are designed. In order to show the effectiveness of the designed $\mathcal{H}_\infty$ controller, we apply the developed theoretical results to a numerical example.**

***Index Terms*—Finite-time, $\mathcal{H}_\infty$ controller design, quantization-error-dependent Lyapunov function, quantized signal.**

I. Introduction

THE past decades have witnessed a rapid advance in studies of networked control systems (NCSs) which are widely applied in power networks [1], fuzzy systems [2], fault detection [3], etc. NCSs, which consist of dispersing system components and signal transmission networks, have more compatibility and application diversity compared with integrated control systems whose system components, such as actuators, controllers, and sensors are located at the same place. Since information exchange between system components heavily rely on the performance of communication networks, many efforts have been devoted to this field, which is seen in [4]–[6] and the references therein.

Although various advantages such as increased flexibility and reduced cost are associated with NCSs, the applicability of communication networks can be seriously affected by limited network capacity, which is generally caused by network congestion. In order to reduce the amount of data transmission, signals should be quantized before transmitted. In practice, network throughput may vary in order to improve system performance, and as a result quantization density should also vary, which may lead to a varying quantization error. Such variation can be modeled via switched system theory, i.e., each quantization density can be regarded as a subsystem mode and the overall networked control system is therefore regarded as a class of switched system. One can address the variation of quantization density using persistent dwell-time (PDT), since its actual variation sequence can hardly be obtained. A PDT switching signal refers to a class of switching signal composed of infinitely many dispersed intervals in which the subsystem mode remains stationary. In the intermissions of such intervals, however, the subsystem mode can randomly switch. Compared with other kinds of switching signal [7]–[9], only a small amount of the current literature has addressed the problems of PDT switching signal.

The controller design problem has been extensively probed for conventional dynamic control systems, and various control strategies are extended to network-based case [10]–[12]. It should be noted that the majority of existing works are based on the hypothesis that a quantizer is associated with only control input or measurement output, and the quantization density is assumed to be invariant. Obviously such assumptions are ideal and may result in an increased quantization error or degraded system performance. Although some efforts have been devoted to addressing the problem of variable quantization densities [13], [14], to the best of our knowledge, the controller design problem for NCSs with quantized control input and measurement output subject to variable quantization densities remains open.

In order to prevent saturation caused by excessive state value of the system, the concept of finite-time stability was introduced in 1953 [15]. Compared with conventional asymptotic stability in Lyapunov sense, finite-time stability is concerned with the state of the systems at each time instead of the trend of system. Therefore, finite-time stability is of practical significance in many fields such as power electronics [16], networked systems [17], flight control systems [18]. Despite the fact that studies of finite-time stability can be found in many literatures [19]–[22], finite-time stability of NCSs, especially NCSs with variable quantization density is

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Y. M. Cheng, T. H. Liu, and C. H. Wang are with the Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin 150001, China (e-mail: cheng8951@126.com; thliu@stu.hit.edu.cn; cwang@hit.edu.cn).

X. Zhang is with the School of Astronautics, Harbin Institute of Technology, Harbin 150001, China (e-mail: zhangxu@hit.edu.cn).

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seldom addressed.

Motivated by the aforementioned discussions, this paper is concerned with finite-time stability analysis and $\mathcal{H}_\infty$ control problems for a class of NCSs with variable quantization density. The contributions of this paper include: 1) the interested quantization density of networked system is modeled as a class of switched systems with persistent dwell time switching signals; 2) a class of Lyapunov-like functions that are both mode-dependent and quantization density-dependent is developed; 3) the switched system with PDT switching is finite-time bounded and has a prescribed $\mathcal{H}_\infty$ performance.

The remainder of this paper is organized as follows. In Section II, the controller design problem is formulated, and preliminary knowledge is given. Section III investigates the finite-time stability analysis result, which is finite-time bounded with the $\mathcal{H}_\infty$ performance analysis result and controller design method. A numerical simulation is performed in Section IV to illustrate the validity and advantage of developed results. Section V concludes this paper.

Notations: $\mathbb{R}^n$ represents the n-dimensional Euclidean space. The zero matrix and the identity matrix are denoted as 0 and I respectively. The matrix inequalities $P \le 0$ ($P < 0$) means that P is symmetric and semi-negative (negative) definite. The matrix inequalities $P \ge 0$ ($P > 0$) means that P is symmetric and semi-positive (positive) definite. The superscripts "-1" represents inverse of a matrix. We use diag$\{\cdot\}$ as a block-diagonal matrix. The symbol "$*$" is used as an ellipsis for the symmetric term in symmetric matrices or complex matrix expressions. $\lambda_{\max}\{P\}$ and $\lambda_{\min}\{P\}$ represent the maximum and minimum eigenvalues of matrix P, respectively.

II. Problem Formulation and Preliminaries

Consider the following discrete-time linear system:

$$x(k+1) = Ax(k) + B\tilde{u}(k) + E\omega(k) \tag{1}$$

$$z(k) = Cx(k) + D\tilde{u}(k) + F\omega(k) \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$, $\tilde{u}(k) \in \mathbb{R}^{n_u}$ and $z(k) \in \mathbb{R}^{n_y}$ represents system state, control input, and system output, respectively; $\omega(k) \in \mathbb{R}^{n_\omega}$ refers to external disturbance belonging to $l_2[0,\infty)$ and A, B, C, D, E, F represent system matrices.

In practice, it is very common to have signal quantized before transmission in order to mitigate network congestion due to limited communication network capacity. As a sketch of networked system layout is shown in Fig. 1, system state $x(k)$ and control input $u(k)$ should both be quantized. In this paper, we are interested in a class of logarithmic quantized signals with the following form:

$$\tilde{x}(k) = Q_{x,\sigma_k}(x(k)),\ \tilde{u}(k) = Q_{u,\sigma_k}(u(k)) \tag{3}$$

where $\tilde{x}(k) \in \mathbb{R}^{n_x}$ is the input of the controller; $Q_{u,\sigma_k}(\cdot)$ and $Q_{x,\sigma_k}(\cdot)$ represent odd-symmetric logarithmic quantizers of the control input channel and the measurement output channel, respectively; σ_k is the switching signal, which is a piecewise constant function taking value in a finite set $\mathcal{I} = \{1,\ldots,L\}$, where L denotes the number of subsystems. The switching sequence $k_1, k_2, \ldots, k_s, \ldots$ are unknown a priori, but are known instantly, in which the switching instants are denoted as k_s, $s \in \mathbb{Z}_+$. When $k \in [k_s, k_{s+1})$, it is said that σ_kth system is active for $k_{s+1} - k_s$. It is assumed that the quantization density may change, and each quantization density corresponds to a subsystem mode. The set of logarithmic quantization levels is depicted as

$$\mathcal{U}_{v,\sigma_k} = \Big\{\pm\mu_{v,\sigma_k,q} | \mu_{v,\sigma_k,q} = \rho^q_{v,\sigma_k}\mu_{v,0},\ q = 0,\ \pm1, \pm2, \ldots\Big\} \cup \{0\} \tag{4}$$

where $v \in \{x, u\}$ indicates the system or controller, with which the quantizer is associated; $\mu_{v,\sigma_k,q} > 0$ represents a quantization level for a corresponding segment that is mapped to this quantization level by the logarithmic quantizer; $\rho_{v,\sigma_k} \in (0,1)$ can be regarded as the quantization density of the quantizer $Q_{v,\sigma_k}(\cdot)$. The associated quantizer $Q_{v,\sigma_k}(\cdot)$ is defined as [10]

$$Q_{v,\sigma_k}(v) = \begin{cases} \mu_{v,\sigma_k,q}, & v_{\min} < v \le v_{\max} \\ 0, & v = 0 \\ -Q_{v,\sigma_k}(-v), & v < 0 \end{cases} \tag{5}$$

where δ_{v,σ_k} is the sector bound of $v(k)$,

$$\delta_{v,\sigma_k} = \frac{1-\rho_{v,\sigma_k}}{1+\rho_{v,\sigma_k}}$$
$$v_{\min} = \frac{\mu_{v,\sigma_k,q}}{1+\delta_{v,\sigma_k}}$$
$$v_{\max} = \frac{\mu_{v,\sigma_k,q}}{1-\delta_{v,\sigma_k}}.$$

Define the quantization errors as

$$e_v(k) = Q_{v,\sigma_k}(v(k)) - v(k) = \Delta_{v,\sigma_k} v(k) \tag{6}$$

where $\Delta_{v,\sigma_k} \in [-\delta_{v,\sigma_k}, \delta_{v,\sigma_k}]$ and the quantizer can therefore be given by

$$\tilde{u}(k) = Q_{u,\sigma_k}(u(k)) = \Big(1 + \Delta_{u,\sigma_k}\Big)u(k) \tag{7}$$

$$\tilde{x}(k) = Q_{x,\sigma_k}(x(k)) = \Big(1 + \Delta_{x,\sigma_k}\Big)x(k). \tag{8}$$

In this paper, we are interested in a class of state feedback controller as follows:

$$u(k) = K_{\sigma_k}\tilde{x}(k) \tag{9}$$

where K_{σ_k} is the controller gain matrix. The resulting closed-loop system can be given by

$$x(k+1) = \tilde{A}_i x(k) + Ew(k) \tag{10}$$

$$z(k) = \tilde{C}_i x(k) + Fw(k) \tag{11}$$

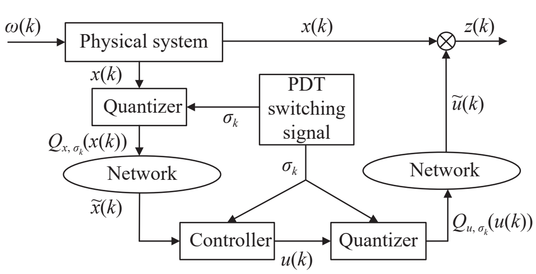

Fig. 1. The quantized networked control system.

where $\tilde{A}_{\sigma_k}(k) = A + \Delta_{u,x,\sigma_k} BK_{\sigma_k}$, $\tilde{C}_{\sigma_k}(k) = C + \Delta_{u,x,\sigma_k} DK_{\sigma_k}$, $\Delta_{u,x,k} = (1+\Delta_{u,\sigma_k})(1+\Delta_{x,\sigma_k})$.

Some definitions should be introduced before proceeding further.

Definition 1 [23]: Consider the switching instants $k_1, k_2, \ldots, k_s, \ldots$ with $k_1 = 0$. A positive constant τ is said to be the persistent dwell-time (PDT) if there exists an infinite number of disjoint intervals of length no smaller than τ on which σ is constant at subsystem Ω_i, and consecutive intervals with this property are separated by no more than T, where T is the period of persistence.

Remark 1: According to the above definition, a PDT switching signal is composed of infinitely many consecutive switching stages. Each stage includes a period with length at least τ and a period with length no greater than T. The former period is called the τ-portion, in which subsystem switching is prohibited, and the latter period is regarded as the T-portion, in which no constraint is applied to the sequence and frequency of subsystem switching.

Remark 2: Some notations for PDT switching signal should be introduced for the sake of conciseness. Let k_p^n denote the actual running time of the T-portion of the pth stage, and $T^{(p)}$ denotes the actual running time of entire T-portion. It follows that $T^{(p)} = \sum_{n=1}^{S[k_p^1,k_{p+1})} T_{\sigma(k_p^n)} \leq T$ where $S[k_p^1,k_{p+1})$ denote the switching times within $[k_p^1,k_{p+1})$. Additionally, k_p indicates the instant entering pth stage and k_p^i is the ith switching instant within T-portion.

Definition 2 [24]: Given positive constants c_1, c_2, N with $c_1 < c_2$, and a positive definite matrix R, consider a finite interval $[k_1,k_N]$ and a certain switching signal $\sigma(k)$, where systems (10) and (11) is finite-time (FT) stable with respect to (c_1,c_2,R,N,σ), if $\{x^T(k_1)Rx(k_1)\} \leq c_1 \Longrightarrow x^T(k)Rx(k) \leq c_2$ for any $k \in [k_1,k_N]$.

Definition 3 [24]: Given positive constants c_1, c_2, d, N with $c_1 < c_2$, and a positive definite matrix R, consider a finite interval $[k_1,k_n]$ and a certain switching signal $\sigma(k)$, where systems (10) and (11) is finite-time bounded with respect to (c_1,c_2,R,d,N,σ), if $\forall \omega(k): \sum_{k=k_1}^{k_n} \omega^T(k)\omega(k) \leq d$, $\{x^T(k_1) Rx(k_1)\} \leq c_1 \Rightarrow x^T(k)Rx(k) \leq c_2$ for any $k \in [k_1,k_n]$.

As a consequence, the main objective of this paper is to determine a set of controllers with appropriate PDT switching signals such that the closed-loop systems (10) and (11) is finite-time bounded under the condition of the quantized signal (3).

III. Main Results

In this section, we present the finite-time stability criteria and the finite-time bounded $\mathcal{H}_\infty$ performance analysis result. Based on the analysis result, a controller design method is proposed under the condition of quantized signal with PDT switching.

Lemma 1: Consider a class of discrete-time switched system $x(k+1) = f_{\sigma(k)}(x(k))$, and c_1, c_2, N, μ, α, T are given positive constants with $c_1 < c_2$, $\mu > 1$, $\alpha \geq 1$. For $\forall(\sigma(k)\times\sigma(k-1)) = (i\times j) \in \mathcal{I}\times\mathcal{I}$, $i \neq j$, suppose that there exists a family of functions $V_{\sigma(k)}: (\mathbb{R}^{n_x}, \mathbb{Z}_+) \to \mathbb{R}$, positive definite matrices R and $\bar{P}_i$, $\bar{P}_i = R^{1/2}P_iR^{1/2}$, such that

$$V_i(x(k+1),k+1) \leq \alpha V_i(x(k),k) \tag{12}$$

$$V_i(x(k),k) \leq \mu V_j(x(k),k) \tag{13}$$

$$c_2\lambda_2\alpha^{-N} > c_1\lambda_1. \tag{14}$$

Then the system is finite-time stable with respect to (c_1,c_2,R,N,σ) for the PDT switching signals satisfying

$$\tau \geq \frac{N(T+1)\ln\mu}{\ln\varphi_1 - \ln\varphi_2 - N\ln\alpha} - T \tag{15}$$

where $\lambda_1 = \max_{\forall i\in\mathcal{I}}(\lambda_{\max}(P_i))$, $\lambda_2 = \min_{\forall i\in\mathcal{I}}(\lambda_{\min}(P_i))$, $\varphi_1 = c_2\lambda_2$, and $\varphi_2 = c_1\lambda_1$.

Proof: Suppose that $\sigma(k_p) = i$, $\sigma(k_p^1 + T^{(p)}) = j$ are the modes of the τ_i portion and the mode at $k_p^1 + T^{(p)}$ in the pth stage of switching, respectively. It then follows from (12)–(14) that

$$\begin{aligned}
&V_j\Big(x(k_{p+1}),k_{p+1}\Big)\\
&\quad\leq \alpha V_j\Big(x(k_{p+1}-1),k_{p+1}-1\Big)\\
&\quad\leq \mu\alpha^{T_l}V_l\Big(x(k_{p+1}-T_l),k_{p+1}-T_l\Big)\\
&\quad\quad\vdots\\
&\quad\leq \mu^{S[k_p^1,k_{p+1})}\prod_{i=1}^{n}\alpha^{T[k_p^i,k_p^{i+1})}V_{\sigma(k_p^1)}\Big(x(k_p^1),k_p^1\Big)\\
&\quad\leq \mu^{S[k_p^1,k_{p+1})}\prod_{i=1}^{n}\alpha^{T[k_p^i,k_p^{i+1})}\times\mu\alpha^{T(k_p,k_p^1)}V_i\Big(x(k_p),k_p\Big)\\
&\quad\leq \mu^{S[k_p^1,k_{p+1})}\alpha^{T}\times\mu\alpha^{\tau}V_i\Big(x(k_p),k_p\Big)
\end{aligned}\tag{16}$$

where T_l denotes the actual running time of the subsystem in the T portion of the pth stage.

For the entire stage, it holds that

$$\begin{aligned}
&V_{\sigma(k_{p+1})}\Big(x(k_{p+1}),k_{p+1}\Big)\\
&\quad\leq \mu^{(S[k_p^1,k_{p+1})+1)p}\alpha^{(T+\tau)p}V_{\sigma(k_1)}(x(k_1),k_1).
\end{aligned}\tag{17}$$

Considering $\bar{P}_i = R^{1/2}P_iR^{1/2}$, it can be derived that

$$\begin{aligned}
&V_{\sigma(k_1)}(x(k_1),k_1)\\
&\quad= x^T(k_1)\bar{P}_{\sigma(k_1)}x(k_1)\\
&\quad\leq \lambda_{\max}\Big(P_{\sigma(k_1)}\Big)x^T(k_1)Rx(k_1) \leq \lambda_1c_1
\end{aligned}\tag{18}$$

and

$$\begin{aligned}
&V_{\sigma(k_{p+1})}\Big(x(k_{p+1}),k_{p+1}\Big)\\
&\quad= x^T(k_{p+1})\bar{P}_{\sigma(k_{p+1})}x(k_{p+1})\\
&\quad\geq \lambda_{\min}\Big(P_{\sigma(k_{p+1})}\Big)x^T(k_{p+1})Rx(k_{p+1})\\
&\quad\geq \lambda_2x^T(k_{p+1})Rx(k_{p+1}).
\end{aligned}\tag{19}$$

From (14), one can obtain that

$$\ln\frac{c_2\lambda_2}{c_1\lambda_1} - N\ln\alpha > 0. \tag{20}$$

Therefore, according to (15) and (20), one can conclude that

$$\ln\frac{\lambda_1}{\lambda_2} + \frac{N(T+1)}{T+\tau}\ln\mu + N\ln\alpha < \ln\frac{c_2}{c_1}. \tag{21}$$

Due to the fact that

$$S\left[k_p^1,k_{p+1}\right)\le T, N=\left(k_{p+1}-k_1\right),\qquad p\le\frac{k_{p+1}-k_1}{T+\tau}$$

one can obtain that

$$\ln\frac{\lambda_1}{\lambda_2}+\left(S\left[k_p^1,k_{p+1}\right)+1\right)p\ln\mu+(T+\tau)p\ln\alpha<\ln c_2-\ln c_1. \tag{22}$$

Based on (17)–(19) and (22), it follows that

$$x^T\left(k_{p+1}\right)Rx\left(k_{p+1}\right)\le\frac{\lambda_1}{\lambda_2}\mu^{\left(S\left[k_p^1,k_{p+1}\right)+1\right)p}\alpha^{(T+\tau)p}c_1<c_2. \tag{23}$$

According to Definition 2, the system is finite-time stable with respect to (c_1,c_2,R,N,σ) for PDT switching signals (15). ■

Remark 3: Due to the difference between globally uniformly asymptotically stability and finite-time stability, the PDT signal obtained in this paper distinguishes from the one in [23]. To be specific, in this paper the PDT is associated with matrix eigenvalues of Lyapunov function. It is noted that in the case of zero initial condition, i.e., $c_1=0$, the inequality (14) is tenable and the PDT switching signal is unrelated to the maximum eigenvalue of a matrix.

It can be seen that the sufficient conditions of finite-time stability is proposed without exogenous disturbances in Lemma 1. In order to suppress disturbance and achieve $\mathcal{H}_\infty$ performance at the same time, we give Lemma 2 as follows.

Lemma 2: Consider a class of discrete-time systems $x(k+1)=f_{\sigma(k)}(x(k))$, and c_2, N, μ, α, d, T are given positive constants with $\mu>1$, $\alpha\ge1$. For $\forall(\sigma(k)\times\sigma(k-1))=(i\times j)\in\mathcal{I}\times\mathcal{I}$, $i\ne j$, suppose that there exists a family of functions $V_{\sigma(k)}:(\mathbb{R}^{n_x},\mathbb{Z}_+)\to\mathbb{R}$, positive definite matrices R and $\bar{P}_i$, $\bar{P}_i=R^{1/2}P_iR^{1/2}$, such that

$$V_i(x(k+1),k+1)\le\alpha V_i(x(k),k)-\Gamma(k) \tag{24}$$

$$V_i(x(k),k)\le\mu V_j(x(k),k) \tag{25}$$

$$c_2\lambda_2\alpha^{-N}>\gamma^2d. \tag{26}$$

Then the system is finite-time bounded with respect to $(0,c_2,R,d,\gamma_l,N,\sigma)$ for PDT switching signals satisfying

$$\tau\ge\max\{\tau_1,\tau_2\} \tag{27}$$

where $\Gamma(k)=z^T(k)z(k)-\gamma^2\omega(k)\omega(k)$, $\tau_1=\frac{N(T+1)\ln\mu}{\ln\kappa_1-\ln\kappa_2}-T$, $\tau_2=\frac{(T+1)\ln\mu}{\ln\alpha}-T$, $\lambda_1=\max_{\forall i\in\mathcal{I}}(\lambda_{\max}(P_i))$, $\lambda_2=\min_{\forall i\in\mathcal{I}}(\lambda_{\min}(P_i))$, $\kappa_1=c_2\lambda_2(\mu-1)+\gamma^2d\alpha^N$, $\kappa_2=\gamma^2d\alpha^N$, $\gamma_l=\gamma\alpha^N\sqrt{\mu^{(T+1)/(T+\tau)}}$.

Proof: The inequality (24) implies that

$$\begin{aligned}V_i&(x(k+1),k+1)\\&\le\alpha V_i(x(k),k)-\left(z^T(k)z(k)-\gamma^2\omega(k)\omega(k)\right)\\&\le\alpha V_i(x(k),k)+\gamma^2\omega(k)\omega(k).\end{aligned} \tag{28}$$

According to (28), one can obtain that

$$\begin{aligned}V_{\sigma(k_p^l)}&\left(x(k_{p+1}),k_{p+1}\right)\\&\le\mu\alpha^{T_l}V_{\sigma(k_p^m)}\left(x(k_p^l),k_p^l\right)+\alpha^{T_l-1}\gamma^2\omega^T\left(k_p^l\right)\omega\left(k_p^l\right)\\&\quad+\cdots+\alpha\gamma^2\omega^T\left(k_{p+1}-2\right)\omega\left(k_{p+1}-2\right)\\&\quad+\gamma^2\omega^T\left(k_{p+1}-1\right)\omega\left(k_{p+1}-1\right)\\&\quad\vdots\\&\le\mu^{S\left[k_p^1,k_{p+1}\right)}\alpha^T\times\mu\alpha^\tau V_{\sigma(k_p)}\left(x(k_p),k_p\right)\\&\quad+\gamma^2d\left[\prod_{i=1}^{l}\mu^i\alpha^{T(k_p^i,k_p^{i+1})}+\cdots+\alpha^{T\left(k_p^l,k_{p+1}\right)}\mu+1\right]\\&\le\mu^{N(T+1)/(T+\tau)}\alpha^NV_{\sigma(k_1)}(x(k_1),k_1)\\&\quad+\gamma^2d\alpha^N\frac{\mu^{(T+1)N/(T+\tau)}-1}{\mu-1}.\end{aligned} \tag{29}$$

At the initial time k_1, one can obtain that

$$V_{\sigma(k_1)}(x(k_1),k_1)=0. \tag{30}$$

At the final time k_{p+1}, one can get that

$$V_{\sigma(k_p^l)}\left(x(k_{p+1}),k_{p+1}\right)\ge\lambda_2x^T(k_{p+1})Rx(k_{p+1}) \tag{31}$$

which implies that

$$x^T(k_{p+1})Rx(k_{p+1})\le\frac{1}{\lambda_2}V_{\sigma(k_p^l)}\left(x(k_{p+1}),k_{p+1}\right). \tag{32}$$

Therefore, according to (27), one can conclude that

$$T+\tau\ge\frac{N(T+1)\ln\mu}{\ln\left(c_2\lambda_2(\mu-1)+\gamma^2d\alpha^N\right)-\ln\left(\gamma^2d\alpha^N\right)}. \tag{33}$$

From (33), it is derived that

$$\mu^{\frac{N(T+1)}{T+\tau}}\le\frac{c_2\lambda_2(\mu-1)+\gamma^2d\alpha^N}{\gamma^2d\alpha^N}$$

which results in

$$\frac{\gamma^2d\alpha^N\left[\mu^{\frac{N(T+1)}{T+\tau}}-1\right]}{\mu-1}<c_2\lambda_2. \tag{34}$$

Based on (30)–(32) and (34), under the zero initial condition, one can conclude

$$x^T(k_{p+1})Rx(k_{p+1})\le\frac{\gamma^2d\alpha^N}{\lambda_2}\frac{\mu^{\frac{N(T+1)}{T+\tau}}-1}{\mu-1}<c_2. \tag{35}$$

As Definition 3 stated, the system is finite-time bounded with respect to $(0,c_2,R,d,N,\sigma)$ for PDT switching signals. Furthermore, considering the H_∞ performance, from (17), it follows that

$$\begin{aligned}V_{\sigma(k_n)}&(x(k_n),k_n)\\&\le\mu^{S(k_1,k_n)}\alpha^{k_n-k_1}V_{\sigma(k_1)}(x(k_1),k_1)\\&\quad-\sum_{l=k_1}^{k_n-1}\mu^{S(l,k_n)}\alpha^{k_n-1-l}\Gamma(l).\end{aligned}$$

As a consequence, one has $V_{\sigma(k_1)}(x(k_1),k_1)=0$ under the zero initial condition. Due to the fact that

$$V_{\sigma(k_n)}(x(k_n),k_n)\ge0$$

and

$$\Gamma(l) = z^T(l)z(l) - \gamma^2\omega(l)\omega(l)$$

it follows that

$$\sum_{l=k_1}^{k_n-1} \mu^{S(l,k_n)}\alpha^{k_n-1-l}\left(z^T(l)z(l) - \gamma^2\omega(l)\omega(l)\right) \le 0.$$

Considering $n \in \mathbb{Z}_{\ge 2}$, it is derived that

$$\begin{aligned}
&\sum_{l=k_1}^{k_n-1} \mu^{S(l,k_n)}\alpha^{k_n-1-l}z^T(l)z(l) \\
&\le \sum_{l=k_1}^{k_n-1} \mu^{\frac{k_n-l}{T+\tau}S_{\max}}\alpha^{k_n-1-l}\gamma^2\omega(l)\omega(l) \\
&\le \sum_{l=k_1}^{k_n-1} \mu^{\frac{1}{T+\tau}S_{\max}}\mu^{\frac{k_n-1-l}{T+\tau}S_{\max}}\alpha^{k_n-1-l}\gamma^2\omega(l)\omega(l) \\
&\le \mu^{\frac{1}{T+\tau}S_{\max}}\sum_{l=k_1}^{k_n-1} \mu^{\frac{k_n-1-l}{T+\tau}S_{\max}}\alpha^{k_n-1-l}\gamma^2\omega(l)\omega(l) \\
&\le \mu^{\frac{1}{T+\tau}S_{\max}}\sum_{l=k_1}^{k_n-1} \left(\mu^{S_{\max}}\alpha^{T+\tau}\right)^{\frac{k_n-1-l}{T+\tau}}\gamma^2\omega(l)\omega(l)
\end{aligned}$$

where $S_{\max} = T+1$. From (27), it implies that $\alpha^{T+\tau} \ge \mu^{S_{\max}}$. Thus, it satisfies that

$$\begin{aligned}
&\sum_{l=k_1}^{k_n-1} z^T(l)z(l) \\
&\le \sum_{l=k_1}^{k_n-1} \mu^{S(l,k_n)}\alpha^{k-1-l}z^T(l)z(l) \\
&\le \gamma^2\mu^{\frac{1}{T+\tau}S_{\max}}\sum_{l=k_1}^{k_n-1}\alpha^{2(k_n-1-l)}\omega^T(l)\omega(l).
\end{aligned}$$

Suppose that $k_1 = 0$, $k_n = N+1$, it follows that

$$\begin{aligned}
&\sum_{l=0}^{N}\alpha^{2(k_n-1-l)}\omega^T(l)\omega(l) \\
&= \alpha^{2N}\omega(0)\omega(0) + \alpha^{2(N-1)}\omega(1)\omega(1) \\
&\quad + \cdots + \omega(N)\omega(N) \\
&\le \alpha^{2N}\sum_{l=0}^{N}\omega(l)\omega(l).
\end{aligned}$$

Therefore, one can conclude that

$$\sum_{l=0}^{N} z^T(l)z(l) \le \gamma^2\mu^{\frac{1}{T+\tau}S_{\max}}\alpha^{2N}\sum_{l=0}^{N}\omega(l)\omega(l) \le \gamma_l^2\sum_{l=0}^{N}\omega(l)\omega(l) \tag{36}$$

where

$$\gamma_l = \gamma\alpha^N\sqrt{\mu^{(T+1)/(T+\tau)}}.$$

■

Remark 4: It is evident that the $\mathcal{H}_\infty$ performance index γ_l is affected by the parameters of the PDT signal and γ. Moreover, γ_l grows with the increase of α and μ.

It can be seen that the finite-time stability and the finite-time boundness with $\mathcal{H}_\infty$ performance are considered in Lemma 1 and Lemma 2, respectively. A quantization-error dependent (QED) Lyapunov function which is distinguished from the one in [10] consists of a class of multiple Lyapunov-like functions dependent on both system mode and quantization error $\Delta_{x,u,k}$. Multiple Lyapunov-like functions can effectively reduce the analysis conservatism compared with the common Lyapunov function and the QED Lyapunov function can overcome the effect of signal quantization error. The QED Lyapunov function is proposed as follows:

$$V_i(x(k),k) = x^T(k)\bar{P}_i(\Delta_{x,u,k})x(k)$$

where $\bar{P}_i(\Delta_{x,u,k}) = \beta_{i,1}\bar{P}_{i,1} + \beta_{i,2}\bar{P}_{i,2} + \beta_{i,3}\bar{P}_{i,3} + \beta_{i,4}\bar{P}_{i,4}$, with $\bar{P}_{i,b} > 0, \forall b \in \{1,2,3,4\}$

$$\begin{aligned}
\beta_{i,1} &= \frac{(\delta_{x,i}+\Delta_{x,i,k})}{2\delta_{x,i}}\frac{(\delta_{u,i}+\Delta_{u,i,k})}{2\delta_{u,i}} \\
\beta_{i,2} &= \frac{(\delta_{x,i}+\Delta_{x,i,k})}{2\delta_{x,i}}\frac{(\delta_{u,i}-\Delta_{u,i,k})}{2\delta_{u,i}} \\
\beta_{i,3} &= \frac{(\delta_{x,i}-\Delta_{x,i,k})}{2\delta_{x,i}}\frac{(\delta_{u,i}+\Delta_{u,i,k})}{2\delta_{u,i}} \\
\beta_{i,4} &= \frac{(\delta_{x,i}-\Delta_{x,i,k})}{2\delta_{x,i}}\frac{(\delta_{u,i}-\Delta_{u,i,k})}{2\delta_{u,i}}
\end{aligned}$$

and $\sum_{r=1}^{4}\beta_r = 1$.

Lemma 3: Consider discrete-time switched system (10)-(11), and c_1, c_2, N, μ, α, T are given positive constants with $c_1 < c_2$, $\mu > 1$, $\alpha \ge 1$. For $\forall(\sigma(k)\times\sigma(k-1)) = (i\times j) \in \mathcal{I}\times\mathcal{I}$, $i \ne j$, $\forall a,b \in \{1,2,3,4\}$, suppose that there exists positive definite matrix R, $\bar{P}_{i,a} = R^{1/2}P_{i,a}R^{1/2}$, if there exists a set of matrices $\bar{P}_{i,a} > 0$, such that (14) is satisfied, and

$$\begin{bmatrix} -\bar{P}_{i,a} & \bar{P}_{i,a}\tilde{A}_{i,b} \\ * & -\alpha_i\bar{P}_{i,b} \end{bmatrix} < 0 \tag{37}$$

$$\sum_{r=1}^{4}\beta_{i,r}\bar{P}_{i,r} \le \mu\sum_{r=1}^{4}\beta_{j,r}\bar{P}_{j,r} \tag{38}$$

where

$$\begin{aligned}
\tilde{A}_{i,1} &= A + (1+\delta_{x,i})(1+\delta_{u,i})BK_i \\
\tilde{A}_{i,2} &= A + (1+\delta_{x,i})(1-\delta_{u,i})BK_i \\
\tilde{A}_{i,3} &= A + (1-\delta_{x,i})(1+\delta_{u,i})BK_i \\
\tilde{A}_{i,4} &= A + (1-\delta_{x,i})(1-\delta_{u,i})BK_i.
\end{aligned}$$

Then, the switched system is finite-time stable with respect to (c_1,c_2,R,N,σ) for PDT switching signals satisfying (15) where $\lambda_1 = \max_{\forall i\in\mathcal{I}}(\lambda_{\max}(P_{i,a}))$, $\lambda_2 = \min_{\forall i\in\mathcal{I}}(\lambda_{\min}(P_{i,a}))$.

Proof: if (37) is satisfied, for $\forall b \in \{1,2,3,4\}$ one can obtain that

$$\begin{bmatrix} -\bar{P}_{i,1} & \bar{P}_{i,1}\tilde{A}_{i,b} \\ * & -\alpha\bar{P}_{i,b} \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} -\bar{P}_{i,2} & \bar{P}_{i,2}\tilde{A}_{i,b} \\ * & -\alpha\bar{P}_{i,b} \end{bmatrix} < 0 \tag{40}$$

$$\begin{bmatrix} -\bar{P}_{i,3} & \bar{P}_{i,3}\tilde{A}_{i,b} \\ * & -\alpha\bar{P}_{i,b} \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} -\bar{P}_{i,4} & \bar{P}_{i,4}\tilde{A}_{i,b} \\ * & -\alpha\bar{P}_{i,b} \end{bmatrix} < 0. \tag{42}$$

By multiplying both sides of (39)–(42) by β_1, β_2, β_3 and β_4, respectively, and summing up their results, it can be obtained that

$$\begin{bmatrix} -\bar{P}_i(\Delta_{x,u,k}) & \bar{P}_i(\Delta_{x,u,k})\tilde{A}_{i,b} \\ * & -\alpha\bar{P}_{i,b} \end{bmatrix} < 0. \tag{43}$$

By the Schur complement, it is easy to show

$$\tilde{A}_{i,b}^T\bar{P}_i(\Delta_{x,u,k})\tilde{A}_{i,b} - \alpha\bar{P}_i(\Delta_{x,u,k}) < 0. \tag{44}$$

Hence, (44) and (38) implies that (12) and (13) are satisfied, which guarantees the systems (10) and (11) is finite-time stable. ■

Remark 5: By setting $\bar{P}_{i,m} = \bar{P}_{i,n}$, $\forall m,n \in \{1,2,3,4\}$ in Lemma 3, the Lyapunov function is reduced to conventional multiple Lyapunov-like functions, which is of greater conservatism. Since it is difficult to obtain the quantization error, one can assume that the ratio of quantization error to quantization bound is the same at the instant just before and the instant just after the switching, which means that $\Delta_{x,i,k}/\delta_{x,i} = \Delta_{x,j,k}/\delta_{x,j}$ and $\Delta_{u,i,k}/\delta_{u,i} = \Delta_{u,j,k}/\delta_{u,j}$. It is noted that the assumption is only applied in the switching instant and there are no restrictions on the quantization error at other instants. Therefore, the theoretical analysis based on the assumption is believable. The inequalities (38) can be simplified as

$$\bar{P}_{i,a} - \mu\bar{P}_{j,a} \leq 0. \tag{45}$$

The following lemma and theorem are based on the assumption that the ratio of quantization error to quantization bound is same at different modes in switching adjoining times. In order to suppress disturbance and achieve $\mathcal{H}_\infty$ performance, Lemma 4 is proposed as follows.

Lemma 4: Consider discrete-time switched system (10,11), where c_2, N, μ, α, d, T are given positive constants with $\mu > 1$, $\alpha \geq 1$.For$\forall(\sigma(k)\times\sigma(k-1)) = (i\times j) \in \mathcal{I}\times\mathcal{I}, \forall a,b \in \{1,2,3,4\}$, suppose that there exists a positive definite matrix R, $\bar{P}_{i,a} = R^{1/2}P_{i,a}R^{1/2}$, if there exists a set of matrices $\bar{P}_{i,a} > 0$, such that (26) is satisfied

$$\Xi_i < 0 \tag{46}$$

$$\bar{P}_{i,a} - \mu\bar{P}_{j,a} \leq 0 \tag{47}$$

where

$$\Xi_i = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix}$$

$$\Xi_{11} = -\mathrm{diag}\{\bar{P}_{i,a}, I\}$$

$$\Xi_{12} = \begin{bmatrix} \bar{P}_{i,a}\tilde{A}_{i,b} & \bar{P}_{i,a}E \\ \tilde{C}_{i,b} & F \end{bmatrix}$$

$$\Xi_{22} = -\mathrm{diag}\{\alpha_i\bar{P}_{i,b}, \gamma^2\}.$$

Then, the corresponding system is finite-time bounded with respect to $(0,c_2,R,d,N,\sigma)$ for the PDT switching signal satisfying (27) and has an $\mathcal{H}_\infty$ performance index no greater than γ_l.

Proof: By defining $\xi(k) = \left[x^T(k)\; w^T(k)\right]^T$, one can obtain that

$$V_i(x(k+1),k+1) - \alpha V_i(x(k),k) + \Gamma(k) = \xi^T(k)\Phi_i\xi(k)$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{48}$$

with

$$\Phi_{11} = \tilde{A}_i^T\bar{P}_i\tilde{A}_i + \tilde{C}_i^T\tilde{C}_i - \alpha_i\bar{P}_i$$
$$\Phi_{12} = \tilde{A}_i^T\bar{P}_iE + \tilde{C}_i^TF$$
$$\Phi_{21} = E^T\bar{P}_i\tilde{A}_i + F^T\tilde{C}_i$$
$$\Phi_{22} = E^T\bar{P}_iE + F^TF - \gamma^2.$$

If (46) holds, by applying the same approach in Lemma 3, it can be implied that

$$\begin{bmatrix} -\bar{P}_i & 0 & \bar{P}_i\tilde{A}_i & P_iE \\ * & -I & \tilde{C}_i & F \\ * & * & -\alpha\bar{P}_i & 0 \\ * & * & * & -\gamma^2 \end{bmatrix} \leq 0. \tag{49}$$

By the Schur complement, one can observe that $\Phi_i \leq 0$. Furthermore, one can obtain that (24) holds. Similarly, (47) implies that (25) holds. Hence, (46) and (47) guarantee that the systems (10) and (11) is finite-time bounded. ■

Obviously there exists cross couplings of matrices which have different modes as shown in (46), Lemma 4 can not be used for controller design. Therefore, we present the following controller design method.

Theorem 1: Consider the discrete-time switched systems (10) and (11), where c_2, N, d, μ, α, T are given positive constants with $\mu > 1$, $\alpha \geq 1$. For $\forall(\sigma(k)\times\sigma(k-1)) = (i\times j) \in \mathcal{I}\times\mathcal{I}$, $\forall a,b \in \{1,2,3,4\}$, suppose that there exists a positive definite matrix R, $\bar{Z}_{i,a} = Y^T\bar{P}_{i,a}Y$, $W_{i,a} = \bar{Z}_{i,a} - Y^T - Y$ and $\bar{P}_{i,a} = R^{1/2}P_{i,a}R^{1/2}$; if there exists a set of matrices X_i, Y, $\bar{Z}_{i,a} > 0$ such that (26) is satisfied

$$\Omega_i < 0 \tag{50}$$

$$W_{i,a} - \mu W_{j,a} < 0 \tag{51}$$

where $\varpi_1 = (1+\delta_x)(1+\delta_u)$, $\varpi_2 = (1+\delta_x)(1-\delta_u)$, $\varpi_3 = (1-\delta_x)(1+\delta_u)$, $\varpi_4 = (1-\delta_x)(1-\delta_u)$ and

$$\Omega_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix}$$

$$\Omega_{11} = -\mathrm{diag}\{W_{i,a}, I\}$$

$$\Omega_{12} = \begin{bmatrix} A_iY + \varpi_bB_iX_i & E \\ C_iY + \varpi_bD_iX_i & F \end{bmatrix}$$

$$\Omega_{22} = -\mathrm{diag}\{\alpha_i\bar{Z}_{i,b}, \gamma^2I\}$$

then there exists a set of controllers such that the system is finite-time bounded with respect to $(0, c_2, R, d, N, \sigma)$ for the PDT switching signal satisfying (27) with the $\mathcal{H}_\infty$ performance index no greater than γ_l where $\lambda_1 = \max_{\forall i\in\mathcal{I}}(\lambda_{\max}(P_{i,a}))$, $\lambda_2 = \min_{\forall i\in\mathcal{I}}(\lambda_{\min}(P_{i,a}))$. Moreover, if (50) and (51) have a solution, the admissible controller can be given by

$$K_i = X_i Y^{-1}. \tag{52}$$

Proof: Since $\bar{P}_{i,a} > 0$, it follows that

$$\left(Y - \bar{P}_{i,a}^{-1}\right)^T \bar{P}_{i,a}\left(Y - \bar{P}_{i,a}^{-1}\right) > 0$$

which implies $W_{i,a} > -\bar{P}_{i,a}^{-1}$. With $X_i = K_i Y$, it implies that, from (50)

$$\begin{bmatrix} -\bar{P}_{i,a}^{-1} & 0 & (A_i + \varpi_b B_i K_i)Y & E \\ * & -I & (C_i + \varpi_b D_i K_i)Y & F \\ * & * & -\alpha Y^T \bar{P}_{i,b} Y & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{53}$$

Performing congruence transformations to (53) by $\mathrm{diag}\{I, I, V^{-1}, I\}$, one can obtain that

$$\begin{bmatrix} -\bar{P}_{i,a}^{-1} & 0 & A_i + \varpi_b B_i K_i & E \\ * & -I & C_i + \varpi_b D_i K_i & F \\ * & * & -\alpha \bar{P}_{i,b} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{54}$$

If (54) holds, by applying the same approach in Lemma 3 and Schur complement, one can finally obtain $\Theta_i < 0$ in Lemma 4. From (51), one can imply that (47) is satisfied. Hence, (50) and (51) guarantee the systems (10) and (11) is finite-time bounded. ■

Remark 6: It is worth noting that the PDT switching signal (27) can not be calculated, since the parameter γ is not given in this paper which distinguishes from the one in [22]. We can get the minimum value of parameter γ and the matrix eigenvalues by solving matrix inequalities (50) and (51).

IV. Numerical Example

A numerical example is presented to show the validity of the obtained theoretical results. Consider a class of NCSs (1) and (2) given by

$$A = \begin{bmatrix} 0.6 & 0.24 \\ 1.5 & 0.6 \end{bmatrix}, B = \begin{bmatrix} -0.42 & -2.4 \end{bmatrix}^T$$
$$E = \begin{bmatrix} 0.48 & 0.84 \end{bmatrix}^T, C = \begin{bmatrix} 0.18 & 0.12 \end{bmatrix}$$
$$D = 0.18, \ F = 0.3$$

and a zero initial condition with the following exogenous disturbance :

$$\omega(k) = 0.04\cos(k)e^{-0.5k}. \tag{55}$$

Assigning associated parameters $\alpha = 1.01$, $\mu = 1.03$, $c_1 = 0$, $c_2 = 200$, $R = I$, $N = 40$, $d = 0.018$, and period of persistence $T = 3$. Suppose that the quantization density may vary between two modes and the variation is subject to PDT switching signal. The maximum error bounds in Q_{x,σ_k} and Q_{u,σ_k} are assigned to be

$$\begin{aligned} &\text{Mode 1}: \ \delta_{x,1} = 0.02, \ \delta_{u,1} = 0.04 \\ &\text{Mode 2}: \ \delta_{x,2} = 0.05, \ \delta_{u,2} = 0.07. \end{aligned} \tag{56}$$

One can obtain that $\lambda_1 = 0.2976$, $\lambda_2 = 0.00043$, $\gamma = 0.5751$, $\tau_1 = 15.3581$, $\tau_2 = 8.8825$ and the minimal PDT $\tau = 16 \geq \max\{\tau_1, \tau_2\}$. In order to verify the correctness of the developed results, the state response of the open-loop system under the zero initial condition is depicted in Fig. 2, from which can be clearly seen that the uncontrolled system diverges. One can obtain a set of controllers and the performance index $\gamma_l = 0.8590$ based on Theorem 1. Fig. 3 demonstrates the performance of the closed-loop system with controllers obtained by Theorem 1. Compared with the open-loop system in Fig. 2, the state response of the closed-loop system converges in Fig. 3. Therefore, the designed controller which is against the signal quantization error in the networked channel is effective. As shown in Fig. 4, it can be seen that $x^T(k)Rx(k) < 3\times 10^{-4}$ which means that the closed-loop system is finite-time bounded.

For the definition of the actual $\mathcal{H}_\infty$ performance index γ_{real} and the obtained $\gamma_l = \gamma\sqrt{\Psi}$ in Lemma 2, one can obtain that $\gamma_{\text{real}} = 0.7024 < \gamma_l = 0.8590$. It suggests that the obtained $\mathcal{H}_\infty$ performance can be well guaranteed for these different maximum quantization errors in (56); thus the effectiveness of

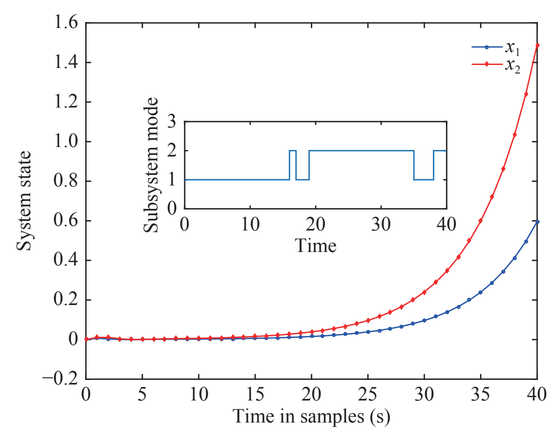

Fig. 2. State response of the open-loop system.

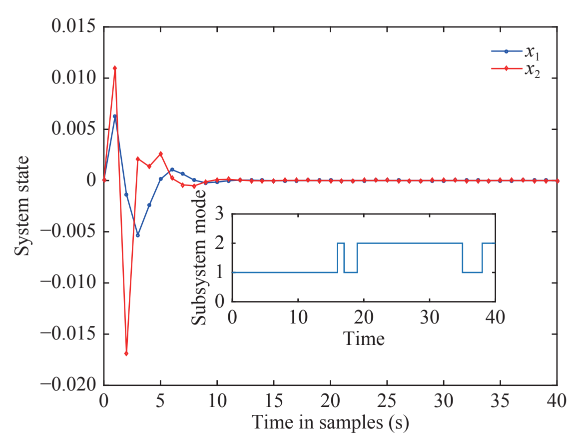

Fig. 3. State response of the closed-loop system.

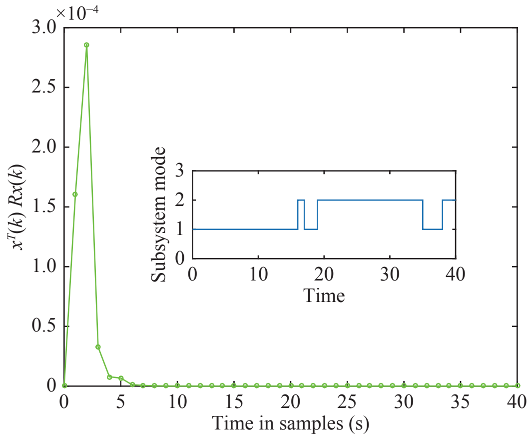

Fig. 4. $x^T(k)Rx(k)$ of the closed-loop system.

the designed $\mathcal{H}_\infty$ controller has been manifested.

To elucidate the influence of the obtained $\mathcal{H}_\infty$ performance index γ_l in different quantization error bounds of the two modes, one assumes that all parameters remain the same, moreover

$$\begin{aligned}\delta_1 &= \delta_{x,1} = \delta_{u,1}\\ \delta_2 &= \delta_{x,2} = \delta_{u,2}.\end{aligned}\tag{57}$$

By simulation and calculation, one can obtain the $\mathcal{H}_\infty$ performance index γ_l in different quantization error bounds of two modes as shown in Fig. 5. It is easy to see that γ_l grows with the increase of δ_1 and δ_2. Moreover, there exists upper bound $\delta^{\max}$ to δ_1 and δ_2. As shown in Fig. 5, the rate of γ_l growth is faster when δ_1 or δ_2 is close to the upper bound. In addition, there is no feasible solution, if δ_1 or δ_2 is greater than $\delta^{\max} = 0.54$. Fig. 6 is given to illustrate that γ_l grows with the increase of α in different quantization error bounds.

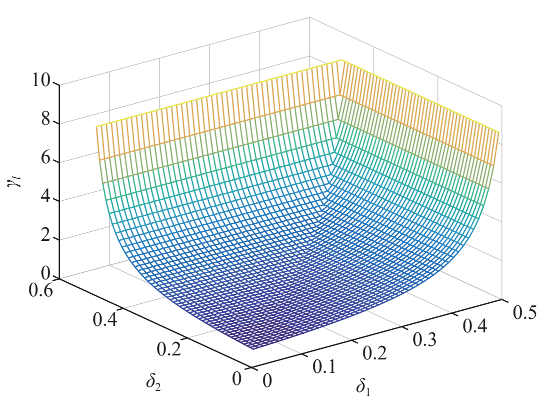

Fig. 5. γ_l in different quantization error bounds of two modes.

V. Conclusion

This paper investigates the finite-time control problem for a class of NCSs with signal quantization density variation. A quantization error dependent Lyapunov function is adopted, and the finite-time bounded analysis and $\mathcal{H}_\infty$ performance analysis are carried out. Based on the analysis results, a set of $\mathcal{H}_\infty$ controllers suitable for the interested NCSs are designed to guarantee finite-time boundedness along with $\mathcal{H}_\infty$

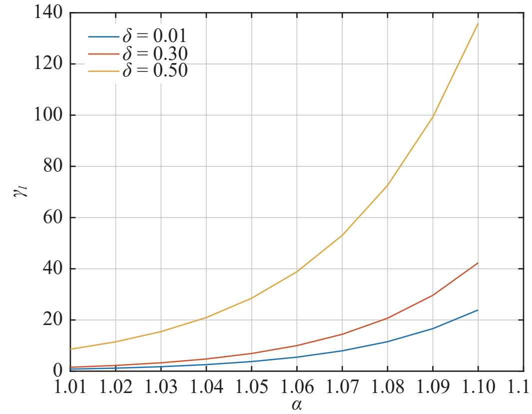

Fig. 6. γ_l for different α in different quantization error bounds.

performance. A numerical example is provided to illustrate the validity and potential of the developed results. We will carry out practical systems in order to support theoretical results in the future work.

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Yiming Cheng received the B.S. and M.S degrees in control science and engineering from Harbin Institute of Technology in 2012 and 2014, respectively. He is currently a Ph.D. candidate at Harbin Institute of Technology. His research interests include switched system control and spacecraft control.

Xu Zhang received the B.S. and M.S degrees in mechanical and electronic engineering from Harbin Institute of Technology in 2005 and 2010, respectively. His research interests include intelligent systems and control and switched system control.

Tianhe Liu received the B.S. and M.S degrees in electronic engineering from Purdue University and Georgia Institute of Technology in 2012 and 2013, respectively. He is currently a Ph.D. candidate at Harbin Institute of Technology. His research interests include switched system control and integrated navigation systems.

Changhong Wang received the B.S., M.E., and Ph.D. degrees in 1983, 1986, and 1991, respectively, all from Harbin Institute of Technology (HIT), Harbin, China. In 1986, he joined the Department of Control Science and Control Engineering, HIT, where he is currently a Full Professor. He served as the Director of Control Theory and Application for five years, and as the Chairman of Department of Control Science and Control Engineering for three years. He is currently the Director of the Space Control and Inertial Technology Research Center, and also the Deputy Dean of HIT (Anshan) Institute of Industrial Technology. His research interests include intelligent systems and control, inertial technology and its testing equipment, and spacecraft control.