Stabilization Parametric Region of Distributed PID Controllers for General First-Order Multi-Agent Systems With Time Delay

Xinyi Yu, Fan Yang, Chao Zou, and Linlin Ou

***Abstract*—The stabilization problem of distributed proportional-integral-derivative (PID) controllers for general first-order multi-agent systems with time delay is investigated in the paper. The closed-loop multi-input multi-output (MIMO) framework in frequency domain is firstly introduced for the multi-agent system. Based on the matrix theory, the whole system is decoupled into several subsystems with respect to the eigenvalues of the Laplacian matrix. Considering that the eigenvalues may be complex numbers, the consensus problem of the multi-agent system is transformed into the stabilizing problem of all the subsystems with complex coefficients. For each subsystem with complex coefficients, the range of admissible proportional gains is analytically determined. Then, the stabilizing region in the space of integral gain and derivative gain for a given proportional gain value is also obtained in an analytical form. The entire stabilizing set can be determined by sweeping proportional gain in the allowable range. The proposed method is conducted for general first-order multi-agent systems under arbitrary topology including undirected and directed graph topology. Besides, the results in the paper provide the basis for the design of distributed PID controllers satisfying different performance criteria. The simulation examples are presented to check the validity of the proposed control strategy.**

***Index Terms*—Consensus, frequency domain, multi-agent systems, stability, time delay.**

I. Introduction

In recent years, the distributed cooperative control of multi-agent systems has attracted extensive attention due to their wide applications in many areas such as formation control [1], distributed computing [2] and sensor networks [3]. One critical issue arising from multi-agent systems is to design distributed control protocols based on local information that enable all agents to reach an agreement on certain quantities of interest, which is known as the consensus problem.

So far, increasing results of consensus problems for multi-agent systems have been obtained. In [4], the consensus problem of first-order integrator multi-agent system was discussed originally. The dynamical consensus algorithm for the second-order multi-agent system was proposed in [5], where all agents achieved the same dynamical value. Sampled-data consensus of the second-order multi-agent systems with time delay was investigated in [6]. The necessary and sufficient conditions for consensus in third order multi-agent systems were obtained in [7]. In addition, other work about the distributed consensus problem for multi-agent systems with different dynamics can be seen in [8]–[11]. In real applications, the agents are generally described by first-order dynamic model. In [12], the model predictive control scheme was applied for multi-agent systems with discrete-time single-integrator dynamics under switching directed interaction graphs. The cluster lag consensus for multi-agent systems with a time-varying communication topology and heterogeneous multi-agent systems with a directed topology was studied in [13]. Hence, the consensus problem of first-order multi-agent systems is very important. It was concluded that the condition for the single-integrator system to achieve consensus was that the network communication topology has a spanning tree. However, such the condition may not ensure the general first-order multi-agent systems to reach consensus.

In addition, in practical applications, time delays often appear when local information data travel along in a large-scale network. Delay effect is an important issue on consensus problems since it may affect the control performance and even its stability [14]. Therefore, it is desirable to design the distributed control protocol for the general first-order multi-agent systems with time delay. Up to date, some literatures have been presented to design distributed control protocols for multi-agent systems with time delay in time domain. In [15], the consensus protocol based on the low gain solution of a parametric algebraic Riccati equation was designed for the multi-agent systems with time-varying communication delay. The effect of quantized dwell times in solving consensus problems in time-delayed multi-agent systems was investigated in [16]. In [17], a predictive tracking controller is proposed to compensate the negative effects caused by bilateral time-delays in a wireless network. Recently, some researches have been devoted to multi-agent systems with time delay in frequency domain. The frequency domain method is proved to be effective for the time-delay issue

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because the time delay is a non-minimum phase term whose amplitude is always equal to 1. In [18], an analytical approach to design H_2 controller of multi-agent systems with time delays was presented. In [19], the H_2 disturbance rejection controller was proposed for synchronized output rejection of multi-agent systems with time delay. The above-mentioned consensus protocols are effective for small-scale multi-agent systems with time delay. When the number of the agents increases, these methods become complicated. Besides, these methods cannot be utilized to design multi-agent systems under directed topology graph since the Laplacian matrix has complex eigenvalues. For multi-agent systems with time delay under directed topology, the design of the distributed controllers is still challenging.

Due to the advantages of the proportional-integral-differential (PID) controllers in control engineering and application, it is desirable to introduce the distributed PID controller into the multi-agent system to improve the consensus performance. A distributed PID protocol was designed for the consensus of homogeneous and heterogeneous networks using appropriate state transformations and Lyapunov functions [20]. In [21], the H_∞ PID feedback for arbitrary-order delayed multi-agent systems was investigated. In the actual engineering application, the distributed PID controllers are generally required to meet several performance criteria simultaneously. To reach this purpose, a natural idea is to first present the stabilizing regions of the distributed PID controllers and then design the PID controller by finding the intersection of the control parameters meeting each required criterion. Thus, the problem of determining the stabilizing PID region is important from a practical viewpoint. Some effective approaches on presenting the stabilizing region of the PID controller have been reported for linear systems with time delay [22], [23]. To the best of the authors' knowledge, there are few literatures on presenting the stabilizing region of the distributed PID controllers for multi-agent systems under directed topology.

Motivated by the above-mentioned discussion, the stabilizing region of the distributed PID controllers is derived for the general first-order multi-agent systems with time delay under fixed topology. Firstly, a multi-input multi-output (MIMO) framework is introduced to uniformly describe the multi-agent systems with time delay in frequency domain. Then based on the matrix theory and graph theory, the multi-agent system is decoupled into several subsystems with respect to the eigenvalues of the Laplacian matrix. Since the eigenvalues may be complex numbers, the consensus problem of the multi-agent systems is transferred into the stabilizing problem of the subsystems with complex coefficients. For each subsystem, the range of admissible proportional gains (k_{P}) is analytically determined. Then, the stabilizing region with respect to the integral gain (k_{I}) and derivative gain (k_{D}) for a given k_{P} value in the range can be also obtained in an analytical form. By computing the intersection of the resultant regions for all the subsystems, the stabilizing (k_{I}, k_{D}) region of the distributed PID controller is derived. The entire set can be determined by sweeping k_{P} in the allowable range. The parameters chosen in the resultant set can guarantee the consensus of the multi-agent system with time delay. The proposed method is conducted for general first-order multi-agent systems with time delay under arbitrary topology, including undirected and directed topologies. Further, the results in the paper solve the stabilization problem of the systems with complex coefficients and provide the basis for the design of distributed PID controllers satisfying different performance criteria.

The paper is organized as follows. In Section II, some basic concepts about the graph theory and the stabilization for the systems with time delay are introduced. The problem statement and the design objective are presented in Section III. The approach for determining the stabilizing set of distributed PID controllers is proposed in Section IV. Numerical examples are provided to demonstrate the validity of the main results in Section V. Finally, the conclusion is given in Section VI.

Notations: We denote the $n\times n$ dimensional identity matrix by I_n. I_n^m refers to $\mathrm{diag}(I_m, 0_{n-m})$. $\mathbf{1}$ represents a column vector with all entries equal to one. The symbols $\otimes$ and $\oplus$ denote the Kronecker product and direct sum, respectively. $\det A$ refers to the determinant of the matrix A. $\deg(F(s))$ represents the degree of the polynomial $F(s)$.

II. Preliminaries

In this section, some basic concepts about the graph theory and the stabilization of time-delayed systems are introduced.

A. Graph Theory

A multi-agent system is assumed to have n agents. The communication topology between agents can be modeled by a graph $G(V,E)$, where $V=\{v_1,\ldots,v_n\}$ is the set of agents and $E\in V\times V$ is the edge set. For a given graph, its adjacency matrix is defined by $A=(a_{ij})_{n\times n}$ where $a_{ij}=1$ if $(v_i,v_j)\in E$ and $a_{ij}=0$ otherwise. The Laplacian matrix of the graph is defined as $L=(l_{ij})_{n\times n}$, where $l_{ij}=-a_{ij}$ if $i\neq j$ and $l_{ii}=\sum_{j=1,\, j\neq i}^{n} a_{ij}$. If $a_{ij}=a_{ji}$, the graph $G(V,E)$ is undirected, otherwise the graph is directed. If there exists a directed path from Node i to any other node, the graph contains a directed spanning tree with the Node i as the root, which is usually regarded as the leader in leader-following topology.

Lemma 1 [24]: The Laplacian matrix L has a simple eigenvalue 0 with the vector $\mathbf{1}$ as the corresponding eigenvector, and all the other eigenvalues have positive parts if and only if the directed network has a spanning tree. Furthermore, the rank of the Laplacian matrix is $n-1$ if the network has a spanning tree.

B. Stabilization of the System With Time Delay

Many problems in control engineering involve time delays. These time delays lead to dynamic models with characteristic equations of the form

$$\delta(s)=D(s)+e^{-T_1 s}N_1(s)+e^{-T_2 s}N_2(s)+\cdots+e^{-T_p s}N_p(s) \quad (1)$$

where $D(s)$, $N_i(s)$ for $i=1,2,\ldots,p$ are the polynomials and T_i for $i=1,2,\ldots,p$ represent the time delays. In this paper, we make the following assumption.

Assumption 1: For the characteristic equation (1), we assume that

1) $\deg(D(s)) = q$ and $\deg(N_i(s)) \le q$ for $i = 1,2,\ldots,p$;

2) $0 < T_1 < T_2 < \cdots < T_p$.

The following lemmas are presented to give sufficient and necessary conditions for the stability of $\delta(s)$.

Lemma 2 [25]: Let $\delta(s)$ be given by (1) and write

$$\delta(j\omega) = \delta_{\rm r}(\omega) + j\delta_{\rm i}(\omega) \tag{2}$$

where $\delta_{\rm r}(\omega)$ and $\delta_{\rm i}(\omega)$ respectively represent the real and imaginary part of $\delta(j\omega)$. Under Assumption 1, $\delta(s)$ is stable if and only if

1) $\delta_{\rm r}(\omega)$ and $\delta_{\rm i}(\omega)$ have only simple real roots and they are interlaced;

2) $\delta_{\rm i}'(\omega_o)\delta_{\rm r}(\omega_o) - \delta_{\rm i}(\omega_o)\delta_{\rm r}'(\omega_o) > 0$ for some $\omega_o \in (-\infty, +\infty)$, where $\delta_{\rm r}'(\omega)$ and $\delta_{\rm i}'(\omega)$ denote the first derivative with respect to ω of $\delta_{\rm r}(\omega)$ and $\delta_{\rm i}(\omega)$, respectively.

Lemma 3 [25]: Let ξ and ζ denote the highest powers of s and e^s in $\delta(s)$. Let η be an appropriate constant such that the coefficients of terms of the highest degree in $\delta_{\rm r}(\omega)$ and $\delta_{\rm i}(\omega)$ do not vanish at $\omega = \eta$. Then for the equations $\delta_{\rm r}(\omega) = 0$ or $\delta_{\rm i}(\omega) = 0$ to have only real roots, it is necessary and sufficient that in the intervals $-2l\pi + \eta \le \omega \le 2l\pi + \eta$, $l = 1,2,3,\ldots$, $\delta_{\rm r}(\omega)$ and $\delta_{\rm i}(\omega)$ have exactly $4l\zeta + \xi$ roots, starting with sufficiently large l.

III. Problem Statement

The topology structure of the multi-agent systems is shown in Fig. 1. We consider the consensus problem of n identical first-order agents. The transfer function of each agent is

$$G_1(s) = \cdots = G_n(s) = G(s) = \frac{K}{1+Ts}e^{-\theta s} \tag{3}$$

where θ is the input time delay of the agent, K is the positive steady-state gain of the agent and T represents the time constant of the agent. The distributed PID controllers imposed on each agent has the following form:

$$C_1(s) = \cdots = C_n(s) = C(s) = k_{\rm P} + \frac{k_{\rm I}}{s} + k_{\rm D}s \tag{4}$$

where $k_{\rm P}$, $k_{\rm I}$, $k_{\rm D}$ are the proportional, integral and differential gains, respectively. The agents are divided into two groups. The first m agents have access to the same reference signal, $r_1(s) = \cdots = r_m(s) = r(s) \ne 0$. The external input can be regarded as the target state. The following $n-m$ agents are only accessible to the relative state errors with respect to their neighboring agents, $r_{m+1}(s) = \cdots = r_n(s) = 0$. According to the definition in [10], the dynamic of each agent can be described as

$$y_i(s) = G(s)u_i(s) = G(s)C(s)e_i(s) \tag{5}$$

where y_i, u_i and e_i are the state, the control input and the system error of Agent i, respectively. The coupling between the agents is caused via the communication channels:

$$\begin{cases} e_i(s) = r(s) - y_i(s) + \sum\limits_{v_i\in N_i} a_{ij}[y_j(s) - y_i(s)], \\ \qquad\qquad \text{for } i = 1,2,\ldots,m \\ e_i(s) = \sum\limits_{v_i\in N_i} a_{ij}[y_j(s) - y_i(s)], \\ \qquad\qquad \text{for } i = m+1, m+2, \ldots, n. \end{cases} \tag{6}$$

From (6), we can rewrite $E(s)$ in the vector form,

$$E(s) = R(s) - I_n^m Y(s) - LY(s) \tag{7}$$

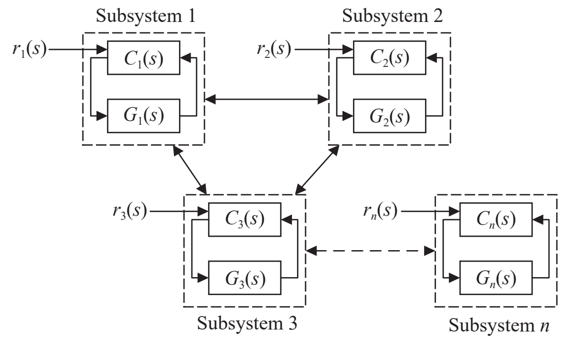

Fig. 1. The control block diagram for the subsystem.

where $R(s) = [r_1(s), \ldots, r_n(s)]$ denote the respected consensus value, L is the Laplacian matrix and $Y(s) = [y_1(s), \ldots, y_n(s)]$ are the output states of n agents. Now, the closed-loop MIMO representation of the networked multi-agent systems shown in Fig. 2 can be established, where $\hat{G}(s) = \oplus\sum_{i=1}^n G(s)$, $\hat{C}(s) = \oplus\sum_{i=1}^n C(s)$ imply that all the agents are identical and each controller has the same structure. The information exchange among these agents can be represented as the matrix $\tilde{L} = L + I_n^m$. The transfer function from $R(s)$ to $Y(s)$ is given by

$$\Delta(s) = [I + \tilde{L}\hat{C}(s)\hat{G}(s)]^{-1}\hat{G}(s)\hat{C}(s). \tag{8}$$

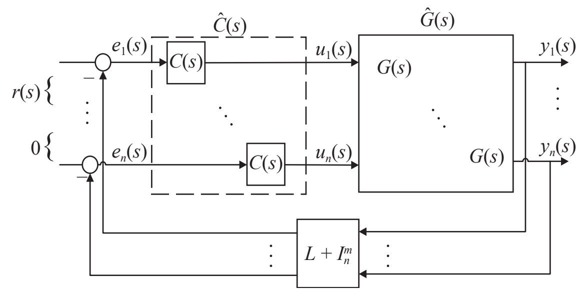

Fig. 2. The MIMO model of the multi-agent system.

For the known agent dynamics and the fixed communication topology, the objective of the paper is to obtain the complete stabilizing set of the distributed PID controllers analytically for general first-order multi-agent systems.

IV. Stabilization Analysis of Distributed PID Controllers

From (8), it can be derived that the characteristic equation of the multi-agent systems is

$$P(s) = \det[I + \tilde{L}\hat{G}(s)\hat{C}(s)]. \tag{9}$$

Define a transform V such that $\Lambda = V^{-1}\tilde{L}V$ is upper triangular with the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of $\hat{L}$ on the diagonal. From Lemma 1, it can be derived that if the graph has a spanning tree, the corresponding Laplacian matrix is diagonalizable. Thus the following equation is presented:

$$\begin{aligned}\det(I + \tilde{L}\hat{G}(s)\hat{C}(s)) &= \det(VV^{-1} + V\Lambda\hat{G}(s)\hat{C}(s)V^{-1}) \\ &= \prod_{i=1}^n \det(1 + \lambda_i C(s)G(s)).\end{aligned} \tag{10}$$

It is seen that all the roots of the characteristic equation (9) are also the roots of the characteristic equation (10). In other word, in order to ensure the stability of the multi-agent systems, all the roots of the characteristic equation $p_i(s) = 1+\lambda_i C(s)G(s)$ must locate on the open left-half complex plane. Therefore, the whole system in Fig. 2 can be decomposed into n isolated subsystems with respect to the eigenvalues of $\tilde{L}$. The structure of each subsystem is shown in Fig. 3.

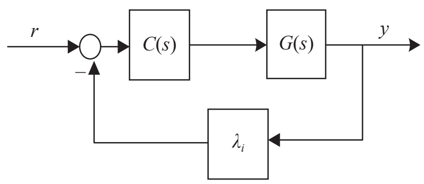

Fig. 3. The control block diagram for the subsystem.

Next, we state the procedure to determine the stabilizing set of distributed PID controllers for the multi-agent system with time delay. The first step of the procedure is to determine the $k_{\rm P}$ range of the stabilizing distributed PID controller. Then for a fixed $k_{\rm P}$ within this range, the stabilizing ($k_{\rm I}$, $k_{\rm D}$) region is derived in the second step.

A. Determination of Admissible $k_{\rm P}$ Range

Assume that the eigenvalues of the Laplacian matrix is

$$\lambda_i = a_i + jb_i = |\lambda_i| e^{j\varphi_i} \tag{11}$$

where a_i, b_i respectively represent real and imaginary part of λ_i, $|\lambda_i|$ is the modulus of λ_i and φ_i is the argument. In following analysis, we suppose that the eigenvalues satisfy the following condition:

$$0 = |\lambda_1| < |\lambda_2| < \cdots < |\lambda_n|. \tag{12}$$

From (10), the stabilizing range of $k_{\rm P}$ can be determined for each decomposed subsystem. The intersection of all the resultant ranges is the stabilizing $k_{\rm P}$ range for the multi-agent system. The closed-loop characteristic equation of each decomposed subsystem is

$$p_i(s) = \lambda_i(Kk_{\rm I} + Kk_{\rm P}s + Kk_{\rm D}s^2)e^{-\theta s} + (1+Ts)s. \tag{13}$$

For the multi-agent systems with undirected topology, all the eigenvalues λ_i are real numbers. The admissible range of $k_{\rm P}$ values can be determined in terms of Theorem 2.1 in [15]. However, for the multi-agent systems with directed topology, the eigenvalues may be complex numbers. In this case, the subsystem will contain complex coefficients such that the stabilizing PID parameters cannot be directly determined.

Rewrite the quasipolynomial $p_i^*(s)$ as

$$\begin{aligned} p_i^*(s) =& \lambda_i(Kk_{\rm I} + Kk_{\rm P}s + Kk_{\rm D}s^2) + (1+Ts)se^{\theta s} \\ =& (\cos\varphi_i + j\sin\varphi_i)\Big(|\lambda_i|(Kk_{\rm I} + Kk_{\rm P}s + Kk_{\rm D}s^2) \\ &+(1+Ts)se^{\theta s - j\varphi_i}\Big). \end{aligned} \tag{14}$$

Denote that

$$\hat{p}_i^*(s) = |\lambda_i|(Kk_{\rm I} + Kk_{\rm D}s^2 + sKk_{\rm P}) + (s+Ts^2)e^{\theta s - j\varphi_i}. \tag{15}$$

Suppose that there is a root $s = \sigma + j\omega$ for $p_i^*(s) = 0$. Substituting $s = \sigma + j\omega$ into (14), it can be derived that

$$\begin{aligned} p_i^*(\sigma + j\omega) =& (\cos\varphi_i + j\sin\varphi_i)\Big(\mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right] \\ &+ j\mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right]\Big) \end{aligned} \tag{16}$$

where $\mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right]$ and $\mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right]$ are the real part and imaginary part of $\hat{p}_i^*(\sigma + j\omega)$, respectively. For the directed topology, it is known that $\varphi_i \neq 0$. Suppose that $\mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right] \neq 0$ and $\mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right] \neq 0$. In this case, if the characteristic equation $p_i^*(s) = 0$ has roots, we have

$$\cos\varphi_i \mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right] - \sin\varphi_i \mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right] = 0 \tag{17}$$

$$\cos\varphi_i \mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right] + \sin\varphi_i \mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right] = 0. \tag{18}$$

Combining (17) and (18), it can be derived that

$$\mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right](\sin^2\varphi_i + \cos^2\varphi_i) = 0. \tag{19}$$

However, from the definition of the argument, it is known that $\sin^2\varphi_i + \cos^2\varphi_i = 1$. Therefore, if $\mathrm{Re}\left[\hat{p}_i^*(\sigma + j\omega)\right] \neq 0$ and $\mathrm{Im}\left[\hat{p}_i^*(\sigma + j\omega)\right] \neq 0$, the equation $p_i^*(s)$ has no roots. To guarantee that $p_i^*(s) = 0$ has roots, the real part and the imaginary part of $\hat{p}_i^*(\sigma + j\omega)$ must be equal to zero. In other words, the stability of the characteristic equation $p_i^*(s)$ is equivalent to the stability of the equation $\hat{p}_i^*(s)$. In what follows, we will discuss the stability of $\hat{p}_i^*(s)$ based on Lemmas 2 and 3.

Substituting $s = j\omega$ into (15), we have

$$\hat{p}_i^*(j\omega) = \mathrm{Re}\left[\hat{p}_i^*(\omega)\right] + j\mathrm{Im}\left[\hat{p}_i^*(\omega)\right] \tag{20}$$

where

$$\begin{aligned} \mathrm{Re}\left[\hat{p}_i^*(\omega)\right] =& K_i k_{\rm I} - K_i k_{\rm D}\omega^2 - \omega\sin(\theta\omega - \varphi_i) \\ &- T\omega^2\cos(\theta\omega - \varphi_i) \end{aligned} \tag{21}$$

and

$$\begin{aligned} \mathrm{Im}\left[\hat{p}_i^*(\omega)\right] =& \omega[K_i k_{\rm P} + \cos(\theta\omega - \varphi_i) \\ &- T\omega\sin(\theta\omega - \varphi_i)] \end{aligned} \tag{22}$$

where, $K_i = |\lambda_i|K$. From (21) and (22), it is seen that the parameter $k_{\rm P}$ only affects the imaginary part of $\hat{p}_i^*(j\omega)$ whereas the parameters $k_{\rm I}$ and $k_{\rm D}$ affect the real part. We now have the following theorem.

Theorem 1: The imaginary part of $\hat{p}_i^*(j\omega)$ has only simple real roots if and only if

$$\begin{cases} -\dfrac{1}{K_n} < k_{\rm P} < \dfrac{1}{K_n}\left[\dfrac{T}{\theta}\alpha_1\sin(\alpha_1 - \varphi_n) - \cos(\alpha_1 - \varphi_n)\right], \\ \qquad\qquad\text{for } T > 0 \\ \dfrac{1}{K_n}\left[\dfrac{T}{\theta}\alpha_1\sin(\alpha_1 - \varphi_n) - \cos(\alpha_1 - \varphi_n)\right] < k_{\rm P} < -\dfrac{1}{K_n}, \\ \qquad\qquad\text{for } T < 0 \text{ and } \left|\dfrac{T}{\theta}\right| > 0.5 \end{cases} \tag{23}$$

where $K_n = K|\lambda_n|$ and α_1 is the solution of the equation

$$\tan(\alpha - \varphi_n) = -\frac{T}{T+\theta}\alpha \tag{24}$$

in the interval $(\varphi_n, \varphi_n + 2\pi)$. For the $k_{\rm P}$ values outside the range provided in (23), there are no stabilizing PID controllers.

Proof: Two cases are considered as follows.

Case 1: $T > 0$

Substituting $z = \theta\omega$ into (21) and (22), we have

$$\mathrm{Re}\left[\hat{p}_i^*(z)\right] = K_i k_\mathrm{I} - \frac{K_i k_\mathrm{D}}{\theta^2} z^2 - \frac{z}{\theta}\sin(z-\varphi_i) - \frac{T}{\theta} z^2 \cos(z-\varphi_i) \quad (25)$$

$$\mathrm{Im}\left[\hat{p}_i^*(z)\right] = \frac{z}{L}\left[K_i k_\mathrm{P} + \cos(z-\varphi_i) - \frac{T}{\theta} z \sin(z-\varphi_i)\right]. \quad (26)$$

From (26), it is clear that $\omega_0 = 0$ is one of the solutions of $\mathrm{Im}\left[\hat{p}_i^*(z)\right] = 0$. Then, we have

$$K_i k_\mathrm{P} + \cos(z-\varphi_i) - \frac{T}{\theta} z \sin(z-\varphi_i) = 0. \quad (27)$$

The other roots are difficult to find from (27). To overcome this problem, we can plot the terms involved in (27) and examine the nature of the solution graphically. Denote the positive root of (27) by $z_t, t = 1, 2, \ldots$, arranged in increasing order of magnitude. Now use Lemma 3 to check if $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ has only real roots. From (22), it is derived that $\xi = 2$ and $\zeta = 1$. Next, choose $\eta = \pi/4$ to satisfy that $\sin(\eta) \neq 0$ and $\cos(\eta) \neq 0$. If we take $l = 1$, $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ must satisfy the condition that there are 6 real roots in $[-7\pi/4, 9\pi/4]$. Denote that

$$f(z) = \frac{K_i k_\mathrm{P} + \cos(z-\varphi_i)}{\sin(z-\varphi_i)}. \quad (28)$$

It can be easily obtained that $f(z)$ is a periodic function with a period of 2π. Differentiating $f(z)$ with respect to z, we have

$$f'(z) = \frac{-1 - K_i k_\mathrm{P}\cos(z-\varphi_i)}{\sin^2(z-\varphi_i)}. \quad (29)$$

There are three cases to consider according to the k_P values.

a) $k_\mathrm{P} < -1/K_i$. In this case, there exists one root for $f'(z) = 0$ in the interval $(\varphi_i, \varphi_i + \pi)$ and one root in $(-\pi + \varphi_i, \varphi_i)$. Considering the characteristic of $\cos(z - \varphi_i)$, it can be derived that there is a maximum in the interval $(\varphi_i, \varphi_i + \pi)$ and a minimum in $(-\pi + \varphi_i, \varphi_i)$. Thereby, the plots of $f(z)$ and $(T/\theta)z$ are shown in Fig. 4.

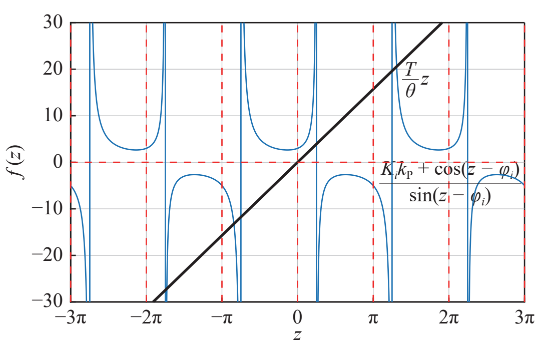

Fig. 4. Plots of terms involved in (27) for $k_\mathrm{P} < -(1/K_i)$.

b) $-1/K_i \le k_\mathrm{P} \le 1/K_i$. In this case, the value of (29) is always small than 0 in one period, which indicates that $f(z)$ is a decreasing function in one period. Hence, we can draw the plots of $f(z)$ and $(T/\theta)z$ in Fig. 5.

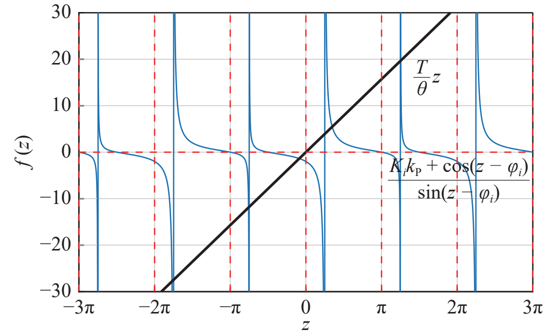

Fig. 5. Plots of terms involved in (27) for $-(1/K_i) \le k_\mathrm{P} \le (1/K_i)$.

c) $1/K_i < k_\mathrm{P}$. The characteristic of $f(z)$ is similar to that in the case a), that is, there is one root for $f'(z) = 0$ in the interval $(\varphi_i, \varphi_i + \pi)$ and one root in $(-\pi + \varphi_i, \varphi_i)$. It is worth noting that, with the increase of k_P, the maximum in the interval $(-\pi + \varphi_i, \varphi_i)$ also increases, which may lead to the result that the curve $f(z)$ will not intersect the line $(T/\theta)z$ twice in this interval. Thus, there are two different plots of $f(z)$ and $(T/\theta)z$ for $1/K_i < k_\mathrm{P}$, which are shown in Fig. 6. The plot in Fig. 6(a) corresponds to the case where $1/K_i < k_\mathrm{P} < \bar{K}_i$, and $\bar{K}_i$ is the largest number so that $f(z)titz)$ intersects $(T/\theta)z$ twice in the interval $(-\pi + \varphi_i, \varphi_i)$. The plot in Fig. 6(b) corresponds to the case where $k_\mathrm{P} \ge \bar{K}_i$ and the plot of $f(z)$ intersects the line $(T/\theta)z$ once in the interval $(-\pi + \varphi_i, \varphi_i)$.

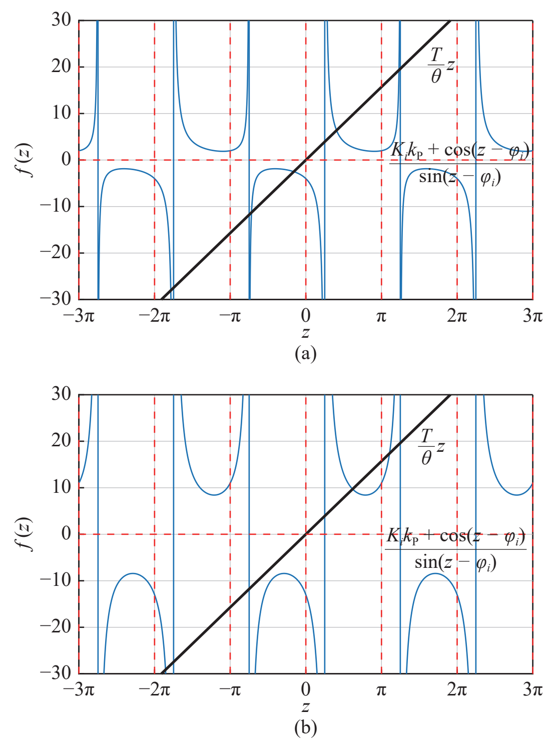

Fig. 6. Plots of terms involved in (27) for $(1/K_i) < k_\mathrm{P}$. (a) $(1/K_i) < k_\mathrm{P} < (1/\bar{K}_i)$; (b) $(1/\bar{K}_i) \le k_\mathrm{P}$.

From Figs. 5 and 6(a), it is seen that for $-1/K_i < k_\mathrm{P} < \bar{K}_i$, $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ has 6 real roots in the interval $[-7\pi/4, 9\pi/4]$. Moreover, it is clear from Figs. 5 and 6(a) that $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ has 4 real roots in each of the intervals $[2l\pi + \pi/4, 2(l+1)\pi + \pi/4]$ and $[-2(l+1)\pi + \pi/4, -2l\pi + \pi/4]$ for $l = 1, 2, \ldots$. Thus, from

Lemma 3, we can conclude that for $-1/K_i < k_P < \bar{K}_i$, $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ has only real roots. Besides, from Figs. 4 and 6(b), it is clear that all the roots of $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ will not be real for the cases $-1/K_i > k_P$ and $k_P > \bar{K}_i$. Next, we should determine the upper bound $\bar{K}_i$ on the allowable value of k_P. From the definition of $\bar{K}_i$, it follows that if $k_P = \bar{K}_i$, $f(z)$ intersects $(T/\theta)z$ once in the interval $(0,\pi)$. Denote the value of z for which this intersection occurs by α_1. Thus, we have the following equations:

$$\frac{K_i\bar{K}_i + \cos(\alpha_1 - \varphi_i)}{\sin(\alpha_1 - \varphi_i)} = \frac{T}{\theta}\alpha_1 \tag{30}$$

$$f'(\alpha_1) = \frac{T}{\theta} \Rightarrow 1 + K_i\bar{K}_i\cos(\alpha_1 - \varphi_i) = -\frac{T}{\theta}\sin^2(\alpha_1 - \varphi_i). \tag{31}$$

Eliminating $K_i\bar{K}_i$ between (30) and (31), we conclude that α_1 can be obtained as a solution of the following equation:

$$\tan(\alpha_1 - \varphi_i) = -\frac{T}{T+\theta}\alpha_1.$$

Once α_1 is determined, the parameter $\bar{K}_i$ is determined by

$$\bar{K}_i = \frac{1}{K_i}\left[\frac{T}{\theta}\alpha_1\sin(\alpha_1 - \varphi_i) - cos(\alpha_1 - \varphi_i)\right]. \tag{32}$$

Thus, for each decomposed subsystem of the multi-agent system, there is a corresponding admissible range of k_P, i.e.,

$$-\frac{1}{K_i} < k_P < \frac{1}{K_i}\left[\frac{T}{\theta}\alpha_1\sin(\alpha_1 - \varphi_i) - cos(\alpha_1 - \varphi_i)\right]. \tag{33}$$

From (12), the following inequality can be got

$$\frac{1}{K|\lambda_2|} > \frac{1}{K|\lambda_3|} > \cdots > \frac{1}{K|\lambda_n|}. \tag{34}$$

Therefore, when $T > 0$, the stabilizing range of k_P values for a given first-order multi-agent system is given by

$$-\frac{1}{K_n} < k_P < \frac{1}{K_n}\left[\frac{T}{\theta}\alpha_1\sin(\alpha_1 - \varphi_n) - cos(\alpha_1 - \varphi_n)\right].$$

Case 2: $T < 0$

In this case, the proof follows along the same lines as that of Case 1. The only nonobvious change is that in the case that $-0.5 < T/\theta < 0$, the curves $(K_ik_P + \cos(z - \varphi_i))/\sin(z - \varphi_i)$ and $(T/\theta)z$ do not intersect in the interval $(0,\pi)$ regardless of the value of k_P in $(-\infty, -(1/K_i))$. ■

B. Determination of the Stabilizing Set for the Distributed PID Controllers

From (25), for each subsystem, $\mathrm{Re}\left[\hat{p}_i^*(z)\right]$ can be rewritten as

$$\mathrm{Re}\left[\hat{p}_i^*(z)\right] = \frac{K_i}{\theta^2}z^2\left[-k_D + M_i(z)k_I + B_i(z)\right] \tag{35}$$

where

$$M_i(z) = \frac{\theta^2}{z^2} \tag{36}$$

$$B_i(z) = -\frac{\theta}{K_iz}\left[\sin(z - \varphi_i) + \frac{T}{\theta}z\cos(z - \varphi_i)\right]. \tag{37}$$

To get the stabilizing set for the distributed PID controllers, we have the following theorem:

Theorem 2: When k_P is fixed, the stabilizing (k_I, k_D) region for the subsystem with respect to the eigenvalue λ_i is determined by the following inequalities:

$$\begin{cases} k_D > M_i(z_1)k_I + B_i(z_1) \\ k_D < M_i(z_2)k_I + B_i(z_2) \\ \dfrac{T}{K_i} > k_D > -\dfrac{T}{K_i} & \text{if } T > 0 \\ k_I > 0 \end{cases}$$
$$\begin{cases} k_D < M_i(z_1)k_I + B_i(z_1) \\ k_D > M_i(z_2)k_I + B_i(z_2) \\ -\dfrac{T}{K_i} > k_D > \dfrac{T}{K_i} & \text{if } T < 0 \\ k_I < 0 \end{cases} \tag{38}$$

where z_1 is the first solution of $\mathrm{Im}\left[\hat{p}_i^*(z)\right] = 0$ and z_2 is the second solution of $\mathrm{Im}\left[\hat{p}_i^*(z)\right] = 0$ arranged in increasing order of magnitude.

Proof: According to different values of T, we now consider the following two cases.

Case 1: $T > 0$

To ensure the stability of the quasipolynomial $\hat{p}_i^*(s)$, we need to check the two conditions given in Lemma 2.

Step 1: We first check Condition 2) of Lemma 2. Denote $\mathrm{Re}'\left[\hat{p}_i^*(\omega)\right]$ and $\mathrm{Im}'\left[\hat{p}_i^*(\omega)\right]$ as the first derivative with respect to ω of $\mathrm{Re}\left[\hat{p}_i^*(\omega)\right]$ and $\mathrm{Im}\left[\hat{p}_i^*(\omega)\right]$, respectively. If Condition 2) is satisfied, we have

$$\begin{aligned} E(\omega_0) = {} & \mathrm{Im}'\left[\hat{p}_i^*(\omega_0)\right]\mathrm{Re}\left[\hat{p}_i^*(\omega_0)\right] \\ & - \mathrm{Im}\left[\hat{p}_i^*(\omega_0)\right]\mathrm{Re}'\left[\hat{p}_i^*(\omega_0)\right] > 0 \end{aligned} \tag{39}$$

for some ω_0 in $(-\infty, +\infty)$. By taking $\omega_0 = 0$, it is derived that $\mathrm{Re}\left[\hat{p}_i^*(\omega)\right] = K_ik_I$ and $\mathrm{Im}\left[\hat{p}_i^*(\omega)\right] = 0$. We also have

$$\begin{aligned} \mathrm{Im}'\left[\hat{p}_i^*(\omega)\right]|_{\omega=0} &= \frac{K_ik_P}{\theta} + \frac{1}{\theta}\cos(\varphi_i) \\ \Rightarrow E(0) &= (\frac{K_ik_P + \cos(\varphi_i)}{\theta})(K_ik_I). \end{aligned}$$

Recall that $K_i > 0$ and $\theta > 0$. Thus, if we pick

$$\begin{aligned} & k_I > 0 \text{ and } k_P > -\frac{\cos(\varphi_i)}{K_i} \text{ or} \\ & k_I < 0 \text{ and } k_P < -\frac{\cos(\varphi_i)}{K_i} \end{aligned} \tag{40}$$

we have $E(0) > 0$.

Step 2: Next, we check Condition 1) of Lemma 2, i.e., $\mathrm{Re}\left[\hat{p}_i^*(\omega)\right]$ and $\mathrm{Im}\left[\hat{p}_i^*(\omega)\right]$ have only simple real roots and these interlace. From Theorem 1, we know that the roots of $\mathrm{Im}\left[\hat{p}_i^*(\omega)\right]$ are all real if and only if the parameter k_P satisfies the inequality (23). Moreover, based on the results in [15], the necessary and sufficient condition on k_I and k_D for the roots of $\mathrm{Re}\left[\hat{p}_i^*(z)\right]$ and $\mathrm{Im}\left[\hat{p}_i^*(z)\right]$ to interlace is given by

$$k_I > 0 \tag{41}$$

$$(-1)^tk_D < (-1)^tM_i(z_t)k_I + (-1)^tB_i(z_t) \tag{42}$$

where z_t is the tth solution of $\mathrm{Im}\left[\hat{p}_i^*(z)\right] = 0$ and $t = 1, 2, \ldots$. It can be easily obtained that for a given value of k_P, the value of

$M_i(z_t)$ satisfies

$$M_i(z_t) > M_i(z_{t+1}) > \cdots > M_i(z_\infty) = 0 \tag{43}$$

and the value of $B_i(z_t)$ satisfies

1) If $-1/K_i < k_P < 1/K_i$, then

$$\begin{cases} B_i(z_t) < B_i(z_{t+2}) < -\dfrac{T}{K_i}, \\ \qquad\qquad\qquad \text{for odd value of } t \\ B_i(z_t) > \dfrac{T}{K_i} \text{ and } B_i(z_t) \to \dfrac{T}{K_i} \text{ as } t \to \infty, \\ \qquad\qquad\qquad \text{for even value of } t. \end{cases} \tag{44}$$

2) If $1/K_i < k_P$, then

$$\begin{cases} B_i(z_t) > B_i(z_{t+2}) > -\dfrac{T}{K_i}, \text{ for odd value of } t \\ B_i(z_t) < B_i(z_{t+2}) < \dfrac{T}{K_i}, \quad \text{for even value of } t \\ B_i(z_1) < B_i(z_2). \end{cases} \tag{45}$$

From (43)–(45), one can draw a conclusion that for the subsystem with complex coefficients, the boundaries of the stabilizing (k_I, k_D) region for the subsystem are simplified as:

$$\begin{cases} k_D > M_i(z_1)k_I + B_i(z_1) \\ k_D < M_i(z_2)k_I + B_i(z_2) \\ \dfrac{T}{K_i} > k_D > -\dfrac{T}{K_i} \\ k_I > 0. \end{cases} \tag{46}$$

Case 2: $T < 0$ and $|T/\theta| > 0.5$

From Theorem 1, it is seen that there is no admissible range for k_P in the case $T < 0$ and $|T/\theta| > 0.5$. In other words, there is no stabilizing region for the distributed PID controller in this case. Hence, we assume that the system satisfies the condition that $T < 0$ and $|T/\theta| > 0.5$. Following along the same lines as that of Case 1, the stabilizing region of the distributed PID controllers in this case can be calculated. ■

According to (38), when k_P is fixed, the stabilizing (k_I, k_D) region for each subsystem can be easily obtained. The stabilizing (k_I, k_D) region for the multi-agent system can be derived by intersecting the resultant stabilizing regions for all λ_i values. The results reveal that the stabilizing (k_I, k_D) region has the linear programming characterization and is a union of convex sets for a fixed k_P gain. By sweeping over k_P in the allowable range, the entire stabilizing set of the distributed PID controllers can be derived.

Remark 1: From (23), it is seen that the range of k_P is given in terms of time delay θ. For $\theta > 0$, i.e., no time delay, one can solve (24) analytically to obtain α_1. Using this value, the upper bound in (23) evaluates out to ∞, which means that there is no upper bound for the time-free case. Also, it is easy to see that as T/θ decreases, the value α_1 approaches π. By substituting for T/θ from (24) into the upper bound in (23) and differentiating with respect to α, it can be shown that as α approaches π, the upper bound in (23) monotonically decreases to $1/K_i$. This shows that as θ increases, the range of k_P shrinks. When the time delay is sufficiently large, the range of k_P belongs to $(-1/K_i, 1/K_i)$. However, although the admissible range of k_P can be obtained, the corresponding stabilizing (k_I, k_D) region can be quite tiny for $\theta \to \infty$.

Remark 2: The main results solve the stabilization problem of systems with complex coefficients. Besides, the resultant stabilizing region provides the basis for both the tuning of the distributed PID controller for multi-agent system in practice and the design of PID controller satisfying different performance criteria. For example, to get good tracking performance of the multi-agent system, one can choose integrated time absolute error (ITAE) index as the optimization function and search the optimal parameters in the resultant stabilizing region.

C. Algorithm for Determining Stabilizing PID Parameters

In terms of Theorems 1–3, the algorithm to determine the complete set of the distributed PID controllers for the general first-order multi-agent system is shown as follows:

Step 1: Observe the topology structure of the multi-agent system and compute the eigenvalues $\lambda_2, \lambda_3, \ldots, \lambda_n$ of the corresponding Laplacian matrix.

Step 2: Determine the allowable range of k_P according to (23).

Step 3: Pick a k_P in the range and find the roots z_1 and z_2 of (27) for each eigenvalue λ_i.

Step 4: Compute the parameters $M_i(z)$ and $B_i(z)$ for each eigenvalue from (36) and (37).

Step 5: Determine the (k_I, k_D) region for each eigenvalue according to (38). The intersection of all the regions is the stabilizing (k_I, k_D) region for the multi-agent system.

Step 6: By sweeping over k_P in the allowable range, repeat Steps 3–5 to determine the complete set of the distributed PID controllers.

V. Simulation Examples

Example 1: Consider a multi-agent system with 10 identical vessels. The dynamic equation of each vessel with time delay is given as [26]

$$M\dot{v}(t) + cv(t) = u(t - \theta)$$

where M, v, u and θ are the inertia term, velocity, external control force and input time delay, respectively. c is the Coriolis and centripetal term including the damping constant. Obviously, the transfer function of each vessel is the typical first-order model if the velocity is regarded as the output state and the force is regarded as the control input

$$G(s) = \frac{v(s)}{u(s)} = \frac{1}{Ms + c} e^{-\theta s}.$$

Set $M = c = 1$ and $\theta = 0.2$. The initial values of each vessel are randomly selected in the interval [–2, 2]. The information flow is shown in Fig. 7. The objective is to determine the complete set of the distributed PID controller for the stabilization of the multi-agent system.

From Fig. 7, the nonzero eigenvalues of the Laplacian matrix are: 0.8299, 2, 2.6889, 3.4796, 4.4812, 0.7322 + 0.7132j, 0.7322–0.7132j, 1.5281+0.645j and 1.5281–0.645j. According to Theorem 1, the admissible range of k_P is $(-0.2232, 2.1354)$. Then, for a fixed k_P value in the admissible range, such as $k_P = 1$, the stabilizing (k_I, k_D) region can be determined based on Theorem 2, which is shown in Fig. 8. To check the effectiveness of the resultant stabilizing region, the step response curves of the multi-agent system are shown in Fig. 9 for different values of k_I and k_D. From Fig. 9, it is seen that, when the value of (k_I, k_D) is chosen inside the stabilizing region, for example, (1, 0.1), the multi-agent system can reach consensus. When it is chosen to be (2.497, –0.05457), which is located on the boundary of the stabilizing region, the system is critically unstable. When it is chosen outside the stabilizing region, for example, (2, –0.1), the system becomes unstable.

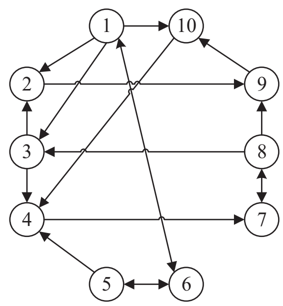

Fig. 7. The direct topology for Example 1.

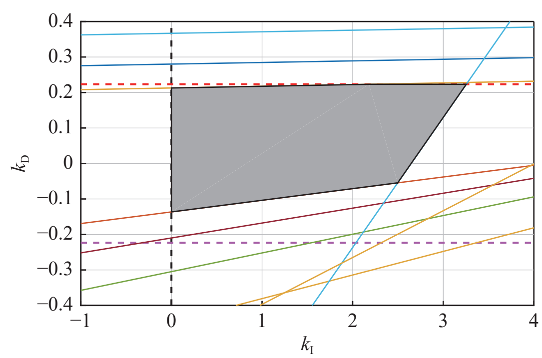

Fig. 8. The stabilizing (k_I, k_D) region for $k_P = 1$.

According to the algorithm in Section IV-C, the complete stabilizing set of the distributed PID controllers is presented as a 3D plot which is shown in Fig. 10.

Example 2: Consider a consensus tracking problem with 6 unstable agents studied in [16]. The topology is shown in Fig. 11. Only agent 1 is accessible to the target state $r(s) = 1$. By calculation, all the non-zero eigenvalues are 1. The agent dynamics are the first order unstable processes with time delays which can be found in pitch control of ship course control. The initial value for each agent is set randomly. The agent model is shown as follow:

$$G(s) = \frac{1}{s-1} e^{-0.1s}.$$

Obviously, although the topology has a spanning tree, the multi-agent system cannot achieve consensus. According to Theorem 1, the admissible range of k_P is (1, 17.7702). Then by fixing $k_P = 2.9819$, based on (38), the boundaries of the stabilizing (k_I, k_D) region are calculated as

$$\begin{cases} k_D > 0.0462 k_I - 0.7972 \\ k_D < 0.0011 k_I + 0.9956 \\ -1 < k_D < 1 \\ k_I > 0. \end{cases} \tag{47}$$

The stabilizing region is shown in Fig. 12. The step

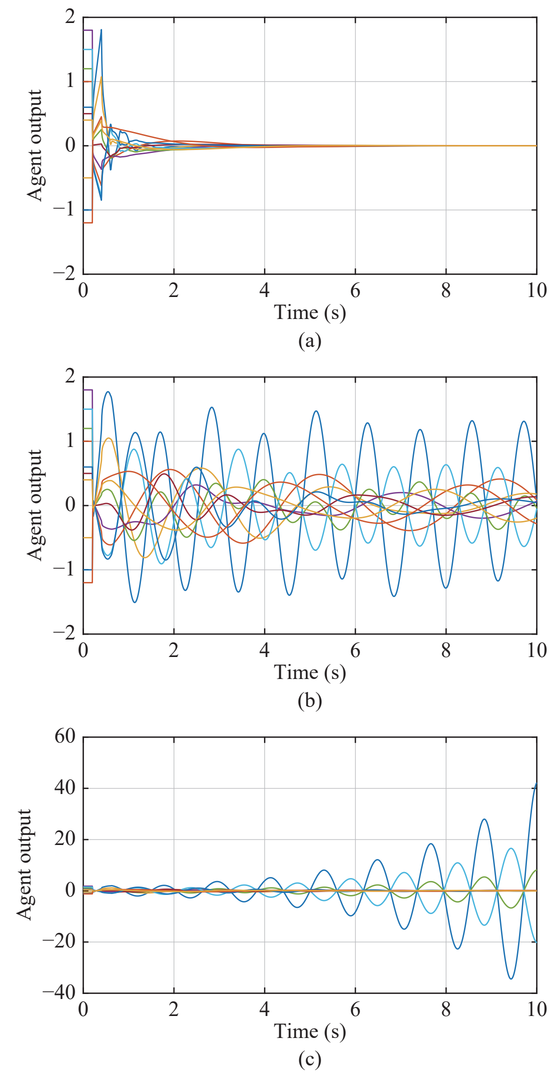

Fig. 9. The output response for $k_P = 1$: (a) $(k_I, k_D) = (1, 0.1)$; (b) $(k_I, k_D) = (2.497, -0.05457)$; (c) $(k_I, k_D) = (2, -0.1)$.

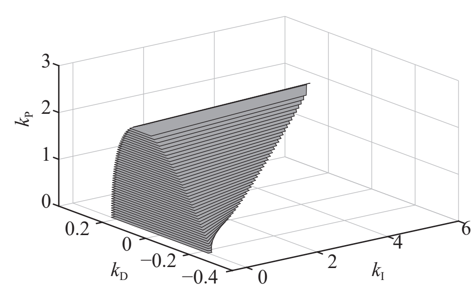

Fig. 10. The stabilizing parametric region for the multi-agent systems.

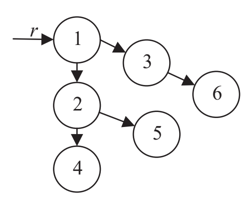

Fig. 11. The direct topology for Example 2.

response curves shown in Fig. 13 for different (k_I, k_D) values demonstrate the validity of the proposed method. Note that the controller parameters $(k_I, k_D) = (2.538, 0.724)$ derived in [11] are also located in the stabilizing region, which indicate that the proposed method can provide the basis for the tuning of the distributed PID controller. Furthermore, the stabilizing region makes it more flexible to choose the optimal controller parameters satisfying different performance criteria.

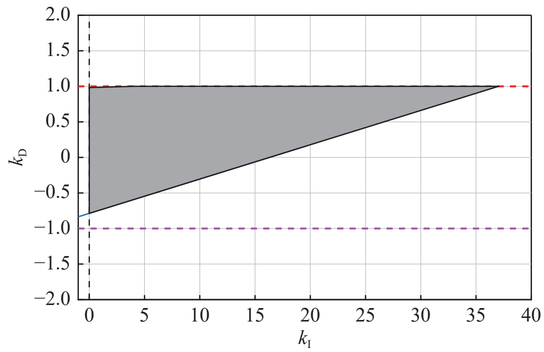

Fig. 12. The stabilizing (k_I, k_D) region for $k_P = 2.9819$.

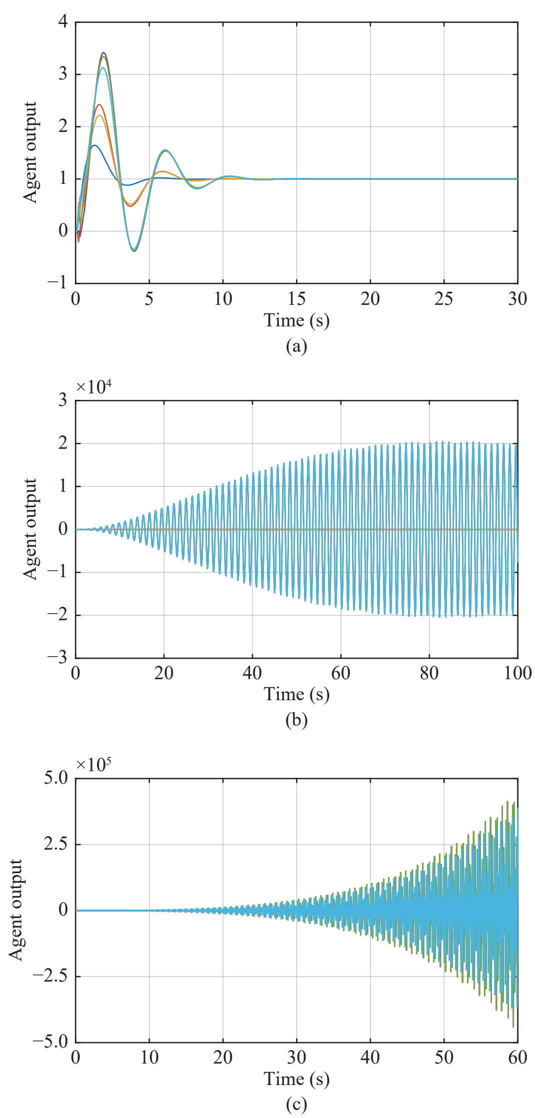

Fig. 13. The output response for $k_P = 2.9819$: (a) $(k_I, k_D) = (3, 0.4)$; (b) $(k_I, k_D) = (10, -0.335)$; (c) $(k_I, k_D) = (3, 1.01)$.

VI. Conclusion

A comprehensive method to compute the entire set of stabilizing distributed PID controllers for general first-order multi-agent systems under arbitrary fixed topology is presented in this paper. All the parameters chosen in the resultant stabilizing region can guarantee the consensus of the given multi-agent system. The results of the paper provide insight into designing and analysing of the distributed PID controller for general first-order multi-agent systems under fixed topology including the undirected and directed topology. Further, the results in the paper solve the stabilization problem of the systems with complex coefficients.

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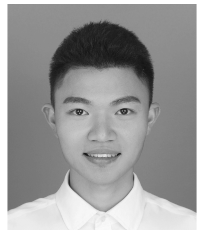

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