Distributed Asymptotic Consensus in Directed Networks of Nonaffine Systems With Nonvanishing Disturbance

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***Abstract*—In this paper the distributed asymptotic consensus problem is addressed for a group of high-order nonaffine agents with uncertain dynamics, nonvanishing disturbances and unknown control directions under directed networks. A class of auxiliary variables are first introduced which forms second-order filters and induces all measurable signals of agents' states. In view of this property, a distributed robust integral of the sign of the error (DRISE) design combined with the Nussbaum-type function is presented that guarantees not only the desired asymptotic consensus, but also the uniform boundedness of all closed-loop variables. Compared with the traditional sliding mode control (SMC) technique, the main feature of our approach is that the integral operation in the proposed control algorithm is designed to be adopted in a continuous manner and ensures less chattering behavior. Simulation results for a group of Duffing-Holmes chaotic systems are employed to verify our theoretical analysis.**

***Index Terms*—Asymptotic consensus, nonaffine systems, nonvanishing disturbance, Nussbaum-type functions, unknown control directions.**

I. Introduction

THE cooperative control of multi-agent systems (MASs) has received a large amount of attentions in the last several decades, where the design of distributed control algorithms for consensus of MASs has been the fundamental task due to its considerable applications [1]–[6]. Besides, consensus control for nonlinear MASs is technically important since the unknown nonlinear dynamics usually exist in practical applications, where many important works have been reported [7]–[13].

Recently, the researchers have mainly focused on the distributed consensus problem of nonaffine systems, and several control algorithms have been developed [14]–[17]. A distributed adaptive consensus tracking algorithm was proposed for the MASs in nonaffine pure-feedback form by using the backstepping technique, neural networks (NNs), and the dynamic surface control method in [14]. In [15], a class of distributed consensus tracking algorithms with the continuous control action were developed for networked nonaffine MASs to ensure that the tracking error can converge to a small region within a prescribed finite time. In [16], by using output constrained feedback, a distributed consensus algorithm was proposed for nonaffine MASs subject to partially unknown control directions to drive the agents' states to the region of predefined constraints. An adaptive filtered backstepping control method is employed to achieve the cooperative tracking of uncertain nonaffine MASs with input nonlinearities and unknown control directions in [17]. It is worth noting that most existing results have widely utilized the technique of NNs to cope with the unknown system dynamics, since NNs have the ability of approximating smooth, unknown nonlinear dynamics. However, NNs based approaches mainly focus on the process of choosing units and neurons for the NNs, which increases the burden in adjusting parameters. Furthermore, it is known that NNs can not approximate nonlinear dynamics that depend on the time variable.

Another challenge for consensus algorithms is the distributed control design of MASs with nonvanishing disturbance, where it requires the use of an advanced robust control scheme. When addressing nonvanishing disturbances, the NNs based approaches are not capable of providing satisfactory performance, such as the asymptotic convergence of system states. Therefore, disturbance observers should be designed to reject the unknown disturbance [18], [19]. As an alternative approach, the sliding mode control (SMC) method is well known for its disturbance rejection property [20]. Sliding mode observers and a distributed fuzzy logic control method were adopted to approximate unknown dynamics and drive nonlinear MASs to consensus in [21]. Event-triggered adaptive fuzzy controllers with SMC were employed in [22]. One problem of the aforementioned works is that by using the SMC technique, the high-frequency chattering phenomenon was not addressed [23], which causes wear and tear to the actuator and might lead to the possibility of system destabilization. The better solution for alleviating the chattering phenomenon, while maintaining the invariance property of the sliding mode against uncertainties involves the use of high-order SMC [24]–[26]. However, the sliding surface and the adaptive estimation error are shown to be

Manuscript received November 9, 2020; revised January 4, 2021; accepted March 2, 2021. This work was supported in part by the National Natural Science Foundation of China (61973074, 61921004, U1713209). Recommended by Associate Editor Tengfei Liu. *(Corresponding author: Qingling Wang.)*

Citation: Q. L. Wang and C. Y. Sun, "Distributed asymptotic consensus in directed networks of nonaffine systems with nonvanishing disturbance," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 6, pp. 1133–1140, Jun. 2021.

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Digital Object Identifier 10.1109/JAS.2021.1004021

uniformly ultimate bound, and the above designed approaches are based on the dynamical systems with known control directions.

From the above literature, it can be observed that the issues of uncertain dynamics, nonvanishing disturbance, unknown control directions, and the reduction in the chattering for MASs have not been explored all together in a single paper. In this paper we mainly concentrate on the control design of nonaffine agents with a new asymptotically consensus algorithm that can deal with nonlinear dynamics, nonvanishing disturbance, and unknown control directions. As in most of the related works, we first transform the nonaffine agent into the affine dynamics, and then we take its first-order time derivative to raise the order of the dynamics where the derivative of control input for each agent contains affinely. Next, we incorporate the robust integral of the sign of the error (RISE) scheme [27] into the proposed consensus algorithms to cope with the problem of novanishing disturbance in each agent. Furthermore, a mechanism is designed to cope with unknown control directions in each agent. Inspired by the work in [28], the Nussbaum-type function is employed to be included in the RISE scheme. Therefore, in this paper a distributed robust integral of the sign of the error (DRISE) structure is developed, which to the authors' best knowledge has not been studied, for the high-order nonaffine agents with nonvanishing disturbance. The main contributions of the paper can be summarized as follows:

1) This is the first work to consider the leaderless consensus problem of high-order nonaffine agents with uncertain dynamics, nonvanishing disturbance, and unknown control directions, in contrast to existing results for consensus of nonaffine agents.

2) A new DRISE design combined with the Nussbaum-type function is first proposed to guarantee not only the desired asymptotic consensus, but also the uniform boundedness of all closed-loop variables.

3) Compared to the related literature [16], where the control directions are identical and partially known for agents with undirected graphs, in this work the unknown control directions can be nonidentical for agents with a directed graph having a spanning tree. Furthermore, different from existing results on consensus of agents with nonvanishing disturbance, in this work the integral operation in the proposed control algorithm is adopted to incorporate in a continuous manner and ensure less chattering behavior.

We organize the rest of this study as follows. Preliminaries and problem formulation are described in Section II. In Section III, auxiliary variables and the technical development for open-loop dynamics are presented, and a new control algorithm with the Nussbaum-type function is proposed. Then, the main results on asymptotic consensus are illustrated. Simulation results on Duffing-Holmes chaotic systems are presented in Section IV. Finally, Section V concludes this work with some remarks.

II. Preliminaries and Problem Formulation

Notations: $\mathcal{L}_\infty$ denotes the space of bounded function on $[0,\infty)$. $|\cdot|$ represents the absolute value, $\|\cdot\|$ represents the Euclidean norm of a vector, and $\|\cdot\|_\infty$ represents the infinity norm. The notation sgn $(\cdot)$ is the signum function.

A. Preliminaries

Definition 1 [29]: $\chi(\cdot)$ is regarded as the Nussbaum-type function when it has

$$\begin{cases} \lim_{\kappa\to\infty}\sup\left(\frac{1}{\kappa}\int_0^\kappa \chi(\tau)\,d\tau\right) = +\infty \\ \lim_{\kappa\to\infty}\inf\left(\frac{1}{\kappa}\int_0^\kappa \chi(\tau)\,d\tau\right) = -\infty. \end{cases} \tag{1}$$

Remark 1: In view of the Nussbaum-type function $\chi(p)$, if $s(\cdot)$ denotes a bounded function within $I_v = [b^-, b^+]$, where b^- and b^+ are unknown constants, and the value $0 \notin I_v$, $\bar{\chi}(p) = s(\cdot)\chi(p) + h(\cdot)$ is also the Nussbaum-type function, where $h(\cdot) \in \mathcal{L}_\infty$ is a continuous function [28].

The basic concepts of directed graphs are omitted in this paper, and the detailed introduction can be found in [28]. Some related definitions on directed graphs are revisited in the following. A directed graph (digraph) $\mathcal{G}$ is defined as $\mathcal{G} = (\mathcal{V}, E, \mathcal{A})$ where $\mathcal{V}$ is the set of vertices, E is the set of edges, and $\mathcal{A}$ is the adjacency matrix of the graph. Assume a graph with N agents $\mathcal{V} = \{1,2,\ldots,N\}$ and denote by a_{iv} the (i,v)-th element of $\mathcal{A}$. If there is an edge from v to i, i.e., $(v,i)\in E$, then a_{iv} is positive. The in-degree of agent i is also defined by $d_i := \sum_{v=1}^N a_{iv}$ with in-degree matrix given by $\mathcal{D} = \mathrm{diag}\{d_1,\ldots,d_N\}$. A spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has a spanning tree if a subset of the edges forms a spanning tree. A center of a graph is a node from which other nodes are reachable. If the graph $\mathcal{G}(t)$ has a center, we say $\mathcal{G}(t)$ is quasi-strongly connected. If we give a constant $\delta > 0$, the definition of δ-arc is the arc with the property that $\int_{t_1}^{t_2} a_{ji}(t)dt \ge \delta$ in the interval $[t_1,t_2)$. The definition of δ-path is a path in the interval $[t_1,t_2)$ which contains the δ-arc.

Definition 2 [30]: The definition of uniformly quasi-strongly δ-connected graphs is that for a constant $T>0$, the δ-arcs of $\mathcal{G}(t)$ induce a quasi-strongly connected graph in the interval $[t, t+T)$ with any $t \ge 0$.

Remark 2: It is shown [28] that the directed graph having a spanning tree can be regarded as a special case of the uniformly quasi-strongly δ-connected graphs.

Lemma 1 [30]: For a group of single-integrator dynamics, where the i-th one of the dynamics are

$$\dot{\xi}_i(t) = -\sum_{k=1}^N a_{ik}(t)\left(\xi_i(t)-\xi_k(t)\right) + \varrho_i(t) \tag{2}$$

for each $i = 1,2,\ldots,N$, in which the function $\varrho_i(t)$ is continuous in the interval of $[0,\infty)$, excluding some set with measure zero. Suppose the graph is the uniformly quasi-strongly δ-connected graphs. If $\varrho_i \in \mathcal{L}_\infty$ with $\lim_{t\to\infty}\varrho_i(t) = 0$, then we have $\lim_{t\to\infty}(\xi_i(t)-\xi_k(t)) = 0$ for $i,k = 1,2,\ldots,N$.

Definition 3 [31]: For a function $\dot{x} = f(x,t)$ with $[0,\infty)$, the vector $x:[0,\infty)\to\mathbb{R}^n$ is a solution of $\dot{x} = f(x,t)$, if $x(t)$ is continuous and it is satisfied that

$$\dot{x} \in K[f](x,t) \tag{3}$$

for $t \in [0,\infty)$, in which the Lebesgue measurable $f: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is locally bounded, and $K[f](x,t)$ is defined as

$$K[f](x,t) \equiv \cap_{\delta>0}\left(\cap_{\mu M=0}\mathrm{co} f(B(x,\delta)-N,t)\right) \tag{4}$$

and $\cap_{\mu M=0}$ is the intersection where all sets M of Lebesgue measurable are zero. The notation co represents the convex closure, and $B(x,\delta) = \{v \in \mathbb{R} \mid \|x-v\| < \delta\}$.

Lemma 2 [32]: Suppose $x(t)$ is the Filippov solution of $\dot{x} = f(x,t)$ and V is locally Lipschitz. Then, V is continuous, and $\frac{d}{dt}V(x(t),t)$ exists almost everywhere for $t \in [0,\infty)$, and $\dot{V}(x,t) \in \dot{\tilde{V}}(x,t)$ with

$$\dot{\tilde{V}}(x,t) = \underset{\xi \in V(z,t)}{\cap} \xi^T \begin{pmatrix} K[f](x,t) \\ 1 \end{pmatrix}. \tag{5}$$

B. Problem Formulation

We consider N nonaffine agents of the form:

$$x_i^{(n)}(t) = f_i(\bar{x}_i(t), u_i(t)) + d_i(t) \tag{6}$$

with $n \geq 1$ and $\bar{x}_i(t) = [x_i(t), \dot{x}_i(t), \ldots, x_i^{(n-1)}(t)]^T$, where $x_i(t) \in \mathbb{R}$ and $x_i^{(m)}(t) \in \mathbb{R}$, $m = 1,2,\ldots,n-1$. $f_i(\bar{x}_i(t), u(t))$ is the unknown, smooth nonaffine nonlinear dynamics for each agent, $d_i(t) \in \mathbb{R}$ is the nonvanishing disturbance, and $u_i(t) \in \mathbb{R}$ denotes the control input.

Assumption 1: The disturbance $d_i(t)$ and its first-order derivative and second-order derivative are bounded, that is, $d_i^{(j)}(t) \in \mathcal{L}_\infty$ for $j = 0,1,2$.

Assumption 2: Suppose a constant b_i satisfies $0 < b_i \leq |(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t)|$ for $\bar{x}_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}$.

Remark 3: Since the term $f_i(\bar{x}_i(t), u(t))$ is assumed to be unknown, the values of b_i and $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t)$ and the sign of the vector $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t)$ are all unknown.

Assumption 3: If $\bar{x}_i(t) \in \mathcal{L}_\infty$, $\dot{\bar{x}}_i(t) \in \mathcal{L}_\infty$ and $\ddot{\bar{x}}_i(t) \in \mathcal{L}_\infty$, all elements of $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial x_i(t) \,|_{u_i(t)=\vartheta\left(\bar{x}_i(t), x_i^{(n)}(t)-d_i(t)\right)}$ are bounded, and its time derivative of each element is bounded.

The control objective is to design distributed control algorithms for agents (6) under Assumptions 1–3 and communication topology described by directed graphs in order to achieve asymptotic consensus in the sense that

$$\begin{cases} \lim\limits_{t\to\infty} (x_i(t) - x_k(t)) = 0 \\ \lim\limits_{t\to\infty} x_i^{(m)}(t) = 0 \end{cases} \tag{7}$$

for all agents with $m = 1,2,\ldots,n-1$, $i,k \in \{1,2,\cdots,N\}$. Furthermore, all signals of the closed-loop systems are bounded.

III. Main Results

A. Auxiliary Variables and the Technical Development

In this subsection, auxiliary variables will be introduced to facilitate the algorithms design, and the technical development for open-loop dynamics will be illustrated. The following lemma will be first introduced to derive the main technical development:

Lemma 3 [33]: Let $x(t)$ be the smooth function, and suppose the initial conditions $x(0)$, $x^{(m)}(0)$, $m = 1,2,\ldots,n-1$ are all bounded. Let

$$s(x,t) = \left(\gamma + \frac{d}{dt}\right)^{n-1} x(t) \tag{8}$$

where the constant $\gamma > 0$. If $|s(x,t)| \leq k$ with $k > 0$ and $t \geq 0$, then it is satisfied that

$$\left\|x^{(m)}(t)\right\| \leq \frac{2^m k}{\gamma^{n-m-1}}, \quad t \geq T_0 \tag{9}$$

for $m = 1,2,\ldots,n-1$, in which the finite time T_0 depends on the initial conditions $x(0)$, $x^{(m)}(0)$, $m = 1,2,\ldots,n-1$. Moreover, if $\lim_{t\to\infty} s(x,t) = s^* \in \mathcal{L}_\infty$, then $\lim_{t\to\infty} x(t) = x^* \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} x^{(m)}(t) = x_m^* \in \mathcal{L}_\infty$. In particular, if $\lim_{t\to\infty} s(x,t) = 0$, then $\lim_{t\to\infty} x(t) = 0$ and $\lim_{t\to\infty} x^{(m)}(t) = 0$, $m = 1,2,\ldots,n-1$.

To facilitate the technical development, we first define the following states:

$$\begin{aligned} \eta_i(t) &= \left(\gamma + \frac{d}{dt}\right)^{n-1} x_i(t) \\ &= C_{n-1}^0 \gamma^{n-1} x_i(t) + C_{n-1}^1 \gamma^{n-2} \dot{x}_i(t) + \cdots \\ &\quad + C_{n-1}^{n-2} \gamma x_i^{(n-2)}(t) + C_{n-1}^{n-1} x_i^{(n-1)}(t) \end{aligned} \tag{10}$$

and

$$\begin{aligned} \varphi_i(t) &= C_{n-1}^0 \gamma^{n-1} \dot{x}_i(t) + C_{n-1}^1 \gamma^{n-2} \ddot{x}_i(t) + \cdots \\ &\quad + C_{n-1}^{n-3} \gamma^2 x_i^{(n-2)}(t) + C_{n-1}^{n-2} \gamma x_i^{(n-1)}(t) \end{aligned} \tag{11}$$

where $\gamma > 0$, and the notations C_i^j in (10) and (11) are coefficients of the binomial expansion. Then, we define

$$\phi_i(t) = \dot{\eta}_i(t) + z_i(t) \tag{12}$$

with

$$z_i(t) = \sum_{k=1}^{N} a_{ik} (\eta_i(t) - \eta_k(t)) \tag{13}$$

and define

$$r_i(t) = \phi_i(t) + \int_0^t \phi_i(\tau) d\tau. \tag{14}$$

It is seen from (14) that $r_i(t)$ cannot be used in the feedback control since the term $\dot{\eta}_i(t)$ in $\phi_i(t)$ is unmeasurable. Taking the first-order derivative of $r_i(t)$, we have

$$\begin{aligned} \dot{r}_i(t) &= \frac{\partial f(\bar{x}_i, u_i)}{\partial \bar{x}_i} \dot{\bar{x}}_i(t) + \frac{\partial f(\bar{x}_i, u_i)}{\partial u_i} v_i(t) + \dot{d}_i(t) \\ &\quad + \dot{\varphi}_i(t) + \dot{z}_i(t) + \dot{\eta}_i(t) + z_i(t) \end{aligned} \tag{15}$$

with

$$v_i(t) = \dot{u}_i(t) \tag{16}$$

where the auxiliary signal $v_i(t)$ can be regarded as a new system state. It is seen that to design the controller of (16) the derivatives of agents' states are required [27], which can not be measurable because of the lack of sensors. Thus, extra feedback signals are needed to update the control signal $v_i(t) = \dot{u}_i(t)$.

To overcome this issue, we introduce the following auxiliary signals $r_{ia}(t)$ and $\phi_{ia}(t)$ to replace the unmeasurable signals $r_i(t)$ and $\phi_i(t)$:

$$\begin{cases}\dot{\phi}_{ia}(t)=r_{ia}(t)-\phi_{ia}(t)\\ \dot{r}_{ia}(t)=-r_{ia}(t)-(\beta_i+1)r_{is}(t)-\phi_{ia}(t)\end{cases} \tag{17}$$

with $\phi_{ia}(0)=0$, $r_{ia}(0)=0$ and

$$r_{is}(t)=r_i(t)+r_{ia}(t). \tag{18}$$

We define

$$\phi_{is}(t)=\int_0^t \phi_i(\tau)d\tau+\phi_{ia}(t). \tag{19}$$

Then, we have from (18) that

$$r_{is}(t)=\phi_{is}(t)+\dot{\phi}_{is}(t). \tag{20}$$

Integrating the first entry of (17), we obtain

$$\phi_{ia}(t)=\int_0^t r_{ia}(\tau)d\tau-\int_0^t \phi_{ia}(\tau)d\tau. \tag{21}$$

Integrating the second entry of (17), we obtain

$$\begin{aligned}r_{ia}(t)=&-(\beta_i+2)\int_0^t r_{ia}(\tau)d\tau-\int_0^t\phi_{ia}(\tau)d\tau\\ &-(\beta_i+1)\int_0^t r_i(\tau)d\tau\\ =&-(\beta_i+2)\int_0^t r_{ia}(\tau)d\tau-(\beta_i+1)\eta_i(t)\\ &+(\beta_i+1)\eta_i(0)-(\beta+1)\int_0^t z_i(\tau)d\tau\\ &-(\beta_i+1)\int_0^t(\eta_i(\tau)-\eta_i(0))\,d\tau\\ &-(\beta_i+1)\int_0^t\left(\int_0^t z_i(\tau)d\tau\right)d\tau\\ &-\int_0^t\phi_{ia}(\tau)d\tau.\end{aligned} \tag{22}$$

It is observed that each term of the right-hand of the above two equations (21) and (22) is available for calculating $\phi_{ia}(t)$ and $r_{ia}(t)$ online.

In what follows, taking the derivative of $r_{is}(t)$ and substituting (15), we obtain the open-loop dynamics as

$$\begin{aligned}\dot{r}_{is}(t)=&\frac{\partial f(\bar{x}_i,u_i)}{\partial\bar{x}_i}\dot{\bar{x}}_i(t)+\frac{\partial f(\bar{x}_i,u_i)}{\partial u_i}v_i(t)+\dot{d}_i(t)\\ &+\dot{\varphi}_i(t)+\dot{z}_i(t)+\dot{\eta}_i(t)+z_i(t)-r_{ia}(t)\\ &-(\beta+1)r_{is}(t)-\phi_{ia}(t).\end{aligned} \tag{23}$$

For the design of control algorithms, we define

$$E_{id}(t)=\left[\frac{\partial f(\bar{x}_i,t)}{\partial\bar{x}_i}\,|_{\bar{x}_i(t)=x_{id},\dot{\bar{x}}_i(t)=\dot{x}_{id}}\right]^T\dot{x}_{id}+\dot{d}_i(t) \tag{24}$$

and

$$\tilde{E}_i(t)=-r_{ia}(t)-\phi_{ia}(t)+\int_0^t\phi_i(\tau)d\tau+\dot{\varphi}_i(t)+\dot{z}_i(t)+\phi_i(t). \tag{25}$$

Therefore, the open-loop dynamics can be rewritten as

$$\begin{aligned}\dot{r}_{is}(t)=&-(\beta_i+1)r_{is}(t)+\frac{\partial f(\bar{x}_i,u_i)}{\partial u_i}v_i(t)\\ &-\int_0^t\phi_i(\tau)d\tau+E_{id}(t)+\tilde{E}_i(t).\end{aligned} \tag{26}$$

Remark 4: In view of Assumptions 1 and 3, we obtain that $\|E_{id}(t)\|_\infty\le\varsigma_{i1}$ and $\left\|\dot{E}_{id}(t)\right\|_\infty\le\varsigma_{i2}$, where ς_{i1} and ς_{i2} are two positive constants.

Remark 5: Note that $\tilde{E}_i(t)$ is continuously differentiable with respect to $\bar{x}_i(t)$, $\dot{\bar{x}}_i(t)$, $r_{ia}(t)$ and $\phi_{ia}(t)$. It is known [27] that $\tilde{E}_i(t)$ is upper bounded by

$$\left|\tilde{E}_i(t)\right|\le\rho_i\left(\|e_i(t)\|\right)\|e_i(t)\| \tag{27}$$

where the vector $e_i(t)=\left[\int_0^t\phi_i(\tau)d\tau,\phi_{ia}(t),r_{ia}(t),r_{is}(t)\right]^T$, and the function $\rho_i(\cdot)$ is invertible and nondecreasing.

B. Distributed Control Algorithms Design

According to Assumption 2, since the sign of $\partial f(\bar{x}_i,u_i)/\partial u_i$ in (26) is unknown, a Nussbaum-type function should be employed to deal with the unknown control directions. Thus, in this subsection, a new control algorithm with the Nussbaum-type function will be presented to cope with the nonlinear affine dynamics and unknown control directions that achieves asymptotic consensus.

In this subsection, the virtual control input $v_i(t)$ is proposed by

$$\begin{cases}v_i(t)=\chi(p_i(t))\left[-(\beta_i+1)r_{ia}(t)+\psi_i\text{sgn}(\phi_{is}(t))\right]\\ \dot{p}_i(t)=\left[-(\beta_i+1)r_{ia}(t)+\psi_i\text{sgn}(\phi_{is}(t))\right]r_{is}(t)\end{cases} \tag{28}$$

where $\beta_i>0$ and $\psi_i>0$ are two positive constants. It is seen from the second entry of (28) that

$$\begin{aligned}p_i(t)=&-(\beta_i+1)\int_0^t r_{ia}(\tau)r_{is}(\tau)d\tau\\ &+\psi_i\int_0^t\text{sgn}(\phi_{is}(\tau))r_{is}(\tau)d\tau\\ =&\int_0^t r_{ia}(\tau)\left[(\dot{r}_{ia}(\tau)+r_{ia}(\tau)+\phi_{ia}(\tau))\right]d\tau\\ &+\psi_i\int_0^t\text{sgn}(\phi_{is}(\tau))\dot{\phi}_{is}(\tau)d\tau+\psi_i\int_0^t|\phi_{is}(\tau)|\,d\tau\\ =&\frac{1}{2}r_{ia}^2(t)-\frac{1}{2}r_{ia}^2(0)+\int_0^t r_{ia}^2(\tau)d\tau+\frac{1}{2}\phi_{ia}^2(t)\\ &-\frac{1}{2}\phi_{ia}^2(0)+\int_0^t\phi_{ia}^2(\tau)d\tau+\psi_i|\phi_{is}(t)|\\ &-\psi_i|\phi_{is}(0)|+\psi_i\int_0^t|\phi_{is}(\tau)|\,d\tau\end{aligned} \tag{29}$$

in which each term is available for calculating $p_i(t)$. From (16), (28), and (29), we know that the control input $u_i(t)$ can be implemented since all the terms are measurable signals.

In view of (26) and (28), the closed-loop system is

$$\begin{aligned}\dot{r}_{is}(t)=&-(\beta_i+1)r_{is}(t)+\tilde{E}_i(t)+(\beta_i+1)\,r_{ia}(t)\\ &+E_{id}(t)-\psi_i\text{sgn}(\phi_{is}(t))-\phi_i(t)\\ &+\left[\frac{\partial f(\bar{x}_i,t)}{\partial u_i}\chi(p_i(t))+1\right]\frac{\dot{p}_i(t)}{r_{is}(t)}\end{aligned} \tag{30}$$

where the term $-(\beta_i+1)r_{is}(t)$ is proposed to counteract the effect of $\tilde{E}_i(t)$, and the term $\psi_i\text{sgn}(\phi_{is}(t))$ is a key design to cope with the bounded term $E_{id}(t)$. It is observed from (28) that although the discontinuous function exists in the signal $v_i(t)$, the control input $u_i(t)$ is actually continuous since we have $v_i(t)=\dot{u}_i(t)$.

C. Boundedness and Asymptotic Consensus Results

In this subsection, we will prove that if the control algorithms are designed as described in the previous subsections, then all closed-loop variables remain bounded and asymptotic consensus is achieved.

The main result is summarized as follows:

Theorem 1: Suppose Assumptions 1–3 hold. Considering a group of N nonaffine agents in the form (6) with the proposed control algorithms (28), (29), and (17) under the communication topology described by a directed graph having a spanning tree, all closed-loop variables are uniformly ultimately bounded and semiglobal asymptotic consensus is achieved providing that the following condition is satisfied:

$$\psi_i > 2(\varsigma_{i1} + \varsigma_{i2}) \tag{31}$$

where ς_{i1} and ς_{i2} are two parameters defined in Remark 4. Furthermore, all signals of the closed-loop systems are bounded.

Proof: Let the Lyapunov function candidate be

$$V_i(t) = \frac{1}{2}\left(\int_0^t \phi_i(\tau)d\tau\right)^2 + \frac{1}{2}r_{is}^2(t) + \frac{1}{2}\phi_{ia}^2(t) + \frac{1}{2}r_{ia}^2(t) + \psi_i\,|\phi_{is}(t)| - E_{id}\phi_{is}(t). \tag{32}$$

In view of (31) and Remark 4, since $\psi_i > 2(\varsigma_{i1}+\varsigma_{i2}) > \varsigma_{i1}$, we can obtain that $0 < \psi_i\,|\phi_{is}(t)| - E_{id}\phi_{is}(t) \le (\psi_i + \varsigma_{i1})\,|\phi_{is}(t)|$. Thus, it can be proved that $V_i(t) > 0$ is upper bounded as

$$V_i(t) \le \frac{1}{2}\,\|e_i(t)\|^2 + (\psi_i + \varsigma_{i1})\,|\phi_{is}(t)|$$

where $e_i(t)$ is defined in Remark 5, and $V_i(t)$ is continuous, differentiable everywhere except on the set $\Phi = \{e_i(t) \mid \phi_{is}(t) = 0\}$. Since the Lyapunov function is not directly employed, we use the tool in Lemma 2, that is

$$\dot{V}_i(t) \in \dot{\tilde{V}}_i(e_i(t),t) = \bigcap_{\xi\in\partial V(e_i(t),t)} \xi^T \begin{pmatrix} K[\dot{e}_i(t)](e_i(t),t) \\ 1 \end{pmatrix}$$

where $\partial V(e_i(t),t)$ is defined as

$$\partial V(e_i(t),t) = \Big[\int_0^t \phi_i(\tau)d\tau, r_{is}(t), \phi_{ia}(t), r_{ia}(t), -\dot{E}_{id}(t)\phi_{is}(t), \psi_i\mathrm{sgn}\,(\phi_{is}(t)) - E_{id}(t)\Big]. \tag{33}$$

To simplify the analysis, the above $\partial V(e_i(t),t)$ can be rewritten as

$$\partial V(e_i(t),t) = \Big[\int_0^t \phi_i(\tau)d\tau, r_{is}(t), \phi_{ia}(t), r_{ia}(t), -\dot{E}_{id}(t)\phi_{is}(t)\psi_i\xi_{\phi_{is}} - E_{id}(t), \mid \xi_{\phi_{is}} \in \mathrm{sgn}\,(\phi_{is}(t))\Big]. \tag{34}$$

Therefore, we have

$$\begin{aligned}
&\dot{\tilde{V}}_i(e_i(t),t)\\
&= \bigcap_{\xi\in\partial V(e_i(t),t)} \xi^T \begin{pmatrix} K[\dot{e}_i(t)](e_i(t),t) \\ 1 \end{pmatrix}\\
&= \bigcap_{\xi_{\phi_{is}}\in\mathrm{sgn}(\phi_{is}(t))} \Big\{\phi_i(t)\int_0^t \phi_i(\tau)d\tau + r_{is}(t)\dot{r}_{is}(t)\\
&\quad + \phi_{ia}(t)\dot{\phi}_{ia}(t) + r_{ia}(t)\dot{r}_{ia}(t) + \xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t)\\
&\quad - E_{id}(t)\dot{\phi}_{is}(t) - \dot{E}_{id}(t)\phi_{is}(t)\Big\}\\
&= -\left(\int_0^t \phi_i(\tau)d\tau\right)^2 - r_{ia}(t)\int_0^t \phi_i(\tau)d\tau + r_{is}(t)\tilde{E}_i(t)\\
&\quad - (\beta_i+1)\,r_{is}^2(t) + (\beta_i+1)\,r_{is}(t)r_{ia}(t) + r_{is}(t)E_{id}(t)\\
&\quad - r_{ia}^2(t) - r_{ia}(t)\phi_{ia}(t) - \phi_{ia}^2(t) + r_{ia}(t)\phi_{ia}(t)\\
&\quad + \left[\frac{\partial f(\bar{x}_i,t)}{\partial u_i}\chi(p_i(t)) + 1\right]\dot{p}_i(t) - E_{id}(t)\dot{\phi}_{is}(t)\\
&\quad - (\beta_i+1)\,r_{is}(t)r_{ia}(t) - \dot{E}_{id}(t)\phi_{is}(t) + \bigcap_{\xi_{\phi_{is}}\in\mathrm{sgn}(\phi_{is}(t))}\\
&\quad \Big\{\xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t) - \psi_i\mathrm{sgn}\,(\phi_{is}(t))\,r_{is}(t)\Big\}.
\end{aligned} \tag{35}$$

where $c_1 = \psi_i/2(\psi_i+\varsigma_{i1})$ and $\bar{\chi}(p_i(t)) = (\partial f(\bar{x}_i,t))/\partial u_i\chi(p_i(t)) + 1$. When $\phi_{is}(t) \ne 0$, then we have $\xi_{\phi_{is}} = \mathrm{sgn}\,(\phi_{is}(t))$ such that the last term of the above inequality (38) is eliminated. When $\phi_{is}(t) = 0$, then it is not difficult to check that the last term of the above inequality (38) is also zero. Thus, we have

Since $-r_{ia}(t)\int_0^t \phi_i(\tau)d\tau \le (1/2)r_{ia}^2(t) + (1/2)\left(\int_0^t \phi_i(\tau)d\tau\right)^2$ and $r_{is}(t)\tilde{E}_i(t) - \beta_i r_{is}^2(t) \le \rho(\|e(t)\|)\,\|e(t)\|\,|r_{is}(t)| - \beta_i r_{is}^2(t) \le (1/4\beta_i)\times \rho^2(\|e(t)\|)\,\|e(t)\|^2$, then, we have

$$\begin{aligned}
&\dot{\tilde{V}}_i(e_i(t),t)\\
&\le -\frac{1}{2}\left(\int_0^t \phi_i(\tau)d\tau\right)^2 - r_{is}^2(t) - \beta_i r_{is}^2(t)\\
&\quad - \frac{1}{2}r_{ia}^2(t) + \Big(E_{id}(t) - \dot{E}_{id}(t)\Big)\phi_{is}(t)\\
&\quad - \phi_{ia}^2(t) + r_{is}(t)\tilde{E}_i(t) - \psi_i\,|\phi_{is}(t)|\\
&\quad + \left[\frac{\partial f(\bar{x}_i,t)}{\partial u_i}\chi(p_i(t)) + 1\right]\dot{p}_i(t) + \bigcap_{\xi_{\phi_{is}}\in\mathrm{sgn}(\phi_{is}(t))}\\
&\quad \Big\{\xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t) - \psi_i\mathrm{sgn}\,(\phi_{is}(t))\,\dot{\phi}_{is}(t)\Big\}\\
&\le -\frac{1}{4}\,\|e(t)\|^2 - \frac{1}{2}\psi_i\,|\phi_{is}(t)|\\
&\quad - \left(\frac{1}{2}\psi_i - \varsigma_{i1} - \varsigma_{i2}\right)|\phi_{is}(t)|\\
&\quad - \left(\frac{1}{4} - \frac{1}{4\beta_i}\rho^2(\|e(t)\|)\,\|e(t)\|^2\right)\\
&\quad + \left[\frac{\partial f(\bar{x}_i,t)}{\partial u_i}\chi(p_i(t)) + 1\right]\dot{p}_i(t) + \bigcap_{\xi_{\phi_{is}}\in\mathrm{sgn}(\phi_{is}(t))}\\
&\quad \Big\{\xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t) - \psi_i\mathrm{sgn}\,(\phi_{is}(t))\,\dot{\phi}_{is}(t)\Big\}.
\end{aligned} \tag{36}$$

In view of (31), if we define the following condition, i.e.,

$$e(t)\in\Omega=\left\{e(t)\mid\|e(t)\|\le\rho^{-1}\left(\sqrt{\beta_i}\right)\right\} \tag{37}$$

then

$$\begin{aligned}\dot{\tilde{V}}_i&(e_i(t),t)\\ &\le -\min\left\{\frac{1}{2},\frac{\psi_i}{2(\psi_i+\varsigma_{i1})}\right\}V_i(t)\\ &\quad+\left[\frac{\partial f(\bar{x}_i,t)}{\partial u_i}\chi(p_i(t))+1\right]\dot{p}_i(t)+\bigcap_{\xi_{\phi_{is}}\in\operatorname{sgn}(\phi_{is}(t))}\\ &\quad\left\{\xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t)-\psi_i\operatorname{sgn}(\phi_{is}(t))\dot{\phi}_{is}(t)\right\}\\ &=-c_1V_i(t)+\bar{\chi}(p_i(t))\dot{p}_i(t)+\bigcap_{\xi_{\phi_{is}}\in\operatorname{sgn}(\phi_{is}(t))}\\ &\quad\left\{\xi_{\phi_{is}}\psi_i\dot{\phi}_{is}(t)-\psi_i\operatorname{sgn}(\phi_{is}(t))\dot{\phi}_{is}(t)\right\}\end{aligned} \tag{38}$$

$$\dot{V}_i(t)\in\dot{\tilde{V}}_i(e_i(t),t)\le -c_1V_i(t)+\bar{\chi}(p_i(t))\dot{p}_i(t). \tag{39}$$

That is

$$\dot{V}_i(t)\le -c_1V_i(t)+\bar{\chi}(p_i(t))\dot{p}_i(t) \tag{40}$$

for $e(t)\in\Omega$. According to Remark 1, we know that the function $\bar{\chi}(p_i(t))$ is also the Nussbaum-type function. Integrating both sides of (40), we have

$$V_i(t)-V_i(0)+c_1\int_0^t V_i(\tau)d\tau\le\int_0^t\bar{\chi}(p_i(\tau))\dot{p}_i(\tau)d\tau \tag{41}$$

for $e(t)\in\Omega$. Since $V_i(t)\ge 0$, the following inequality can be obtained:

$$-V_i(0)\le\int_0^t\bar{\chi}(p_i(\tau))\dot{p}_i(\tau)d\tau\quad\text{for}\quad e(t)\in\Omega$$

that is

$$-V_i(0)\le\int_0^{p_i(t)}\bar{\chi}(p_i(\tau))d\tau-\int_0^{p_i(0)}\bar{\chi}(p_i(\tau))d\tau \tag{42}$$

for $e(t)\in\Omega$. Let the above inequality (42) be divided by $p_i(t)$, then, we have

$$\frac{1}{p_i(t)}\left(\int_0^{p_i(0)}\bar{\chi}(p_i(\tau))d\tau-V_i(0)\right)\le\frac{1}{p_i(t)}\int_0^{p_i(t)}\bar{\chi}(p_i(\tau))d\tau. \tag{43}$$

If $p_i(t)\to\infty$, we have

$$0\le\frac{1}{p_i(t)}\int_0^{p_i(t)}\bar{\chi}(p_i(\tau))d\tau \tag{44}$$

which contradicts the definition of the Nussbaum-type function in Definition 1. Therefore, we can obtain that $p_i(t)$ is bounded with $e(t)\in\Omega$, which implies $\int_0^t\bar{\chi}(p_i(\tau))\dot{p}_i(\tau)d\tau=\int_{p_i(0)}^{p_i(t)}\bar{\chi}(p_i(\tau))d\tau$ is bounded as

$$\left|\int_0^t\bar{\chi}(p_i(\tau))\dot{p}_i(\tau)d\tau\right|\le c_p\ \text{for}\ e(t)\in\Omega \tag{45}$$

in which $c_p>0$ is a constant. In view of (41), it is obtained that

$$\frac{1}{2}\|e_i(t)\|^2\le V_i(t)\le V_i(0)+c_p. \tag{46}$$

Therefore, the region of attraction can be calculated as

$$D=\left\{\|e_i(t)\|\le\sqrt{2\left(V_i(0)+c_p\right)}\right\}\subset\Omega$$

In view of (37), if β_i can be chosen as

$$\beta_i\ge\rho_i^2\left(\sqrt{2\left(V_i(0)+c_p\right)}\right) \tag{47}$$

then in the attraction D, the trajectories of $e_i(t)$ remain in Ω with $t>0$ for any initial states. Therefore, the signal $e_i(t)$ is uniformly ultimately bounded stability. Note from (37) that by increasing the value of β_i, the region of attraction can be made arbitrary large to include any initial states. Thus the semiglobal performance is achieved.

According to (32) and (46), we obtain that $r_{ia}(t)$, $\phi_{ia}(t)$, and $\phi_{is}(t)$ are all bounded. In view of (29), we have

$$\begin{aligned}p_i(t)&-\frac{1}{2}r_{ia}^2(t)+\frac{1}{2}r_{ia}^2(0)-\frac{1}{2}\phi_{ia}^2(t)\\ &+\frac{1}{2}\phi_{ia}^2(0)-\psi_i|\phi_{is}(t)|+\psi_i|\phi_{is}(0)|\\ &=\int_0^t r_{ia}^2(\tau)d\tau+\int_0^t\phi_{ia}^2(\tau)d\tau\\ &+\psi_i\int_0^t|\phi_{is}(\tau)|d\tau.\end{aligned} \tag{48}$$

Since $p_i(t)$ is bounded, it is obtained that the left side of the above equation (48) is bounded. Thus, we can conclude that $\int_0^t r_{ia}^2(\tau)d\tau$, $\int_0^t\phi_{ia}^2(\tau)d\tau$ and $\int_0^t|\phi_{is}(\tau)|d\tau$ are all bounded. From (17), (19), and (20), we have $\dot{r}_{ia}(t)$, $\dot{\phi}_{ia}(t)$ and $\dot{\phi}_{is}(t)$ are bounded. According to Barbalat's lemma, $r_{ia}(t)$, $\phi_{ia}(t)$, and $\phi_{is}(t)$ are asymptotically converging to zero. In view of (19), we conclude that $\int_0^t\phi_i(\tau)d\tau$, $i=1,2,\ldots,N$ asymptotically converges to zero.

According to (12) and (13), we have

$$z_i(t)=-\sum_{k=1}^N a_{ik}\left(\int_0^t z_i(\tau)d\tau-\int_0^t z_k(\tau)d\tau\right)+\phi_{id}(t) \tag{49}$$

with

$$\phi_{id}(t)=\sum_{k=1}^N a_{ik}\left(\int_0^t\phi_i(\tau)d\tau-\int_0^t\phi_k(\tau)d\tau\right). \tag{50}$$

Since $\int_0^t\phi_i(\tau)d\tau$, $i=1,2,\ldots,N$ converges to zero, we have $\phi_{id}(t)\to 0$ as $t\to\infty$ in (50). Since the communication topology is a directed graph having a spanning tree, which can be regarded as a special case of the uniformly quasi-strongly δ-connected graphs; according to Lemma 1, we have from (49) that $z_i(t)\to 0$ as $t\to\infty$. According to Lemma 3 with (10) and (13), it can be obtained that $x_i(t)-x_k(t)\to 0$ and $x_i^{(m)}(t)\to 0$, $m=1,2,\ldots,n-1$ as $t\to\infty$. This implies that the control objective (7) is achieved. Furthermore, it is not difficult to check that all signals of the closed-loop systems are bounded. ■

IV. Simulation Examples

In this section, a group of four Duffing-Holmes chaotic systems is considered. The agent model is adopted from [18], and the dynamics of each agent are described as follows:

$$\ddot{x}_i=-p_1x_i-p_2\dot{x}_i-x_i^3+h_i(x_i,u_i)+\bar{d}_i(t) \tag{51}$$

where $\bar{d}_i(t) = q\cos(\varpi t) + d_i^s(t)$, $p_1 = 0.3 + 0.2\sin(10t)$, $p_2 = 0.2 + 0.2\cos(5t)$, $q = 5 + 0.1\cos(t)$, $\varpi = 0.5 + 0.1\sin(t)$, $h_i(x_i, u_i) = (-1)^i u_i + 0.9/\sqrt{1+x_i^2}\cos(u_i)$, and $d_i^s(t) = 0.4\sin(0.2\pi t) + 0.3\sin(x_i\dot{x}_i)$.

It is clear that the disturbance $\bar{d}_i(t)$ and its first-order derivative and second-order derivative are bounded, which means Assumption 1 is satisfied. We know from (51) that $f_i(\bar{x}_i(t), u_i(t)) = -p_1 x_i - p_2\dot{x}_i - x_i^3 + h_i(x_i, u_i)$. Therefore, we have $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t) = (-1)^i - 0.9/\sqrt{1+x_i^2}\sin(u_i)$. It can be verified that $|(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t)| = \left|(-1)^i - 0.9/\sqrt{1+x_i^2}\sin(u_i)\right| < 2$ is bounded, which implies Assumption 2 is satisfied. Furthermore, the control directions for agent 1 and agent 3 are $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t) = -1 - 0.9/\sqrt{1+x_i^2}\sin(u_i) < 0$, and the control directions for agent 2 and agent 4 are $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial u_i(t) = 1 - 0.9/\sqrt{1+x_i^2}\sin(u_i) > 0$. This implies that the control directions of agents are nonidentical. In addition, we have $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial x_i(t) = -p_1 - p_2\ddot{x}_i - 3x_i^2 - x_i(1+x_i^2)^{-\frac{3}{2}}\cos(u_i)$. If $\bar{x}_i(t) \in \mathcal{L}_\infty$, $\dot{\bar{x}}_i(t) \in \mathcal{L}_\infty$ and $\ddot{\bar{x}}_i(t) \in \mathcal{L}_\infty$, it can be verified that all elements of $(\partial f_i(\bar{x}_i(t), u_i(t)))/\partial x_i(t)$ are bounded, and its time derivative of each element is bounded, meaning Assumption 3 is satisfied.

The initial conditions are given as $[x_1(0), \dot{x}_1(0)]^T = [2, -1]^T$, $[x_2(0), \dot{x}_2(0)]^T = [-4, 1]^T$, $[x_3(0), \dot{x}_3(0)]^T = [1, 2]^T$, $[x_4(0), \dot{x}_4(0)]^T = [-2, 0.5]^T$. Furthermore, the control algorithms are designed by (28), (29), and (17), where the Nussbaum-type function is $\chi(p_i(t)) = p_i^2(t)\cos(p_i(t))$. The communication topology among four agents is described by a directed graph having a spanning tree shown in Fig. 1. For this simulation the parameters are designed as $\beta_i = 2$ and $\psi_i = 15$ for all $i = 1, \ldots, 4$.

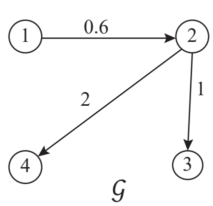

Fig. 1. The communication graph having a spanning tree.

The simulation results are illustrated in Figs. 2–4. It is observed from Fig. 2 that asymptotic consensus is obtained. Fig. 3 shows the performance of control input. At the beginning of the first 10 s, it is shown that the control input contains some frequent oscillations when the Nussbaum-type function works and the directions of the control inputs are determined. Furthermore, it is seen that the values of $u_i(t)$ are within a reasonable range. It is also observed from Fig. 3 that after the first 10 s the chattering phenomenon exists. The reason is that the proposed algorithm in (28) has discontinuous terms, and its integral implies a continuous and chattering control input. As we know, although the control algorithms are smooth, the existence of chattering phenomenon is very common, and the robust control algorithms may be able to eliminate the chattering phenomenon.

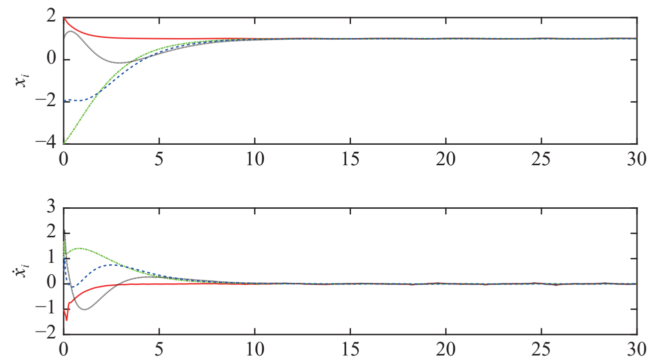

Fig. 2. Time responses of the agent states $x_i(t)$ and $\dot{x}_i(t)$ $(1 \le i \le 4)$.

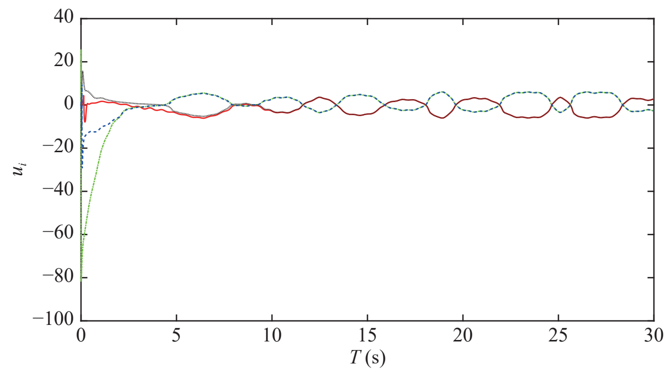

Fig. 3. Time responses of the control inputs $u_i(t)$ $(1 \le i \le 4)$.

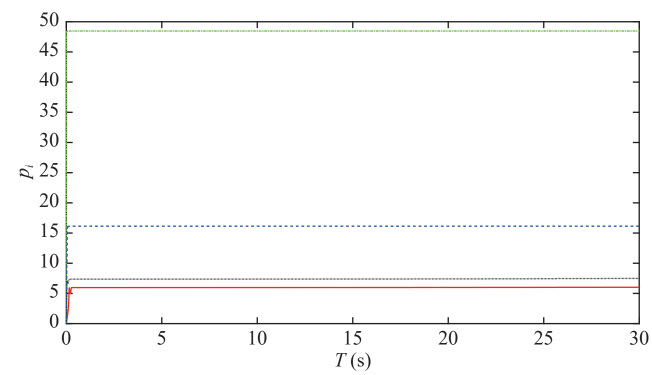

Fig. 4. Time responses of the parameters $p_i(t)$ $(1 \le i \le 4)$.

V. Conclusions

In this paper, we designed asymptotic consensus algorithms for networked nonaffine systems with nonvanishing disturbance under directed graphs. By using the DRISE design with the Nussbaum-type function, a new class of continuous algorithms with disturbance rejection has been developed. The corresponding asymptotic consensus results relax the chattering phenomenon since the proposed control algorithms are continuous. Future research directions may focus on designs of algorithms for output feedback systems, event-triggered control systems, and time-delay systems.

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