Finite-Time Fuzzy Sliding Mode Control for Nonlinear Descriptor Systems

Zhixiong Zhong, *Member, IEEE*, Xingyi Wang, and Hak-Keung Lam, *Fellow, IEEE*

Abstract—This article addresses the finite-time boundedness (FTB) problem for nonlinear descriptor systems. Firstly, the nonlinear descriptor system is represented by the Takagi-Sugeno (T-S) model, where fuzzy representation is assumed to be appearing not only in both the state and input matrices but also in the derivative matrix. By using a descriptor redundancy approach, the fuzzy representation in the derivative matrix is reformulated into a linear one. Then, we introduce a fuzzy sliding mode control (FSMC) law, which ensures the finite-time boundedness (FTB) of closed-loop fuzzy control systems over the reaching phase and sliding motion phase. Moreover, by further employing the descriptor redundancy representation, the sufficient condition for designing FSMC law, which ensures the FTB of the closed-loop control systems over the entire finite-time interval, is derived in terms of linear matrix inequalities (LMIs). Finally, a simulation study with control of a photovoltaic (PV) nonlinear system is given to show the effectiveness of the proposed method.

Index Terms—Finite-time boundedness (FTB), fuzzy sliding mode control (FSMC), Takagi-Sugeno (T-S) fuzzy descriptor system.

I. INTRODUCTION

THE fuzzy control algorithm consists of a set of fuzzy logic, fuzzy sets, and heuristic control rules [1]–[3], and it has been used for the effective handling of control of complex nonlinear systems including robotic teleoperations [4], surgical robotics [5], and multiple robots [6]. Among these fuzzy control methods, Takagi-Sugeno (T-S) fuzzy model uses linear equations to represent each local system corresponding to their local rules, and then employs fuzzy reasoning to blend local linearity for implementing total nonlinearity. Nowadays, the T-S model has been very popular in the control society because of its ability to provide good approximation. Therefore, over the past few decades, the

Z. X. Zhong is with the Key Laboratory of Information Processing and Intelligent Control, Minjiang University, Fuzhou 350121, China (e-mail: zhixiongzhong2012@126.com).

X. Y. Wang is with the College of Mechanical and Electronic Engineering, Fujian Agriculture and Forestry University, Fuzhou 350108, China (e-mail: xingyiwang2019@126.com).

H.-K. Lam is with the Department of Engineering, King's College London, London, WC2R 2LS, United Kingdom (e-mail: hak-keung.lam@kcl.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

problems of stability analysis and control synthesis have been investigated for the T-S fuzzy model more frequently [7]–[9].

Sliding mode control (SMC) has been regarded as one of the most powerful nonlinear control methods, and has been widely used due to its quick response and strong robustness in practical applications. The essence of SMC is to drive state trajectories toward the switching manifold. Such motion is motivated by imposing disruptive control actions, commonly in the form of switching control strategies. An ideal sliding mode exists only when the system state satisfies the dynamic equation that governs the sliding mode for all time. This requires an infinite switching, in general, to ensure the sliding motion. The past decades have witnessed the successfully practical application of SMC in several areas (see [10]–[13]). In addition, descriptor systems are referred to as implicit/singular systems, which enable describing a larger class of systems than the normal model representation [14]. More recently, stability results of fuzzy descriptor systems using the SMC method have been reported in [15]–[17]. However, note that the aforementioned works only highlighted the asymptotic behavior of the fuzzy control system over an infinite working time interval, and all aforementioned works of the SMC consider system dynamics within a sufficiently long time interval. In fact, in many practical applications, a finite-time stability (FTS) may be required when facing the prescribed restraints on transient dynamics, such as, for example, dual-arm robots [18], [19], input-delay systems [20], Markovian jump cyber-physical systems [21], multi-input and multi-output (MIMO) nonlinear systems [22], [23], and nonlinear systems with positive powers of odd rational numbers [24]. Both FTS and practical stability (PS) have a similar definition for stability analysis, and they work on the boundary of state trajectories starting from a desired initial region. However, the main distinction between FTS and PS is that FTS works with a finite period while PS works for an infinite amount of time [25]. When taking into account norm bounded disturbances, the concept should be changed from FTS to finite time boundedness (FTB). FTB ensures FTS, but its converse is not true [26]. We are aware that the finite-time SMC of fuzzy descriptor systems is of the wide practical applicability. However, few works studied the FTB of the FSMC descriptor system in both the reaching phase and the sliding one. It reflects the following two important control problems. One is determining how to partition the specified finite time S into two subintervals $[0, S^*]$ and $[S^*, S]$, which ensures the FTB of the corresponding FSMC descriptor system over the reaching

phase and sliding motion phase. The other is determining how to design the FSMC law via linear matrix inequalities (LMIs), which ensure the FTB of the closed-loop fuzzy descriptor system over the whole finite-time interval $[0, S]$.

Motivated by the aforementioned considerations, this paper proposes a novel fuzzy sliding mode control strategy for nonlinear descriptor systems using a FTB method. Firstly, the nonlinear descriptor system is represented by the T-S model, where fuzzy representation is assumed to be appearing not only in both the state and input matrices but also in the derivative matrix, and the derivative matrix is assumed to not always be nonsingular. By using a descriptor redundancy formulation, the fuzzy representation in the derivative matrix is reformulated into one that is linear. Then, we introduce the fuzzy sliding mode control (FSMC) law, which ensures the FTB of the closed-loop fuzzy control systems through the reaching phase and sliding motion phase. Moreover, by employing the descriptor redundancy reformulation, it is shown that a sufficient condition for designing FSMC law, which ensures the FTB of the closed-loop control systems through the entire finite-time interval, is derived in terms of LMIs. Finally, a simulation study for the control of the photovoltaic (PV) nonlinear system is provided to show the effectiveness of the proposed method. The main contributions of this paper are summarized as follows: 1) For a specified time interval $[0, S]$, we partition it into two subintervals $[0, S^*]$ and $[S^*, S]$, where the proposed FSMC law ensures the FTB of the corresponding FSMC descriptor system over reaching phase and sliding motion phase. 2) Sufficient conditions for designing the proposed FSMC law, which ensures the FTB of the fuzzy descriptor system over the whole finite-time interval $[0, S]$, are derived in terms of LMIs.

Notations: $\mathfrak{R}^n$ is the n-dimensional Euclidean space and $\mathfrak{R}^{n \times m}$ is the set of $n \times m$ matrices. Sym$\{A\}$ denotes $A + A^T$, where A^T is the transpose of the matrix A. $\|A\| = \sqrt{\lambda_{\max}[A^T A]}$ denotes the square-root of the maximum eigenvalue of the matrix $A^T A$. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix and $0_{m \times n}$ denotes the $m \times n$ zero matrix. $\mathbb{N}$ is the set $[0, 1, \ldots]$. The subscripts n and $m \times n$ are deleted when their size is irrelevant or can be inferred from the context. For a matrix $A \in \mathfrak{R}^{n \times n}$, A^{-1} denotes the inverse of the matrix A. diag$\{\cdots\}$ is a block-diagonal matrix. The notation $\|x\|$ denotes its Euclidean vector norm, and the notation $\star$ is its symmetric term.

II. PROBLEM FORMULATION AND PRELIMINARIES

Descriptor systems are referred to as implicit/singular systems, which enables us to describe a larger class of systems with normal model representation [14]. This paper considers a class of nonlinear descriptor systems

$$E(\zeta(t))\dot{x}(t) = A(\zeta(t))x(t) + B(\zeta(t))u(t) + D(\zeta(t))\omega(t) \quad (1)$$

where $x(t) \in \mathfrak{R}^{n_x}$, $u(t) \in \mathfrak{R}^{n_u}$, and $\omega(t) \in \mathfrak{R}^{n_\omega}$ are the system state, the control input, and the external disturbance, respectively; $\zeta(t)$ denotes the measurable variables; $\{E(\zeta(t)), A(\zeta(t)), B(\zeta(t)), D(\zeta(t))\}$ are the system matrices.

Currently, the most attention is given to nonlinear systems with "sector nonlinearities" [8]. Thus, the considered nonlinear system (1) can be described by the following form

of the T-S model:

$$E(h)\dot{x}(t) = A(\mu)x(t) + B(\mu)u(t) + D(\mu)\omega(t) \quad (2)$$

where $E(h)$ is nonsingular and satisfies that $E(h) := \sum_{s=1}^{r_e} h_s[\zeta(t)]E_s$, where $\sum_{s=1}^{r_e} h_s[\zeta(t)] = 1$; $A(\mu) := \sum_{l=1}^{r} \mu_l[\zeta(t)]A_l$, $B(\mu) := \sum_{l=1}^{r} \mu_l[\zeta(t)]B_l$, $\sum_{l=1}^{r} \mu_l[\zeta(t)] = 1$; r_e and r represent the number of inference rules in the left- and right-hand sides, respectively; $h_s[\zeta(t)]$ and $\mu_l[\zeta(t)]$ are normalized membership functions, which satisfy the following conditions:

$$h_s[\zeta(t)] := \frac{\prod_{\phi=1}^{g} h_{s\phi}[\zeta_\phi(t)]}{\sum_{\varsigma=1}^{r_e} \prod_{\phi=1}^{g} h_{\varsigma\phi}[\zeta_\phi(t)]} \geq 0, \sum_{s=1}^{r_e} h_s[\zeta(t)] = 1$$

$$\mu_l[\zeta(t)] := \frac{\prod_{\phi=1}^{g} \mu_{l\phi}[\zeta_\phi(t)]}{\sum_{\varsigma=1}^{r} \prod_{\phi=1}^{g} \mu_{\varsigma\phi}[\zeta_\phi(t)]} \geq 0, \sum_{l=1}^{r} \mu_l[\zeta(t)] = 1 \quad (3)$$

where $h_{s\phi}[\zeta_\phi(t)]$ and $\mu_{l\phi}[\zeta_\phi(t)]$ are the grade of membership. For simplicity of narration, we will define $h_s := h_s[\zeta(t)]$ and $\mu_l := \mu_l[\zeta(t)]$.

Remark 1: Since $E(h)$ is nonsingular, we can perform its inverse operation in the descriptor fuzzy system (2). In this case, the descriptor fuzzy system can be transformed into one that is nominal (nondescriptor). As pointed out in [27], when considering the T-S descriptor representation, the number of fuzzy inference rules will decrease so that the number of LMIs to controller design is remarkably reduced.

Here, without loss of generality, we only consider that the class of norm-bounded square integrable disturbance acts over the time interval $[t_1, t_2]$, which is defined as below [28]:

$$\mathbb{W}_{[t_1, t_2], \delta} \triangleq \left\{ \omega \in L_2[t_1, t_2] : \int_{t_1}^{t_2} \omega^T(s)\omega(s)ds \leq \delta \right\} \quad (4)$$

where δ is a positive scalar.

Before moving on, we extend the definition of the FTB in [29]–[33] to the fuzzy descriptor system (2) as follows:

Definition 1: For a time interval $[t_1, t_2]$, and two scalars c_1, c_2 subject to $0 < c_1 < c_2$, the symmetrical matrix satisfies $R > 0$. Then, the fuzzy descriptor system (2) with $u(t) = 0$ is the FTB subject to $(c_1, c_2, [t_1, t_2], R, \mathbb{W}_{[t_1, t_2], \delta})$, if it satisfies

$$x^T(t_1)Rx(t_1) \leq c_1 \implies x^T(t_2)Rx(t_2) < c_2$$

$\forall t \in [t_1, t_2]$ for all $\omega(t) \in \mathbb{W}_{[t_1, t_2], \delta}$.

This paper aims at to design a FSMC law, which can drive the system trajectories of the considered fuzzy descriptor model into the sliding surface function within a finite time, where the FTB subject to $(c_1, c_2, [0, S], R, \mathbb{W}_{[0, S], \delta})$. Furthermore, sufficient conditions for designing the proposed FSMC law is derived in the form of LMIs.

III. DESIGN OF FSMC LAW BASED ON FTB

In this section, for the specified finite time and the initial state, we will perform the FTB of the FSMC descriptor system in both the reaching and sliding phases, and it will be shown that sufficient conditions for designing the proposed FSMC law is given in the form of LMIs.

A. Model Transformation

Firstly, motivated by [27] we can rewrite the T-S fuzzy descriptor model in (2) as below:

$$\bar{E}\dot{\bar{x}}(t) = \bar{A}(h,\mu)\bar{x}(t) + \bar{B}(\mu)u(t) + \bar{D}(\mu)\omega(t) \qquad (5)$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \bar{E} = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{A}(h,\mu) = \sum_{s=1}^{r_e}\sum_{l=1}^{r} h_s \mu_l \bar{A}_{sl}, \bar{A}_{sl} = \begin{bmatrix} 0 & \mathbf{I} \\ A_l & -E_s \end{bmatrix}$$

$$\bar{B}(\mu) = \sum_{l=1}^{r} \mu_l \bar{B}_l, \bar{B}_l = \begin{bmatrix} 0 \\ B_l \end{bmatrix}$$

$$\bar{D}(\mu) = \sum_{l=1}^{r} \mu_l \bar{D}_l, \bar{D}_l = \begin{bmatrix} 0 \\ D_l \end{bmatrix}. \qquad (6)$$

Remark 2: Note that, by using a descriptor redundancy approach, the fuzzy representation in the derivative matrix (2) is reformulated into the linear approach as shown in (5). In that case, it is easy to choose the Lyapunov matrix as below:

$$\bar{P} = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

where $0 < P_1^T = P_1 \in \mathfrak{R}^{n_x \times n_x}$, $\{P_2, P_3\} \in \mathfrak{R}^{n_x \times n_x}$, and it is easy to see that $\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \geq 0$.

B. Design of FSMC Law

Firstly, based on the fuzzy descriptor system (5), an integral-type sliding surface function is constructed as below [15]:

$$s(t) = \bar{G}\bar{E}\bar{x}(t) - \int_0^t \bar{G}\left[\bar{A}(h,\mu) + \bar{B}(\mu)\bar{K}(\mu)\right]\bar{x}(s)ds \qquad (7)$$

where $\bar{G}$ is a specified matrix, which ensures that $\bar{G}\bar{B}_l$ is the nonsingular matrix, $\bar{B}(\mu) = \sum_{l=1}^{r}\mu_l\bar{B}_l$, $\bar{B}_l = \begin{bmatrix} 0 \\ B_l \end{bmatrix}$, $\bar{K}(\mu) = \sum_{l=1}^{r}\mu_l\bar{K}_l$, $\bar{K}_l = \begin{bmatrix} K_l & 0 \end{bmatrix}$, where K_l is the fuzzy controller gain to be determined.

A framework figure on the fuzzy sliding mode control of T-S fuzzy descriptor system is shown in Fig. 1. In the following, based on the sliding surface function (7), we design a FSMC law $u(t)$, which can drive the system trajectories of the fuzzy descriptor system (5) into the sliding surface function $s(t) = 0$ in a finite time $S^* \leq S$.

Theorem 1: For a specified finite time S, the reachability of the sliding surface function (7) can be satisfied within a finite time $S^* \leq S$ by using the FSMC law
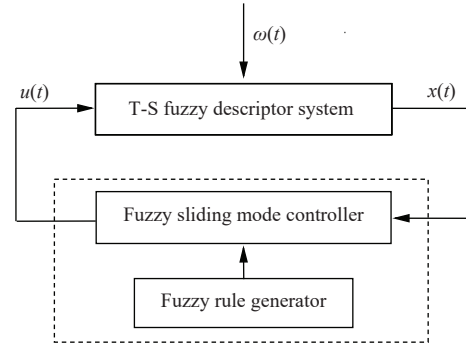
$$u(t) = u_b(t) + u_c(t) \qquad (8)$$

with

Fig. 1. Fuzzy sliding mode control of T-S fuzzy descriptor system.

$$u_b(t) = \sum_{l=1}^{r} \mu_l \bar{K}_l \bar{x}(t)$$

$$u_c(t) = -\sum_{l=1}^{r} \mu_l \left[\bar{G}\bar{B}_l\right]^{-1} \bar{\rho}(t)\,\text{sgn}\,(s(t)) \qquad (9)$$

where $\bar{K}_l = \begin{bmatrix} K_l & 0 \end{bmatrix}$, K_l is the fuzzy controller gain, $\bar{\rho}(t) = \frac{\bar{\varrho} + \left\|\sum_{l=1}^{r}\mu_l\bar{G}\bar{D}_l\right\|\|\omega(t)\|}{\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\lambda_{\min}\left(\bar{G}\bar{B}_l[\bar{G}\bar{B}_p]^{-1}\right)}$, $\bar{\varrho} \geq \frac{1}{S}\left\|\bar{G}\bar{E}\bar{x}(0)\right\|$, and $\text{sgn}(\star)$ denotes the switching sign function.

Proof: Based on the relations (7)–(9), we have

$$s^T(t)\dot{s}(t) = -s^T(t)\bar{G}\bar{B}(\mu)\sum_{l=1}^{r}\mu_l\left[\bar{G}\bar{B}_l\right]^{-1}\bar{\rho}(t)\,\text{sgn}\,(s(t))$$

$$+ s^T(t)\bar{G}\bar{D}(\mu)\omega(t)$$

$$\leq -\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\lambda_{\min}\left(\bar{G}\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1}\right)\bar{\rho}(t)\|s(t)\|$$

$$+ \left\|\sum_{l=1}^{r}\mu_l\bar{G}\bar{D}_l\right\|\|\omega(t)\|\,\|s(t)\|$$

$$= -\bar{\varrho}\|s(t)\|. \qquad (10)$$

Now, let us define

$$V_1(t) = \frac{1}{2}s^T(t)s(t). \qquad (11)$$

It has

$$\dot{V}_1(t) \leq -\bar{\varrho}\|s(t)\| = -\sqrt{2}\bar{\varrho}\sqrt{V_1(t)}. \qquad (12)$$

Based on the virtue of [34], it yields

$$S^* \leq \frac{\sqrt{2}}{\bar{\varrho}}\sqrt{V_1(0)}. \qquad (13)$$

Besides, it follows from (11) that:

$$V_1(0) = \frac{1}{2}\|s(0)\|^2. \qquad (14)$$

After substituting (14) into (13), we have

$$S^* \leq \frac{1}{\bar{\varrho}}\left\|\bar{G}\bar{E}\bar{x}(0)\right\|. \qquad (15)$$

It follows from $\bar{\varrho} \geq \frac{1}{S}\left\|\bar{G}\bar{E}\bar{x}(0)\right\|$ in (9) that:

$$S^* \leq S \qquad (16)$$

which implies that, the FSMC law (8) can drive the system trajectories of the fuzzy descriptor model (5) into the sliding surface function $s(t) = 0$ within a finite time $S^* \le S$. ∎

Remark 3: Note that the FSMC law in (8) becomes a non-sliding-mode fuzzy controller when $u_c(t) = 0$. Moreover, it tends to become a linear controller $u(t) = \bar{K}\bar{x}(t)$ when further ignoring the fuzzy premise variables.

C. Reaching Phase of FTB Within $[0, S^*]$

During the finite-time interval $[0, S^*]$ of the reaching phase, the system trajectories are outside of the sliding surface function (7), that is $s(t) \ne 0$. After substituting the FSMC law (8) into (5), the resulting closed-loop control system is obtained as below:

$$\bar{E}\dot{\bar{x}}(t) = \sum_{s=1}^{r_e} \sum_{l=1}^{r} \sum_{p=1}^{r} h_s \mu_l \mu_p \bar{\mathcal{A}}_{slp} \bar{x}(t) + \sum_{l=1}^{r} \mu_l \bar{D}_l \omega(t)$$
$$- \sum_{l=1}^{r} \sum_{p=1}^{r} \mu_l \mu_p \bar{B}_l \left[\bar{G}\bar{B}_p\right]^{-1} \bar{\rho}(t) \mathrm{sgn}(s(t)) \qquad (17)$$

where $\bar{\mathcal{A}}_{slp} = \bar{A}_{sl} + \bar{B}_l \bar{K}_p$.

Define $\tilde{\rho}(t) = \bar{\rho}(t)\mathrm{sgn}(s(t))$, $\tilde{\varrho} = \dfrac{\bar{\varrho}}{\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\lambda_{\min}\left(\bar{G}\bar{B}_l[\bar{G}\bar{B}_p]^{-1}\right)}$, and $\varepsilon = \dfrac{\sum_{l=1}^{r}\mu_l\|\bar{G}\bar{D}_l\|}{\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\lambda_{\min}\left(\bar{G}\bar{B}_l[\bar{G}\bar{B}_p]^{-1}\right)}$. Due to $s(t) \ne 0$, we have

$$\tilde{\rho}^2(t) = \bar{\rho}^2(t) = \left[\tilde{\varrho} + \varepsilon\|\omega(t)\|\right]^2$$
$$= \tilde{\varrho}^2 + 2\tilde{\varrho}\varepsilon\|\omega(t)\| + \varepsilon^2\|\omega(t)\|^2$$
$$\le \left(1 + \varepsilon^2\right)\tilde{\varrho}^2 + \left(1 + \varepsilon^2\right)\|\omega(t)\|^2. \qquad (18)$$

In the following, a sufficient condition for the FTB of the closed-loop control system (17) within the finite-time interval $[0, S^*]$ is established as below.

Theorem 2: For the specified finite time S, the resulting closed-loop control system in (17) is the FTB toward $(c_1, c^*, [0, S^*], R, \mathbb{W}_{[0, S^*], \delta})$, if there exist the matrices $\bar{X} \in \mathfrak{R}^{n_{2x} \times n_{2x}}, 0 < X_1^T = X_1 \in \mathfrak{R}^{n_x \times n_x}, \{X_2, X_3\} \in \mathfrak{R}^{n_x \times n_x}$, and the controller gain $\tilde{K}_l \in \mathfrak{R}^{n_u \times 2n_x}$, and the positive scalars $\{c_1, c^*, c_2, \eta\}, c_1 < c^* < c_2$, such that for all the index $s \in \{1, 2, \ldots, r_e\}$ the following LMIs hold:

$$\bar{\Theta}_{sll} < 0, \ 1 \le l \le r \qquad (19)$$

$$\bar{\Theta}_{slp} + \bar{\Theta}_{spl} < 0, \ 1 \le l < p \le r \qquad (20)$$

where

$$\Theta_{slp} = \begin{bmatrix} \Theta_{slp}^{(1)} & \bar{D}_l & -\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1} \\ \star & -\eta\mathbf{I} & 0 \\ \star & \star & -\eta\mathbf{I} \end{bmatrix}$$

$$\Theta_{slp}^{(1)} = \mathrm{Sym}\left\{\bar{A}_{sl}\bar{X} + \bar{B}_l\tilde{K}_p\right\} - \eta\bar{E}\bar{X}, \bar{X} = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}. \qquad (21)$$

Furthermore, the bounding is calculated as below:

$$\frac{\bar{\sigma}_{P1}c_1 + \left(\eta S \tilde{\varrho}^2 + \eta\delta\right)\left(1 + \varepsilon^2\right) + \eta\delta}{e^{-\eta S}\underline{\sigma}_{P1}} < c^*. \qquad (22)$$

Proof: Consider the Lyapunov function in the descriptor-

system domain,

$$V_2(t) = \bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t), \forall t \in [0, S^*] \qquad (23)$$

where $\bar{P} \in \mathfrak{R}^{2n_x \times 2n_x}$ and $\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \ge 0$.

Along the state trajectory of the fuzzy control system (17), we have

$$\dot{V}_2(t) = 2\left[\bar{E}\dot{\bar{x}}(t)\right]^T \bar{P}\bar{x}(t)$$
$$= 2\left[\sum_{s=1}^{r_e}\sum_{l=1}^{r}\sum_{p=1}^{r} h_s\mu_l\mu_p\bar{\mathcal{A}}_{slp}\bar{x}(t)\right]^T \bar{P}\bar{x}(t)$$
$$- 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1}\tilde{\rho}(t)\right]^T \bar{P}\bar{x}(t)$$
$$+ 2\sum_{l=1}^{r}\mu_l\left[\bar{D}_l\omega(t)\right]^T \bar{P}\bar{x}(t). \qquad (24)$$

An index function is introduced as below:

$$J_1(t) = \dot{V}_2(t) - \eta V_2(t) - \eta\omega^T(t)\omega(t) - \eta\tilde{\rho}^2(t) \qquad (25)$$

where η is a positive scalar.

Based on the relations (24) and (25), we have

$$J_1(t) = 2\left[\sum_{s=1}^{r_e}\sum_{l=1}^{r}\sum_{p=1}^{r} h_s\mu_l\mu_p\bar{\mathcal{A}}_{slp}\bar{x}(t)\right]^T \bar{P}\bar{x}(t)$$
$$- 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1}\tilde{\rho}(t)\right]^T \bar{P}\bar{x}(t)$$
$$+ 2\sum_{l=1}^{r}\mu_l\left[\bar{D}_l\omega(t)\right]^T \bar{P}\bar{x}(t) - \eta\bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t)$$
$$- \eta\omega^T(t)\omega(t) - \eta\tilde{\rho}^2(t)$$
$$= \sum_{s=1}^{r_e}\sum_{l=1}^{r}\sum_{p=1}^{r} h_s\mu_l\mu_p\chi^T(t)\Theta_{slp}\chi(t) \qquad (26)$$

where

$$\chi(t) = \begin{bmatrix} \bar{x}^T(t) & \omega^T(t) & \tilde{\rho}^T(t) \end{bmatrix}^T$$

$$\Theta_{slp} = \begin{bmatrix} \Theta_{slp}^{(1)} & \bar{P}^T\bar{D}_l & -\bar{P}^T\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1} \\ \star & -\eta\mathbf{I} & 0 \\ \star & \star & -\eta\mathbf{I} \end{bmatrix}$$

$$\Theta_{slp}^{(1)} = \mathrm{Sym}\left\{\bar{P}^T\bar{A}_{sl} + \bar{P}^T\bar{B}_l\bar{K}_p\right\} - \eta\bar{P}^T\bar{E}. \qquad (27)$$

To cast the inequality $\sum_{s=1}^{r_e}\sum_{l=1}^{r}\sum_{p=1}^{r} h_s\mu_l\mu_p\Theta_{slp} < 0$ into LMIs, we have

$$\bar{P}^{-1} = \bar{X} = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix} \qquad (28)$$

where $0 < X_1 = X_1^T \in \mathfrak{R}^{n_x \times n_x}, \{X_2, X_3\} \in \mathfrak{R}^{n_x \times n_x}$. It is easy to see that the inequality $\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \ge 0$ holds.

Now, we define $\Gamma = \mathrm{diag}\{\bar{X}, \mathbf{I}, \mathbf{I}\}$, and $\tilde{K}_l = \bar{K}_l\bar{X}$, and use a congruent transformation to $\sum_{s=1}^{r_e}\sum_{l=1}^{r}\sum_{p=1}^{r} h_s\mu_l\mu_p\bar{\Theta}_{slp} < 0$ by Γ. After extracting the fuzzy premise variables, the

inequalities in (19) and (20) can be directly obtained.

It can be seen from (19) and (20) that $J_1(t) < 0$, which implies that

$$\dot{V}_2(t) < \eta V_2(t) + \eta \omega^T(t)\omega(t) + \eta \tilde{\rho}^2(t). \tag{29}$$

Pre- and post-multiplying both sides of (29) by $e^{-\eta t}$ and integrating the successive inequality from 0 to t, where $t \in [0, S^*]$, we have

$$
\begin{aligned}
e^{-\eta t} V_2(t) &< V_2(0) + \eta \int_0^t e^{-\eta s} \tilde{\rho}^2(s) ds \\
&\quad + \eta \int_0^t e^{-\eta s} \omega^T(s)\omega(s) ds \\
&\leq \bar{x}^T(0)\bar{E}^T \bar{P}\bar{x}(0) + \eta S\left(1 + \varepsilon^2\right)\tilde{\varrho}^2 \\
&\quad + \eta\left(1 + \varepsilon^2\right)\delta + \eta\delta.
\end{aligned} \tag{30}
$$

In addition, it follows from (23) that:

$$e^{-\eta t} V_2(t) \geq e^{-\eta S} \bar{x}^T(t)\bar{E}^T \bar{P}\bar{x}(t) \tag{31}$$

which implies that

$$
\begin{aligned}
e^{-\eta S} \bar{x}^T(t)\bar{E}^T \bar{P}\bar{x}(t) &< \bar{x}^T(0)\bar{E}^T \bar{P}\bar{x}(0) + \eta S\left(1 + \varepsilon^2\right)\tilde{\varrho}^2 \\
&\quad + \eta\left(1 + \varepsilon^2\right)\delta + \eta\delta.
\end{aligned} \tag{32}
$$

Furthermore, we specify the matrix $\bar{P}$ as follows:

$$\bar{P} = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \tag{33}$$

where $0 < P_1^T = P_1 \in \mathfrak{R}^{n_x \times n_x}$, $\{P_2, P_3\} \in \mathfrak{R}^{n_x \times n_x}$, and it is easy to see that $\bar{E}^T \bar{P} = \bar{P}^T \bar{E} \geq 0$.

Now, due to $\bar{x}(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$, it follows from (30)–(33) that:

$$
\begin{aligned}
e^{-\eta S} x^T(t) P_1 x(t) &< x^T(0) P_1 x(0) + \eta S\left(1 + \varepsilon^2\right)\tilde{\varrho}^2 \\
&\quad + \eta\left(1 + \varepsilon^2\right)\delta + \eta\delta.
\end{aligned} \tag{34}
$$

Now, we introduce the matrix $0 < R_1^T = R_1 \in \mathfrak{R}^{n_x \times n_x}$, and define

$$c_1 = x^T(0) R_1 x(0)$$

$$\bar{\sigma}_{P1} = \lambda_{\max}\left(R_1^{-\frac{1}{2}} P_1 R_1^{-\frac{1}{2}}\right), \quad \underline{\sigma}_{P1} = \lambda_{\min}\left(R_1^{-\frac{1}{2}} P_1 R_1^{-\frac{1}{2}}\right). \tag{35}$$

Based on the relationship (34) and (35), we have

$$x^T(t) R_1 x(t) < \frac{\bar{\sigma}_{P1} c_1 + \left(\eta S \tilde{\varrho}^2 + \eta\delta\right)\left(1 + \varepsilon^2\right) + \eta\delta}{e^{-\eta S} \underline{\sigma}_{P1}}. \tag{36}$$

From Definition 1, the fuzzy descriptor system in (17) is the FTB subject to $(c_1, c^*, [0, S^*], R, \mathbb{W}_{[0, S^*], \delta})$. ∎

D. Sliding Motion Phase of FTB Within $[S^*, S]$

During the finite-time interval $[S^*, S]$ of the sliding phase, we will derive the sufficient conditions to ensure the FTB of the FSMC descriptor system. When the system trajectories arrive at the sliding surface, it has that $\dot{s}(t) = 0$. Thus, the equivalent controller $u_{eq}(t)$ is obtained as below:

$$\bar{G}\bar{B}(\mu) u_{eq}(t) = \bar{G}\bar{B}(\mu)\bar{K}(\mu)\bar{x}(t) - \bar{G}\bar{D}(\mu)\omega(t) \tag{37}$$

where $\bar{G}$ is a given matrix so that $\bar{G}\bar{B}(\mu)$ is nonsingular.

Motivated by [35], [36], by augmenting the system (5) and the equivalent controller (37), it yields

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}(h, \mu)\tilde{x}(t) + \tilde{D}(\mu)\omega(t) \tag{38}$$

where

$$\tilde{E} = \begin{bmatrix} \bar{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{x}(t) = \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t) \\ u_{eq}(t) \end{bmatrix}$$

$$\tilde{D}(\mu) = \begin{bmatrix} \bar{D}(\mu) \\ 0 \\ -\bar{G}\bar{D}(\mu) \end{bmatrix}$$

$$\tilde{A}(h, \mu) = \begin{bmatrix} 0 & \bar{A}(h, \mu) & \bar{B}(\mu) \\ \mathbf{I} & -\mathbf{I} & 0 \\ \bar{G}\bar{B}(\mu)\bar{K}(\mu) & 0 & -\bar{G}\bar{B}(\mu) \end{bmatrix}. \tag{39}$$

Remark 4: Here, by further employing the descriptor redundancy representation we can avoid the inverse operation in the equivalent controller (37).

In the following, we will derive a sufficient condition to ensure the FTB of the FSMC descriptor system (38) within the finite-time interval $[S^*, S]$.

Theorem 3: For the specified finite time S, the resulting closed-loop control system in (38) is the FTB toward $(c^*, c_2, [S^*, S], R, \mathbb{W}_{[0, S], \delta})$, if there exist the matrices $0 < X_1 = X_1^T \in \mathfrak{R}^{n_x \times n_x}$, $\{X_2, X_3\} \in \mathfrak{R}^{n_x \times n_x}$, $\{X_{(21)}, X_{(22)}\} \in \mathfrak{R}^{2n_x \times 2n_x}$, $\{X_{(23)}, X_{(24)}\} \in \mathfrak{R}^{n_u \times 2n_x}$, $X_{(25)} \in \mathfrak{R}^{n_u \times n_u}$, and the controller gain $\tilde{K}_l \in \mathfrak{R}^{n_u \times 2n_x}$, and the positive scalars $\{c^*, \eta\}$, such that for all the index $s \in \{1, 2, \ldots, r_e\}$ the following LMIs hold:

$$\bar{\Phi}_{sll} < 0, \quad 1 \leq l \leq r \tag{40}$$

$$\bar{\Phi}_{slp} + \bar{\Phi}_{spl} < 0, \quad 1 \leq l < p \leq r \tag{41}$$

where

$$\bar{\Phi}_{slp} = \begin{bmatrix} \text{Sym}\left(\Phi_{slp(1)}\right) - \eta\tilde{E}\tilde{X} & \tilde{D}_l \\ \star & -\eta\mathbf{I} \end{bmatrix}$$

$$\tilde{X} = \begin{bmatrix} \bar{X} & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}$$

$$\Phi_{slp(1)} = \begin{bmatrix} \Phi_{slp(11)} & \Phi_{slp(12)} & \bar{B}_l X_{(25)} \\ \bar{X} - X_{(21)} & -X_{(21)} & 0 \\ \Phi_{lp(31)} & -\bar{G}\bar{B}_l X_{(24)} & -\bar{G}\bar{B}_l X_{(25)} \end{bmatrix}$$

$$\Phi_{slp(11)} = \bar{A}_{sl} X_{(21)} + \bar{B}_l X_{(23)}$$

$$\Phi_{slp(12)} = \bar{A}_{sl} X_{(22)} + \bar{B}_l X_{(24)}$$

$$\Phi_{lp(31)} = \bar{G}\bar{B}_l \tilde{K}_p - \bar{G}\bar{B}_l X_{(23)}. \tag{42}$$

Furthermore, the bounding is calculated as below:

$$\frac{\bar{\sigma}_{P1} c^* + \eta\delta}{e^{-\eta S} \underline{\sigma}_{P1}} < c_2. \tag{43}$$

Proof: Consider the following Lyapunov function:

$$V_3(t) = \tilde{x}^T(t)\tilde{E}^T \tilde{P}\tilde{x}(t), \quad \forall t \in [S^*, S] \tag{44}$$

where $\tilde{E}^T \tilde{P} = \tilde{P}^T \tilde{E} \geq 0$.

Along the trajectory of system (38), we have

$$\dot{V}_3(t) = 2\left[\tilde{A}(h,\mu)\tilde{x}(t) + \tilde{D}(\mu)\omega(t)\right]^T \tilde{P}\tilde{x}(t). \qquad (45)$$

An index function is introduced as follows:

$$J_2(t) = \dot{V}_3(t) - \eta V_3(t) - \eta\omega^T(t)\omega(t) \qquad (46)$$

where η is a positive scalar.

It follows from (44)–(46) that:

$$J_2(t) = 2\left[\tilde{A}(h,\mu)\tilde{x}(t) + \tilde{D}(\mu)\omega(t)\right]^T \tilde{P}\tilde{x}(t)$$
$$- \eta\tilde{x}^T(t)\tilde{E}^T \tilde{P}\tilde{x}(t) - \eta\omega^T(t)\omega(t)$$
$$= \tilde{\chi}^T(t)\Phi(h,\mu)\tilde{\chi}(t) \qquad (47)$$

where $\tilde{\chi}(t) = \left[\begin{array}{cc} \tilde{x}^T(t) & \omega^T(t) \end{array}\right]^T$, and

$$\Phi(h,\mu) = \left[\begin{array}{cc} \mathrm{Sym}\left(\tilde{A}^T(h,\mu)\tilde{P}\right) - \eta\tilde{E}^T\tilde{P} & \tilde{P}^T\tilde{D}(\mu) \\ \star & -\eta\mathbf{I} \end{array}\right].$$

It is easy to see that $\Phi(h,\mu) < 0$, which implies that $J_2(t) < 0$. Now, by extracting the fuzzy premise variables, it yields

$$\Phi_{sll} < 0, \ 1 \le l \le r \qquad (48)$$

$$\Phi_{slp} + \Phi_{spl} < 0, \ 1 \le l < p \le r \qquad (49)$$

where

$$\Phi_{slp} = \left[\begin{array}{cc} \mathrm{Sym}\left(\tilde{A}_{slp}^T\tilde{P}\right) - \eta\tilde{E}^T\tilde{P} & \tilde{P}^T\tilde{D}_l \\ \star & -\eta\mathbf{I} \end{array}\right]$$

$$\tilde{A}_{slp} = \left[\begin{array}{ccc} 0 & \bar{A}_{sl} & \bar{B}_l \\ \mathbf{I} & -\mathbf{I} & 0 \\ \bar{G}\bar{B}_l\bar{K}_p & 0 & -\bar{G}\bar{B}_l \end{array}\right]. \qquad (50)$$

To cast the conditions (48) and (49) into LMIs, we define

$$\tilde{P}^{-1} = \tilde{X} \qquad (51)$$

where $\tilde{P} = \left[\begin{array}{ccc} \bar{P} & 0 & 0 \\ P_{(21)} & P_{(22)} & 0 \\ P_{(23)} & P_{(24)} & P_{(25)} \end{array}\right], \bar{P} = \left[\begin{array}{cc} P_1 & 0 \\ P_2 & P_3 \end{array}\right],$

$\tilde{X} = \left[\begin{array}{ccc} \bar{X} & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{array}\right], \ \bar{X} = \left[\begin{array}{cc} X_1 & 0 \\ X_2 & X_3 \end{array}\right], 0 < P_1 = P_1^T \in$

$\mathfrak{R}^{n_x \times n_x}, \ 0 < X_1 = X_1^T \in \mathfrak{R}^{n_x \times n_x}, \{P_2,P_3,X_2,X_3\} \in \mathfrak{R}^{n_x \times n_x}. \{P_{(21)},$
$P_{(22)},X_{(21)},X_{(22)}\} \in \mathfrak{R}^{n_x \times n_x}, \ \{P_{(23)},P_{(24)},X_{(23)},X_{(24)}\} \in \mathfrak{R}^{n_u \times n_x},$
$\{P_{(25)},X_{(25)}\} \in \mathfrak{R}^{n_u \times n_u}.$

Now, we define $\Gamma = \mathrm{diag}\{\tilde{X},\mathbf{I}\}$, and use a congruent transformation to (48) and (49) by Γ. The inequalities in (40) and (41) can be directly obtained.

In addition, $J_2(t) < 0$ implies that

$$\dot{V}_3(t) < \eta V_3(t) + \eta\omega^T(t)\omega(t). \qquad (52)$$

Pre- and post-multiplying both sides of (52) by $e^{-\eta t}$ and integrating the successive inequality from S^* to t with $t \in [S^*, S]$, we have

$$e^{-\eta t} V_3(t) < V_3(S^*) + \eta\int_{S^*}^t e^{-\eta s}\omega^T(s)\omega(s)ds$$
$$\le \tilde{x}^T(S^*)\tilde{E}^T \tilde{P}\tilde{x}(S^*) + \eta\delta$$
$$= x^T(S^*)P_1 x(S^*) + \eta\delta < \bar{\sigma}_{P1}c^* + \eta\delta. \qquad (53)$$

In addition, it can be seen from (44) that

$$e^{-\eta t}V_3(t) \ge e^{-\eta S}\tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{x}(t)$$
$$= e^{-\eta S}x^T(t)P_1 x(t)$$
$$> e^{-\eta S}\underline{\sigma}_{P1}x^T(t)R_1 x(t) \qquad (54)$$

which implies that

$$x^T(t)R_1 x(t) < \frac{\bar{\sigma}_{P1}c^* + \eta\delta}{e^{-\eta S}\underline{\sigma}_{P1}}. \qquad (55)$$

From Definition 1, the fuzzy descriptor system in (17) is the FTB. ∎

E. Design of Controller Gain $\bar{K}_l$

In the following, we will design the controller gain $\bar{K}_l$ in the sliding surface function (7), which guarantees that the conditions in Theorems 2 and 3 are feasible synchronously. The corresponding result can be summarized as follows.

Theorem 4: Consider the T-S fuzzy descriptor system (7) and the FSMC law (8). For the specified finite time S, the resulting FSMC descriptor system is the FTB toward $(c_1,c_2,[0,S],R,\mathbb{W}_{[0,S],\delta})$, if there exist matrices $0 < X_1 = X_1^T \in \mathfrak{R}^{n_x \times n_x}, \{X_2, X_3\} \in \mathfrak{R}^{n_x \times n_x}, \{X_{(21)}, X_{(22)}\} \in \mathfrak{R}^{2n_x \times 2n_x}, \{X_{(23)}, X_{(24)}\} \in \mathfrak{R}^{n_u \times 2n_x}, X_{(25)} \in \mathfrak{R}^{n_u \times n_u}$, the controller gain $\tilde{K}_l \in \mathfrak{R}^{n_u \times 2n_x}$, and the positive scalars η, such that for all the index $s \in \{1,2,\ldots,r_e\}$ the following LMIs hold:

$$\Theta_{sll} < 0, \ 1 \le l \le r \qquad (56)$$

$$\bar{\Phi}_{sll} < 0, \ 1 \le l \le r \qquad (57)$$

$$\Theta_{slp} + \Theta_{spl} < 0, \ 1 \le l < p \le r \qquad (58)$$

$$\bar{\Phi}_{slp} + \bar{\Phi}_{spl} < 0, \ 1 \le l < p \le r \qquad (59)$$

where

$$\Theta_{slp} = \left[\begin{array}{ccc} \Theta_{slp}^{(1)} & \bar{D}_l & -\bar{B}_l\left[\bar{G}\bar{B}_p\right]^{-1} \\ \star & -\eta\mathbf{I} & 0 \\ \star & \star & -\eta\mathbf{I} \end{array}\right]$$

$$\Theta_{slp}^{(1)} = \mathrm{Sym}\left\{\bar{A}_{sl}\bar{X} + \bar{B}_l\tilde{K}_p\right\} - \eta\bar{E}\bar{X}, \bar{X} = \left[\begin{array}{cc} X_1 & 0 \\ X_2 & X_3 \end{array}\right]$$

$$\bar{\Phi}_{slp} = \left[\begin{array}{cc} \mathrm{Sym}\left(\Phi_{slp(1)}\right) - \eta\tilde{E}\tilde{X} & \tilde{D}_l \\ \star & -\eta\mathbf{I} \end{array}\right]$$

$$\tilde{X} = \left[\begin{array}{ccc} \bar{X} & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{array}\right], \tilde{D}_l = \left[\begin{array}{c} \bar{D}_l \\ 0 \\ -\bar{G}\bar{D}_l \end{array}\right]$$

$$\Phi_{slp(1)} = \left[\begin{array}{ccc} \Phi_{slp(11)} & \Phi_{slp(12)} & \bar{B}_l X_{(25)} \\ \bar{X} - X_{(21)} & -X_{(21)} & 0 \\ \Phi_{lp(31)} & -\bar{G}\bar{B}_l X_{(24)} & -\bar{G}\bar{B}_l X_{(25)} \end{array}\right]$$

$$\Phi_{slp(11)} = \bar{A}_{sl} X_{(21)} + \bar{B}_l X_{(23)}$$

$$\Phi_{slp(12)} = \bar{A}_{sl} X_{(22)} + \bar{B}_l X_{(24)}$$

$$\Phi_{lp(31)} = \bar{G}\bar{B}_l\tilde{K}_p - \bar{G}\bar{B}_l X_{(23)}. \qquad (60)$$

Furthermore, the controller gain can be obtained as below:

$$\bar{K}_l = \tilde{K}_l \bar{X}^{-1}. \qquad (61)$$

F. Design Procedure for the FTB Algorithm

The detailed calculation steps of the proposed FTB algorithm for the FSMC descriptor system are summarized as follows:

1) Use the T-S fuzzy model method to describe the nonlinear descriptor system as shown in (1), and rewrite the T-S fuzzy descriptor model as shown in (5).

2) Choose a suitable matrix $\bar{G}$, so that $\bar{G}\bar{B}(\mu)$ is nonsingular, and solve Theorem 4 to obtain the controller gain K_l. Given the initial state $x(0)$, and the finite-time S, and construct a FSMC law $u(t)$ as shown in (8) and (9);

3) Given the matrix R_1 and calculate the bounding c_1. Solve Theorem 2 to obtain the bounding c^*, and solve Theorem 3 to obtain the bounding c_2.

Remark 5: It is easy to see from (56)–(59) that treating the matrix $\bar{G}$ and the scalar η as constant parameters, (56)–(59) become a set of LMIs.

Remark 6: It is noted that $\bar{G}$ can be chosen so that $\bar{G}\bar{B}(\mu)$ is nonsingular, and it is easy to see from (9) that a smaller $\|\bar{G}\bar{B}_l\|$ can enhance the effect on $u_c(t)$. However, oppositely, we can ignore $u_c(t)$ if a sufficient large $\|\bar{G}\bar{B}_l\|$ is chosen. Moreover, one can find an appropriated scalar η by the linear searching so that the obtained boundary c^* is the minimum.

IV. SIMULATION STUDY

The PV systems are built to transform sun irradiance into electrical power. However, building such systems come at a relatively high cost. All work done in the published literature focuses on increasing the efficiency of such systems and decreasing their cost. In order to show the effectiveness of the proposed control method, we consider a maximum power point tracking (MPPT) problem for a solar PV power system using a DC/DC boost converter as shown in Fig. 2, which consists of a solar PV array, an inductor L, a capacitor C_0, and a load. Its dynamic model can be represented by the following differential equations [37]:

$$\begin{cases} L\dot{\phi}_{pv} = -(1-u)v_{dc} + v_{pv} \\ C_0\dot{v}_{dc} = (1-u)\phi_{pv} - \phi_0 \end{cases} \qquad (62)$$

where $u \in [0, 1]$ represents the duty ratio; ϕ_{pv} and v_{dc} stand for the inductor current and the capacitor voltage, respectively; ϕ_0 and v_0 denote the load current and the load voltage, respectively. It should be noted that the duty ratio u carries out the switching action by using PWM.
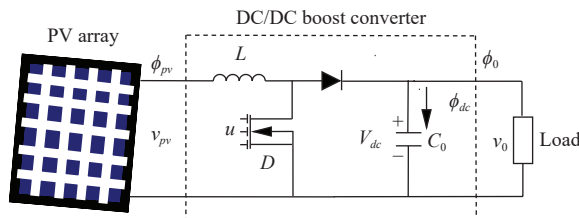

Fig. 2. A solar PV power with DC/DC boost converter.

In order to maximize the efficiency of PV power-generation systems, the electric characteristic of PV arrays is considered as follows [37]:

$$\begin{cases} \phi_{pv} = n_p I_{ph} - n_p I_{rs} (e^{\gamma v_{pv}} - 1) \\ P_{pv} = \phi_{pv} v_{pv} \end{cases} \qquad (63)$$

where n_p and n_s are the number of the parallel and series cells, respectively; $\gamma = q/(n_s \phi K H)$ with the electronic charge $q = 1.6 \times 10^{-19}$ C, Boltzmann's constant $K = 1.3805 \times 10^{-23}$ J/°K, cell temperature H; I_{ph} and I_{rs} are the light-generated current and the reverse saturation current, respectively. Here, series resistances and their intrinsic shunt are neglected. According to the representation of array power in (63) and by taking the partial derivative of P_{pv} with respect to the PV voltage v_{pv}, one gets [37]

$$\frac{dP_{pv}}{dv_{pv}} = \phi_{pv} - n_p \gamma I_{rs} v_{pv} e^{\gamma v_{pv}}. \qquad (64)$$

1) Here, we let $\frac{dP_{pv}}{dv_{pv}} = 0$ for obtaining the MPPT performance, and define $x(t) = \begin{bmatrix} \phi_{pv} & v_{dc} & P_{pv} \end{bmatrix}^T$, and choose $\zeta_1 = \frac{v_{pv}}{\phi_{pv}}$, $\zeta_2 = \frac{\phi_0}{v_{dc}}$, $\zeta_3 = \frac{v_{pv}}{P_{pv}} e^{\gamma v_{pv}}$, $\zeta_4 = v_{dc}$, and $\zeta_5 = \phi_{pv}$ as fuzzy premise variables. Now, assume that the disturbance $\omega(t)$ acts on the output voltage v_{dc}, and the system parameters are $L = 0.015$ H, $C_0 = 0.01$ F, $n_p = 36$, $\gamma = 0.3863$, $I_{rs} = 4$. We further linearize the above mentioned nonlinear system around the operation points $\zeta_1 = (5, 7.6923)$, $\zeta_2 = (0.25, 0.50)$, $\zeta_3 = (23.8040, 49.4831)$, $\zeta_4 = (10, 16)$, and $\zeta_5 = (1.56, 2)$. Then, the nonlinear PV system (62) with the MPPT problem (64) is represented by the following T-S fuzzy descriptor model:

$$E\dot{x}(t) = A(\mu)x(t) + B(\mu)u(t) + D\omega(t) \qquad (65)$$

where $A(\mu) := \sum_{l=1}^{32} \mu_l[\zeta(t)]A_l$, $B(\mu) := \sum_{l=1}^{32} \mu_l[\zeta(t)]B_l$, $\{E, A_{1\sim31}, B_{1\sim31}, D\}$ is listed in Appendix A.

The normalized membership functions are given in Fig. 3, and we rewrite the T-S fuzzy descriptor model of the nonlinear PV system as below:

$$\bar{E}\dot{\bar{x}}(t) = \bar{A}(\mu)\bar{x}(t) + \bar{B}(\mu)u(t) + \bar{D}\omega(t) \qquad (66)$$
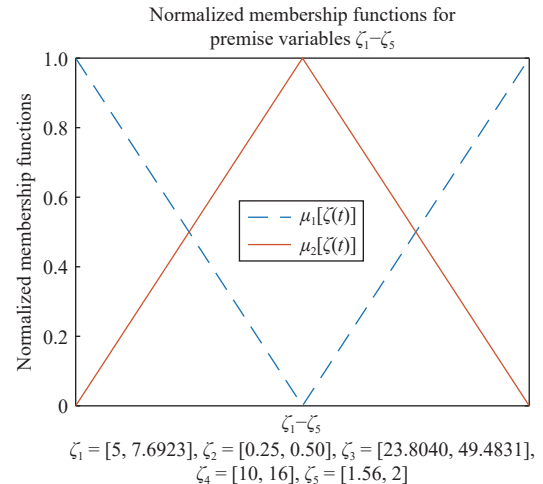

Fig. 3. Normalized membership functions.

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}(\mu) = \begin{bmatrix} 0 & I \\ A(\mu) & -E \end{bmatrix}$$

$$\bar{B}(\mu) = \begin{bmatrix} 0 \\ B(\mu) \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} 0 \\ D \end{bmatrix}.$$

2) Choose a suitable matrix $\bar{G} = \begin{bmatrix} 0 & 1 & -1 & 0 & 1 & 0 \end{bmatrix}$, $\eta = 0.3$, and solve Theorem 4 to obtain the desired controller gains, which are listed in Appendix B.

Given $\omega(t) = 0.5\sin 500t$, the finite time $S = 1$ s, and the initial state $x(0) = \begin{bmatrix} -8 & 1 & 2 \end{bmatrix}^T$. We calculate $\bar{\varrho} = 2$, and $\bar{\rho}(t) = \frac{2+\|\omega(t)\|}{0.9}$. Now, we construct the fuzzy sliding mode controller as follows:

$$u(t) = u_b(t) + u_c(t) \tag{67}$$

with

$$u_b(t) = \sum_{l=1}^{r} \mu_l \bar{K}_l \bar{x}(t)$$

$$u_c(t) = -\sum_{l=1}^{r} \mu_l \left[\bar{G}\bar{B}_l\right]^{-1} \frac{2+\|\omega(t)\|}{0.9} \text{sgn}(s(t))$$

where the sgn($\star$) is a switching sign function.

3) Given the matrix $R_1 = \text{diag}\{1,1,1\}$, and calculate the bounding $c_1 = 69$. Solve Theorem 2 to obtain the bounding $c^* > 3.9033 \times 10^6$, and solve Theorem 3 to obtain the bounding $c_2 > 2.0521 \times 10^{11}$.

With the above solution, the response of the sliding surface function is shown in Fig. 4. It is easy to see that the proposed FSMC can force PV system states around the sliding surface within $S = 0.3$ s, which is less than the pre-specified finite time $S = 1$ s. The responses of PV system states by the proposed FSMC control strategy are shown in Fig. 5. It can be seen that the approximated MPPT of the PV nonlinear system can be achieved within $S = 0.05$ s. Moreover, we further compare with non-fuzzy sliding mode control, and the corresponding results are respectively given in Fig. 6. It is easy to see that the proposed fuzzy sliding mode control achieves better control performance in comparison with non-fuzzy sliding mode control. Note that the state trajectories of
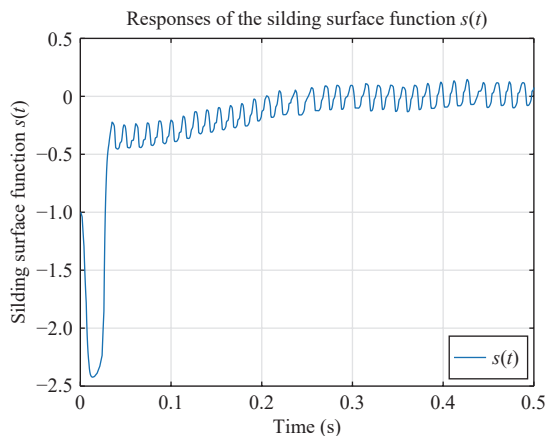

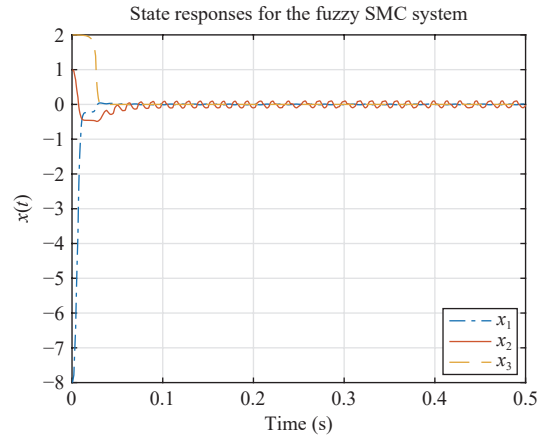Fig. 4.   Response of the sliding surface function.

Fig. 5.   State responses for the fuzzy SMC system.
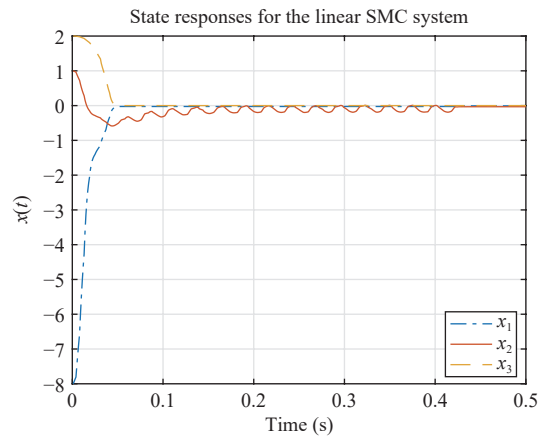

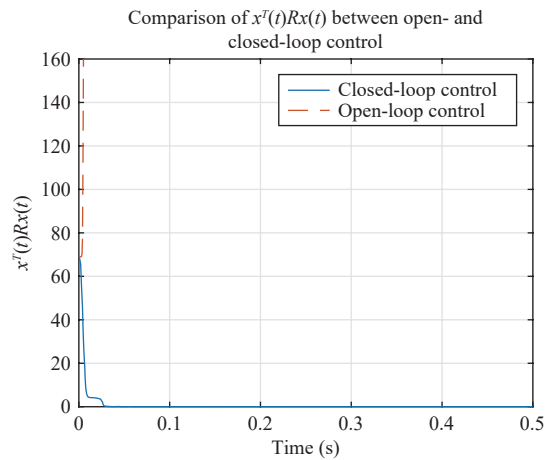Fig. 6.   State responses for the linear SMC system.

Fig. 7.   Comparison of $x^T(t)Rx(t)$ between open- and closed-loop control.

open-loop PV system are unbounded. However, the proposed FSMC control strategy ensures the state trajectories boundness, and the comparison of $x^T(t)Rx(t)$ between the open-loop system and the closed-loop one is given in Fig. 7. Responses of the derivative of the state $x(t)$ and control input $u(t)$ are respectively given in Figs 8 and 9.

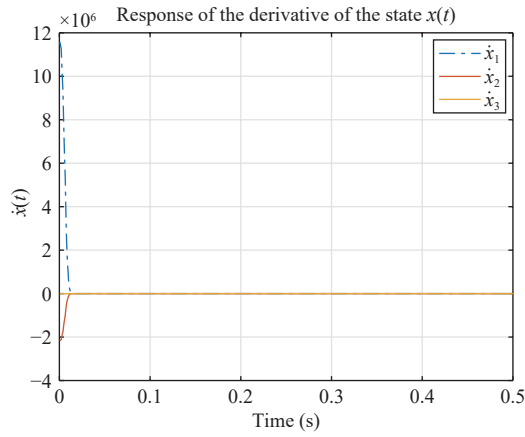*Remark 7:* It is worth to point out that the proposed FSMC

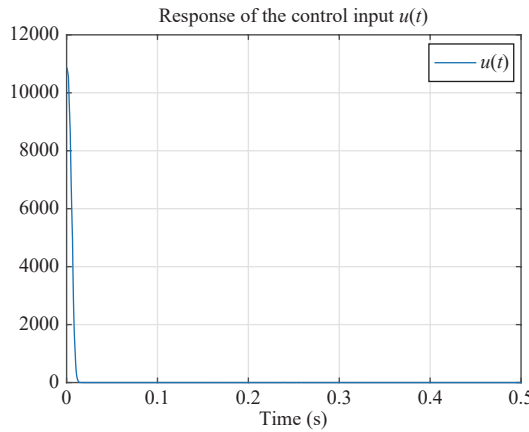Fig. 8.   Response of the derivative of the state $x(t)$.

Fig. 9.   Response of the control input $u(t)$.

in (67) carries the advantages of both the fuzzy method and the sliding mode technique at the same time. The fuzzy method can be regarded as a powerful and flexible approximator, and the main feature of sliding mode approach is its fast response and robustness against uncertainties or disturbances. Figs. 5 and 6 have shown that the proposed FSMC achieves fast response against disturbances in comparison with non-fuzzy sliding mode control.

Remark 8: It is noted that all computations in the sequel were done in MATLAB R2018b running under Windows 10 PC. The computer used was equipped with Intel Xeon E-2276M 2.8 GHz CPU and 16 GB RAM. First, the desired SMC controller gains are solved off-line. The computational time using the FSMC design proposed in Theorem 4 is 218.5 s while the times using the linear SMC result are within 2.8 s. Then, after the off-line controller gains are obtained, for the considered fuzzy system, the SMC is implemented on-line. The computational time of the FSMC is 3.52×10^{-4} s in each iteration while the computational time of the linear SMC is 3.05×10^{-4} s. Moreover, the number of total decision variables using the FSMC design in Theorem 4 is 205 but the number of total decision variables on the linear SMC result is 116. Therefore, it is a trade-off between design complexity and desired control performance when considering with the

applications of the FSMS and linear SMC.

Remark 9: Note that the choices of fuzzy premise variables and fuzzy rules have a great impact on control performance and computational complexities. Since the authors have tried different rules for this example, the selected premise variables are 5 and the selected fuzzy rules are 32, which have taken into account both the control performance and computational complexities. Thus, it will avoid the overfitting problem.

V. CONCLUSIONS

This paper proposes a novel fuzzy sliding mode control strategy to T-S fuzzy descriptor systems using a FTB method. By using a descriptor redundancy approach, the fuzzy representation in the derivative matrix is reformulated into a linear one. We introduce a fuzzy sliding mode control (FSMC) law, and it is shown that the proposed FSMC law ensures the FTB of the closed-loop fuzzy control systems over the reaching phase and sliding motion phase. Sufficient conditions for designing the proposed FSMC law is derived in terms of LMIs. The simulation study shows that the MPPT control of the PV nonlinear system can be achieved within a specified finite time.

APPENDIX A

$$E = \begin{bmatrix} 0.015 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 5 & -1 & 0 \\ 1 & -0.25 & 0 \\ 1 & 0 & -1324.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 5 & -1 & 0 \\ 1 & -0.25 & 0 \\ 1 & 0 & -2752.6 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 5 & -1 & 0 \\ 1 & -0.5 & 0 \\ 1 & 0 & -1324.1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 5 & -1 & 0 \\ 1 & -0.5 & 0 \\ 1 & 0 & -2752.6 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 7.6923 & -1 & 0 \\ 1 & -0.25 & 0 \\ 1 & 0 & -1324.1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 7.6923 & -1 & 0 \\ 1 & -0.25 & 0 \\ 1 & 0 & -2752.6 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 7.6923 & -1 & 0 \\ 1 & -0.5 & 0 \\ 1 & 0 & -1324.1 \end{bmatrix}$$

$$A_8 = \begin{bmatrix} 7.6923 & -1 & 0 \\ 1 & -0.5 & 0 \\ 1 & 0 & -2752.6 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 10 \\ -1.56 \\ 0 \end{bmatrix}, \quad B_9 = \begin{bmatrix} 16 \\ -1.56 \\ 0 \end{bmatrix}$$

$$B_{17} = \begin{bmatrix} 10 \\ -2 \\ 0 \end{bmatrix}, \quad B_{25} = \begin{bmatrix} 16 \\ -2 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$A_1 = A_9 = A_{17} = A_{25}, A_2 = A_{10} = A_{18} = A_{26}$

$A_3 = A_{11} = A_{19} = A_{27}, A_4 = A_{12} = A_{20} = A_{28}$

$A_5 = A_{13} = A_{21} = A_{29}, A_6 = A_{14} = A_{22} = A_{30}$

$A_7 = A_{15} = A_{23} = A_{31}, A_8 = A_{16} = A_{24} = A_{32}$

$B_1 = B_2 = B_3 = B_4 = B_5 = B_6 = B_7 = B_8$

$B_9 = B_{10} = B_{11} = B_{12} = B_{13} = B_{14} = B_{15} = B_{16}$

$B_{17} = B_{18} = B_{19} = B_{20} = B_{21} = B_{22} = B_{23} = B_{24}$

$B_{25} = B_{26} = B_{27} = B_{28} = B_{29} = B_{30} = B_{31} = B_{32}.$

APPENDIX B

$K_1 = \begin{bmatrix} -1727.3 & -190.5 & -245.8 \end{bmatrix}$

$K_2 = \begin{bmatrix} -1629.3 & -179.7 & -235.5 \end{bmatrix}$

$K_3 = \begin{bmatrix} -1901.7 & -209.7 & -260.7 \end{bmatrix}$

$K_4 = \begin{bmatrix} -1720.9 & -189.8 & -249.1 \end{bmatrix}$

$K_5 = \begin{bmatrix} -1729.6 & -190.7 & -243.1 \end{bmatrix}$

$K_6 = \begin{bmatrix} -1721.2 & -189.8 & -246.6 \end{bmatrix}$

$K_7 = \begin{bmatrix} -1743.1 & -192.2 & -244.2 \end{bmatrix}$

$K_8 = \begin{bmatrix} -1742.8 & -192.2 & -252.4 \end{bmatrix}$

$K_9 = \begin{bmatrix} -1211.2 & -133.6 & -173.7 \end{bmatrix}$

$K_{10} = \begin{bmatrix} -1206.2 & -133.0 & -174.2 \end{bmatrix}$

$K_{11} = \begin{bmatrix} -1177.5 & -129.9 & -167.0 \end{bmatrix}$

$K_{12} = \begin{bmatrix} -1171.9 & -129.3 & -170.7 \end{bmatrix}$

$K_{13} = \begin{bmatrix} -1234.6 & -136.1 & -176.3 \end{bmatrix}$

$K_{14} = \begin{bmatrix} -1229.2 & -135.5 & -179.4 \end{bmatrix}$

$K_{15} = \begin{bmatrix} -1214.7 & -133.9 & -171.4 \end{bmatrix}$

$K_{16} = \begin{bmatrix} -1211.7 & -133.6 & -177.8 \end{bmatrix}$

$K_{17} = \begin{bmatrix} -1642.2 & -181.1 & -232.6 \end{bmatrix}$

$K_{18} = \begin{bmatrix} -1656.3 & -182.7 & -237.0 \end{bmatrix}$

$K_{19} = \begin{bmatrix} -1697.9 & -187.3 & -238.5 \end{bmatrix}$

$K_{20} = \begin{bmatrix} -1692.4 & -186.6 & -244.3 \end{bmatrix}$

$K_{21} = \begin{bmatrix} -1633.2 & -180.1 & -228.4 \end{bmatrix}$

$K_{22} = \begin{bmatrix} -1623.8 & -179.1 & -230.9 \end{bmatrix}$

$K_{23} = \begin{bmatrix} -1722.3 & -189.9 & -240.0 \end{bmatrix}$

$K_{24} = \begin{bmatrix} -1715.6 & -189.2 & -246.8 \end{bmatrix}$

$K_{25} = \begin{bmatrix} -1179.7 & -130.1 & -168.6 \end{bmatrix}$

$K_{26} = \begin{bmatrix} -1171.5 & -129.2 & -169.6 \end{bmatrix}$

$K_{27} = \begin{bmatrix} -1141.7 & -125.9 & -161.5 \end{bmatrix}$

$K_{28} = \begin{bmatrix} -1137.1 & -125.4 & -166.0 \end{bmatrix}$

$K_{29} = \begin{bmatrix} -1200.5 & -132.4 & -170.7 \end{bmatrix}$

$K_{30} = \begin{bmatrix} -1194.3 & -131.7 & -173.8 \end{bmatrix}$

$K_{31} = \begin{bmatrix} -1232.8 & -135.9 & -172.1 \end{bmatrix}$

$K_{32} = \begin{bmatrix} -1242.5 & -137.0 & -193.3 \end{bmatrix}.$

REFERENCES

[1] J. Garibaldi, "The need for fuzzy AI," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 3, pp. 610–622, 2019.

[2] W. He, L. Kong, Y. Dong, Y. Yu, C. Yang, and C. Sun, "Fuzzy tracking control for a class of uncertain MIMO nonlinear systems with state constraints," *IEEE Trans. Systems, Man, and Cybernetics: Systems*, vol. 49, no. 3, pp. 543–554, 2019.

[3] Z. Zhang, H. Li, C. Wu, and Qi Zhou, "Finite frequency fuzzy H_∞ control for uncertain active suspension systems with sensor failure," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 4, pp. 777–786, 2018.

[4] T. Haidegger, L. Kovacs, S. Preitl, R. Precup, B. Benyo, and Z. Benyo,

"Controller design solutions for long distance telesurgical applications," *Int. J. Artificial Intelligence*, vol. 6, no. S11, pp. 48–71, 2011.

[5] A. Takacs, L. Kovacs, I. Rudas, R. Precup, and T. Haidegger, "Models for force control in telesurgical robot systems," *Acta Polytechnica Hungarica*, vol. 12, no. 8, pp. 95–114, 2015.

[6] L. Kong, W. He, C. Yang, Z. Li, and C. Sun, "Adaptive fuzzy control for coordinated multiple robots with constraint using impedance learning," *IEEE Trans. Cybernetics*, vol. 49, no. 8, pp. 3052–3063, 2019.

[7] S. Sun, H. Zhang, Z. Qin, and R. Xi, "Delay-dependent H_∞ guaranteed cost control for uncertain switched T-S fuzzy systems with multiple interval time-varying delays," *IEEE Trans. Fuzzy Systems*, 2020. DOI: 10.1109/TFUZZ.2020.2968877

[8] X. Zhao, L. Zhang, P. Shi, and H. Karimi, "Novel stability criteria for T-S fuzzy systems," *IEEE Trans. Fuzzy Systems*, vol. 22, no. 2, pp. 313–323, 2014.

[9] Z. Zhong, Y. Zhu, M.V. Basin, and H.K. Lam, "Event-based multirate control of large-scale distributed nonlinear systems subject to time-driven zero order holds," *Nonlinear Analysis*: *Hybrid Systems*, vol. 36, no. 100864, pp. 1–18, 2020.

[10] W. He, X. Mu, L. Zhang, and Y. Zou, "Modeling and trajectory tracking control for flapping-wing micro aerial vehicles," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 1, pp. 148–156, Jan. 2021.

[11] H. Gao, W. He, L. Zhang, and C. Sun, "Neural-network control of a stand-alone tall building-like structure with an eccentric load: An experimental investigation," *IEEE Trans. Cybernetics*, 2020. DOI: 10.1109/TCYB.2020.3006206

[12] M. Basin, "Applying sliding mode technique to optimal filter and controller design," *Lecture Notes in Control & Information Sciences*, vol. 412, pp. 325–349, 2011.

[13] H. Karimi, "A sliding mode approach to H_∞ synchronization of master-slave time-delay systems with Markovian jumping parameters and nonlinear uncertainties," *Journal of the Franklin Institute*, vol. 349, no. 4, pp. 1480–1496, 2012.

[14] G. Liu, S. Xu, Y. Wei, Z. Qi, and Z. Zhang, "New insight into reachable set estimation for uncertain singular time-delay systems," *Applied Mathematics and Computation*, vol. 320, pp. 769–780, 2018.

[15] J. Li, Q. Zhang, X. Yan, and S. Spurgeon, "Observer-based fuzzy integral sliding mode control for nonlinear descriptor systems," *IEEE Trans. Fuzzy Systems*, vol. 26, no. 5, pp. 2818–2832, 2018.

[16] Y. Wang, H. Karimi, H. Shen, Z. Fang, and M. Liu, "Fuzzy-model-based sliding mode control of nonlinear descriptor systems," *IEEE Trans. Cybernetics*, vol. 49, no. 9, pp. 3409–3419, 2018.

[17] Y. Wang, J. Zhang, H. Zhang, and J. Sun, "A new stochastic sliding-mode design for descriptor fuzzy systems with time-varying delay," *IEEE Trans. Cybernetics*, 2019. DOI: 10.1109/TCYB.2019.2945795

[18] C. Yang, Y. Jiang, J. Na, Z. Li, L. Cheng, and C. Su, "Finite-time convergence adaptive fuzzy control for dual-arm robot with unknown kinematics and dynamics," *IEEE Trans. Fuzzy Systems*, vol. 27, no. 3, pp. 574–588, 2019.

[19] C. Yang, Y. Jiang, W. He, J. Na, Z. Li, and B. Xu, "Adaptive parameter estimation and control design for robot manipulators with finite-time convergence," *IEEE Trans. Industrial Electronics*, vol. 65, no. 10, pp. 8112–8123, 2018.

[20] X. Lin, S. Huang, W. Zhang, and S. Li, "Finite-time feedback stabilization of a class of input-delay systems with saturating actuators via digital control," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 5, pp. 1281–1290, 2019.

[21] Z. Cao, Y. Niu, and J. Song, "Finite-time sliding mode control of

Markovian jump cyber-physical systems against randomly occurring injection attacks," *IEEE Trans. Automatic Control*, vol. 65, no. 3, pp. 1264–1271, 2020.

[22] Y. Li, K. Li, and S. Tong, "Finite-time adaptive fuzzy output feedback dynamic surface control for MIMO nonstrict feedback systems," *IEEE Trans. Fuzzy Systems*, vol. 27, no. 1, pp. 96–110, 2019.

[23] Y. Li, K. Li, and S. Tong, "Adaptive Neural network finite-time control for multi-input and multi-output nonlinear systems with positive powers of odd rational numbers," *IEEE Trans. Neural Networks and Learning Systems*, vol. 31, no. 7, pp. 2532–2543, 2020.

[24] Z. Zhong, Y. Zhu, C. Lin, and T. Huang, "A fuzzy control framework for interconnected nonlinear power networks under TDS attack: Estimation and compensation," *Journal of the Franklin Institute*, vol. 358, no. 1, pp. 74–88, 2021.

[25] Z. Su, Q. Zhang, and J. Ai, "Practical and finite-time fuzzy adaptive control for nonlinear descriptor systems with uncertainties of unknown bound," *Int. J. Systems Science*, vol. 44, no. 12, pp. 2223–2233, 2013.

[26] M. Basin, "Finite- and fixed-time convergent algorithms: Design and convergence time estimation," *Annual Reviews in Control*, vol. 48, pp. 209–221, 2019.

[27] T. Taniguchi, K. Tanaka, and H. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Trans. Fuzzy Systems*, vol. 8, no. 4, pp. 442–452, 2000.

[28] R. Kavikumar, R. Sakthivel, O.M. Kwon, and B. Kaviarasan, "Finite-time boundedness of interval type-2 fuzzy systems with time delay and actuator faults R," *Journal of the Franklin Institute*, vol. 356, pp. 8296–8324, 2019.

[29] H. Shen, F. Li, H. Yan, H. Karimi, and H. Lam, "Finite-time event-triggered H_∞ control for T-S fuzzy markov jump systems," *IEEE Trans. Fuzzy Systems*, vol. 26, no. 5, pp. 3122–3135, 2018.

[30] Z. Xiang, C. Qiao, and M. Mahmoud, "Finite-time analysis and H_∞ control for switched stochastic systems," *Journal of the Franklin Institute*, vol. 349, no. 3, pp. 915–927, 2012.

[31] B. Xiao, Q. Hu, and Y. Zhang, "Finite-time attitude tracking of spacecraft with fault-tolerant capability," *IEEE Trans. Control Systems Technology*, vol. 23, no. 4, pp. 1338–1350, 2015.

[32] S. Huang, Z. Xiang, and H. Karimi, "Input-output finite-time stability of discrete-time impulsive switched linear systems with state delays," *Circuits Systems & Signal Processing*, vol. 33, no. 1, pp. 141–158, 2014.

[33] J. Song, Y. Niu, and Y. Zou, "Finite-time stabilization via sliding mode control," *IEEE Trans. Automatic Control*, vol. 62, no. 3, pp. 1478–1483, 2017.

[34] W. Haddad and A. L'Afflitto, "Finite-time stabilization and optimal feedback control," *IEEE Trans. Automatic Control*, vol. 61, no. 4, pp. 1069–1074, 2016.

[35] M. Chadli and H. Karimi, "Robust observer design for unknown inputs Takagi-Sugeno models," *IEEE Trans. Fuzzy Systems*, vol. 21, no. 1, pp. 158–164, 2013.

[36] K. Tanaka, H. Ohtake, and H. Wang, "A descriptor system approach to fuzzy control system design via fuzzy Lyapunov functions," *IEEE Trans. Fuzzy Systems*, vol. 15, no. 3, pp. 333–341, 2007.

[37] Z. Zhong, C. Lin, Z. Shao, and M. Xu, "Decentralized event-triggered control for large-scale networked fuzzy systems," *IEEE Trans. Fuzzy Systems*, vol. 26, no. 1, pp. 29–45, 2018.

Zhixiong Zhong (M'18) received the B.S. degree in mechatronics from Fuzhou University in 2008, and the M.S. degree in control theory and control engineering from Fuzhou University in 2012. In 2013 he was a visiting student with the Department of Mechanical Engineering, University of Victoria, Canada. In 2015 he received the Ph.D. degree in the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology and jointed in the School of Computer and Control Engineering, Minjiang University in 2018. He is currently a Professor in Minjiang University. He has published more than 40 papers in journals, conferences, and authored two books: Modeling, Control, Estimation, and Optimization for Microgrids: A Fuzzy-Model-Based Method (CRC, 2019) and Large-Scale Fuzzy Interconnected Control Systems Design and Analysis (IGI Global, 2017). His research interests include fuzzy control, robust filtering, and large-scale control.

Xingyi Wang received M.S. degree in control theory and control engineering from Fuzhou University in 2011. She is currently an Associate Professor in Minjiang University, and working toward the Ph.D. degree at the College of Mechanical and Electronic Engineering, Fujian Agriculture and Forestry University. Her research interests include fuzzy logic systems, control with neural networks.

Hak-Keung Lam (M'98–SM'12–F'20) received the B.Eng. (Hons.) and Ph.D. degrees from the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, China, in 1995 and 2000, respectively. During the period of 2000 and 2005, he worked with the Department of Electronic and Information Engineering at The Hong Kong Polytechnic University as Post-Doctoral Fellow and Research Fellow, respectively. He joined as a Lecturer at King's College London in 2005 and is currently a Reader. His current research interests include intelligent control systems and computational intelligence. He has served as a program committee member and international advisory board member for various international conferences and a reviewer for various books, international journals and international conferences. He is an Associate Editor for *IEEE Transactions on Fuzzy Systems*, *IEEE Transactions on Circuits and Systems II: Express Briefs*, *IET Control Theory and Applications*, *International Journal of Fuzzy Systems*, *Neurocomputing*, and *Nonlinear Dynamics*; and Guest Editor for a number of international journals, and is in the editorial board of for a number of international journals. He was named as Highly Cited Researcher and is an IEEE Fellow. He is the coeditor for two edited volumes: Control of Chaotic Nonlinear Circuits (World Scientific, 2009) and Computational Intelligence and Its Applications (World Scientific, 2012), and the coauthor of the monographs: Stability Analysis of Fuzzy-Model-Based Control Systems (Springer, 2011), Polynomial Fuzzy Model Based Control Systems (Springer, 2016), and Analysis and Synthesis for Interval Type-2 Fuzzy-Model-Based Systems (Springer, 2016).