Reduced-Order Observer-Based Leader-Following Formation Control for Discrete-Time Linear Multi-Agent Systems

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***Abstract*—Formation control of discrete-time linear multi-agent systems using directed switching topology is considered in this work via a reduced-order observer, in which a formation control protocol is proposed under the assumption that each directed communication topology has a directed spanning tree. By utilizing the relative outputs of neighboring agents, a reduced-order observer is designed for each following agent. A multi-step control algorithm is established based on the Lyapunov method and the modified discrete-time algebraic Riccati equation. A sufficient condition is given to ensure that the discrete-time linear multi-agent system can achieve the expected leader-following formation. Finally, numerical examples are provided so as to demonstrate the effectiveness of the obtained results.**

***Index Terms*—Discrete-time systems, formation control, leader-following, multi-agent system, reduced-order observer.**

I. Introduction

FOR the last few years, a great deal of researchers have studied distributed coordination of groups of agents for their broad applications in many domains. As an important fundamental problem in cooperative control, the purpose of consensus control is to design appropriate control protocols such that all agents can reach the agreement value of their common states asymptotically or in finite time. Much effort has been taken to solve all sorts of distributed coordination control problems in the early literature, such with formation control [1]–[3], containment control [4]–[7], output regulation [8], the adaptive consensus problem [9]–[10], the state constraint consensus problem [11]–[12] etc.

In the existing literature, Lyapunov-based methods and stochastic matrices were commonly used in solving first-order [13]–[16] and second-order [17]–[22] consensus problems. In addition, the consensus problem of linear multi-agent systems has also been investigated in many literature [23]–[27]. Compared to systems with continuous-time dynamics, discrete-time systems are much more suitable for realization using computer. Moreover, many practical systems cannot be characterized by continuous dynamics. Some interesting issues on consensus stability of discrete-time multi-agent systems have been investigated in [28]–[32].

Most literature mentioned above assume that agents could measure the state information of their neighbours through communication channels unerringly. However, for many practical systems, the agents can only measure the output information of their neighbors due to an unreliable communication environment or other reason, full state information is thus not always available. For such consensus problems, a common method is to propose suitable observers so as to estimate these unmeasurable variables. In [17], owing to the unpredictable speed of the leader, an estimation law was adopted using first-order followers to estimate the leader's state. An observer-based consensus strategy for a second-order system lacking a velocity measurement was proposed in [18]. Based on the relative outputs of the neighbours, observer-type consensus protocols were presented in [23], which can solve consensus problem for linear networked systems. In many practical systems, switching is a common phenomenon. For example, those systems with abrupt parameter variations can be modelled as switched systems. The leader-following consensus problem of linear systems with state-observer under one group of directed switching topologies was investigated in [24], and the conclusion of [24] was then extended in [30] to discrete-time systems. In [31], two consensus problems for discrete-time multi-agent systems with switching network topology were studied, and the consensus problem for discrete-time linear multi-agent systems under directed switching networks was investigated in [32]. In [33], the authors investigated the consensus problem with linear multi-agent systems by adopting a new reduced-order observer. Based on [33], the leader-following multi-agent consensus problem was investigated in [34] by designing a reduced-order observer for each following agent. For a system with discrete-time linear dynamics, a distributed reduced-order observer-based consensus control law was given in [35].

With the development of consensus theory, its application by researchers trying to solve formation control issues has increased. By converting formation control problems for networked systems into consensus-like problems, the tools in consensus theory can be used in dealing with subsequent

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problems. It has been proposed in [36] that many existing leader-following, virtual leader, and behavioural formation control methods can be integrated in the general framework of consensus protocol establishment. Some necessary and sufficient graphical conditions for formation control of unicycles were obtained in [37]. In [38], a behavior-based method was proposed to realize complex formation for multi-robots. A new formation control approach based on the distances among the networked robots modeled as single integrators was provided in [39]. For general linear multi-agent systems with switching directed topologies, the time-varying formation control was investigated in [40] and the time-varying output formation control was studied in [41]. By using an adaptive based method, a distributed time-varying formation control strategy for multi-agent systems with high-order linear dynamics was investigated in [42]. For leader-to-formation stability of multi-agent systems, an adaptive optimal control approach was proposed in [43]. Considering the same communication problems as above-mentioned, many issues of observer-based formation control problems have been studied recently. A learning-based model predictive control (LBMPC) algorithm was presented in [3] for formation flight control of multiple vehicles systems. In [2], by introducing linear extended state observer, a formation control strategy was provided for the case where the velocity of the neighboring agent is unmeasurable. The leader-following formation problem for a multi-robot system was studied in [1], in which the leader agent is unknown to the followers.

So far, the bulk of existing literature on observer-based formation control has been largely focused on continuous-time systems. Motivated by the above works, especially by [33] and [34], we consider the leader-following formation control problem for discrete-time linear multi-agent systems through using a reduced-order observer-based strategy. The main contributions of this work are summarized as follows: 1) Under directed switching topology, the leader-following formation control problem for discrete-time linear multi-agent systems is first considered in this work; 2) A novel reduced-order observer is designed for each following agent based on the relative output information, which can estimate the state effectively; 3) Based on the Lyapunov method and the modified discrete-time algebraic Riccati equation, a multi-step control algorithm is established for achieving the expected leader-following formation.

A. Notation

$\mathbb{R}^{m\times n}$ (or $\mathbb{C}^{m\times n}$) denotes the set of $m\times n$ real (or complex) matrices. A^T represents the transpose of matrix A. I_n is the $n\times n$ identity matrix, and $1_n=[1,1,\ldots,1]^T$. For a symmetric matrix P, when $P>0\,(<0,\geq 0,\leq 0)$, we say it is positive definite (negative definite, positive semi-definite, or negative semi-definite). $\|\cdot\|$ denotes the Euclidean norm. $|\cdot|$ denotes the module of a complex number or the absolute value of a real number. $\otimes$ denotes the Kronecker product.

II. Preliminaries and Problem Formulation

Using graph theory, the interaction relationship among N agents of a multi-agent system can be described by a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$. Here, $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$ denotes the set of nodes and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set. $\mathcal{A}=\left[a_{ij}\right]\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix, which is defined such that $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$ and $a_{ij}=0$ otherwise. It is assumed that $a_{ii}=0$, $i\in S_r\triangleq\{1,2,\ldots,N\}$. The Laplacian matrix $\mathcal{L}$ is defined as $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$, $i\in S_r$, and $l_{ij}=-a_{ij}$, $i\neq j$. A directed tree of a directed graph that is formed by the graph edges that connect from the root node to every other node in the graph is called a directed spanning tree. According to the definition, we can easily obtain that the Laplacian matrix $\mathcal{L}$ satisfies $\mathcal{L}1_N=0$. Now some basic lemmas are introduced in the following.

Lemma 1 [13]: The Laplacian matrix $\mathcal{L}$ is positive semi-definite and satisfies $\mathcal{L}1_N=0$. If a weighted digraph $\mathcal{G}$ contains a directed spanning tree, then the corresponding Laplacian matrix $\mathcal{L}$ has exactly one zero eigenvalue.

Lemma 2 [44]: A symmetric matrix S can be partitioned into the following block form

$$S=\begin{bmatrix} S_{11} & S_{12}\\ S_{12}^T & S_{22}\end{bmatrix}$$

where $S_{11}=S_{11}^T$ and $S_{22}=S_{22}^T$. Then, $S<0$ if and only if

$$S_{11}<0,\quad S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$$

or

$$S_{22}<0,\quad S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0.$$

Then, the discrete-time modified algebraic Riccati equation is given as follow:

$$A^TPA-P-\delta A^TPB(I+B^TPB)^{-1}B^TPA+Q=0 \tag{1}$$

where matrix $Q=Q^T>0$ is a given matrix and $(A,Q^{\frac{1}{2}})$ is detectable.

The following lemma is given to address the existence of solutions for the modified discrete-time algebraic Ricatti equation.

Lemma 3 [45]–[46]: If (A,B) is stabilizable and $(A,Q^{\frac{1}{2}})$ is detectable, there exists a scalar $\delta_c\in[0,1)$ such that the discrete-time modified algebraic Riccati equation (1) has a unique positive definite solution P for any $\delta_c<\delta\leq 1$. Meanwhile, for any initial condition $P_0\geq 0$, $P=\lim_{k\to\infty}P_k$ holds if P_k satisfies

$$P_{k+1}=A^TP_kA+Q-\delta A^TP_kB(I+B^TP_kB)^{-1}B^TP_kA. \tag{2}$$

Remark 1: It is not hard to see that the Riccati equation (1) is reduced to a Lyapunov equation if $\delta=0$. If $\delta=1$, the equation (1) is degenerated to the common discrete-time Riccati equation.

Lemma 4 [47]: For the Laplacian matrix $\mathcal{L}$, the following $N\times N$ nonsingular matrix U can be found

$$U=\begin{bmatrix} 1_N & U_1\end{bmatrix},\quad U^{-1}=\begin{bmatrix} \upsilon^T\\ U_2\end{bmatrix}$$

where $U_1\in\mathbb{R}^{N\times(N-1)}$, $U_2\in\mathbb{R}^{(N-1)\times N}$ and $\upsilon^T1_N=1$, $\upsilon^T\mathcal{L}=0$, such that

$$U^{-1}\mathcal{L}U=\begin{bmatrix}0 & 0\\ 0 & D_{\mathcal{L}}\end{bmatrix} \tag{3}$$

where $D_{\mathcal{L}}$ is an upper-triangular matrix, and the diagonal elements λ_i of which satisfies $\mathrm{Re}(\lambda_i)>0$, $i=2,3,\ldots,N$.

Consider a leader-following multi-agent system consisting of one leader and N followers. Let $\mathcal{G}'$ be a directed graph with these $N+1$ nodes. $\mathcal{G}'$ is used to model the communicating topology of the multi-agent system, and $\mathcal{G}'$ contains v_0 (denoting the leader) and a subgraph $\mathcal{G}$. It is supposed in this work that at least one following agent of $\mathcal{G}$ is connected to the leader v_0 via a direct edge, and graph $\mathcal{G}'$ contains a directed spanning tree rooted at the leader.

In discussing time-varying interaction topology, the set of graphs for the existing interaction topologies is given as $H=\left\{\mathcal{G}'_1,\mathcal{G}'_2,\ldots,\mathcal{G}'_m\right\}$ with index set $\mathcal{P}=\{1,2,\ldots,m\}$. Let $\sigma:[0,\infty)\to P$ be a switching signal used to describe the topology switching between subintervals. $\mathcal{L}^{\sigma}$ denotes the Laplacian of graph $\mathcal{G}'_{\sigma}$ with switching signal σ. The dynamics of the following agents are described by the following discrete-time linear systems

$$\begin{cases}x_i(k+1)=Ax_i(k)+Bu_i(k)\\ y_i(k)=Cx_i(k), \qquad i\in S_r\end{cases} \tag{4}$$

where $x_i\in\mathbb{R}^n$, $u_i\in\mathbb{R}^p$ and $y_i\in\mathbb{R}^q$ are the state, control input and measured output of agent i, respectively. $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times p}$ and $C\in\mathbb{R}^{q\times n}$ are constant matrices. Based on the above dynamics equation and supposing C is full row rank, it would not be difficult to select a matrix $T\in\mathbb{R}^{(n-q)\times n}$ such that $\left[C^T,\ T^T\right]^T$ is non-singular.

The leader's dynamics is described by

$$\begin{cases}x_0(k+1)=Ax_0(k)+Bu_0(k)\\ y_0(k)=Cx_0(k)\end{cases} \tag{5}$$

where $x_0\in\mathbb{R}^n$ is the leader's state, $y_0\in\mathbb{R}^q$ is the output of leader and $u_0\in\mathbb{R}^p$ is the input of leader.

Remark 2: It is assumed in this work that $u_0(k)$ is known by all followers, here we consider the case that the leader's control input is nonzero. The coefficient matrices for the leader are supposed to be the same as those of the followers due to practical backgrounds including birds, insects, etc.

Assumption 1: Each directed communication topology $\mathcal{G}'$ has a directed spanning tree.

Assumption 2: The matrix pair (A,B) is stabilizable, (A,C) is detectable. C is a full row rank matrix, namely $rank(C)=q$.

Assumption 3: The followers can only receive the relative output measurements with their neighbors directly.

The leader-following system (4) and (5) is said to achieve consensus, if

$$\lim_{k\to\infty}\|x_i(k)-x_0(k)\|=0,\quad i\in S_r. \tag{6}$$

The purpose of this paper is to design a distributed reduced-order observer-based control protocol $u_i(k)$ using only the relative output feedback information for formation control problems of a leader-following discrete-time linear system. Specifying a formation by a vector $h=\left[h_1^T,\ldots,h_N^T\right]^T\in\mathbb{R}^{Nn}$, in which $h_i\in\mathbb{R}^n$ is the desired relative place of agent i to the leader, then the leader-following system is said to achieve formation h if

$$\lim_{k\to\infty}\|x_i(k)-x_0(k)-h_i\|=0,\quad i\in S_r. \tag{7}$$

III. Main Results

A. Generalized Reduced-Order Observer

In this subsection, a distributed reduced-order observer is introduced for each following agent in (4):

$$\begin{cases}z_i(k+1)=W\left(\bar{N}z_i(k)+Ky_i(k)+HBu_i(k)\right)\\ \mu_i(k)=z_i(k)+Ey_i(k)=T\hat{x}_i(k)\end{cases} \tag{8}$$

where $z_i\in\mathbb{R}^{\hat{n}}$ is the observer state and $\mu_i\in\mathbb{R}^{\hat{n}}$ is the observer output of agent i, $i\in S_r$. $W\in\mathbb{R}^{\hat{n}\times n}$, $\bar{N}\in\mathbb{R}^{n\times\hat{n}}$, $K\in\mathbb{R}^{n\times q}$, $H\in\mathbb{R}^{n\times n}$, $E\in\mathbb{R}^{\hat{n}\times q}$ and $T\in\mathbb{R}^{(n-q)\times n}$ are given gain matrices. Define the estimation error as follows:

$$e_i(k)=Tx_i(k)-T\hat{x}_i(k),\quad i\in S_r \tag{9}$$

where $\hat{x}_i(k)$ is the estimation of $x_i(k)$. Then, we give the following theorem.

Remark 3: For consensus control of linear multi-agent systems, the distributed continuous-time reduced observers were designed in [33] and [34]. Motivated by the works [33] and [34], a novel distributed discrete-time reduced-order observer is designed based on the relative output information. Based on the Lyapunov method and the modified discrete-time algebraic Riccati equation, a multi-step control algorithm is established for leader-following formation control of discrete-time linear multi-agent systems.

Theorem 1: Consider a general discrete-time system model (4), and suppose that Assumptions 1–3 hold. If the following conditions (10) of coefficient matrices hold, and $W\bar{N}$ is Schur stable, the estimation error $e_i(k)$ converge to zero exponentially asymptotically, which means that the observer (8) can estimate the state of Tx_i effectively.

$$\begin{cases}TA-ECA-W\bar{N}T+W\bar{N}EC-WKC=0\\ TB-ECB-WHB=0\end{cases} \tag{10}$$

where matrices $E,W,\bar{N},K,T,H$ are given in (8).

Proof: With the general system model (4) and estimation law (8), it can be obtained that

$$\begin{aligned}e_i(k+1)=&Tx_i(k+1)-(z_i(k+1)+ECx_i(k+1))\\ =&(T-EC)(Ax_i(k)+Bu_i(k))\\ &-W(\bar{N}z_i(k)+Ky_i(k)+HBu_i(k))\\ =&W\bar{N}e_i(k)+[TA-ECA-W\bar{N}T\\ &+W\bar{N}EC-WKC)]x_i(k)\\ &+[TB-ECB-WHB]u_i(k).\end{aligned} \tag{11}$$

From (10) and (11), we have

$$e_i(k+1)=W\bar{N}e_i(k). \tag{12}$$

Since the matrix $W\bar{N}$ is Schur stable, we can find positive definite matrices $P=P^T$ and $Q=Q^T$ satisfying the discrete Lyapunov equation

$$(W\bar{N})^TP(W\bar{N})-P=-Q. \tag{13}$$

Let $e(k)=[e_1^T(k),\ldots,e_N^T(k)]^T$, and consider the following Lyapunov candidate function for the dynamic system (12)

$$V_e(k)=e^T(k)(I_N\otimes P)e(k). \tag{14}$$

Then, we can get

$$\begin{aligned}\Delta V(k) &= V_e(k+1)-V_e(k)\\ &= e^T(k)\left[I_N\otimes\left((W\bar{N})^T P(W\bar{N})-P\right)\right]e(k)\\ &= -e^T(k)(I_N\otimes Q)e(k)\\ &\le -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V_e(k).\end{aligned}\tag{15}$$

Based on matrix theory [44] and the Lyapunov equation $(W\bar{N})^T P(W\bar{N}) = P-Q$, it is easy to obtain that $0<\chi<1$, where $\chi := \lambda_{\min}(Q)/\lambda_{\max}(P)$.

Moreover, from (14) and (15), we have $V_e(k)\le(1-\chi)^k V_e(0)$. Thus, the estimation error $e(k)$ converges to zero exponentially asymptotically, i.e., $\lim_{k\to\infty} e_i(k)=0,\ i\in S_r$. ■

Now the idea of the observer design can be applied to the formation control of leader-following systems in next subsection.

B. Discrete-Time Formation Control of Linear Systems

In this subsection, a distributed control protocol is proposed, utilizing merely the relative output feedback information for leader-following formation control. Considering the formation definition (7), a new state vector can be designed as follow:

$$\psi_i(k) = \sum_{j=1}^{N}\left[l_{ij}^{\sigma}\left(x_j(k)-h_j\right)+g_i^{\sigma}\left(x_i(k)-h_i-x_0(k)\right)\right]\tag{16}$$

where g_i^{σ} is the connection weight between the i-th agent with the leader of graph $\mathcal{G}'_{\sigma}$.

Let $\psi(k)=[\psi_1^T(k),\ldots,\psi_N^T(k)]^T$, $\Sigma^{\sigma}=\text{diag}\left\{g_1^{\sigma},\ldots,g_N^{\sigma}\right\}$. From (16) and Lemma 1, we have

$$\psi(k) = \left((\mathcal{L}^{\sigma}+\Sigma^{\sigma})\otimes I_n\right)\left(x(k)-h-1_N\otimes x_0(k)\right)\tag{17}$$

and

$$\begin{aligned}\psi(k+1) =& \left[(\mathcal{L}^{\sigma}+\Sigma^{\sigma})\otimes I_n\right]\\ &\times\left[x(k+1)-h-1_N\otimes x_0(k+1)\right]\\ =& \left[(\mathcal{L}^{\sigma}+\Sigma^{\sigma})\otimes I_n\right]\left[(I_N\otimes A)x(k)\right.\\ &\left.+(I_N\otimes B)u(k)-h-1_N\otimes x_0(k+1)\right]\\ =&(I_N\otimes A)\psi(k)\\ &+\left[(\mathcal{L}^{\sigma}+\Sigma^{\sigma})\otimes B\right]\left[u(k)-1_N\otimes u_0(k)\right]\\ &+\left[(\mathcal{L}^{\sigma}+\Sigma^{\sigma})\otimes(A-I_n)\right]h\end{aligned}\tag{18}$$

where $u(k)=[u_1^T(k),\ldots,u_N^T(k)]^T$. Moreover, define the relative output vector

$$\xi(k) = (I_N\otimes C)\psi(k).\tag{19}$$

A new reduced-order observer to estimate the state of ψ_i is designed as follow:

$$\begin{cases} z_i'(k+1) = W\Big[\bar{N}z_i'(k)+K\xi_i(k)\\ \qquad\qquad +B\sum_{j=1}^{N} l_{ij}u_j(k)\\ \qquad\qquad +g_i\left(u_i(k)-u_0(k)\right)\\ \qquad\qquad +(A-I_n)\sum_{j=1}^{N} l_{ij}h_j+g_ih_i\Big]\\ \varepsilon_i'(k) = z_i'(k)+E\xi_i(k)\end{cases}\tag{20}$$

where z_i' is the observer state and ε_i' is the observer output of agent i, $i\in S_r$, and c is a positive gain to be determined. Then, a novel control protocol for agent i is designed as follow:

$$u_i(k) = -cMSR_1\xi_i(k)+u_0(k)-cMSR_2\varepsilon_i'(k)+\Theta_i\tag{21}$$

where Θ_i is any vector satisfying $B\Theta_i=(I_n-A)h_i$, $M\in\mathbb{R}^{q\times n}$ is a given gain matrix, and $S\in\mathbb{R}^{n\times n}$, $R_1\in\mathbb{R}^{m\times q}$, $R_2\in\mathbb{R}^{m\times\hat{n}}$ are the given coefficient matrices. Based on the works [9] and [11], the coefficient matrices $W,\bar{N},K$ are designed as follow:

$$\begin{cases} W = T-EC\\ \bar{N} = ASR_2\\ K = ASR_2E+ASR_1\end{cases}\tag{22}$$

where $\begin{bmatrix} C\\ T\end{bmatrix}^{-1} = \begin{bmatrix} SR_1 & SR_2\end{bmatrix}$, $R_1\in\mathbb{R}^{m\times q}$, $R_2\in\mathbb{R}^{m\times\hat{n}}$.

Algorithm 1: Under the Assumptions 1–3, we do the following steps.

1) By Lemma 3, there exist positive definite matrices P_1, $Q_1\in\mathbb{R}^{n\times n}$ satisfying the modified discrete-time algebraic Riccati equation (23). Then, we can obtain the matrices P_1,Q_1.

$$A^TP_1A-P_1+Q_1-\delta A^TP_1B\left(I_p+B^TP_1B\right)^{-1}B^TP_1A=0.\tag{23}$$

2) Select c such that

$$\begin{cases} \dfrac{\min\text{Re}(\lambda_{i\sigma})}{\max\text{Re}(\lambda_{i\sigma})} > \dfrac{1-\sqrt{\vartheta}}{1+\sqrt{\vartheta}}\\ \dfrac{1-\sqrt{\vartheta}}{\min\text{Re}(\lambda_{i\sigma})} < c < \dfrac{1+\sqrt{\vartheta}}{\max\text{Re}(\lambda_{i\sigma})}\\ \vartheta = 1-(c\max|\text{Im}(\lambda_{i\sigma})|)^2-\delta_c\end{cases}\tag{24}$$

with $\lambda_{i\sigma}\neq 0$ being the i–th eigenvalue of Laplacian matrix $L^{\sigma}+\Sigma^{\sigma}$, and $\delta_c\in[0,1)$, $\delta_c<\delta<1$.

3) Obtain the positive define matrices $P_2,Q_2\in\mathbb{R}^{\hat{n}\times\hat{n}}$ and a gain matrix E satisfying the following condition:

$$(TASR_2-ECASR_2)^T\times P_2\times(TASR_2-ECASR_2)-P_2=-Q_2.\tag{25}$$

Remark 4: Consider the following equality

$$\begin{bmatrix} sI-CASR_1 & -CASR_2\\ -TASR_1 & sI-TASR_2\\ I & 0\end{bmatrix} = \begin{bmatrix}\begin{bmatrix} C\\ T\\ 0\end{bmatrix} & \begin{matrix}0\\ I\end{matrix}\end{bmatrix}\begin{bmatrix} SR_1 & SR_2\end{bmatrix}$$

and suppose (A,C) is detectable, it's easy to obtain

$$Rank\left(\begin{bmatrix} sI-CASR_1 & -CASR_2\\ -TASR_1 & sI-TASR_2\\ I & 0\end{bmatrix}\right)=n$$

for any $\text{Re}(s)\ge 0$. Then, we have

$$Rank\left(\begin{bmatrix} sI-TASR_2\\ -CASR_2\end{bmatrix}\right)=n-q$$

for any s satisfying $\text{Re}(s)\ge 0$, which means $(TASR_2, CASR_2)$ is detectable. Owing to its detectable properties, we can always find a matrix E satisfying the condition 3) in Algorithm 1.

Theorem 2: Consider the leader-following systems (4) and (5) with the control protocol (21) and the reduced-order observer (20), and suppose that Assumptions 1–3 hold. Then, the considered system can achieve formation control if condition (22) holds. The gain matrix M is chosen as $M=(I_p+B^TP_1B)^{-1}B^TP_1A$, and the constant c, matrices P_1, E are obtained via Algorithm 1.

Proof: A new error vector can be designed as

$$e'(k)=(I_N\otimes T)\psi(k)-\varepsilon'(k) \tag{26}$$

where $e'(k)=[e_1'^T(k),\ldots,e_N'^T(k)]^T$. Then, the dynamics of $e'(k)$ is obtained

$$\begin{aligned}e'(k+1)=&(I_N\otimes T)\psi(k+1)-\varepsilon'(k+1)\\
=&(I_N\otimes T)\psi(k+1)-z'(k+1)\\
&-(I_N\otimes EC)\psi(k+1)\\
=&(I_N\otimes TA)\psi(k)+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes T(A-I_n))h\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes TB)(u(k)-1_N\otimes u_0(k))\\
&-(I_N\otimes(T-EC))[((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(A-I_n))h\\
&+(I_N\otimes\bar{N})z'(k)+(I_N\otimes KC)\psi(k)\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes B)(u(k)-1_N\otimes u_0(k))]\\
&-(I_N\otimes EC)[(I_N\otimes A)\psi(k)\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes B)(u(k)-1_N\otimes u_0(k))\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(A-I_n))h]\\
=&[I_N\otimes(TA-ECA-W\bar{N}T\\
&+W\bar{N}EC-WKC)]\psi(k)\\
&+[(\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(TB-ECB-WB)]\\
&\times(u(k)-1_N\otimes u_0(k))\\
&+[((T-EC)(A-I_n)-W(A-I_n))\\
&\otimes(\mathcal{L}^\sigma+\Sigma^\sigma)]h+(I_N\otimes(WN))e'(k)\\
=&(I_N\otimes(TASR_2-ECASR_2))e'(k).\end{aligned} \tag{27}$$

Combining with (18) and (27), we get that

$$\begin{aligned}\psi(k+1)=&(I_N\otimes A)\psi(k)\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(A-I_n))h\\
&+((\mathcal{L}^\sigma+\Sigma^\sigma)\otimes B)(u(k)-1_N\otimes u_0(k))\end{aligned} \tag{28}$$

$$e'(k+1)=(I_N\otimes(TASR_2-ECASR_2))e'(k). \tag{29}$$

Let $\eta(k)=[\psi^T(k),e'^T(k)]^T$, we have

$$\eta(k+1)=F_\sigma\eta(k) \tag{30}$$

where

$$F_\sigma=\begin{bmatrix}D_1 & D_2\\ 0 & D_3\end{bmatrix}$$

$$\begin{cases}D_1=I_N\otimes A-(\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(cBM)\\
D_2=(\mathcal{L}^\sigma+\Sigma^\sigma)\otimes(cBMSR_2)\\
D_3=I_N\otimes(TASR_2-ECASR_2).\end{cases}$$

Define

$$\zeta(k)=(U^{-1}\otimes I)\psi(k) \tag{31}$$

where $\zeta(k)=[\zeta_1^T(k),\zeta_2^T(k),\ldots,\zeta_N^T(k)]^T$, and define $\tilde{\eta}_i=[\zeta_i^T, e_i'^T]^T$. According to Lemma 4, we can obtain

$$\tilde{\eta}_i(k+1)=\tilde{F}_\sigma\tilde{\eta}_i(k),\quad i\in\{2,\ldots,N\} \tag{32}$$

where $\tilde{F}_\sigma=\begin{bmatrix}A-c\lambda_{i\sigma}BM & c\lambda_{i\sigma}BMSR_2\\ 0 & TASR_2-ECASR_2\end{bmatrix}$. Then, we can determine that $\lim_{k\to\infty}\tilde{\eta}_i(k)=0, i\in S_r$, if each subsystem in (32) converges to zero.

Construct a common Lyapunov function for dynamic system (30)

$$V(\tilde{\eta}(k))=\tilde{\eta}(k)^T\tilde{P}\tilde{\eta}(k) \tag{33}$$

where $\tilde{P}=\begin{bmatrix}I_N\otimes P_1 & 0\\ 0 & \omega I_N\otimes P_2\end{bmatrix}$ is the parameter-dependent matrix and ω is a given positive parameter. Then, we have

$$\begin{aligned}\Delta V_k&=V(\tilde{\eta}(k+1))-V(\tilde{\eta}(k))\\
&=\tilde{\eta}^T(k)\left(\tilde{F}_\sigma^T\tilde{P}\tilde{F}_\sigma-\tilde{P}\right)\tilde{\eta}(k).\end{aligned} \tag{34}$$

According to Lemma 3 and Algorithm 1, P_1 is the unique positive-definite solution of (23). Then, we have

$$\begin{aligned}&(A-c\lambda_{i\sigma}BM)^TP_1(A-c\lambda_{i\sigma}BM)-P_1\\
&\quad=A^TP_1A-P_1-2c\mathrm{Re}(\lambda_{i\sigma})A^TP_1BM\\
&\qquad+c^2|\lambda_{i\sigma}|^2(BM)^TP_1BM.\end{aligned} \tag{35}$$

As we know $M=(I+B^TP_1B)^{-1}B^TP_1A$, by using the transformation formula:

$$\begin{aligned}B^TP_1B\left(I_p+B^TP_1B\right)^{-1}-I_p=&B^TP_1B\left(I_p+B^TP_1B\right)^{-1}\\
&-\left(I_p+B^TP_1B\right)\left(I_p+B^TP_1B\right)^{-1}\\
=&-\left(I_p+B^TP_1B\right)^{-1}.\end{aligned}$$

We can get

$$\begin{aligned}&A^TP_1A-P_1-2c\mathrm{Re}(\lambda_{i\sigma})A^TP_1BM+c^2|\lambda_{i\sigma}|^2(BM)^TP_1BM\\
&=A^TP_1A-P_1+c^2|\lambda_{i\sigma}|^2(BM)^TP_1BM\\
&\quad-2c\mathrm{Re}(\lambda_{i\sigma})A^TP_1B\left(I_p+B^TP_1B\right)^{-1}B^TP_1A\\
&=A^TP_1A-P_1-\left(2c\mathrm{Re}(\lambda_{i\sigma})-c^2|\lambda_{i\sigma}|^2\right)\\
&\quad\times A^TP_1B\left(I_p+B^TP_1B\right)^{-1}B^TP_1A\\
&\quad+c^2|\lambda_{i\sigma}|^2A^TPB\left(I_p+B^TP_1B\right)^{-T}\\
&\quad\times\left[B^TP_1B\left(I_p+B^TP_1B\right)^{-1}-I_p\right]\times B^TP_1A\\
&=A^TP_1A-c^2|\lambda_{i\sigma}|^2M^TM-\left(2c\mathrm{Re}(\lambda_{i\sigma})-c^2|\lambda_{i\sigma}|^2\right)\\
&\quad\times A^TP_1B\left(I_p+B^TP_1B\right)^{-1}B^TP_1A-P_1\\
&\le A^TP_1A-P_1-\left(2c\mathrm{Re}(\lambda_{i\sigma})-c^2|\lambda_{i\sigma}|^2\right)\\
&\quad\times A^TP_1B\left(I_p+B^TP_1B\right)^{-1}B^TP_1A.\end{aligned} \tag{36}$$

By Algorithm 1, we know that there exists $\delta\in(\delta_c,1]$ satisfying the following condition:

$$\frac{\min\mathrm{Re}(\lambda_{i\sigma})}{\max\mathrm{Re}(\lambda_{i\sigma})}>\frac{1-\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}}{1+\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}}. \tag{37}$$

So, we have

$$\begin{aligned}&\frac{1-\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}}{\min\mathrm{Re}(\lambda_{i\sigma})}\\
&\qquad<\frac{1+\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}}{\max\mathrm{Re}(\lambda_{i\sigma})}.\end{aligned} \tag{38}$$

Then, we can obtain

$$\begin{aligned}&1-\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}\\&\quad< c\min(\mathrm{Re}(\lambda_{i\sigma}))\\&\quad\le c\max(\mathrm{Re}(\lambda_{i\sigma}))\\&\quad< 1+\sqrt{1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta}.\end{aligned}\tag{39}$$

According to the fact that $c\min(\mathrm{Re}(\lambda_{i\sigma}))\le c\mathrm{Re}(\lambda_{i\sigma})\le c\max(\mathrm{Re}(\lambda_{i\sigma}))$, we can get that $c\mathrm{Re}(\lambda_{i\sigma})\le 1+\sqrt{\vartheta}$. Similarly, it is easy to obtain that $1-\sqrt{\vartheta}\le c\mathrm{Re}(\lambda_{i\sigma})$, where $\vartheta=1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta_c$, $\delta_c<\delta<1$. Then, from (39) the two inequalities $c\mathrm{Re}(\lambda_{i\sigma})-1\le\sqrt{\vartheta}$ and $1-c\mathrm{Re}(\lambda_{i\sigma})\le\sqrt{\vartheta}$, we have

$$(c\mathrm{Re}(\lambda_{i\sigma})-1)^2\le 1-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2-\delta.\tag{40}$$

Then, we get

$$\begin{aligned}1&\ge 2c\mathrm{Re}(\lambda_{i\sigma})-(c|\lambda_{i\sigma}|)^2\\&\ge 2c\mathrm{Re}(\lambda_{i\sigma})-(c\mathrm{Re}(\lambda_{i\sigma}))^2-(c\max|\mathrm{Im}(\lambda_{i\sigma})|)^2\\&\ge\delta.\end{aligned}\tag{41}$$

From (41) and Lemma 3 we have

$$\begin{aligned}&A^TP_1A-P_1-\left(2c\mathrm{Re}(\lambda_{i\sigma})-c^2|\lambda_{i\sigma}|^2\right)\\&\quad\times A^TP_1B\left(I_p+B^TP_1B\right)^{-1}\times B^TP_1A\\&\quad\le -Q_1.\end{aligned}\tag{42}$$

According to condition 3) of Algorithm 1, we can choose a positive definite matrix P_2 satisfying

$$(TASR_2-ECASR_2)^TP_2(TASR_2-ECASR_2)-P_2-Q_2.\tag{43}$$

In summary, we can get that

$$\begin{aligned}\Delta V_k&=V(\tilde\eta(k+1))-V(\tilde\eta(k))\\&=\tilde\eta^T(k)\left(\tilde F_\sigma^T\tilde P\tilde F_\sigma-\tilde P\right)\tilde\eta(k)\\&\le-\tilde\eta^T(k)Q_\sigma\tilde\eta(k)\end{aligned}\tag{44}$$

where

$$Q_\sigma=\begin{bmatrix}I_N\otimes Q_1 & \Phi_\sigma\\ * & \Lambda_\sigma\end{bmatrix}$$

$$\begin{cases}\Phi_\sigma=-cH_\sigma\otimes A^TP_1BMSR_2\\ \qquad +c^2{H_\sigma}^TH_\sigma\otimes(BM)^TP_1BMSR_2\\ \Lambda_\sigma=\omega I_N\otimes Q_2-c^2{H_\sigma}^TH_\sigma\otimes(BMSR_2)^TP_1BMSR_2.\end{cases}$$

Note that the following inequality holds if ω is large enough

$$\omega I_N\otimes Q_2>c^2{H_\sigma}^TH_\sigma\otimes(BMSR_2)^TP_1BMSR_2+\Phi^TQ_1^{-1}\Phi.\tag{45}$$

From Lemma 2, we can easily conclude that Q_σ is positive definite if (45) is satisfied. Furthermore, it is known that the Lyapunov function $V(\eta(k))$ satisfies

$$\lambda_{\min}(\tilde P)\|\tilde\eta\|^2\le V(\tilde\eta)\le\lambda_{\max}(\tilde P)\|\tilde\eta\|^2.\tag{46}$$

Together with the inequality

$$\tilde\eta^TQ_\sigma\tilde\eta\ge\frac{\lambda_{\min}(Q_\sigma)}{\lambda_{\max}(\tilde P)}\tag{47}$$

and similarly letting $\chi:=\lambda_{\min}(Q_\sigma)/\lambda_{\max}(\tilde P)$. From (15), it is easy to know that $0<\chi<1$. Then, we can get that

$$\begin{cases}\Delta V(\tilde\eta)\le-\tilde\eta^TQ_\sigma\tilde\eta\le-\chi\tilde\eta^T\tilde P\tilde\eta\\ V(\tilde\eta(k))\le(1-\chi)^kV(\tilde\eta(0)).\end{cases}\tag{48}$$

Thus, the system (30) is exponentially asymptotically stable, the formation problem is solved by the control protocol (21) and the reduced-order observer (20), i.e., $\lim_{k\to\infty}\|x_i(k)-x_0(k)-h_i\|=0\ \ \forall i\in S_r$. ■

Remark 5: It can be seen that if we choose the formation vector $h=0$, the consensus problem of multi-agent system (4) and (5) can be solved if (7) holds. Thus, the consensus problem is a special case of the formation problem. The observer-based consensus problem of discrete-time linear systems has been studied in [10]. The common Lyapunov function is available for all interaction topologies, and under certain conditions the switching topology can be extended to the jointly connected topology case that each of the subsystems is unstable.

Remark 6: Compared with most of current works on consensus control with undirected switching topology, leader-following formation control for discrete-time linear multi-agent systems with directed switching topology is first considered in this work. Under directed topology, the Laplacian matrix associated with the communication graph is asymmetrical. The eigenvalues of the matrix have an imaginary part because the Laplacian matrix associated with communication topology is asymmetric. Comparing the controller and observer design with undirected switching topology then becomes difficult. Moreover, using switching communication, the description and certification of theoretical results becomes complicated and difficult for a system with directed topology.

IV. Simulations

A numerical simulation is provided in this section to illustrate the effectiveness of the theoretical results. Consider a group of agents consisting of four followers and one leader. Fig. 1 shows the switching communication topology. The coefficient matrices of both leader and follower dynamics are given as

$$A=\begin{bmatrix}0&2&0\\0&0&1.5\\-0.2&0.2&1.1\end{bmatrix}$$

$$B=\begin{bmatrix}1&0\\1&1\\1&1\end{bmatrix},\quad C=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}.$$

Consider the following Laplacian matrices $\mathcal{L}^i(i=1,2,3)$ for graphs $\mathcal{G}^i$

$$\mathcal{L}^1=\begin{bmatrix}2&-1&0&-1\\0&1&-1&0\\-1&0&1&0\\-1&0&0&1\end{bmatrix}$$

$$\mathcal{L}^2=\begin{bmatrix}3&-1&0&-2\\-1.5&3.5&-2&0\\0&-1&1&0\\-1&0&0&1\end{bmatrix}$$

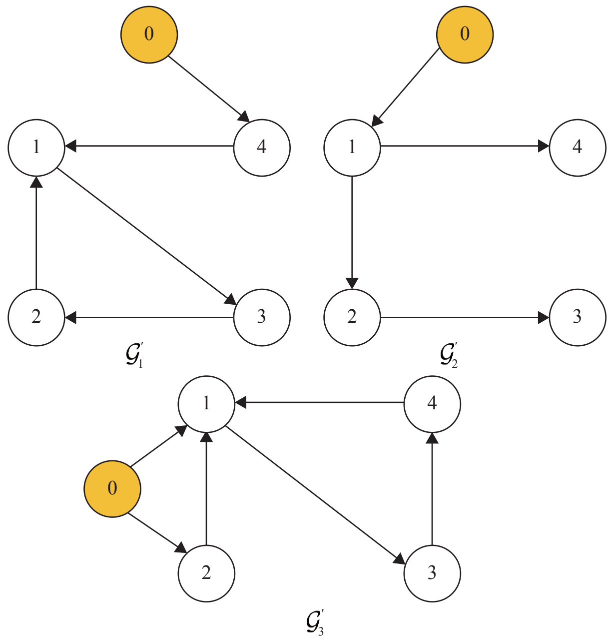

Fig. 1. Switching communication topology graph.

$$\mathcal{L}^3 = \begin{bmatrix} 3.5 & -1 & -2 & 0 \\ -1 & 2.5 & 0 & -1.5 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

and the diagonal matrices

$$\begin{cases} G^1 = \mathrm{diag}\{1,1,1,0\} \\ G^2 = \mathrm{diag}\{0,2,2,2\} \\ G^3 = \mathrm{diag}\{0,0,2,2\}. \end{cases}$$

The topologies are arbitrarily switched among the three graphs $\mathcal{G}_i'$, $i = 1,2,3$.

The observer coefficient matrix $T = [0,\ 0,\ 1]^T$, and some coefficient matrices of control protocol (21) are given as follows:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.08 & 0.2 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Through simple calculations, we can get $W\bar{N} = 0.8$. Then, take $\delta = 0.4$ and $c = 0.3$ which satisfies the above conditions. We can obtain

$$M = \begin{bmatrix} -0.0703 & 0.1785 & 0.4262 \\ -0.0977 & -0.1676 & 0.5704 \end{bmatrix}$$

with

$$P_1 = \begin{bmatrix} 0.2311 & -0.0092 & -0.3964 \\ -0.0092 & 1.4861 & -1.3189 \\ -0.3964 & -1.3189 & 4.3495 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 0.0956 & 0.0093 & -0.0633 \\ 0.0093 & 0.7312 & -0.9970 \\ -0.0633 & -0.9970 & 1.4475 \end{bmatrix}.$$

In general, $x_0(0)$ is taken as $[10,-5,15]^T$ and $u_0(k)$ is taken as $[2,1]^T$. The initial states of the following agents are random generated with an item in each dimension in the interval $[-30,30]$. Choosing a simple formation vector $h = [[2,-2,-2]^T,[-2,2,2]^T,[4,3,2]^T,[-4,-3,-2]^T]^T$ for the leader-following system. Figs. 2–3 shows the estimation error $e_i(k)$ and the state $\psi_{i1}(k)$ of four followers. The position trajectories of leaders and followers under the protocol (21) are showed in Figs. 4–7. It is shown that the multi-agent system can achieve the expected formation. Now, the feasibility and effectiveness of protocols (20) and (21) are now verified.

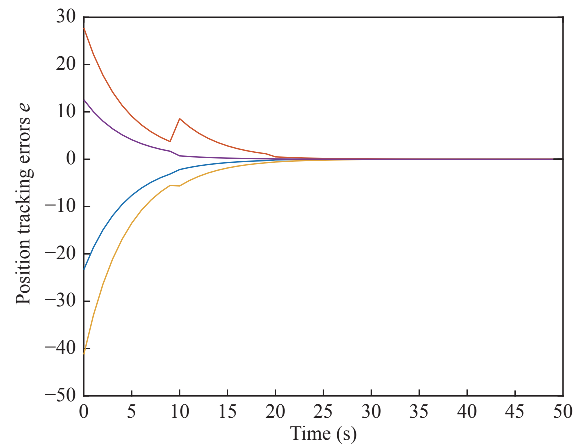

Fig. 2. Estimation error trajectories of four followers.

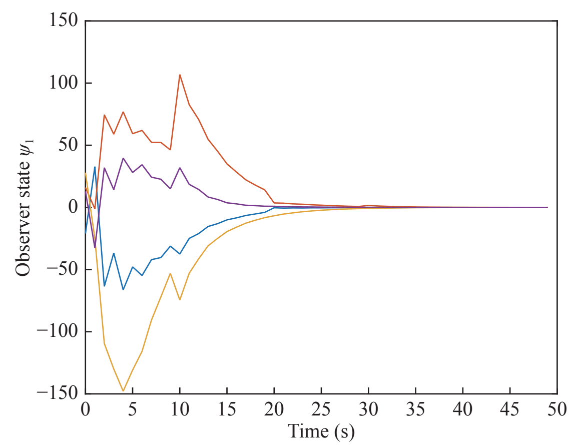

Fig. 3. State trajectories of the observers.

V. Conclusions

In this work, a novel reduced-order observer-based control law is presented for leader-following formation of discrete-time linear multi-agent systems with directed switching topologies by utilizing the relative output information of neighboring agents. Using the model transformation method, the formation control problem is converted into an output feedback control problem with a reduced-order observer of the associated switching system. Based on the Lyapunov method and the modified discrete-time algebraic Riccati equation, a sufficient condition is obtained to ensure that the discrete-time linear multi-agent system can achieve the expected leader-

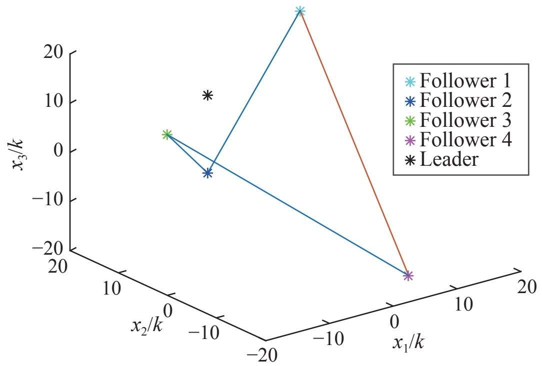

Fig. 4. Position trajectories: $k = 0$.

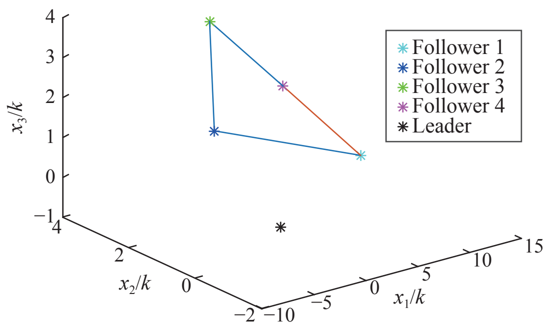

Fig. 5. Position trajectories: $k = 15$.

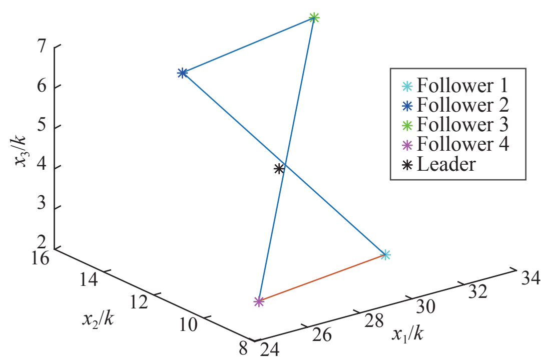

Fig. 6. Position trajectories: $k = 30$.

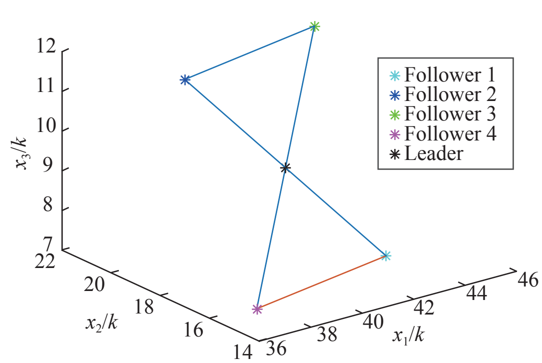

Fig. 7. Position trajectories: $k = 45$.

following formation. Future works will focus on parameter time-changing systems.

References

[1] T. Sun, F. Liu, H. Pei, and Y. He, "Observer-based adaptive leader-following formation control for non-holonomic mobile robots," *IET Control Theory Appl.*, vol. 6, no. 18, pp. 2835–2841, Jun. 2012.

[2] W. Qin, Z. Liu, and Z. Chen, "Formation control for nonlinear multi-agent systems with linear extended state observer," *IEEE/CAA J. Autom. Sinica*, vol. 1, no. 2, pp. 171–179, Feb. 2014.

[3] A. T. Hafez, S. N. Givigi, and S. Yousefi, "Unmanned aerial vehicles formation using learning based model predictive control," *Asian J. Control*, vol. 20, no. 3, pp. 1014–1026, Mar. 2018.

[4] F. Wang, Z. Liu, and Z. Chen, "Distributed containment control for second-order multiagent systems with time delay and intermittent communication," *Int. J. Robust Nonlin. Control*, vol. 28, no. 18, pp. 5730–5746, 2018.

[5] F. Wang, Z. Liu, and Z. Chen, "Containment control for second-order nonlinear multi-agent systems with aperiodically intermittent position measurements," *J. Frankl. Inst.*, vol. 356, no. 15, pp. 8706–8725, 2019.

[6] F. Wang, Y. Ni, Z. Liu, and Z. Chen, "Containment control for general second-order multi-agent systems with switched dynamics," *IEEE Trans. Cybern.*, vol. 50, no. 2, pp. 550–560, Feb. 2020.

[7] F. Wang, Y. Ni, Z. Liu, and Z. Chen, "Fully distributed containment control for second-order multi-agent systems with communication delay," *ISA Transactions*, vol. 99, pp. 123–129, Apr. 2020.

[8] A. Odekunle, W. Gao, M. Davari, and Z. P. Jiang, "Reinforcement learning and non-zero-sum game output regulation for multi-player linear uncertain systems, " *Automatica*, vol. 112, Aiticle No. 108672, 2020. DOI: 10.1016/j.automatica.2019.108672.

[9] H. Cai and J. Huang, "Leader-following adaptive consensus of multiple uncertain rigid spacecraft systems," *Sci. China Inf. Sci.*, vol. 59, no. 1, pp. 1–13, 2016.

[10] W. Wang, C. Wen, and J. Huang, "Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances," *Automatica*, vol. 77, no. 3, pp. 133–142, 2017.

[11] Z. Liu and Z. Chen, "Discarded consensus of network of agents with state constraint," *IEEE Trans. Autom. Control*, vol. 57, no. 11, pp. 2869–2874, Nov. 2012.

[12] D. H. Nguyen, T. Narikiyo, and M. Kawanishi, "Robust consensus analysis and design under relative state constraints or uncertainties," *IEEE Trans. Autom. Control*, vol. 63, no. 6, pp. 1784–1790, Jul. 2017.

[13] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[14] F. Xiao and L. Wang, "State consensus for multi-agent systems with switching topologies and time-varying delays," *Int. J. Control*, vol. 79, no. 10, pp. 1277–1284, 2006.

[15] Y. Sun and L. Wang, "Consensus of multi-agent systems in directed networks with nonuniform time-varying delays," *IEEE Trans. Autom. Control*, vol. 54, no. 7, pp. 1607–1613, May 2009.

[16] S. Feng, L. Wang, Y. Li, S. Sun, and C. Xia, "A nonlinear merging protocol for consensus in multi-agent systems on signed and weighted graphs," *Physica A: Statistical Mechanics and its Applications*, vol. 490, pp. 653–663, 2018.

[17] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

[18] A. Abdessameud and A. Tayebi, "On consensus algorithms for double-integrator dynamics without velocity measurements and with input constraints," *Syst. Control Lett.*, vol. 59, no. 12, pp. 812–821, 2010.

[19] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, no. 6, pp. 1089–1095, 2010.

[20] L. Wang, S. Sun, C. Xia, "Finite-time stability of multi-agent system in disturbed environment," *Nonlinear Dynamics*, vol. 67, no. 3, pp. 2009–2016, 2012.

[21] F. Wang, Z. Liu, and Z. Chen, "A novel leader-following consensus of multi-agent systems with smart leader," *Int. J. Control Autom. Syst.*, vol. 16, no. 4, pp. 1483–1492, 2018.

[22] F. Wang, Z. Liu, and Z. Chen, "Leader-following consensus of second-

order nonlinear multi-agent systems with intermittent position measurements, " *Sci. China Inf. Sci*., vol. 62, no. 10, Aiticle No. 202204, 2019. DOI: 10.1007/s11432-018-9732-7.

[23] Z. Li, Z. Duan, and G. Chen, "Dynamic consensus of linear multi-agent systems," *IET Control Theory Appl.*, vol. 5, no. 1, pp. 19–28, 2011.

[24] L. Gao, J. Li, X. Zhu, and W. Chen, "Leader-following consensus of linear multi-agent systems with state-observer under switching topologies," *Math. Probl. Eng.*, vol. 2013, pp. 1–13, 2013.

[25] M. Diao, Z. Duan, and G. Wen, "Consensus tracking of linear multi-agent systems under networked observability conditions," *Int. J. Control*, vol. 87, no. 8, pp. 1478–1486, 2014.

[26] M. Hu, L. Guo, A. Hu, and Y. Yang, "Leader-following consensus of linear multi-agent systems with randomly occurring nonlinearities and uncertainties and stochastic disturbances," *Neurocomputing*, vol. 149, pp. 884–890, 2015.

[27] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 148–157, Jan. 2016.

[28] P. Lin and Y. Jia, "Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies," *Automatica*, vol. 45, no. 9, pp. 2154–2158, 2009.

[29] Y. Gao, J. Ma, M. Zuo, T. Jiang, and J. Du, "Consensus of discrete-time second-order agents with time-varying topology and time-varying delays," *J. Frankl. Inst.*, vol. 349, no. 8, pp. 2598–2608, 2012.

[30] X. Xu, S. Chen, W. Huang, and L. Gao, "Leader-following consensus of discrete-time multi-agent systems with observer-based protocols," *Neurocomputing*, vol. 118, pp. 334–341, 2013.

[31] Y. Su and J. Huang, "Two consensus problems for discrete-time multi-agent systems with switching network topology," *Automatica*, vol. 48, no. 9, pp. 1988–1997, 2012.

[32] J. Huang, "The consensus for discrete-time linear multi-agent systems under directed switching networks," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4086–4092, Aug. 2016.

[33] L. Gao, B. Xu, J. Li, and H. Zhang, "Distributed reduced-order observer-based approach to consensus problems for linear multi-agent systems," *IET Control Theory Appl.*, vol. 9, no. 5, pp. 784–792, 2015.

[34] T. Yang, P. Zhang, and S. Yu, "Consensus of linear multi-agent systems via reduced-order observer," *Neurocomputing*, vol. 240, pp. 200–208, 2017.

[35] B. Xu, J. Li, and L. Gao, "Distributed reduced-order observer-based consensus control of discrete-time linear multi-agent systems," *IFAC Proceedings Volumes*, vol. 46, no. 20, pp. 124–129, 2013.

[36] W. Ren, "Consensus strategies for cooperative control of vehicle formations," *IET Control Theory Appl.*, vol. 1, no. 2, pp. 505–512, 2007.

[37] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," *IEEE Trans. Autom. Control*, vol. 50, no. 1, pp. 121–127, Jan. 2005.

[38] J. R. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Trans. Robot. Autom.*, vol. 19, no. 6, pp. 933–941, Jun. 2003.

[39] K. K. Oh and H. S. Ahn, "Formation control of mobile agents based on inter-agent distance dynamics," *Automatica*, vol. 47, no. 10, pp. 2306–2312, 2011.

[40] X. Dong and G. Hu, "Time-varying formation control for general linear multi-agent systems with switching directed topologies," *Automatica*, vol. 73, pp. 47–55, 2016.

[41] X. Dong, C. Sun, and G. Hu, "Time-varying output formation control for linear multi-agent systems with switching topologies," *Int. J. Robust Nonlin. Control*, vol. 26, no. 16, pp. 3558–3579, 2016.

[42] R. Wang, X. Dong, Q. Li, and Z. Ren, "Distributed adaptive time-varying formation for multi-agent systems with general high-order linear time-invariant dynamics," *J. Frankl. Inst.*, vol. 353, no. 10, pp. 2290–2304, 2016.

[43] W. Gao, Z. P. Jiang, F. L. Lewis, and Y. Wang, "Leader-to-formation stability of multiagent systems: An adaptive optimal control approach," *IEEE Trans. Autom. Control*, vol. 63, no. 10, pp. 3581–3587, Oct. 2018.

[44] R. A. Horn and C. R. Johnson. Matrix Analysis. London, UK: Cambridge University Press, 1985.

[45] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1453–1464, Sep. 2004.

[46] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, "Foundations of control and estimation over lossy networks," *Proc. IEEE*, vol. 95, no. 1, pp. 163–187, Jan. 2007.

[47] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. New York, USA: Springer-Verlag London Limited, 2010.

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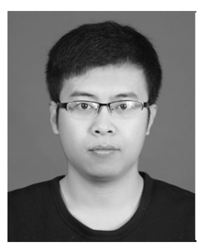

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